The scaling law of piping erosion

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Abtract:

Piping erosion is one of the main cause of serious hydraulic work failure. This phenomenon involves the progression of a continuous tunnel between the upstream and the downstream side. Starting from the bulk equations for diphasic flow with diffusion and from the jump equations with erosion, we propose a two parameters scaling law for piping erosion. The first parameter is the critical shear stress. The second parameter is the erosional characteristic time, which is a function of the initial hydraulic gradient and of the erosion coefficient. The comparison with experimental data has confirmed the validity of the scaling law. We deduce an estimation of the time for breaching in hydraulic works (dams, dykes).

Résumé :

L'érosion par renard hydraulique est l'une des causes de rupture des ouvrages hydrauliques. Elle est liée au développement d'un tunnel continu entre l'amont et l'aval. A partir des équations d'écoulement diphasique avec diffusion, et des équations de saut avec érosion, une loi d'échelle à deux paramètres pour l'érosion par renard est proposée. Le premier paramètre est la contrainte seuil d'érosion. Le second paramètre est le temps caractéristique d'érosion, fonction du gradient hydraulique initial et du coefficient d'érosion. La comparaison avec des résultats expérimentaux valide la loi d'échelle. On en déduit une estimation du temps de rupture d'un ouvrage hydraulique (barrage, digue).

Key-words :

piping erosion; dams ; dykes

1 Introduction

Erosion of soil resulting from piping, seepage or overtopping is the main cause of serious hydraulic work (dykes, dams) failure, in terms of the risk of flooding areas located downstream. The present study concerns the first process: the enlargement of a crack, which leads to an erosion process known as "piping" in soil mechanics. Piping erosion often occurs in hydraulics works, involving the formation and the progression of a continuous tunnel between the upstream and the downstream side. When this phenomenon is suspected of occurring or has already been detected in situ, the time to failure is difficult to predict. To be able to develop effective emergency action plans preventing heavy casualties and damage to property, it is necessary to have a characteristic time to use as a basis. The aim of this study was to draw up a useful model for estimating this time characteristic for breaching.

2 Two-phase flow equations with interface erosion

A large literature on soil erosion also exists in the field of hydraulics and river engineering (Chanson, 1999), and in the field of poromechanics and petroleum engineering (Vardoulakis *et al.*, 1995). In the field of geomechanics, several experimental methods have been developed for simulating the piping erosion process experimentally, with particular attention focussed on the hole erosion test (Wan & Fell, 2002). The experience acquired on more than 200 tests on several soils has confirmed what an excellent tool this test can be for quantifying the rate of piping erosion in a soil. A model for interpreting the hole erosion test has been developed by Bonelli *et al.* (2006).

The soil is eroded by the flow, which then carries away the eroded particles. As long as the particles are small enough in comparison with the characteristic length scale of the flow, this two-phase flow can be said to be a continuum. We take Ω to denote the volume of the two-phase mixture and Γ the fluid/soil interface. For the sake of simplification, sedimentation and deposition processes are neglected, and the soil is taken to be saturated. The mass conservation equations for the water/particles mixture and for the mass of the particles as well as the balance equation of momentum of the mixture within Ω can be written as follows in a Eulerian framework:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0\,, \quad \frac{\partial \rho Y}{\partial t} + \vec{\nabla} \cdot (\rho Y \vec{u}) = -\vec{\nabla} \cdot \vec{J}\,, \quad \frac{\partial \rho \vec{u}}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u} \otimes \vec{u}) = \vec{\nabla} \cdot \boldsymbol{\sigma}\,.$$

In these equations, ρ is the density of the mixture, depending on the particles mass fraction Y, $\vec{u}$ is the mass-weighted average velocity, $\vec{J}$ is the mass diffusion of the flux of particles, and $\boldsymbol{\sigma}$ is the Cauchy stress tensor in the mixture.

As there is a process of erosion, a mass flux crosses the interface Γ. Let us take $\vec{n}$ to denote the normal unit vector of Γ oriented outwards from the soil, and v_Γ to denote the normal velocity of Γ. The jump equations over Γ are

$$\left[\!\left[\rho(\vec{v}_\Gamma - \vec{u}) \cdot \vec{n} \right]\!\right] = 0\,, \quad \left[\!\left[\rho Y(\vec{v}_\Gamma - \vec{u}) \cdot \vec{n} \right]\!\right] = \left[\!\left[\vec{J} \cdot \vec{n} \right]\!\right], \quad \left[\!\left[\rho \vec{u}(\vec{v}_\Gamma - \vec{u}) \cdot \vec{n} \right]\!\right] = -\left[\!\left[\boldsymbol{\sigma} \cdot \vec{n} \right]\!\right],$$

where $[\![a]\!] = a_s - a_w$ is the jump of any physical variable a across the interface, and a_s and a_w stands for the limiting value of a on the solid and fluid sides of the interface, respectively. The soil is assumed to be homogeneous, rigid and devoid of seepage. The co-ordinate system depends on the soil in question. The total flux of eroded material (both particles and water) crossing the interface is therefore $\dot{m} = -\rho_s \vec{v}_\Gamma \cdot \vec{n}$, where ρ_s is the density of the soil.

Erosion laws dealing with soil surface erosion by a tangential flow are often written in the form of threshold laws such as (see Chansonfor a comprehensive review (1999)):

$$\dot{m} = k_{er}(\left|\tau_w\right| - \tau_c) \text{ if } \left|\tau_b\right| > \tau_c, \, 0 \text{ otherwise}\,,$$

where τ_c is the critical shear stress involved in the erosion, k_{er} is the coefficient of soil erosion, and τ_w is the tangential shear stress at the interface defined as follows:

$$\left|\tau_w\right| = \sqrt{(\boldsymbol{\sigma} \cdot \vec{n})^2 - (\vec{n} \cdot \boldsymbol{\sigma} \cdot \vec{n})^2}\,\Big|_w\,.$$

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3 Application to piping erosion, the scaling law

This complete set of equations was previously used to study various situations involving boundary layer and free surface flows over an erodable soil (Brivois, 2005). The use of these equations has been extended to the study of piping erosion (Bonelli *et al.*, 2006), by introducing a spatial integration over a cylinder Ω with radius R (initial value R_0) (Fig. 1).

Reference velocity is $V_0 = Q_0 / \pi R_0^2$, where Q_0 is the initial input flow. The flow time is $t_0 = R_0 / V_0$. The driving pressure is $P = R(p_{in} - p_{out}) / (2L)$ (initial value P_0), where p_{in} and p_{out} are the input and output pressures, respectively ($p_{in} > p_{out}$), and L denotes the characteristic geometrical length of the system (it is not a wave lengthscale).

We take $V_{er} = k_{er} P_0 / \rho_s$ to denote the reference erosion velocity. The erosion time is therefore $t_{er} = R_0 / V_{er}$. The total volume flow of eroded material is $Q_{er} = 2\pi R_0 L V_{er}$.

The flow rate scale ratio is $Q_{er} / Q_0 = 2 L V_{er} / (R_0 V_0)$. We take therefore c_0 to denote the reference volume fraction, defined by $c_0 = (1-n) Q_{er} / Q_0$ where n is the soil porosity. Finally, we define $\tilde{k}_{er} = k_{er} V_0$ as the kinetics of erosion number. These are two key dimensionless numbers in our analysis.

When $c_0 \ll 1 - n$, the amount of eroded material is much more lower than of the water ($Q_{er} \ll Q_0$). In this case, the fluid is a dilued suspension, and the volume fraction does not affect the system. This case arises when the pipe is not too long ($L \ll R_0 \tilde{k}_{er}^{-1}$).

When $\tilde{k}_{er} \ll 1$, the flow time scale is much more lower than of the erosion time scale ($t_0 \ll t_{er}$), and the erosion velocity is much more lower than of the flow velocity ($V_{er} \ll V_0$). Since surface erosion is a slow process, the erosion evolves at a very long scale compared to the flow scale. The flow can be considered as quasisteady, and inertial terms can be neglected.

To settle the equations, we introduce the dimensionless variables:

$$\tilde{t} = \frac{t}{t_{er}},\ \tilde{R} = \frac{R}{R_0},\ \tilde{V} = \frac{V}{V_0},\ \tilde{P} = \frac{P}{P_0},\ \tilde{Q} = \frac{Q}{Q_0},\ \tilde{\tau}_c = \frac{\tau_c}{P_0}$$

Under the dilued suspension situation ($L \ll R_0 \tilde{k}_{er}^{-1}$) and the low kinetics of erosion situation ($\tilde{k}_{er} \ll 1$), obtained equations simplify considerably. The velocity and the flow are related to the pressure and the radius by $\tilde{R}\tilde{P} = \tilde{V}^2$ and $\tilde{R}^5 \tilde{P} = \tilde{Q}^2$. We consider a unit step of driving pressure $\tilde{P}(\tilde{t}) = 0$ if $\tilde{t} < 0$, 1 if $\tilde{t} > 0$. The closed-form solution of the obtained system is entirely given by the following evolution of the radius:

$$\tilde{R}(\tilde{t}) = \begin{cases} 1 \text{ if } \tilde{t} \leq 0 \text{ or } \tilde{\tau}_c \geq 1 \\ \tilde{\tau}_c + (1 - \tilde{\tau}_c)\exp(\tilde{t}) \text{ if } \tilde{t} > 0 \text{ and } \tilde{\tau}_c < 1 \end{cases}$$

This is the scaling law of piping erosion with constant pressure drop. This important result has far-reaching practical consequences. It can be used to fit experimental data, expressing the scaled radius $\tilde{R} - \tilde{\tau}_c$ as a function of the scaling time $\tilde{t} + \ln(1 - \tilde{\tau}_c)$, and making possible a unified description of piping erosion of differents soils, in pipe of various diameters, etc.

4 Comparison with experimental data

The hole erosion test was designed to simulate piping flow erosion in a hole. An eroding fluid is driven through the soil sample to initiate erosion of the soil along a pre-formed hole. The results of the test are given in terms of the flow rate versus time curve with a constant pressure drop. Therefore, the flow rate is used as an indirect measurement of the erosion rate. For further details about this test, see Wan and Fell (2002).

The scaling law is now compared with previously published data (Wan and Fell, 2002). Analysis were performed in 17 tests, using 9 different soils. The initial radius and the length of the pipe were R_0 =3 mm and L =117 mm. Table 1 contains particle size distribution, and critical stress and erosion coefficient. The comparison with experimental data confirms the validity of the scaling law (Fig. 2).

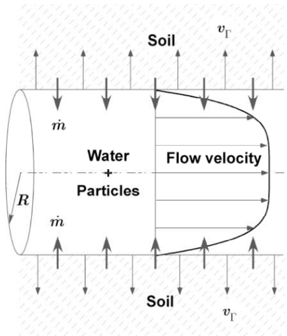

FIG. 1 – Axisymmetrical flow with soil erosion and transport of the particles.

Soil		%Gravel	%Sand	%Fines	%<2µm	τ_c (Pa)	k_{er} (10^{-4} s/m)
Bradys	high plasticity sandy clay	1	24	75	48	50 - 76	3 - 5
Fattorini	medium plasticity sandy clay	3	22	75	14	6	8
Hume	low plasticity sandy clay	0	19	81	51	66 - 92	0.3 - 3
Jindabyne	clayey sand	0	66	34	15	6 - 72	3 - 9
Lyell	silty sand	1	70	29	13	8	140
Matahina	low plasticity clay	7	43	50	25	128	1
Pukaki	silty sand	10	48	42	13	13	10
Shellharbour	high plasticity clay	1	11	88	77	99 - 106	0.5 - 3
Waranga	low plasticity clay	0	21	79	54	106	1

TABLE. 1 – Properties of soils samples, critical stress and erosion coefficient.

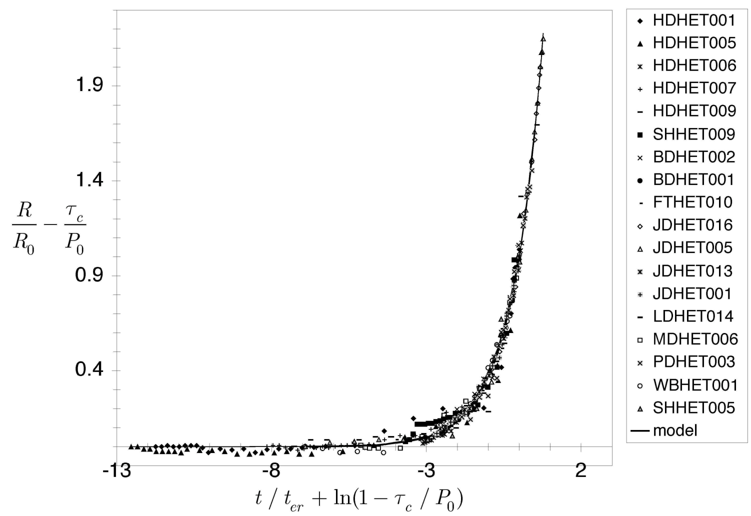

FIG. 2 – Hole Erosion Tests (symbols) versus scaling law (continuous lines). Dimensionless radius is shown as a function of dimensionless scaling time.

It is well known fact that different soils erode at different rate. However, the relationship between the erosion parameters (τ_c, k_{er}) and geotechnical or chemical properties of the soils remain unknown (Wan and Fell, 2002). Therefore, it is recommended to use hole erosion tests in order to evaluate the erosion parameters on any sample of cohesive soil from a site.

5 Possible application to dams and dykes

The rate of erosion has a significant influence on the time for progression of piping and development of a breach in earthdams, dykes or levees. This provides an indication of the amount of warning time available to evacuate the population at risk downstream of the dam, and hence has important implications for the management of dam safety.

Given that erosion has initiated, and the filters are absent or unable to stop erosion, the hydraulics of flow in concentrated leaks are such that erosion will progress to form a continuous tunnel (the pipe, Fig. 3). The rate of which this pipe will enlarge is dependent on the hydraulic gradient and the erodibility of the soil as measured by the coefficient of erosion k_{er}. Gross enlargement of the pipe may form a breach by collapse of the column of soil located above (the roof). The main issue adressed by testing and modelling is: what is the time for progression of piping and breaching ?

The time of piping can be re-written as $t_{er} = 2\gamma_s L / (k_{er} g \gamma_w \Delta H_w)$, therefore a function of well known engineering quantities: the coefficient of erosion k_{er}, the soil density γ_s, the water density γ_w, the head drop ΔH_w between upstream and downstream, the length of the pipe L, and the gravitational constant g. Let us assume that critical stress τ_c and coefficient of erosion k_{er} are known, with preliminary laboratory tests. Let us assume that we know the maximum radius $R_{collapse}$ of a hole in the dam before collapse of the roof, with a preliminary geotechnical analysis. A sketch of the evolution of the pipe as a function of time is represented in Fig. 4.

After initiation by backward erosion, phenomenon which remains far from known, the piping process begins with initial - and unknown - radius R_0. A visual inspection can provide an estimation of the output flow, thus an estimation of the actual radius R_d. The scaling law of piping erosion gives us the following estimation of the remaining time to breaching Δt_u :

$$\Delta t_u \approx t_{er} \ln\left(\frac{R_{collapse} - R_{\min}}{R_d - R_{\min}}\right), \qquad R_{\min} = \frac{2L\tau_c}{\gamma_w \Delta H_w}.$$

Application to Teton dam case gives rough estimation of the remaining time between eyewitnesses observations and breaching. The dam failed during its first filling on June 5, 1976. It was the highest embankment dam that had ever failed catastrophically in the entire history of earth dam construction (Penman, 1987).

Head drop and pipe length can be roughly estimated: $\Delta H_w \approx$30 m and $L \approx$100 m. Laboratory tests gave 20 Pa for the critical stress τ_c and 10^{-3} s/m for the coefficient of erosion k_{er} (Wan and Fell, 2002). Therefore, $R_{\min} \approx$13 cm. The maximum diameter before collapse is unknown. We take $2R_{collapse} = 2H / 3$ where $H \approx$90 m is the relevant dam height.

Eyewitnesses noticed a leak flowing about $Q \approx$1 m/s by 9 :30 AM. We can estimate the corresponding diameter of the hole to be $2R_d \approx$50 cm. The estimation of the remaining time Δt_u between eyewitnesses observations and breaching is therefore about 2:30 hours. This result agrees with reality: at 11h20 AM the eroded hole in the dam was so large that bulldozers sent to fill the hole sank into the flow, and at about 11h55 AM the dam crest was breached as a complete failure occurred.

The coefficient of erosion k_{er} can therefore serve as an indicator of the remaining time to breaching: a value of 10^{-4} s/m would give a remaining time of 20 hours !

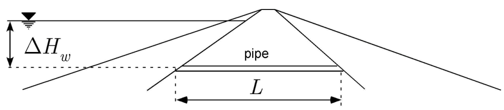

FIG. 3 – Sketch of the piping erosion in a dyke or a dam.

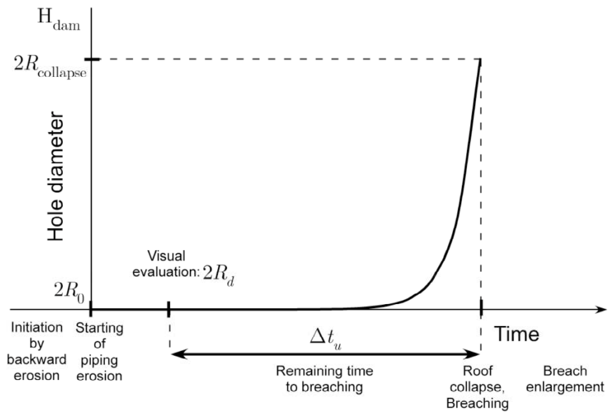

FIG. 4 – Evaluation of remaining time to breaching using the scaling law

6 Conclusion

The hole erosion test appears to be an efficient and simple means of quantifying the rate of piping erosion at laboratory. The scaling law obtained for interpreting the results of this test makes possible a unified description of piping erosion of differents soils, in pipe of various diameters. This description provides an estimation of the remaining time to breaching of hydraulic works (earthdams, dykes) when piping erosion occurs. This is an indication of the amount of warning time available for evacuation. However, two conditions are required: early (visual) detection of piping, and preliminary laboratory (hole erosion) tests.

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