A scaling analysis of ozone photochemistry: I Model development

B. Ainslie, D. G. Steyn

To cite this version:

HAL Id: hal-00302005
https://hal.archives-ouvertes.fr/hal-00302005
Submitted on 19 Dec 2005

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
A scaling analysis of ozone photochemistry: I Model development

B. Ainslie and D. G. Steyn

Atmospheric Science Programme, The University of British Columbia, Vancouver, Canada

Received: 14 October 2005 – Accepted: 23 November 2005 – Published: 19 December 2005

Correspondence to: B. Ainslie (bainslie@eos.ubc.ca)

© 2005 Author(s). This work is licensed under a Creative Commons License.
Abstract

A scaling analysis has been used to capture the integrated behaviour of several photochemical mechanisms for a wide range of precursor concentrations and a variety of environmental conditions. The Buckingham Pi method of dimensional analysis was used to express the relevant variables in terms of dimensionless groups. These grouping show maximum ozone, initial NO\textsubscript{x} and initial VOC concentrations are made non-dimensional by the average NO\textsubscript{2} photolysis rate ($j_{av}$) and the rate constant for the NO-O\textsubscript{3} titration reaction ($k_{NO}$); temperature by the NO-O\textsubscript{3} activation energy ($E_{NO}$) and Boltzmann constant ($k$) and total irradiation time by the cumulative $j_{av}\Delta t$ photolysis rate ($\pi_3$). The analysis shows dimensionless maximum ozone concentration can be described by a product of powers of dimensionless initial NO\textsubscript{x} concentration, dimensionless temperature, and a similarity curve directly dependent on the ratio of initial VOC to NO\textsubscript{x} concentration and implicitly dependent on the cumulative NO\textsubscript{2} photolysis rate. When Weibull transformed, the similarity relationship shows a scaling break with dimensionless model output clustering onto two straight line segments, parameterized using four variables: two describing the slopes of the line segments and two giving the location of their intersection. A fifth parameter is used to normalize the model output. The scaling analysis, similarity curve and parameterization appear to be independent of the details of the chemical mechanism, hold for a variety of VOC species and mixtures and a wide range of temperatures and actinic fluxes.

1. Introduction

The link between NO\textsubscript{x}, mixtures of volatile organic compounds (VOCs) and sunlight in the production of ground level ozone was first recognized in the Los Angeles area in the 1940s (Haagen-Smit, 1952). Since that time, a great deal of effort has been directed at understanding tropospheric photochemistry. This research is complicated by the large number of VOC species found in urban environments and the manifold
reaction paths in which these species can participate – an explicit treatment of ozone formation would require thousands of reactants participating in an even greater number of reactions (Atkinson, 2000). To simplify the representation of these reactions in computer models, different means of condensing the chemistry have been employed, typically yielding chemical mechanisms with 30 to 50 species and between 80 and 200 reactions (Dodge, 2000). While this represents a significant reduction, the resulting mechanisms remain complex.

It is not surprising, then, that various highly idealized models, aimed at encapsulating the most important photochemical processes in as compact form as possible, have been developed alongside these chemical mechanisms. For example Johnson (1984) has developed an analytical model for ozone photochemistry, using smog chamber data, to express the relationship between primary smog products (amount of ozone produced and NO converted to NO$_2$) and initial VOC and NO$_x$ concentrations, in terms of simple formulae. The relationships are valid over a wider range of precursor conditions but appear to provide an incorrect sensitivity to initial NO$_x$ concentration (Blanchard et al., 1994). Statistical and regression techniques have also been used to capture the behaviour of chemical systems: Shenvi et al. (2004) use a neural network to represent the behaviour of an explicit formaldehyde oxidation scheme in terms of a limited number of input variables; Wang et al. (2001) map the output of box model simulations onto a small set of input variables, using an expansion of correlated functions, to develop a high dimensional model representation of a chemical mechanism; Chang and Rudy (1993) develop a relationship for the ozone response surface of a photochemical mechanism in terms of initial VOC, NO$_x$ concentrations and temperature, through a non-linear function minimization routine. A third approach to understanding the behaviour of a complex system is through the use of a simplified physical system: Azzi et al. (1992) develop a highly parameterized set of chemical reactions, involving only a few species, to provide a conceptual framework for ozone photochemistry, though their “mechanism” appears to incorrectly model ozone formation in the low VOC/NO$_x$ regimes (Tonnesen and Jeffries, 1994).
The aim of this paper is to use a scaling-level analysis as a means of developing a simple model for the behaviour of a photochemical mechanism. This type of analysis, which looks for similarities between systems or phenomena which are actually different, has been applied to a wide range of complex systems including: physical (Taylor, 1950), biological (West et al., 1997) and hydrodynamical (Birkhoff, 1950). It can be used to determine the minimum number of essential measurements in experimental work, provide an effective way of displaying large data sets, or reduce complex systems to a simpler form (Bridgman, 1937; Barenblatt, 1996). While the aim of this analysis is to capture essence of ozone photochemistry while excluding a great many details, we are not implying these details are unimportant, but, rather that their influence is often overshadowed by more dominant processes or can be accounted for by using carefully selected variables.

2. Scaling analysis

While photochemical mechanisms consist of hundreds of competing reactions having a variety of timescales, qualitatively they can be understood in terms of an interconnected NOx and radical cycle, each driven by sunlight (Jeffries and Tonnesen, 1994). The first cycle involves the photolysis of NO2 and titration of O3 by NO and consists of relatively fast inorganic reactions. The second cycle involves the slow conversion of NO-to-NO2 via organic peroxy radicals produced in catalytic chain reactions initiated by OH• attack on VOCs. It is the integrated effect of these organic reactions that leads to high ozone concentrations seen in and around many polluted urban environments. Guided by this conceptual model, we use the Buckingham Pi method of scaling analysis – a systematic approach to dimensional analysis in which all relevant physical variables are placed into dimensionless groups (Stull, 1988) – to express the integrated behaviour of a photochemical mechanism in terms of a small set of non-dimensional groups.

Thus, in a hypothetical box of air having volume V, we expect the maximum amount of ozone (NO3) produced from an initial amount of VOC (NVOC) and NOx (NNOX), will
be a function of: the average NO$_2$ photolysis rate constant ($j_{av}$), the length of time the mixture is irradiated ($\Delta t$), the activation energy ($E_{NO}$) and molecular collision rate ($A_{NO}$) for the NO-O$_3$ titration rate. Because OH$^\cdot$ attack represents a rate limiting step in ozone formation, we use the activation energy of the VOC-OH$^\cdot$ reaction ($E_{OH}$), it’s collision rate ($A_{OH}$) and the average molecular translational energy ($kT$) as measures of the importance of the organic reactions.

We have identified eleven different variables having four separate fundamental dimensions of length, time, mass and amount of substance (Table 1 summarizes the relevant variables and their dimensions). Dimensional analysis allows us to select any four of these variables (equal to the number of fundamental dimensions) to non-dimensionalize the remaining variables provided all of the fundamental dimensions are represented by the key variables and the key variables cannot be arranged into a single non-dimensional group. We choose $kT$, $V$, $j_{av}$ and $A_{NO}$ to non-dimensionalize the remaining variables giving the following seven (11 variables – 4 fundamental dimensions) dimensionless groups:

\[
\pi = \frac{N_{O_3}A_{NO}}{Vj_{av}}, \pi'_{1} = \frac{N_{VOC}A_{NO}}{Vj_{av}}, \pi_{2} = \frac{N_{NO_x}A_{NO}}{Vj_{av}}, \\
\pi_{3} = j_{av}\Delta t, \pi'_{4} = \frac{E_{OH}}{kT}, \pi'_{5} = \frac{E_{NO}}{kT}, \pi_{6} = \frac{A_{NO}}{A_{OH}}
\]

(1)

Next, to assist in the analysis and without loss of generality, we form new dimensionless groups from these seven. A new $\pi_1$ is formed by dividing $\pi'_1$ by $\pi_2$ and represents the initial relative abundance of VOC to NO$_x$. Next, since molecular collision theory shows the partitioning of molecules with energy sufficient for bond breaking has an exponential dependence on both the activation and translational energy, $\pi'_4$ and $\pi'_5$ are rewritten using exponentials, $\pi_4 = \exp\{\pi'_4\}$ and $\pi_5 = \exp\{\pi'_5\}$.

The premise of dimensional analysis is that given a complete set of independent variables that account for the behaviour of a system, then a subset of dimensionally independent variables can be found which also captures the system behaviour. Thus,
assuming the original dimensional variables account for all of the important processes, then dimensional analysis suggests the relationship between maximum ozone and its governing variables can be expressed by:

$$\pi = \hat{f}(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6)$$

(2)

where \( \hat{f} \) is an unknown function of only six independent (dimensionless) variables. While this marks the end of the formal Buckingham Pi analysis, we make three further simplifications.

First, for Arrhenius type reactions, \( A_{NO} \) and \( A_{OH} \) are independent of temperature, their ratio becomes a constant and so \( \pi_6 \) will not influence the functional form of the similarity relationship and can be ignored.

Second, we observe for an \( \text{O}_3\)-NO-\( \text{NO}_2 \) system (i.e. one without VOCs), steady state ozone concentration scales with \( j/k_{NO} \):

$$\frac{[\text{O}_3]_{ss}}{j/k_{NO}} = \frac{[\text{NO}_2]_{ss}}{[\text{NO}]_{ss}}$$

(3)

For our \( \text{O}_3\)-\( \text{NO}_x \)-VOC system, a similar scaling for ozone can be achieved by dividing \( \pi \) by \( \pi_5 \):

$$\frac{\pi}{\pi_5} = \frac{N_{O_3}}{V \cdot j_{av}} = \frac{[\text{O}_3]_{max}}{j_{av}/k_{NO}}$$

(4)

where \([\text{O}_3]_{max} (=N_{O_3}/V)\) is the maximum ozone concentration and \( k_{NO} (=A_{NO} \exp\{-E_{NO}/(kT)\}) \) the Arrhenius rate constant for the NO-ozone titration reaction. Accordingly, we scale the dimensionless number of moles of \( \text{NO}_x \) (\( \pi_2 \)) by \( \pi_5 \) and guess the functional dependence of \( \pi_5 \) can be approximated via \( \pi/\pi_5 \) and \( \pi_2/\pi_5 \).

Third, we try a power law for \( \pi_2/\pi_5 \) dependence. This last assumption is guided by the work of Chang and Rudy (1993), Blanchard et al. (1999), and Johnson (1984) who all suggest that maximum ozone concentration scales with a power of initial \( \text{NO}_x \) concentration.
The objective is then to search for empirical relations of the form:

\[ \frac{\pi}{\pi_5} = \left( \frac{\pi_2}{\pi_5} \right)^a f(\pi_1, \pi_3, \pi_4) \]  

(5)

where \(a\) is an unknown exponent and the unknown function \((f)\) is now only dependent on three dimensionless groups.

To start, we use the RADM2 photochemical mechanism (Stockwell et al., 1990) to simulate the photochemical oxidation of the RADM2 propene surrogate (OLT) at constant temperature and for a time varying actinic flux consistent with a single location and date. We use the model output to find the functional form of \(f(\pi_1)\). Then we examine simulations using different VOCs and VOC mixtures. The analysis is then extended to include a range of temperatures \((\pi_3)\) and different total actinic fluxes \((\pi_4)\).

2.1. OZIPR simulations

To generate model output, a matrix of 121 simulations using the OZIPR (Tonnesen, 2000) trajectory model was run using the RADM2 chemical mechanism. In order to facilitate the interpretation of our results, we study the photochemistry in isolation from both mechanical and environmental processes i.e. we simulate a well mixed parcel of air free from dilution, entrainment, deposition or “wall-effects”; which is kept at a constant temperature of 25°C; and exposed to a total dimensionless actinic flux of \(\pi_3=347\) (corresponding to Vancouver B.C. in mid-summer (3 August) with simulation start time at 07:00 a.m. (local solar time (LST)) and end time at 06:00 p.m. LST). We create the test matrix by independently varying the initial OLT and \(\text{NO}_x\) concentrations in 10% increments within a 0.0 and 0.6 ppm OLT range and a was 0.0 to 0.15 ppm \(\text{NO}_x\) range. The simulations produced an overall maximum modeled ozone concentration of 348 ppb.

Model output consisted of maximum ozone concentration along with initial OLT and \(\text{NO}_x\) concentrations. However, before the model output was analyzed, some simulations were excluded. Whenever initial \(\text{NO}_x\) concentration was zero (and \(\pi_1\) infinite), the
RADM2 mechanism produced no ozone and these trivial simulations were removed. In addition, whenever initial OLT concentration were zero ($\pi_1 = 0$), the RADM2 mechanism produced low ozone concentrations, independent of OLT, representing ozone formation in a NO$_x$-only system. Since the scaling analysis is concerned with VOC-NO$_x$ systems, these simulations were also excluded. After these removals, 100 simulations remained to determine $f(\pi_1)$.

2.2. Similarity relationship

Using Eq. (5), modeled maximum ozone concentration was made dimensionless and scaled by a power of dimensionless initial NO$_x$ i.e.:

$$\frac{[O_3]_{\text{max}}}{j_{av}/k_{NO}} = \left(\frac{[\text{NO}_x]_o}{j_{av}/k_{NO}}\right)^a$$

(6)

It was found that with a value of $a = 0.6$, model output, when plotted using Eq. (6), and as a function of $\pi_1$ (conventionally given by the variable $R$), collapsed onto a single common curve (Fig. 1); suggesting success in the scaling analysis. It must be pointed out that this curve does not plot the time evolution of ozone for a single simulation, but rather, it shows scaled maximum ozone concentration (from many different simulations) divided by scaled initial NO$_x$ concentration (raised to a power $a$) versus initial VOC to NO$_x$ concentration from the various simulations. The similarity of this figure to the time evolution of ozone concentration seen in non-diluted and long-irradiated experiments (Jeffries, 1995) is discussed in a companion paper.

The sigmoid shape of this curve hints at a parameterization using a Weibull function. The idea for such parameterization is guided by Chang and Rudy (1993) who using model output, found the function:

$$\phi(R) = 1 - \exp \left\{-aR^b\right\}$$

(7)
fit their NO\textsubscript{x}-scaled, dimensional ozone concentrations well. This expression, chosen because of its simple functional form (Chang and Rudy, 1993), can be expressed as a Weibull function after the transformation \( b = \alpha \) and \( a = \beta^{-\alpha} \).

To test the suitability of the Weibull function, the dimensionless model output was normalized and Weibull transformed:

\[
W(f(R)) = \ln \left( \ln \left( \frac{1}{1 - f(R)/\gamma} \right) \right)
\]  

and then plotted as a function of \( \ln R \) (Fig. 2).

From this figure, it is evident that the dimensionless model output collapses not onto one, but two straight lines, which separates the similarity relationship into two regions characterized by a change in slope at \( \ln R \approx 1.3 \) (\( R \approx 3.7 \)). This shift hints at a change in governing chemical process; perhaps evidence of the change in ozone sensitivity to changing NO\textsubscript{x} concentration across the ridgeline.

The dashed line the Fig. 2 defines two regions: Region I, to the right of the break and regions II to the left. Within each of these regions dotted lines indicate \( \ln R \)-values where more scatter appears. The scatter for low \( \ln R \) values, associated with simulations having low OLT/high NO\textsubscript{x} concentrations, stems from a slight dependence of the NO\textsubscript{x}-scaling exponent (\( a \)) on \( R \). The increased scatter and slight hook in the model output at high \( \ln(R) \) is explored in Sect. 3.2.

A single continuous curve for the similarity relationship, which captures the scaling break, has been developed. It is a composite of two Weibull functions with shape parameter (\( \alpha \)) that varies as a function of \( R \):

\[
f(R)/\gamma = 1 - \exp \left\{ -\lambda \left( R/\beta \right)^{\alpha(R)} \right\}
\]

\[
\alpha(R) = \left( \frac{\alpha_2 - \alpha_1}{2} \right) \tanh(R - \beta) + \left( \frac{\alpha_1 + \alpha_2}{2} \right)
\]

In this function \( \alpha_1 \) and \( \alpha_2 \) are the slopes of line segments fitting the Weibull transformed model output, \( \beta \) and \( \lambda \) give the location of the intersection of the two segments.
When \( \alpha_1 = \alpha_2 \) and \( \lambda = 1 \), a Weibull function is recovered. When \( R \to \infty \) (the relative abundance of VOC to NO\(_x\) is large) \( f(R) \to \gamma \) and maximum ozone concentration is a function of initial NO\(_x\) concentration only (which we call the NO\(_x\)-only scaling (NOS) regime) consistent with Chang and Suzio (1995), Blanchard et al. (1999) and Johnson (1984).

We use Eqs. (9) and (5) to plot predicted and modeled maximum ozone isopleths in Fig. 3. A line of constant \( R \), associated with the scaling break (\( \beta \)), has also been plotted. The ridgeline, as determined by the location where the sensitivity of ozone to changing NO\(_x\) concentrations is zero has also been drawn. It has a value of \( R \approx 4.8 \), and appears to lie below the scaling break. From Fig. 3, we see that the similarity relationship fits the ozone isopleths quite well in most of the domain but with the similarity function diverging from the model output at the lowest [OLT]_o/[NO\(_x\)]_o values.

### 3. Other VOC classes and VOC mixtures

Analyses have been carried out for all RADM2 classes in addition to several VOC mixtures. A selection of the resulting Weibull transformed similarity relationships is given in Fig. 4, and, with the exception of the low reactive ethane surrogate ((ETH) in Fig. 4A) and the pentane-like alkanes (H5C) with moderate OH\(^*\) reactivities in Fig. 4B, all similarity relationships show model output clustering onto two distinct line segments separated by a scaling break. In order to capture the wide range of oxidation behaviours exhibited by the RADM2 classes, initial NO\(_x\) and VOC concentrations, as well as test matrix size, were adjusted for each set of simulations. In general, initial NO\(_x\) concentrations were chosen so that greatest maximum ozone concentration (peak concentration) was between 100 and 300 ppb – a range considered representative of elevated but not unrealistically high ozone pollution concentrations (National Research Council, 1991). Next, initial VOC concentrations were chosen so that there was roughly an equal number of simulations in on either side of the scaling break (\( \beta \)). Finally, the number of simulations was chosen so that the similarity relationship \( f(R) \) exhibited asymptotic
behaviour for large $R$-values. This final constraint required adjusting the increment size for the test matrix; smaller increments giving rise to more simulations over a wider range of $R$-values. For this work only two increments sizes were used: 10% and 2.5% giving 121 and 1681 simulations respectively. This last constraint introduces a difficulty with the analysis: in order to capture the asymptotic behaviour of the similarity relationship, and in order to keep an equal number of simulations on either side of the scaling break, some VOCs were analyzed over a greater range of precursor conditions than others. However, we find the scaling behaviour holds only over a finite range of precursor concentrations (Ainslie, 2004). Thus, the photochemistry of some VOCs, over the chosen range of conditions that covers the VOC- and NO$_x$-limited regions as well as the ridgeline, is easier to parameterize than others and comparisons of results must take this range into account.

3.1. Alkane classes (ETH, HC3, HC5 and HC8) and Non-reactive class (NR)

In general, unreactive VOCs (principally the alkanes) were the most difficult to scale and were poorly fit by the similarity relationship. The ethane surrogate ETH (Fig. 4A) produces a break around $R \approx 670$ but requires $R$-values greater than 22 000 to reach its asymptotic value. Furthermore, the NO$_x$-scaling exponent ($a$) shows an $R$-dependence which leads to increased scatter over the large range of $R$-values shown. The alkane class with the next lowest OH•-reactivities (HC3) produced similar trends (not shown). Surprisingly, the higher alkane classes HC5 (Fig. 4B) and HC8 (not shown) showed poorer scaling than the ethane and HC3 class. The non-reactive class did not appear to scale with any power of [NO$_x$]$^a$ (not shown).

It is likely in the alkane simulations, with the low OH•-reactivities and limited organic radical sources, the conceptual model used to identify key variables is incomplete. For example, the poor scaling shown by the HC8 class likely arises from increased NO$_x$ and peroxy radical losses due to increased organic nitrate formation – reaction pathways whose effects would require the inclusion of new dimensionless groups into the scaling
3.2. Alkene classes (OL2, OLI and OLT)

Of the three alkene classes, the ethene class (OL2) was the hardest to fit, likely due to its low OH\(^{-}\)-reactivity and produced results similar to ETH but with less scatter at low \(\ln(R)\) (not shown). OLI (the surrogate for alkenes with internal double bonds) showed good scaling and, like OLT, shows a distinct second break in the Weibull transformed model output at high \(R\)-values (Fig. 4C). This marks the beginning of a new scaling regime where alkene+O\(_3\) reactions are important. This new trend, difficult to see on an isopleth diagram because these diagrams compress the response surface along the VOC-axis for high \(R\)-values, is not of immediate concern since this regime is greatly removed from the ridgeline where parameterizing the photochemistry is of greatest interest.

3.3. Aromatics (TOLU and XYLE)

Results for the aromatic classes TOLU, representative of toluene and less reactive organics is given in Fig. 4D. It, and the XYLE aromatic class (representing xylenes and more reactive aromatics (not shown)), show more scatter than alkenes but still produce a break in the Weibull transformed model output and are well parameterized by the similarity function.

3.4. Carbonyl (HCHO, ALD and KET)

RADM2 explicitly models formaldehyde and this class (HCHO) proved difficult to scale due the extremely large \(R\)-values required to capture it’s asymptotic behaviour and the modest kink in the Weibull transformed model output (Fig. 4E). The slight break, caused by the shallow slope in the low \(\ln(R)\) line segment, stems from the photolysis of formaldehyde acting as a strong organic radical source and is further discussed in a companion paper. The large range of \(R\)-values required to reach an asymptotic value...
appears to be related to its inability to produce PAN or ONIT precursors; requiring all NO\textsubscript{x} to be converted to HNO\textsubscript{3} before photochemical production ceases (Ainslie, 2004). The higher aldehyde class (ALD) proved to be the best class to model with good agreement between the similarity function and model output everywhere except at the very lowest \(R\)-values (Fig. 4F). The ketone class (KET) was also well modeled (not shown).

3.5. Mixtures

Two VOC mixtures, representative of polluted urban environments, were also modeled using RADM2. The first mixture was an Air Resources Board (ARB) modified version of the (Jeffries et al., 1986) analysis of the Lonneman (Lonneman, 1986) 29 city VOC canister study (Fig. 4G). The second was a mixture used by Stockwell et al. (1997) for his comparison of his RADM2 model and the Regional Atmospheric Chemistry Mechanism (RACM) model (not shown). Both mixtures show good scaling with a distinct scaling break and slight deviations at very low \(R\)-values.

3.6. Other mechanisms

To test whether the scaling is a particularity of the RADM2 mechanism, three additional chemical mechanisms were examined: CB-IV (Gery et al., 1989), SAPRC (Carter, 1990) and the master chemical mechanism (MCM) (Saunders et al., 1997). The first two mechanisms use entirely different approaches to condensing the photochemistry than RADM2 while the third provides a near explicit description of the gas phase tropospheric oxidation of over 120 VOC compounds. Any scaling of \([O_3]_{\text{max}}\) observed from these mechanism strengthens the premise that complex photochemical processes can be described using a small set of variables – at least to the extent that numerical models capture the chemical processes. While only the CB-IV and MCM results are discussed here, the SAPRC model produced results similar to the RADM2 mechanism.

Two simulations were run using CB-IV. The first, using CB-IV classes PAR and OLE
to represent propene, showed good scaling and produced a scaling break (not shown). The second (Fig. 4H) simulated the ARB urban mixture used in the RADM2 simulations and produced a similarity relationship almost identical to the RADM2 ARB mixture (Fig. 4G). The MCM propene results, given in Fig. 4I, show a less distinct break than the RADM2 OLT results (Fig. 2) with the MCM similarity relationship showing a slight downward trend for ln(R)>1, suggesting enhanced ozonolysis in the MCM mechanism. There is also more scatter in the similarity relationship especially near ln(R)=3 and ln(R)=4.

4. Variability of scaling parameters with \( \pi_3 \) and \( \pi_4 \)

To examine the combined effects of varying actinic flux (\( \pi_3 \)), temperature (\( \pi_4 \)) and initial precursor concentration (\( \pi_2 \) and \( \pi_1 \)) on the scaling analysis, OZIPR simulations were performed for an OLT-NO\(_x\)-O\(_3\) system at five different \( \pi_3 \)-values (204, 228, 257, 287 and 319 corresponding to five summer dates encompassing the typical ozone season in Vancouver, B.C.; Pryor and Steyn, 1995) and at four different temperatures (20, 25, 30 and 35\(^\circ\)C). This gave a total of 20 scenarios with each consisting of 121 simulations of varying initial OLT and NO\(_x\) concentrations.

4.1. “Universal” similarity relationship

To find a “universal” similarity relationship which includes the \( \pi_3 \) and \( \pi_4 \) dependencies, modeled maximum ozone and initial NO\(_x\) concentrations, from each of the 2000 (20\( \times \)100) non-trivial simulations, were first made dimensionless by \( k_{NO} \) and \( j_{av} \) (dependent on simulation temperature and actinic flux, respectively). Next a power law was tried for the \( \pi_4 \)-dependence, and dimensionless maximum ozone was scaled by \( \pi_2^a \) and \( \pi_4^b \) (found iteratively and explained below). The resulting largest value for \( \pi/(\pi_2^a \pi_4^b) \) was used as a normalizing constant (\( \gamma \)). The resulting model output was Weibull transformed and plotted as a function of ln\( R \) which revealed a family of separate curves,
distinguished by their $\pi_3$-values and horizontally translated from one another. To get these curves to collapse onto a “universal” curve, each curve was translated so their scaling breaks coincided. This amounted to a translation by $\ln \beta_{av}$ where $\beta_{av}$ is the average $\beta$-value for each $\pi_3$-level (the $\beta$-values showed a slight $\pi_4$ (temperature) dependence). The amount of translation was parameterized using a power law:

$$\beta_{av} = \beta' \pi_3^{-c}$$

where $\beta'$ and $c$ were found by regressing $\beta_{av}$ against $\pi_3$.

Figure 5 shows all the 20 sets of Weibullized transformed model output after translating by $\ln \beta_{av}$. The Figure shows large scatter for both $\ln(\frac{R}{\beta})<-1.0$ and $\ln(\frac{R}{b})>1.0$. One simulation ($J=204$ $T=20^\circ$C) appeared to deviate from the others. This simulation had the lowest $\pi_3$-value and lowest temperature. This low flux/low temperature simulation suggests a limit of applicability to the scaling regime and was not used in the determination of the similarity function. Also plotted in Fig. 5 is the similarity function fitted to the model output from the 19 other scenarios. Values for the scaling powers ($a$ and $b$) as well as the similarity parameters ($\alpha_1$, $\alpha_2$, $\lambda$) were found by minimizing the RMSE between the similarity function and OZIPR model output where the function was forced to fit the model output more closely around the ridgeline area (defined by $\ln(\frac{R}{\beta_{av}})\in[-1.61, 1.61]$).

4.2. Universal propene curve

Based on the above analysis, the “universal” curve OLT has the form:

$$\frac{[O_3]_{max}}{\frac{j_{av}}{k_{NO}}} = \gamma \left( \frac{[NO_x]_o}{\frac{j_{av}}{k_{NO}}} \right)^a \exp \left\{ -\frac{E_{OH}}{kT} \right\}^b \times f(R; \beta_{av}, \alpha_1, \alpha_2, \lambda)$$

and

$$\beta_{av} = \beta' \pi_3^{-c}$$

$\gamma = 3.45 \times 10^5$, $\lambda = 0.62$, $\alpha_1 = 2.5$, $\alpha_2 = 0.69$
$a = 0.65, b = 6.1$

$\beta' = 413, c = -0.81$

This “universal” curve captures the variability of maximum ozone concentration over a wide range of NO$_x$ and OLT concentrations, as well as a range of actinic fluxes and temperatures typical of ozone formation in urban environments (National Research Council, 1991). A total of 8 parameters are needed to fully characterize this variability.

The set of dependent dimensionless groups used to parameterize the behaviour of the various photochemical mechanisms can be compared with findings from a local sensitivity analysis of the RADM2 mechanism (Gao et al., 1995). While the aims of that analysis differs from ours (identification of specific reaction and photolysis rate constants whose uncertainty lead to the largest uncertainty in modeled ozone concentrations versus representing the integrated behaviour of RADM2 by as few variables as possible), they show several common traits. First, the sensitivity analysis shows ozone is relatively insensitive to the parameters of about 90% of the reactions in RADM2, consistent with our scaling of integrated model behaviour using only a few variables. Secondly, their analysis shows ozone sensitivities are strongly dependent on the VOC to NO$_x$ ratio, consistent with the central role played by $\pi_1$ in the scaling. Next, they show the strong dependence of ozone on the NO$_2$, HCHO and O$_3$ photolysis reactions. In the present analysis, $\pi_3$ acts as a surrogate for the total actinic flux and we assume that changes in $\pi_3$ can account for all changes in photolytic activity along the various photolytic pathways. Finally, Gao et al. (1995) show modeled ozone concentrations are more sensitive to the NO$_2$+OH reaction than most of the VOC the OH reactions. This suggests that NO$_2$+OH might have been a better reaction for the OH-pathway reactions than the VOC+OH reaction used here. However, Tonnesen (1999) shows peak ozone concentrations in RADM2 box model simulations are not strongly affected by changes in $k_{HNO3}$ because of strong feedback loops which buffer peak ozone concentrations from changes in the mechanism. Thus the choice of variables used to account for the radical reaction pathways may have little impact on the scaling. It is the integrated behaviour of these feedback loops, and their sensitivity to the relative abundance of VOC
to NO\textsubscript{x}, that the scaling analysis captures. While each chemical mechanism accounts for these feedback loops by different means, the similarity relationships show these loops express themselves in a common manner.

5. Conclusions

Eulerian photochemical models are the principle means used to study tropospheric ozone at urban, regional and continental scales. Central to these models are their photochemical mechanisms. In this paper, we have used dimensional analysis, commonly used in many branches of atmospheric science, to provide simple representations of the behaviour of several photochemical mechanisms. Our research shows dimensionless maximum ozone concentration (\(\pi\)) can be described by a product of powers of dimensionless initial NO\textsubscript{x} concentration (\(\pi_2\)), dimensionless temperature (\(\pi_4\)) and a similarity curve (\(f\)) directly dependent on the ratio of initial VOC to NO\textsubscript{x} concentration (\(\pi_1\)) and internally dependent on the cumulative NO\textsubscript{2} photolysis rate (\(\pi_3\)). The scaled dimensionless model output, when Weibull transformed, exhibits a striking scaling break which suggests the photochemical system switches governing regimes. The break coincides closely (but not exactly) with a change in ozone sensitivity to initial NO\textsubscript{x} concentration (the ridgeline on an isopleth diagram). This close but not exact relationship is also seen between the scaling break and maxima in OH\textsuperscript{*}-production and OH\textsuperscript{*}-chain-length (Ainslie, 2004). In a companion paper, we examine the break using a process based analysis. The regime switch, revealed here through a scaling analysis using model output, can be compared with similar findings using different approaches: Johnson’s (1984) parameterization of ozone formation into light- and NO\textsubscript{x}-limited regimes based on smog chamber data; the Kleinman (2005) radical budget analysis of ozone production (\(d[O_3]/dt\)) sensitivities to [VOC] and [NO] based on a simplified photochemical reaction set; and the Sillman (1995) analysis of a simplified set of reactions to identify NO\textsubscript{x}- and VOC-sensitive regions using observed concentrations of certain long-lived “indicator” species.
The similarity function captures the complex dependence of ozone formation over a wide range of initial precursor concentrations: above, below and around the ridgeline. This represents an improvement over similar efforts by Johnson (1984) and Chang and Rudy (1993). The similarity function predicts, in the NO$_x$-limited region, maximum ozone concentration to be a function of initial NO$_x$ concentration only, consistent with Chang and Suzio (1995), Blanchard et al. (1999) and Johnson (1984). However, we find the dependence on initial NO$_x$ concentration varies with VOC. We find the qualitative features of the similarity relationship hold for a wide range of VOCs, VOC mixtures and for other chemical mechanisms and plotting Weibull transformed dimensionless model output as a function of ln $R$ provides a new way of representing the behaviour of a photochemical mechanism. The scaling methods have allowed us to include temperature and actinic flux dependencies in the development of a “universal” similarity function. Finally, while the sum of ozone produced and NO oxidized provides a more complete description of the photochemical oxidation process, we find the scaling analysis and similarity function are qualitatively the same when using oxidized NO+O$_3$ produced instead of O$_3$.

While our analysis suggests that over a wide range of initial precursor concentrations and environmental conditions, and when viewed from the proper perspective (provided by the scaling analysis), the complex behaviour of a photochemical mechanism can be simply described, it must be stressed that this high-level representation is not intended to accurately describe the internal workings of a photochemical system but rather to accurately reproduce the behaviour of a chemical mechanism (whether or not chemical mechanisms faithfully represent actual ozone formation is a difficult but different question). Furthermore, in order to facilitate the analysis of our results, we have studied photochemical mechanisms in isolation from important physical processes like deposition and entrainment. An investigation of how these physical processes compete with chemical processes could be achieved by a scaling-level analysis of model output from idealized 3-D Eulerian simulations.

**Acknowledgements.** Funding for this research was through the University of British Columbia.
graduate fellowships (UGF), the National Science and Engineering Research Council (NSERC) and the Canadian Foundation for Climate and Atmospheric Sciences (CFCAS). Special thanks to A. Rickard at the University of Leeds for the MCM simulations.

References


Bridgman, P. W.: Dimensional Analysis, Yale University Press, New Haven, CT., USA, 113 pp., 1937. 12960


Sillman, S.: The use of $NO_y$, $H_2O_2$ and $HNO_3$ as indicators for ozone-NOx-hydrocarbon sensitivity in urban locations, J. Geophys. Res., 100, 14 175–14 188, 1995. 12973
Table 1. Physical quantities which affect ozone concentrations in a hypothetical box of air with their SI units and dimensions.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Units</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>Volume</td>
<td>$m^3$</td>
<td>length$^3$</td>
</tr>
<tr>
<td>$N_{O_3}$</td>
<td>Maximum amount of ozone</td>
<td>moles</td>
<td>number</td>
</tr>
<tr>
<td>$N_{NO_x}$</td>
<td>Initial amount of NO$_x$</td>
<td>moles</td>
<td>number</td>
</tr>
<tr>
<td>$j_{av}$</td>
<td>Average NO$_2$ photolysis rate constant</td>
<td>$s^{-1}$</td>
<td>time$^{-1}$</td>
</tr>
<tr>
<td>$A_{NO}$</td>
<td>NO-O$_3$ collision frequency</td>
<td>$cm^3$(molecules $\cdot$ s)$^{-1}$</td>
<td>length$^3$(number $\cdot$ time)$^{-1}$</td>
</tr>
<tr>
<td>$E_{NO}$</td>
<td>Activation energy for O$_3$+NO titration</td>
<td>J molecule$^{-1}$</td>
<td>mass $\cdot$ length$^2$ $\cdot$ time$^{-2}$ $\cdot$ number$^{-1}$</td>
</tr>
<tr>
<td>$N_{VOC}$</td>
<td>Initial amount of VOC</td>
<td>moles</td>
<td>number</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>Length of test</td>
<td>s</td>
<td>time</td>
</tr>
<tr>
<td>$A_{OH}$</td>
<td>VOC-OH collision frequency</td>
<td>$cm^3$(molecules $\cdot$ s)$^{-1}$</td>
<td>length$^3$(number $\cdot$ time)$^{-1}$</td>
</tr>
<tr>
<td>$E_{OH}$</td>
<td>Activation energy for VOC+OH$^\cdot$ reaction</td>
<td>J molecule$^{-1}$</td>
<td>mass $\cdot$ length$^2$ $\cdot$ time$^{-2}$ $\cdot$ number$^{-1}$</td>
</tr>
<tr>
<td>$kT$</td>
<td>Average molecular translational energy</td>
<td>J molecule$^{-1}$</td>
<td>mass $\cdot$ length$^2$ $\cdot$ time$^{-2}$ $\cdot$ number$^{-1}$</td>
</tr>
</tbody>
</table>
Fig. 1. Dimensionless ozone scaled by dimensionless initial NO$_x^a$ versus ratio of initial precursor concentration ($R$). Model output from RADM2 (Stockwell et al., 1990) simulations using OLT as the sole VOC, a constant temperature of 25°C and a total dimensionless actinic flux of $\pi_3$=347. Dashed line shows limiting value ($\gamma$) for the similarity relationship.
Fig. 2. Scaled model output after the Weibull transformation. To highlight the clustering of model output onto two line segments, a pair of solid lines has been included. Also shown are three vertical lines separating the plot into four regions. The dashed line at \( \ln R = 1.3 \) marks the change in slope while the two dotted lines mark the regions of increased scatter.
Fig. 3. Ozone isopleth (in ppb) for the NO\textsubscript{x}-OLT system. Solid lines are isopleths from OZIPR model. Dotted isopleths are based on the similarity function. Also shown is the ridgeline (dashed line labeled Ridge), scaling break (dot-dashed line labeled $R=\beta$) and Regions I and II.
Fig. 4. Weibull transformed similarity relationships for RADM2 classes: (A) ETH (B) HC5 (C) TOLU (D) OLI (E) HCHO (F) ALD. An urban VOC mixture is simulated using (G) the RADM2 mechanism and (H) CB-IV mechanism. Finally, results from MCM using propene are given in (I). In each plot model output is given by “+” signs and the similarity function by solid lines.
Fig. 5. Weibull transformed model output for RADM2 simulations having five different levels of actinic flux and four different temperatures after horizontal translation by $\ln \beta_{av}$. Solid lines is the “universal” similarity function.