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Complex Critical Exponents from Renormalization Group Theory of Earthquakes: Implications for Earthquake Predictions

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Abstract. — Several authors have proposed discrete renormalization group models of earthquakes, viewing them as a kind of dynamical critical phenomena. Here, we propose that the assumed discrete scale invariance stems from the irreversible and intermittent nature of rupture which ensures a breakdown of translational invariance. As a consequence, we show that the renormalization group entails complex critical exponents, describing log-periodic corrections to the leading scaling behavior. We use the mathematical form of this solution to fit the time to failure dependence of the Benioff strain on the approach of large earthquakes. This might provide a new technique for earthquake prediction for which we present preliminary tests on the 1989 Loma Prieta earthquake in northern California and on a recent build-up of seismic activity on a segment of the Aleutian-Island seismic zone. The earthquake phenomenology of precursory phenomena such as the causal sequence of quiescence and foreshocks is captured by the general structure of the mathematical solution of the renormalization group.

1. Introduction

What is the nature of rupture associated with earthquakes? An important working hypothesis [1–6], borrowed from Statistical Physics, is that earthquakes are similar to "critical" points [7,8]: a progressive correlation develops leading to a cascade of events at increasingly large scales culminating in a large earthquake. If true, such a mechanism would constrain drastically the properties of precursory phenomena, and ultimately offer the potential for prediction.

There are basically two approaches to intermediate and short-term earthquake prediction. The first is to search for local precursory phenomena associated directly with the fault instability, such as preseismic creep or changes in the ground water within the fault zone. The second is to look for more spatially extended changes which involve the collective behavior of the entire regional fault network such as changes in the seismic b-value [10] or precursory patterns of seismicity [11].

The problem with the first approach is that local precursory phenomena have not been consistently observed [12]. Ground water changes, for example, have preceded some large events but were absent before others. The second approach looks more promising. Quite a few large earthquakes have been preceded by an increase in the number of intermediate size events [13–19]. The relation between these intermediate events and the subsequent main event has only recently been recognized because the precursory events occur over such a large area. Based on a simple stress analysis, it is difficult to see how such widely separated events could be mechanically related. However, if the seismicity in a given region is viewed as a sequence of seismic cycles, and each cycle is viewed as a progressive cooperative stress build-up culminating in some sort of critical point characterized by global failure in the form of a great earthquake, then the observed increase of activity and long-range correlation between events are expected to precede large earthquakes. The occurrence of such long-range spatial correlations in a self-organized model of the earth crust has been indeed documented recently [20]. In this sense, the occurrence of a large earthquake is similar to an instability, which is intimately associated to a building up of long-range correlations. We should however caution that earthquake data are scarse and that the above statement is still debated since some large earthquakes have been found not to be preceded by an increase of activity for reasons not understood.

Laboratory studies of progressive failure have established a qualitative physical picture for the progressive damage of a system leading to a critical failure point. At first, single isolated microcracks appear and, then with the increase of load or time of loading, they both grow and multiply leading to an increase of the density of cracks per unit volume. As a consequence, microcracks begin to merge until a "critical density" of cracks is reached at which the main fracture is formed. The basic idea is thus that the formation of microfractures prior to a major failure plays a crucial role in the fracture mechanism. These ideas have been formalized in different ways, in percolation models [1, 2, 21], critical branching models [22], hierarchical fiber bundle models [3, 6], euclidean quasi-static [23] and dynamical [24] lattice models of rupture. In these models, criticality can be traced back to the interplay between the preexisting heterogeneity and the correlated growth of cracks mediated by the stress field singularities.

Application of this scenario to the temporal and spatial progression of seismicity raises a few questions. First, what do we mean by "failure" of the earth's crust? In the laboratory, the system is clearly defined as the test specimen and failure is defined by the inability of the specimen to support the applied load [25]. In the crust, each earthquake represents failure of some surrounding region the size of which scales with the size of the event. Each earthquake is, at the same time, the failure of its local region and possibly a part of the precursory failure sequence of an even larger event. Failure in the crust can be thought of as a scaling-up process in which failure at one scale is part of the damage accumulation pattern at a larger scale [26]. This same scaling-up process also occurs in the heterogeneous laboratory samples, but the number of scales over which the process can be observed is severely limited by the size of the test specimen. After only one or two jumps in scale , the rupture reaches the size of the specimen and the failure ensues. In the earth, the maximum scale to which the process extends also appears to be limited to events having a magnitude in the range $8-9$; these are the large global failures which we wish to model. In effect, these events are unique [27] in that they lie at the upper fractal limit of the process and are not precursors to even larger events. The physical explanation of this upper limit is not clear, but probably involves the thickness of the brittle layer and the nature of stress concentration at the plate boundaries. We should caution that it is still a matter of controversy whether there is a genuine cut-off at these magnitudes or if the apparent limit stems from the finiteness of the statistics and much larger earthquakes could occur in the future.

An important assumption of our analysis is that a large earthquake and its procession of

nucleation and precursory phenomena can be singled out and studied essentially as an isolated system. This amounts to the identification of a certain regional domain as the relevant space which can be considered as sufficiently coherent so that the surroundings only affects it in terms of an approximately uniform driving at its boundaries.
It is clear that this hypothesis is questionable :

- earthquakes are correlated in space and time over large distances [28]. Therefore, a partition of a given tectonic plate in sub-domains is bound to retain some arbitrariness.
- Furthermore, the tectonic deformations and fault geometrical structures seem to self-organize themselves over a large distance [29].

However, the physical picture, that the earthquakes and tectonic deformations are globally self-organizing in time and space and present correlations up to the largest continental scales, does not exclude the existence of characteristic time and space scales, due for instance to geometrical scales, to a ductile coupling with the lower crust and upper mantle and to the effect of fluid and chemical reactions in the crust. These mechanisms are very important since they should control the appearance of precursory phenomena, and ultimately our prediction. It is also possible that a future deeper understanding of the self-organization of the earth will reunify these different view points. This will be addressed elsewhere [30] within a general framework based on the renormalization group theory.

If a great earthquake can be viewed as a critical point, precursors of earthquakes should follow characteristic scaling laws [2, 4]. These powerlaws result naturally from the many-body interactions between the small cracks forming before the impending great rupture. For instance, the rate of elastic energy dissipated at constant average applied stress increases as a power law of the time distance to failure time. Furthermore, one expects the rupture patterns to be self-similar. In addition, large fluctuations from systems to systems and their sensitivity to the initial inhomogeneity configuration have been demonstrated in specific models [23, 24] as resulting from the proximity to a critical point. Similar effects are found in laboratory experiments in the acoustic emission associated to the progressive damage of a mechanical system up to global failure [31].

In general, the scaling law should only be observable very near the critical point, in the so-called "critical region", and take the form

$$\frac{\mathrm{d}\epsilon}{\mathrm{d}t} = k \mid t_{\mathrm{f}} - t \mid^{-a}, \tag{1}$$

where ϵ represents regional strain (or any other measure of seismic release), t_{f} is the time of failure (that one wishes to predict), and k and a are constants. a is a critical exponent, which can often be interpreted geometrically in terms of some fractal underlying geometry. In this context, the regional increase of intermediate events which have sometimes been observed to precede a large earthquake comprises the power law increase in seismic release. Integration of equation (1) yields

$$\epsilon = A + B \mid t_{\mathrm{f}} - t \mid^{m}, \tag{2}$$

where A, B and $m = 1 - a$ are constants. Equation (2) has been fit to the accelerating seismicity which preceded the 1989 Loma Prieta earthquake by Bufe and Varnes [32] and to the seismicity increase currently occurring in the Alaska-Aleutian region by Bufe *et al.* [33]. In this later analysis, they predict that one or more $M > 7.5$ events will occur in the time interval between 1994 and 1996. In their analysis, ϵ is the cumulative "Benioff" strain defined as

$$\epsilon(t) = \sum_{n=1}^{N(t)} E_n^q, \tag{3}$$

with $q = \frac{1}{2}$ and where E_n is the energy release during the n^{th} earthquake during the time of observation up to the current time t.

In these papers, equations (1) and (2) are not presented as a consequence of scaling near a critical point, but are justified in terms of run-away crack propagation and empirical expressions for "tertiary creep" which precedes failure in the laboratory [34,35]. The advantage of recognizing these equations as universal scaling laws near a critical point is that we can proceed to derive the leading corrections to scaling, which are expected to be most important for any realistic prediction which must be made at the earliest time possible, i e., long before the impending earthquake. We show below that these corrections take the form of a periodic function of $\log \mid t_f - t \mid$, which is superimposed on the power law in equation (2). This additional structure in the cumulative Benioff strain allows a more accurate prediction of t_f at a significantly earlier time in the sequence.

2. General Renormalization Group Framework for Critical Rupture and Its Universal Leading Correction to Scaling

Our fundamental idea is that the concept of criticality in regional seismicity embodies more usable information than just the power law (1) valid in the asymptotic critical domain. We argue that specific precursors outside the critical regime can lead to "universal" but nevertheless specific recognizable signatures in the regional seismicity which precedes a large event. In order to identify these signatures, we first note that an expression like (1) can be obtained from the solution of a suitable renormalization group (RG) [9]. The RG formalism, introduced in field theory and in critical phase transitions, amounts basically to the decomposition of the general problem of finding the behavior of a large number of interacting elements into a succession of simple problems with a smaller number of elements, possessing effective properties varying with the scale of observation. Its usefulness is based on the existence of a scale invariance or self-similarity of the underlying physics at the critical point, which allows one to define a mapping between physical scale and distance from the critical point in the control parameter axis.

In the real-space version of RG which is the most adapted for rupture and percolation, one translates litterally in the real space the concept that rupture at some scale results from the aggregate response of an ensemble of ruptures at a smaller scale. In the earthquake problem, the seismic release rate $\frac{d\epsilon}{dt}$ at a given time t is related to that at another time t' by the following transformations

$$x' = \phi(x), \tag{4}$$

$$F(x) = g(x) + \frac{1}{\mu} F\Big(\phi(x)\Big), \tag{5}$$

where $x = t_f - t$, t_f is the time of global (regional) failure for the region under consideration, ϕ is called the RG flow map, $F(x) = \epsilon(t_f) - \epsilon(t)$ such that $F = 0$ at the critical point and μ is a constant describing the scaling of the seismic release rate upon the discrete time rescaling (4). The function $g(x)$ represents the non-singular part of the function $F(x)$. We assume as usual that the function $F(x)$ is continuous and that $\phi(x)$ is differentiable.

The critical point(s) is (are) described mathematically as the time(s) at which $F(x)$ becomes singular, i.e., when there exists a finite k^{th} derivative $\mathrm{d}^k F(x)/\mathrm{d}x^k$ which becomes infinite at the singular point(s). The formal solution of equation (5) is obtained by considering the following definitions: $f_0(x) \equiv g(x)$, and $f_{n+1}(x) = g(x) + \frac{1}{\mu} f_n\left[\phi\left(x\right)\right]$,

$n = 0, 1, 2, ..$ It is easy to show (by induction) that $f_n(x) = \sum_{i=0}^{n} \frac{1}{\mu^i} g\left[\phi^{(i)}\left(x\right)\right]$, $n > 0$. Here, we have used superscripts in the form "n" to designate composition, i.e., $\phi^{(2)}(x) = \phi\left[\phi\left(x\right)\right]$; $\phi^{(3)}(x) = \phi\left[\phi^{(2)}\left(x\right)\right]$; etc. It naturally follows that $\lim_{n\to\infty} f_n(x) = F(x)$ assuming that it exists. Note that the power of the RG analysis is to reconstruct the nature of the critical singularities from the embedding of scales, i.e. from the knowledge of the non-singular part $g(x)$ of the observable and the flow map $\phi(x)$ describing the change of scale. The connection between this formalism and the critical point problem stems from the fact that the critical points correspond to the unstable fixed points of the RG flow $\phi(x)$. Indeed, as in standard phase transitions, a singular behavior emerges from the infinite sum of analytic terms, describing the solution for the observable $F(x)$, if the absolute value of the eigenvalue λ defined by $\lambda = \mid \mathrm{d}\phi/\mathrm{d}x \mid_{x=\phi(x)}$ becomes larger than 1, in other words, if the mapping ϕ becomes unstable by iteration at the corresponding (critical) fixed point (the fixed point condition ensuring that it is the same number appearing in the argument of $g(.)$ in the series). In this case, the i^{th} term in the series for the k^{th} derivative of $F(x)$ will be proportional to $(\lambda^k/\mu)^i$ which may become larger than the unit radius of convergence for sufficiently large k, hence the singular behavior.

Thus, the qualitative behavior of the critical points and the corresponding critical exponents can be simply deduced from the structure of the RG flow $\phi(x)$. If $x = 0$ denotes a fixed point ($\phi(0) = 0$) and $\phi(x) = \lambda x + ...$ is the corresponding linearized transformation, then the solution of equation (5) close to $x = 0$ is given by equation (2), i.e., $F(x) \sim x^\alpha$, with α solution of

$$\frac{\lambda^\alpha}{\mu} = 1, \tag{6}$$

which yields $\alpha = \frac{\log \mu}{\log \lambda}$.

To get the leading correction in the critical region, we assume that $F_0(x) \sim x^\alpha$ is a special solution, then the general singular solution $F(x)$ is related to $F_0(x)$ in terms of an *apriori* arbitrary periodic function $p(x)$, with a period $\log \mu$, as

$$F(x) = F_0(x) p(\log F_0(x)). \tag{7}$$

To get this expression, we have neglected the non-singular term $g(x)$ in equation (5). The solution of equation (7) can then be checked by inserting it into the equation $F(\lambda x) = \mu F(x)$ which, because of equation (6), is also obeyed by F_0. Then we get the periodicity requirement $p(\log \mu + \log F_0(x)) = p(\log F_0(x))$ as asserted. Since $F_0(x) \sim x^\alpha$, this introduces a periodic (in $\log x$) correction to the dominating scaling (2) which amounts to considering a complex critical exponent α, since $Re[x^{\alpha' + i\alpha''}] = x^{\alpha'} \cos(\alpha'' \log x)$ gives the first term in a Fourier series expansion of equation (7). Note that one can get directly this complex critical exponent by noting that equation (6) can be rewritten $\lambda^\alpha/\mu = \mathrm{e}^{i2\pi n}$, where n is an integer. Its solution reads $\alpha = \frac{\log \mu}{\log \lambda} + i\frac{2\pi n}{\log \lambda}$ which allows us to recover exactly the previous Fourier series expansion with $\alpha'' = \frac{2\pi n}{\log \lambda}$. A more formal derivation, using Mellin's transform applied to the full equation (5), will be presented elsewhere [30], which furthermore allows us to quantify the effect of randomness on our results presented here.

Expression (7) thus introduces universal oscillations decorating the asymptotic powerlaw (2). Note that this log-periodic corrections have nothing to do with any assumption of periodic occurrence of earthquakes, as has been claimed for instance in the Parkfield region in California [36].

The mathematical existence of such corrections have been identified quite early [37] in RG solutions for the statistical mechanics of critical phase transitions, but have been rejected for translationally invariant systems, since a period (even in a logarithmic scale) implies the existence of one or several characteristic scales, which is forbidden in these ergodic systems in the critical regime. For the rupture of quenched heterogeneous systems, the translational invariance does not hold due to the presence of static inhomogeneities [38] since new damage occurs at specific positions which are not averaged out by thermal fluctuations. Hence, such periodic corrections are allowed and should be looked for in order to embody the physics of damage in the non-critical region. Another equivalent point of view for understanding these Log-periodic correction to scaling is to realize that the RG written with equations (4) and (5) is discrete, i.e., one goes from a time t to another time t' which is at a finite (and not an infinitely small) interval from t. This captures the fact that damage and precursory phenomena occur at particular discrete times and not in a continuous fashion, and these discontinuities reflect the localized and threshold nature of the mechanics of earthquakes and faulting. It is this "punctuated" physics which gives rise to the existence of scaling precursors modelled mathematically by the Log-periodic correction to scaling.

Similar logarithmic periodicities have been found in the rate of acoustic emissions which preceded rupture of pressure vessels composed of carbon fiber-reinforced resin [31]. Identification of this structure in the acoustic emission rate allowed the failure pressure to be predicted within less than 5% error when the test pressure had reached only 85% failure. Other examples which present these log-periodic signatures are discussed in [31]. However, let us stress that our results presented below on these universal periodic corrections are the first ones obtained in natural uncontrolled heterogeneus systems.

We now show that such logarithmic periodicities also occur in regional seismicity data analyzed by Bufe *et al.* [32]. We fit their data for cumulative Benioff strain to the expression

$$\epsilon(t) = A + B(t_f - t)^m \left[1 + C\cos\left(2\pi\frac{\log(t_f - t)}{\log\lambda} + \Psi\right)\right], \qquad (8)$$

and show that equation (8) yields a more accurate prediction of t_f than does the simple powerlaw (2). Note that equation (8) which involves 3 more parameters reduces to equation (2) when $C = 0$. Various criteria exist to quantify the amount of information embedded in a given fit, such as the Akaike information criterion [39] which has been used in many studies of aftershock sequences. A systematic assessment of the relevance of equation (8) in this sense will be reported elsewhere [40]. In this preliminary work, we propose that the ultimate verification of a good theory is its predictive power : if a 7-parameter formula (Eq. (8)) gives a more constrained and precise prediction on the time to rupture that a 4-parameter theory (Eq. (2)), we must conclude that the amount of information gained in the 7-parameter theory is significant.

3. Fit to Regional Seismicity Data

Bufe and Varnes [32] fit equation (2) to the cumulative Benioff strain release for magnitude 5 and greater earthquakes in Northern California for the period 1927 – 1988. They predicted $t_f = 1990$ for the Loma Prieta earthquake, which had a magnitude of 6.7 – 7.1 and occurred on October 18, 1989 (i.e., 1989.8). In Figure 1(a), we have refit their data to equation (2) using a Levenberg-Macquardt algorithm and obtained a similar value of 1990.3 ± 4.1 years (see Table I). Note however that the scatter of their data around the power law is not random,

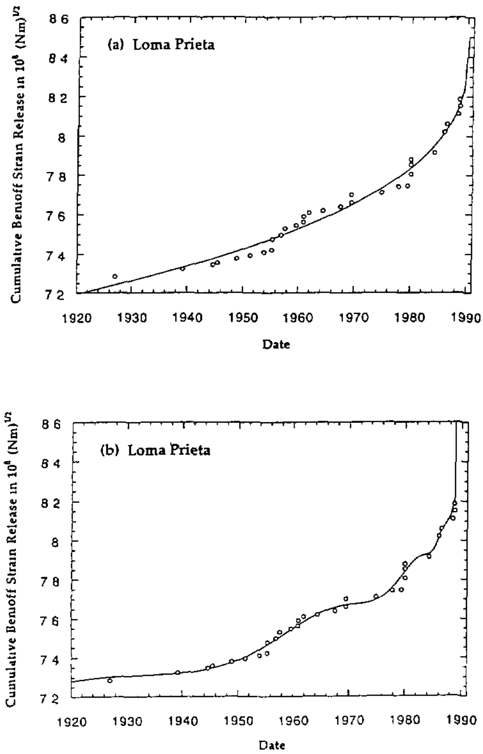

Fig. 1. — Cumulative Benioff strain released by magnitude 5 and greater earthquakes in the San Francisco Bay area prior to the 1989 Loma Prieta eaerthquake (from Ref. [32]). In (a), the data have been fit to the powerlaw equation (2) as in Bufe and Varnes [32]. In (b), the data have been fit to equation (8) which includes the first order correction to scaling. Parameters of both fits are given in Table I.

but appears to fluctuate periodically with a progressively decreasing period as t approaches t_f, exactly as predicted by equation (8). In Figure 1(b), we have fit equation (8) to the same data. Note that the prediction of $t_f = 1989.9 \pm 0.8$ years is closer to the actual value and is more tightly constrained by the additive structure, even though the number of fit parameters has increased from 4 to 7 (see Table I). In Figure 2, we have fit the data prior to the date shown on the abscissa. Note that beginning in 1980, the date of the earthquake is predicted within about a year of its actual occurrence. Examination of the data in Figure 1(b) shows that this is because as early as 1980, enough of the Log-periodic structure has developed to constrain the prediction. We have checked that these results are robust with respect to added noise on the data corresponding to a spread of $0.2 - 0.5$ in magnitude estimates, a not unusual error in earthquake catalogs.

In a subsequent paper [33], Bufe *et al.* performed a similar time-to-failure analysis by fitting equation (2) to the cumulative Benioff strain released by $M > 5.2$ events in several segments Aleutian Islands which are currently experiencing accelerating seismic release. Based on these fits, they predict the occurrence of one or more $M > 7.3$ earthquakes in the Shumagin segment and the Delarof segment sometimes between 1994 and 1996. Although the Kommandorski Island segment also show increasing seismicity, their analysis suggest that culmination is less imminent $(1996 - 1997)$ and that the time to failure is not so well determined by their method.

Table I. — *Parameters found by fitting time-to-failure equations to the cumulative Benioff strain.*

Parameters	Loma Prieta	Kommandorski Island
Power fit Equation (2)		
A	8.50 ± 0.73	6.23 ± 26.9
B	-0.29 ± 0.44	-2.49 ± 19.3
m	0.35 ± 0.23	0.26 ± 1.0
t_f	1990.3 ± 4.1	1998.8 ± 19.7
Log-periodic corrected fit Equation (7)		
A	8.46 ± 0.24	4.95 ± 2.25
B	-0.30 ± 0.16	-1.88 ± 1.77
m	0.34 ± 0.08	0.28 ± 0.14
C	-0.050 ± 0.015	0.040 ± 0.023
λ	3.13 ± 0.14	2.50 ± 0.26
ψ	1.45 ± 0.89	-3.25 ± 2.52
t_f	1989.9 ± 0.8	1996.3 ± 1.1

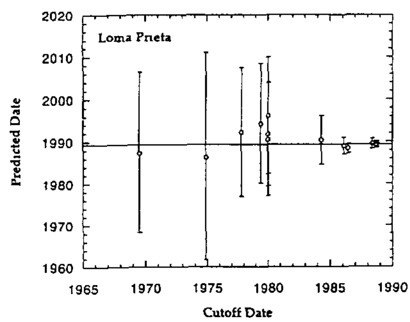

Fig. 2. — Predicted date of the Loma Prieta earthquake using equation (7) to fit all the data prior to the date shown on the abscissa. Error bars are found using a Levenberg-Marcquardt non-linear fitting algorithm. Note that a useful prediction is obtained after 1980, and that the quality of the prediction improves as the date of the earthquake is approached. The horizontal line is the actual date of the Loma Prieta earthquake (October 17, 1989).

In Figure 3(a), we fit equation (2) to the data for the Kommandorski Island segment but, unlike Bufe *et al.*, we did not fix $m = 0.3$ and obtained the solution summarized in Table I. Note that the predicted time of failure has a large uncertainty, and that the data do not scatter randomly about the power law, but show periodic fluctuations. A fit of equation (8) to these data is shown in Figure 3(b) and the fit parameters are summarized in Table I. It is especially interesting that the Log-periodic structure evident in the data from the Kommandorski Island segment allows a prediction which was largely unconstrained by the simpler power law. In addition, we have checked the robustness of the prediction by fitting the data prior to a date in the past as shown in Figure 4. Again, we find a consistent prediction about 10 years in advance of the predicted event, i.e., from 1986 on. The other three segments analyzed by Bufe *et al.*

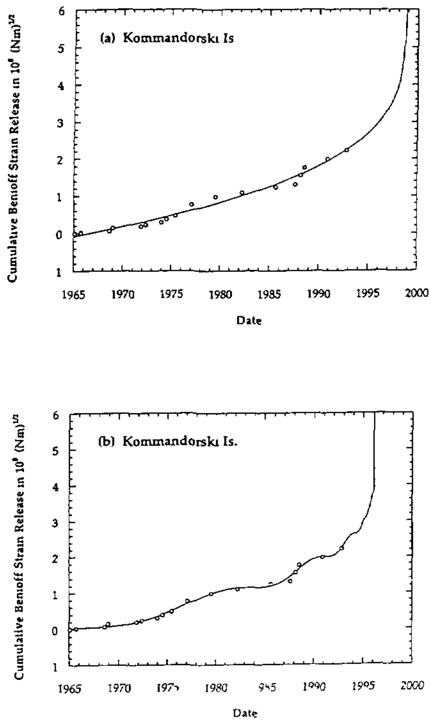

Fig. 3. — Cumulative Benioff strain released by magnitude 5.2 and greater earthquakes in the Kommandorski Island segment of the Aleutian Island seismic zone (from Ref. [33]). In (a), the data have been fit to the powerlaw equation (2). This is similar to the fit in Bufe *et al.* except that we did not fix the exponent $m = 0.3$ as in that paper. In (b), the data have been fit to equation (8) which includes the first order corrections to scaling. Parameters of both fits are given in Table I.

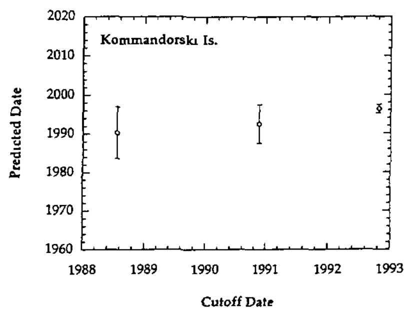

Fig. 4. — Predicted date of an impending earthquake in the Kommandorski segment of the Aleutian islands fitting all the data prior to the date on the abscissa to equation (8). Useful predictions are obtained from 1988 to present.

also show Log-periodic oscillations, but the improvement in the prediction is not as dramatic as for the Kommandorski Island segment shown here.

4. Discussion

Extreme caution should be exercized before concluding that this method is useful for predictive purpose. At present, little is known about earthquake prediction and seismic patterns are sufficiently varied that a devil's advocate could claim that it might be possible to present some evidence of almost any sort of correlation. Therefore, to separate true progress from retroactive fits to data, any new theory of prediction needs a more systematic evaluation using a much larger data set than presented here. Also, with respect to the Loma Prieta data, we are making a "postdiction" instead of a prediction, meaning that we now have a large amount of information to draw the seismicity box in space that is used to construct the cumulative Benioff strain. This is not true for the Aleutian islands, however. These restrictions are important not only with respect to the advancement of scientific knowledge but also due to the political and financial implications : the seismological community has been criticized before, especially in the United States, by repeatedly promising results using various prediction techniques (e.g., dilatancy diffusion, Mogi donuts, pattern recognition algorithms, rainfall, radon, the full moon, etc.) that have not delivered to the expected level [12]. The aim of the present work, which is to present a potential important hypothesis and some encouraging tests, will be complemented by an extensive data analysis to be presented in a more specialized journal [40].

Some readers might find hard to accept the concept of an earthquake as a rupture critical point, since this seems incompatible with the notion that an earthquake is an individual "fluctuation" of the self-organized globally stationary crust. Recently, a new theory of self-organized criticality has been presented [41] which proposes a solution to this paradox. It has been shown that a common feature of systems exhibiting self-organized criticality is that they present a genuine "depinning" dynamical critical transition when forced by a suitable control parameter. Then, self-organized criticality results from the control of the corresponding conjugate *order parameter* at a vanishingly small but *positive* value, which thus automatically ensures that the corresponding control parameter lies exactly at its critical value for the underlying unbinding transition. An important consequence is that each large avalanche or earthquake can be seen in itself as a genuine *truncated* critical point, i.e., endowed with all the characteristics and signature of the global unstable critical point but truncated and round-off by its finite size. This stems simply from the fact that SOC is nothing but "sitting" right on an unstable critical point, controlled by driving the order parameter. As a consequence, fluctuations of all scales appear as the signature of a diverging correlation length. Thus, each large fluctuation is a scaled-down representation of the underlying unstable critical point itself.

The observation that the critical exponent m is found consistently close to 0.3 in the four cases studied is in agreement with, if not a definitive proof of, the concept of a critical point for earthquakes. We note that it is not too far from the mean field value equal to 0.5 [42].

In contrast, the relative weight of the Log-periodic corrections fluctuates from one region to another. Whereas their presence and overall structure are universal, their amplitude are expected to be sensitively dependent upon the specific geometry and heterogeneity of the region under study. In other words, as precursors of the large event, they constitute genuine fingerprints of the specific mechanical structure of the regional crust.

We should also like to stress that their precise Log-periodic structure is intimately related to the simple scaling (1), and defines a more subtle form of scaling, namely the existence of discrete embedding scales that appear to play a similar role as the large event, and it is through this connection that they turn out to be instrumental in constraining the fits.

Similarly to Bufe and Varnes [32], we can also obtain an estimation of the size of the impending earthquake from the difference between the ultimate strain $\epsilon(t = t_f) = A$ and the value at the present time $\epsilon(t)$. In fact, this procedure only provides an upper bound, since further precursors which would give additional Log-periodic oscillations are possible between now and the impending earthquake and will accomodate a fraction of the cumulative Benioff strain that needs to go to its final value $\epsilon(t = t_f) = A$. For the Loma Prieta earthquake, the magnitude is correctly predicted, essentially because the final observation time taken into account is sufficiently close to the main event so that no other significant earthquake occurs before the main event. However, if fitting the data prior to the date shown on the abscissa in Figure 2 allows a good determination of the actual date of the earthquake, its size is less constrained. One possible procedure would be to define the size of the impending earthquake from the cumulative strain integrated from a time $t_f - \tau$ to t_f, where τ denotes a characteristic correlation time scale over which no significant large earthquake (of magnitude say $\geq M_{\rm large} - 2$) would be possible without triggering themselves the great one. This corresponds to the view that during the propagation of the rupture front, an earthquake "does not know" if it will be small or large and this property should be controlled only by the state of stress-stress correlation and the geometrical structure of the region, i.e. the closeness to the critical point. This should be constrained further by the structure of the renormalization group solution with respect to the embedding scaling [30].

The Log-periodic corrections to scaling imply the existence of a hierarchy of characteristic times t_n, determined from the equation $2\pi \frac{\log(t_f - t_n)}{\log \lambda} + \Psi = n\pi$, which yields $t_n = t_f - T\lambda^{\frac{n}{2}}$, with $T = \lambda^{-\frac{\Psi}{2\pi}}$. For the Loma Prieta earthquake, we find $\lambda \simeq 3.3$ and $T \simeq 1.3$ years. As discussed above, we expect a cut-off at short time scales (i.e., above $-n \sim$ a few units) and also at large time scales due to the existence of finite size effects. These time scales $t_f - t_n$ are not universal but depend upon the specific geometry and structure of the region. What is expected to be universal are the ratios $\frac{t_f - t_{n+1}}{t_f - t_n} = \lambda^{\frac{1}{2}}$. These time scales could reflect the characteristic relaxation times associated with the coupling between stress (or strain) and between the brittle and lower ductile crusts.

To finish, we should mention a few open problems. First, what is the theoretical justification for taking the cumulative Benioff strain as the renormalizable variable? Other moments with $q \neq \frac{1}{2}$, as defined in equation (3), could also be used since q controls the relative weight of "small" with respect to "large" earthquakes [40]. Second, the precursors that the present renormalization group method makes use of are localized on faults different from the one which carries the large event. How is it possible to incorporate the actual fault geometry and define objectively a coherent regional domain? Thirdly, our method is based solely on temporal patterns in the increased intensity. Its development should take into account other signatures that have been recognized as potentially important, such as the patterns of increasing fluctuations, clustering in space-time and spatial correlations [11]. Finally, the large event in a given domain is singled out as the critical point. A coherent theory should treat all significant earthquakes on the same footing. We shall come back elsewhere on the development of such a renormalization group theory of complex critical exponents in heterogeneous systems [30].

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