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OFDM Channel Parameters Estimation used for ICI Reduction in time-varying Multipath Channels

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Abstract—In this paper, we present an improved channel matrix estimation algorithm for orthogonal-frequency-division-multiplexing downlink mobile communication systems using comb-type pilot. The channel matrix, which contains the channel frequency response and the coefficients of inter-sub-carrier-interference (ICI), is estimated using parametric multipath channel model. In the algorithm, we assume the delays are time-invariant over several symbols. With delays information, the multipath complex gains time average over the effective duration of each OFDM symbol are estimated using LS or LMMSE criteria. After that, the time-variation of the multipath complex gains within one OFDM symbol are obtained by using low-pass interpolation. Hence, the channel matrix can be calculated and the ICI can be reduced. The simulation results show that the proposed method, with less number of pilots and without suppression of interference, gives a good performance over LS and LMMSE estimators with low-pass interpolation and becomes better with starting interference suppression.

Index Terms—OFDM, ICI, SIS, channel estimation, time-varying channels.

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) is widely known as the promising communication technique in the current broadband wireless mobile communication system due to the high spectral efficiency and robustness to the multipath interference. Currently, OFDM has been adapted to the digital audio and video broadcasting (DAB/DVB) system, high-speed wireless local area networks (WLAN) such as IEEE802.11x, HIPERLAN II and multimedia mobile access communications (MMAC), ADSL, digital multimedia broadcasting (DMB) system and multi-band OFDM type ultra-wideband (MB-OFDM UWB) system, etc. In multi-carrier OFDM system, OFDM system is very vulnerable when the channel changes within one OFDM symbol. In such case, the orthogonality between subcarriers are easily broken down resulting the inter-sub-carrier-interference (ICI) so that system performance may be considerably degraded.

A dynamic estimation of channel is necessary since the radio channel is frequency selective and time-varying for wideband mobile communication systems [1]. In practice, the channel may have significant changes even within one OFDM symbol, therefore it is preferable to estimate channel by inserting pilot tones into each OFDM symbol which called comb-type pilot [2]. The comb-type pilot channel estimation algorithms consist generally to estimate the channel at pilot frequencies and to interpolate the channel frequency response. The estimation of the channel at the pilot frequencies for comb-type channel estimation can be based on Least Square (LS) or Linear Minimum Mean-Square-Error (LMMSE). LMMSE has been shown to have better performance than LS [2]. In [3], the complexity of LMMSE is reduced by deriving an optimal low-rank estimator with singular-value-decomposition. The interpolation techniques used in channel estimation are linear interpolation, second order interpolation, low-pass interpolation, spline cubic interpolation and time domain interpolation. In [4], low-pass interpolation has been shown to perform better than all the interpolation techniques.

In [5] the channel estimator is based on a parametric channel model, which consists to estimate directly the time delays and complex attenuations of the multi-path channel. This estimator yields the best performance among all comb-type pilot channel estimators, with the assumption that the channel is invariant within one OFDM symbol.

In this paper, we present an improved channel matrix estimation algorithm for orthogonal frequency-division multiplexing mobile communication systems using comb-type pilot. The channel matrix, which contains the channel frequency response and the coefficients of ICI, is computed by estimating the parameters of the multipath channel as in [5] but with considering the time variation within one OFDM symbol. Typically in Radio-Frequencies communications, phases and magnitudes of the channels coefficients vary considerably faster than the channel delays. This channel nature can be exploited to design tracking algorithm, as we already did in CDMA context [9] [10]. In the present proposed algorithm, we assume that the delays are invariant (over several OFDM symbols) and perfectly estimated and only the complex gains of the paths have to be estimated. Note that an initial very performant multipath time delays estimation can be obtained by using the ESPRIT (estimation of signal parameters by rotational invariance techniques) method [5] [7]. With the multipath time delays information and for a block of OFDM symbols, the complex gains time average over the effective duration of each OFDM symbol of the different path are estimated using LS or LMMSE criteria. After that, the time variation of the different path complex gains within one OFDM symbol are obtained by using low-pass interpolation. Hence, the matrix channel can be computed and the ICI can be reduced by using the successive interference suppression (SIS). Note that instead of SIS, other detection schemes could be used from the estimated channel matrix.

The authors are with the GIPSA lab. (ex LIS lab.), Grenoble, France (e-mail: hussein.hijazi@lis.inpg.fr, laurent.ros@lis.inpg.fr. † This paper is dedicated to the memory of Genevieve Jourdain, who left us in October 2006.)
This paper is organized as follows. Section II introduces the OFDM baseband model and section III the improved channel matrix estimation. Next, Section IV gives some simulation results that demonstrate the effectiveness of the proposed method. We conclude the paper in Section V.

Notation: Superscripts $(\cdot)^T$ and $(\cdot)^H$ stand for transpose and Hermitian operators, respectively. $\| \cdot \|$ and $(\cdot)^*$ are the magnitude and conjugate of a complex number, respectively. $\mathcal{A}$ and $\mathbb{A}$ denote a vector and a matrix, respectively. $A[m]n$ denotes the $n$th entry of the vector $A$ and $A[m,n]$ denotes the $[m,n]$th entry of the matrix $A$.

II. SYSTEM MODEL

Suppose the symbol duration after serial-to-parallel (S/P) conversion is $T_s$. The entire signal bandwidth is covered by $N$ subcarriers, and the space between two neighboring subcarriers is $1/T_s$. Denote the sampling time by $T_s = T_u/N$, and assume that the length of the cyclic prefix is $T_g = N_g T_s$ with an integer $N_g$. The duration of an OFDM symbol is $T = (N + N_g) T_s$. In an OFDM system, the transmitter usually applies an $N$-point IFFT to data block QAM-symbols $\{X_n[k]\}$, where $n$ and $k$ represent respectively the OFDM symbol index and the subcarrier index, and adds the cyclic prefix (CP), which is a copy of the last samples of the IFFT output, to avoid intersymbol-interference (ISI) caused by multipath fading channels. In order to limit the periodic spectrum of the discrete time signal at the output of the IFFT, we use an appropriate analog transmission filter $G_a(f)$. As a result, the output baseband signal of the transmitter can be represented as [5]:

$$x(t) = \sum_{n=-\infty}^{\infty} \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} X_n[k] \phi_k(t - nT_s) \otimes g_s(t)$$

(1)

where $\otimes$ denotes the convolution, $g_s(t)$ is the impulse response of the transmission analog filter, and $\phi_k(t)$ is the subcarrier pulse that can be described by:

$$\phi_k(t) = \begin{cases} \frac{1}{T_g} e^{j2\pi \frac{k}{T_g} t} & t \in [-T_g, T_u] \\ 0 & \text{otherwise} \end{cases}$$

(2)

It is assumed that the signal is transmitted over a multipath Rayleigh fading channel characterized by:

$$h(t, \tau) = \sum_{l=1}^{L} \alpha_l(t) \delta(\tau - \tau_l T_s)$$

(3)

where $L$ is the total number of propagation paths, $\alpha_l$ is the complex gains of the $l$th path and $\tau_l$ is the $l$th delay normalized by the sampling time (\$\tau_l$ is not necessary an integer number). $\{\alpha_l(t)\}$ are wide-sense stationary (WSS) narrow-band complex Gaussian processes with the so-called Jakes’ power spectrum [6] and the different path gains are uncorrelated with respect to each other.

At the receiver side, after passing to discrete time through low pass filter and A/D and removing the guard time with the assumptions that the CP length is no less than the maximum delay, a $N$-point FFT is applied to transform the sequence into frequency domain. The $k$th subcarrier output of FFT during the $n$th OFDM symbol can be represented by:

$$Y_n[k] = \sum_{m=-\frac{N}{2}}^{\frac{N}{2}-1} X_n[m] G_c[m] G_r[m] H_n[k,m] + W_n[k]$$

(4)

where $W_n[k]$ is a white complex Gaussian noise with variance $\sigma^2$, $G_c[m]$ and $G_r[m]$ are the transmitter and receiver filter frequency response values at the $m$th transmitted subcarrier frequency, and $H_n[k,m]$ are the coefficients of the channel matrix from $m$th transmitted subcarrier frequency to $k$th received subcarrier frequency given by:

$$H_n[k,m] = \frac{1}{N} \sum_{l=1}^{L} \left( e^{-j2\pi \frac{k}{N} \pi m} \sum_{q=0}^{N-1} \alpha_l^q(q T_s)e^{j2\pi \frac{q}{N} \frac{m}{N} - \frac{m}{N}} \right)$$

$$m, k \in \left[ -\frac{N}{2}, \frac{N}{2} - 1 \right]$$

(5)

where $\{\alpha_l^q(q T_s)\}$ is the $T_s$ spaced sampling of $l$th path complex gain during the $n$th OFDM symbol.

If we assume a $N$ transmission subcarriers within the flat region of the transmitter and receiver filter frequency response then, by using the matrix notation and omitting the index time $n$, (4) can be rewritten as:

$$\begin{bmatrix} Y \\ Y' \\ W \end{bmatrix} = H \begin{bmatrix} X \\ Y' \\ W \end{bmatrix}$$

(6)

where $G_c[m]$ and $G_r[m]$ are assumed to be equal to one at the flat region, where $X$, $Y'$, $W$ are $N \times 1$ vectors given by:

$$X = \begin{bmatrix} X[-\frac{N}{2}], X[-\frac{N}{2} + 1], ..., X[\frac{N}{2} - 1] \end{bmatrix}^T$$

$$Y' = \begin{bmatrix} Y[-\frac{N}{2}], Y[-\frac{N}{2} + 1], ..., Y[\frac{N}{2} - 1] \end{bmatrix}^T$$

$$W = \begin{bmatrix} W[-\frac{N}{2}], W[-\frac{N}{2} + 1], ..., W[\frac{N}{2} - 1] \end{bmatrix}^T$$

and $H$ is an $N \times N$ channel matrix, which contains the time average of the channel frequency response $H[k,m]$ on its diagonal and the coefficients of the inter-carrier interference (ICI) $H[k,m]$, $k \neq m$, otherwise.

III. CHANNEL MATRIX ESTIMATION

In this section, we propose a promising technique based on a parametric channel model where the channel matrix $H$ given by (5) is estimated using $L$-path channel. In order to get an estimation of the channel matrix $H$, with the assumption that the delays are invariant and perfectly estimated, we have to estimate the sampled complex gains $\{\alpha_l^q(q T_s)\}$ with sampling period $T_s$.

To estimate the sampled complex gains, we propose a method based on comb-type pilot and multipath time delays information. The $N_p$ pilots are equally spaced inserted into the $N$ subcarriers as shown in Fig 1 where $L_f$ denotes the interval in terms of the number of subcarriers between two adjacent pilots in the frequency domain. $L_f$ can be selected without the need for respecting the sampling theorem as in [5]
[4]. However, for the existence of matrix inverse as we will see with equation (13), \( N_p \) must fulfill the following requirement:

\[
N_p \geq L
\]

(7)

Let \( P \) denote the set that contains the index positions of the \( N_p \) pilot subcarriers. Then

\[
P = \{ p_s \mid p_s = s L_f - \frac{N}{2}, \quad s = 0, \ldots, N_p - 1 \}
\]

(8)

The received pilot subcarriers without ICI components are given by:

\[
Y_p = X_p H_p + W_p
\]

(9)

where \( X_p \) is an \( N_p \times N_p \) diagonal matrix, \( Y_p \) and \( W_p \) are \( N_p \times 1 \) vectors given by:

\[
X_p = diag\{X[p_0], X[p_1], \ldots, X[p_{N_p - 1}]\}
\]

\[
Y_p = [Y[p_0], Y[p_1], \ldots, Y[p_{N_p - 1}]]^T
\]

\[
W_p = [W[p_0], W[p_1], \ldots, W[p_{N_p - 1}]]^T
\]

and \( H_p \) is an \( N_p \times 1 \) vector with elements given by:

\[
H[p_s] = \sum_{l=1}^{L} \sigma_l e^{-j2\pi \frac{p_s}{L} \tau_l}
\]

with \( \sigma_l = \frac{1}{N} \sum_{q=0}^{N-1} \alpha_l[qT_s] \)

\[
(10)
\]

\( \pi_l \) is the time average over the effective duration of OFDM symbol of the \( l \)th complex gain. \( H_p \) can be written as the Fourier transform for the different complex gains time average \( \{\pi_l\} \):

\[
H_p = F_p \pi
\]

(11)

where \( F_p \) and \( \pi \) are respectively the \( N_p \times L \) Fourier transform matrix and the \( N_p \times 1 \) vector given by:

\[
F_p = \begin{bmatrix}
      e^{-j2\pi \frac{p_0}{L} \tau_1} & \ldots & e^{-j2\pi \frac{p_{N_p-1}}{L} \tau_L} \\
      \vdots & \ddots & \vdots \\
      e^{-j2\pi \frac{p_{N_p-1}}{L} \tau_1} & \ldots & e^{-j2\pi \frac{p_{N_p-1}}{L} \tau_L}
\end{bmatrix}
\]

\[
\pi = [\pi_1, \ldots, \pi_L]^T
\]

(12)

By neglecting the ICI contribution, the estimator of \( \pi \) by using LS or LMMSE criteria [4] [5] is:

\[
\hat{\pi}_{LS} = \left( F_p^H F_p \right)^{-1} F_p^H X_p^{-1} Y_p
\]

\[
\hat{\pi}_{LMMSE} = \left( F_p^H F_p + \sigma_p^2 C \right)^{-1} F_p^H X_p^{-1} Y_p
\]

where \( \sigma_p^2 \) is the pilot variance, and \( C = diag(\sigma_1^2, \ldots, \sigma_L^2) \) is the covariance matrix of \( \pi \) where \( \sigma_l^2 \) is variance of \( \pi_l \), which will be estimated later, given by (see Appendix I):

\[
\sigma_l^2 = \frac{\sigma_p^2}{N_p^2} \sum_{q_1=0}^{N_p-1} \sum_{q_2=0}^{N_p-1} J_0[2\pi f_D(q_1 - q_2)]
\]

(14)

The method proceeds as following:

Step 1: We group the OFDM symbol in blocks of \( K \) OFDM symbols each one. Each two consecutive blocks are intersected in two OFDM symbols as shown in Fig. 2.

Step 2: For a block, we estimate the \( K \) complex gains time average of each path over the effective duration by using LS or LMMSE estimator given by (13). The variance of each path can be estimated as:

\[
\hat{\sigma}_{\pi_l} = var(\{\pi_l|_{LS}\})
\]

(15)

where \( \{\pi_l|_{LS}\} \) is the set of \( K \) complex gains time average estimated by LS of the \( l \)th path during one block of \( K \) OFDM symbols and var(\( \cdot \)) is the variance operation.

Step 3: We suppose that the estimated complex gains time average of each path is an estimation of one \( T_s \)-spaced sampling of the complex gain taken in the middle of the effective duration of OFDM symbol. Then by using the low-pass interpolation [4], we interpolate by a factor \( (N + N_p) \) the \( K \) estimated samples of the complex gains for each path. Thus, we obtain the sampled complex gains \( \{\alpha_l[qT_s]\} \) at time \( T_s \) for each path during \( K \) OFDM symbols. Since the complex gain of each path exhibits a significant correlation over an interval of length the coherence time of the channel \( T_{coh} \), so we can choose \( K \) such as \( KT_s \leq T_{coh} \).

Step 4: Hence, the channel matrix for each OFDM symbol can be calculated from (5). This is done without considering
the first and the last OFDM symbols in order to avoid limiting effects of interpolation.

The received data subcarriers without contribution from pilot subcarriers are given by:

\[ Y_d = H_d X_d + W_d \]  

(16)

where \( X_d \) the data transmitted, \( Y_d \) the data received and \( W_d \) the noise at data subcarrier positions are \( (N - N_p) \times 1 \) vectors and \( H_d \) is an \( (N - N_p) \times (N - N_p) \) data channel matrix obtained by elimination rows and columns at \( P \) position in channel matrix \( H \).

By successive interference suppression (SIS) scheme with the optimal ordering and one tap frequency equalizer the data will be estimated. The optimal ordering, which are calculated from the large to the small magnitude of the diagonal elements of the data channel matrix \( H_d \), are given by:

\[ O = \{ O_1, O_2, \ldots, O_{N-N_p} \mid i < j \text{ if } ||H_d(O_i, O_j)|| > ||H_d(O_i, O_j)|| \} \]

(17)

The detection algorithm can now be described as follows:

initialisation :

\[ i \leftarrow 1 \]

\[ O = \{ O_1, O_2, \ldots, O_{N-N_p} \} \]

\[ Y_{d[i]} = Y_d \]

recursion :

\[ X_{d}[O_i] = Y_{d[i]}[O_i]/H_d(O_i, O_i) \]

\[ \hat{X}_d[O_i] = Q(X_{d}[O_i]) \]

\[ Y_{d[i+1]} = Y_{d[i]} - \hat{X}_d[O_i](H_d)[O_i] \]

\[ i \leftarrow i + 1 \]

where \( Q(.) \) denotes the quantization operation appropriate to the constellation in use and \( (H_d)[O_i] \) denotes the \( O_i \)th column of the data channel matrix \( H_d \).

IV. SIMULATION RESULTS

In this section, we test the performance of the improved algorithm in a multipath Rayleigh fading normalized channel. The simulations are concentrated on comparison between the LS and LMMSE channel estimations with low-pass interpolation [11] [4] and proposed method using the LS criteria (we do not need the estimation of \( \sigma_n^2 \) as for LMMSE criteria (14)). The bit error rate (BER) performances in terms of the average signal-to-noise ratio (SNR) [5] [4] for Rayleigh fading channel are examined. In this paper, we use the same simulation environment as in [2]. This scenario corresponds to a 16QAM-OFDM system with carrier frequency of 1GHz and bandwidth of 2 MHz (note that \( (SNR)dB = (\frac{W}{2})dB + 6dB \)). The number of subcarriers is \( N = 1024 \), the length of the cyclic prefix is \( N_g = 56 \) and the number of uniformly-distributed pilot subcarriers is 128 or 64 (i.e., \( L_f = 8 \) or 16). The normalized channel model are Rayleigh as recommended by GSM Recommendations 05.05 [8] [11], with parameters shown in the table below.

<table>
<thead>
<tr>
<th>Path Number</th>
<th>Average Power(dB)</th>
<th>Normalized Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-7.219</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-4.219</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>-6.219</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>-10.219</td>
<td>3.2</td>
</tr>
<tr>
<td>5</td>
<td>-12.219</td>
<td>4.6</td>
</tr>
<tr>
<td>6</td>
<td>-14.219</td>
<td>10</td>
</tr>
</tbody>
</table>

The proposed method is evaluated under fast-fading radio channel to have an important ICI level. So, We choose a fast vehicle speed \( V = 150km/h \), corresponding to the high Doppler spread (normalized by \( 1/T \)) 0.075, and \( K = 14 \) OFDM symbols in each block. The parameter \( R_g \) means that we have estimated \( 2R_g \) diagonals around the main diagonal in channel matrix (in total \( 2R_g + 1 \) diagonals estimated and zeros otherwise).

The legends “suppression interference, inverse diagonal (estimated), inverse diagonal (exact), LS and LMMSE pilot” denote estimation of the channel matrix with SIS and one tap frequency equalizer, estimation only of the diagonal channel matrix by the proposed method with one tap frequency equalizer, the exact diagonal channel matrix with one tap frequency equalizer, LS and LMMSE channel estimations at pilot frequencies with low-pass interpolation [11] [4], respectively.

Fig 3 gives the BER performance for our proposed method with \( R_g = 1 \) (only 3 diagonals of channel matrix are calculated) and \( R_g = 0 \) (only the main diagonal of channel matrix are calculated), LS and LMMSE estimators with \( L_f = 8 \). This result shows that, without interference suppression, our method performs better than LS and LMMSE estimators with frequency interpolation and its performance is closer to the performance of exact estimator and becomes better with starting interference suppression. Fig 4 gives the performance of our proposed method for \( R_g = 1, 4 \) and 12. It is very clear that this method offers an improvement in BER by increasing \( R_g \). The performance improved more when passing from \( R_g = 1 \) to \( R_g = 4 \) than passing from \( R_g = 4 \) to \( R_g = 12 \).
12. This is because the power of the interference which decreases quickly while moving away from the main diagonal as shown in Fig 5 for one OFDM symbol channel realization.

Fig 5. Power of diagonals of the channel matrix

Fig 6 gives the performance of our proposed method for \( L_f = 8 \) (in continuous line) and 16 (in dashed line). The results shows that when using more pilots, performance will be better. Moreover, with less pilots and without suppression interference, our method performs better than LS and LMMSE estimators and becomes better with starting interference suppression.

Fig. 6. Comparison of BERs with \( R_g = 1 \) and \( L_f = 8 \) and 16

For illustration, Fig 7 shows the real and the imaginary part of the exact and estimated complex gain of all paths for one channel realization over 12 OFDM symbols with \( SNR = 20dB \). Because the error of data detection and complex gains estimation, the ICI are not completely eliminated and give rise to an error floor as we see in all performance figures.

Fig. 7. The LS estimated complex gain of six paths over 12 OFDM symbols with \( SNR = 20dB \)

V. CONCLUSION

In this paper, we have presented an improved algorithm to estimate the channel matrix and mitigate the inter-sub-carrier-interference (ICI) for OFDM system by using comb-type pilots. The proposed method, with less number of pilots and without suppression of interference, gives a good performance over LS and LMMSE estimators with low-pass interpolation and becomes better with starting interference suppression. Our further work is to make a theoretical analysis in terms of channel estimation mean square error and to improve the performance of this method by making several iterations between channel estimation and interference suppression.

APPENDIX I

COMPLEX GAIN TIME AVERAGE VARIANCE

The variance of the complex gain time average of the \( l \)th path given by (10) is calculated as:

\[
\sigma^2_{\hat{\alpha}_l} = E\left[\hat{\alpha}_l \hat{\alpha}_l^*\right] = \frac{1}{N^2} \sum_{q_1=0}^{N-1} \sum_{q_2=0}^{N-1} E\left[\alpha_1[q_1T_s]\alpha_1^*[q_2T_s]\right]
\]  

(18)
where $E[\cdot]$ is the expectation operation. Since $\alpha_i(t)$ is wide-sense stationary (WSS) narrow-band complex Gaussian processes with the so-called Jakes’ power spectrum [6] then:

$$E\left[\alpha_i[q_1|T_s]\alpha_i^*[q_2|T_s]\right] = \sigma_{\alpha_i}^2 J_0[2\pi f_D(q_1 - q_2)]$$  \( (19) \)

where $\sigma_{\alpha_i}^2$ is the variance of $i$th path complex gain, $J_0[\cdot]$ denotes the zeroth-order Bessel function of the first kind and $f_D$ is the maximum Doppler frequency normalized by $\frac{1}{T_s}$.

Inserting (19) into (18) yields the expression of the complex gain time average variance of the $i$th path:

$$\sigma_{\alpha_i}^2 = \frac{\sigma_{\alpha_i}^2}{N^2} \sum_{q_1=0}^{N-1} \sum_{q_2=0}^{N-1} J_0[2\pi f_D(q_1 - q_2)]$$  \( (20) \)

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