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Numerical investigation of the asymmetric SK-model with deterministic dynamics

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Abstract. — The dynamics of the asymmetric SK-model with binary couplings $J_{ij} = \pm 1$, which was introduced in the context of neural networks, is investigated numerically at $T = 0$ and with sequential updating (deterministic dynamics). The system sizes are varied up to 8192 neurons. We find a universal algebraic decay of the two-timestep autocorrelation function with exponent $3/2$ for all values of the asymmetry. Furthermore the results suggest a transition (e.g. in the remanent magnetization and the damage spreading behavior) as a function of the asymmetry parameter λ in agreement with previous results by Spitzner and Kinzel.

Disordered spin systems with asymmetric bonds have attracted much attention in recent years not only because of their relevance to neural networks, but also from the viewpoint of nonequilibrium statistical physics. They represent an interesting and up to now not fully understood generalization of spin-glass models [1] which can be described in terms of equilibrium statistical dynamics by means of a Hamiltonian.

The best known example among these spin-glass systems is the famous SK-model [2] whose static and dynamic properties are quite well understood. The allowance of asymmetric bonds within the dynamical formulation of the above models gives rise to a completely new kind of behavior. The meaning of the asymmetric bonds is that the couplings between two spins can act differently in the two directions. The difficulties in treating these models analytically arise due to the fact that one no longer has a Hamiltonian and no detailed balance condition and therefore no fluctuation-dissipation theorem. But the last relation between response and correlation functions is in fact essential for extracting the long-time properties of the dynamics. Whereas in the SK-model the calculation of the EA-order parameter, which is such a long-time quantity, gives the decisive information about a possible spin-glass state, this calculation is in general not possible in the asymmetric generalization.

Only few analytical results are known: in [3] the Langevin-dynamics of the soft-spin version of the asymmetric SK-model was solved in the limit of infinite spin-dimensionality. It was shown

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in this case that for any finite amount of asymmetry no divergence of relaxation times occurs, indicating the absence of a spin-glass phase. A related model is the spherical model considered in [4] arriving at the same conclusion. Both models are infract linear in the spin variables, but the spin-1/2 Ising case is in fact essentially nonlinear because of the hyperbolic tangens occurring in the transition probabilities. Therefore the latter case remains controversial. Whereas to all orders of the perturbation expansion in the nonlinearity parameter of the soft-spin model with double-well potential one cannot detect a divergency of the relaxation times, another perturbation theory (in the asymmetry) yields a nonvanishing EA-order parameter at finite asymmetry and zero temperature [5,6].

Furthermore numerical simulations seem to indicate a transition in the asymmetry parameter at zero temperature: above a certain correlation value between bonds J_{ij} and J_{ji} the remanent magnetization becomes larger than zero [7]. These simulations were performed with continuously (Gaussian) distributed couplings and system sizes up to 2048 spins.

The aim of this letter is to present extensive numerical study of systems with discrete couplings ($\pm J$) and system sizes up to 8192 spins. We are mainly interested in the remanent magnetization, damage spreading and the relaxation behavior of the two-timestep overlap function at its stationary value. We have used sequential updating, but some of the conclusions seem to be true also for parallel dynamics.

The model we are considering is defined as follows: We have a system consisting of N Ising spins $\sigma_i = \pm 1$ interacting with each other *via* the local field $\sum_{j(\neq i)} J_{ij} \sigma_j$. The coupling strengths J_{ij} are random variables obeying the discrete distribution

$$\mathcal{P}_2(J_{ij}, J_{ji}) = \mathcal{P}(J_{ij}) \mathcal{P}(J_{ji}) (p |J_{ij} + J_{ji}| + (1-p) |J_{ij} - J_{ji}|) \quad (1)$$

with

$$\mathcal{P}(J_{ij} = \pm 1) = 1/2. \quad (2)$$

This yields the expectation values

$$\begin{aligned} \overline{J_{ij}} &= \overline{J_{ji}} = 0, \quad \overline{J_{ij}^2} = \overline{J_{ji}^2} = 1 \\ \text{and } \overline{J_{ij} J_{ji}} &= 2p - 1 =: \lambda. \end{aligned} \quad (3)$$

No other correlations between the couplings are present. In the thermodynamic limit the results for the discrete couplings will coincide with those for a continuous (Gaussian) distribution of the J_{ij} in the asymmetric SK-model since only the first two moments contribute.

The update rule for sequential dynamics is given by

$$\sigma_i(t+1) = \text{sign} \left(\sum_{j(<i)} J_{ij} \sigma_j(t+1) + \sum_{j(>i)} J_{ij} \sigma_j(t) \right). \quad (4)$$

The number of neurons is always even in order to avoid vanishing local fields.

The problem of simulating large, fully connected networks lays in generating and storing the coupling matrix J_{ij} . For this reason we chose binary couplings instead of continuous variables, which need 32 or 64 bits for each coupling J_{ij} . The reachable number of neurons therefore grows by a factor $\sqrt{32}$ or $\sqrt{64}$ using multispin coding techniques (see for instance [9]). Thus a network of 8192 neurons requires in principle approximately 8 Megabytes memory.

Using computers with a lower maximal resident memory size produces an extreme time delay while generating the matrix because the rows and columns are passed through simultaneously. An essential improvement of the paging behavior of the computer can be achieved by renumbering the elements of the coupling matrix in a more sophisticated way. By partitioning the matrix into squares aligned to the page size of the underlying virtual memory system you traverse through the rows and columns formed by the squares without paging activity. The allocated virtual memory is still the matrix size, but the references are reordered to minimize page faults. After the initialization we create a disk file with the couplings ordered for maximum update speed and return the virtual memory to the operating system.

Furthermore a multitask system can reduce the real running time of the program by using buffered asynchronous I/O, loading new couplings from the hard disk and doing the calculations for the update process in parallel. Using this programming technique the memory requirement is only a fraction of the whole matrix and increases linearly with N .

That is essentially the way we did the simulations on Sun386 *i* workstations, where we reached a speed of ca. 10^7 coupling evaluation per second. This value can be enhanced by a factor 10^2 - 10^3 using a Cray-YMP, as was shown in [10]; additionally all cycle tests and the damage spreading measurements after each update are done interleaved with I/O and do not contribute to the real time of the job.

In the (symmetric) SK model with parallel updating it is known [8] that the two-timestep autocorrelation function:

$$q(t, t-2) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N \overline{\sigma_i(t) \sigma_i(t-2)}, \quad (5)$$

grows algebraically to its stationary value 1:

$$1 - q(t, t-2) \propto t^{-3/2} \quad \text{for } t \gg 1. \quad (6)$$

This quantity is independent of the starting configuration after averaging over the couplings. (Note that $q(t, t-1)$ is always zero after averaging in parallel dynamics). Stimulated by these investigations we analysed in the full range of the asymmetry parameter λ and for sequential dynamics the two-timestep autocorrelation function and additionally the remanent magnetization

$$M_{\text{rem}}^N = \lim_{t \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N \overline{\sigma_i(t) \sigma_i(0)} \quad (7)$$

and damage spreading, that can be measured by the quantity

$$D^N = \lim_{t \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N \overline{\sigma_i^a(t) \sigma_i^b(t)}, \quad (8)$$

where σ_i^a and σ_i^b are two states of the same network starting with two neighboring initial configurations which differ in a small fraction of spins. Note that it is related to the Hamming distance d via $d = (1 - D)/2$.

All these quantities contain informations about the ergodicity of the system. It is expected that a nonvanishing remanent magnetization indicates a spin glass like phase, where the ergodicity is broken. This fact should also be manifested in the dynamical behavior of the overlap of two initially nearby spin configurations within the same network (i.e. identical couplings): if the overlap of the two configurations vanishes in the long-time limit the system is expected to be in a so called

chaotic phase, whereas a nonvanishing quantity D indicates a partially chaotic and partially frozen phase [11].

We now present the results of the numerical simulations: Firstly we measured the two-timestep autocorrelation function $q(t, t-2)$ for different system sizes N . Its stationary value $q_\infty = \lim_{t \rightarrow \infty} q(t, t-2)$ is depicted in figure 1 in dependence of the asymmetry parameter λ . Apart from the range $p \in [0.95, 1]$ the stationary value of q_∞ decreases monotonically with p .

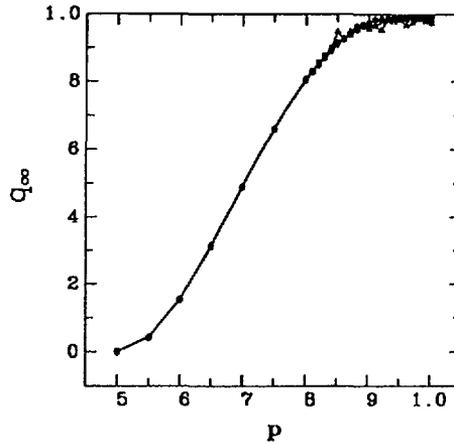


Fig. 1. — Long-time limit of the two-timestep autocorrelation function in dependence of p for $N = 512, 1024, 2048$ and 8192 .

Furthermore we investigated the time dependence of this quantity in order to see, whether the power-law behavior (6) is influenced by the degree of asymmetry. The result is shown in a log-log plot in figure 2. The exponent $3/2$ with which the two-step autocorrelation function approaches

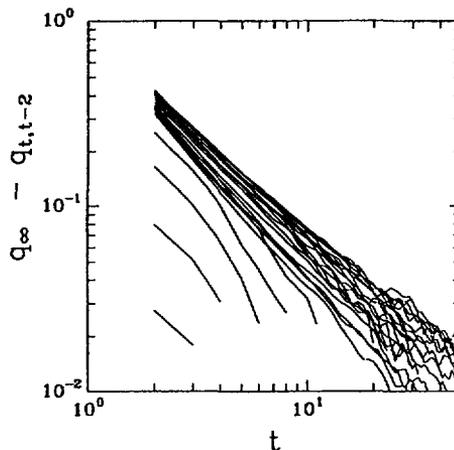


Fig. 2. — Log-log plot of the relaxation behavior of the two-timestep autocorrelation function for different values of p . The $t^{-3/2}$ -behavior is valid for the whole range of $\lambda \in [0, 1]$.

its stationary value algebraically is *the same* for all values of the asymmetry parameter and only the prefactor of the power-law shows an asymmetry dependence. This observation gives no hint for any kind of transition.

In figure 3 the remanent magnetization (7) is shown for different system sizes up to $N = 8192$. It is clear that for high asymmetry ($\lambda < 0.8$) the remanent magnetization vanishes, which can be shown analytically for $\lambda = 0$ [5]. But the remarkable fact is that for $\lambda > \lambda_c(N)$ the remanent magnetization begins to increase with λ . For increasing system sizes the critical value $\lambda_c(N)$ also increases. A finite size analysis (see Fig. 4) yields the critical asymmetry parameter $\lambda_c = \lim_{N \rightarrow \infty} \lambda_c(N) = 0.85$ within a statistical error of 0.02, which is in accordance with previous results [7].

We have also looked for damage spreading (8). The results for two different initial overlaps $D_0 = \frac{1}{N} \sum_{i=1}^N \sigma_i^a(0) \sigma_i^b(0)$ and different system sizes are depicted in figure 5. Again we observe the same phenomena: Exceeding a critical value of λ_c yields a transition from vanishing quantity D_∞ to a nonvanishing value, which means a nonergodic behavior. For different initial overlaps one gets roughly the same values of λ_c as for the remanent magnetization.

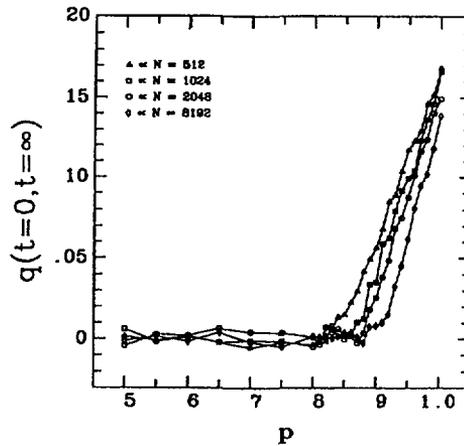


Fig. 3. — Long-time limit of the remanent magnetization *versus* p for different system sizes.

In conclusion we have presented in this paper several numerical criteria for an asymmetry-induced transition in the asymmetric SK-model. This transition does not occur at $\lambda = 1$, i.e. complete symmetry, but at $\lambda_c = 0.83$. This is not in direct contradiction to the results of [3] and [4,12], since they only claimed that for *any* asymmetry of the coupling matrix the relaxation time (of the autocorrelation function) remains finite for nonvanishing temperature. Our results and those previously obtained in [7] refer to other quantities: the remanent magnetization and damage spreading at zero temperature. The analytical investigation of these quantities is not possible (up to now) even in the symmetric case, where one knows the equilibrium distribution. Only $\lambda = 0$ is an exceptional case because it allows for an analytical solution [5,12]. It should be mentioned that a similar transition (in the EA-order parameter) was found within a perturbation expansion in the asymmetry parameter λ [6]. Finally there seems to be a kind of transition in the typical cycle length using deterministic update rules as found in [13] but its connection to the phenomena described in this paper remains to be clarified.

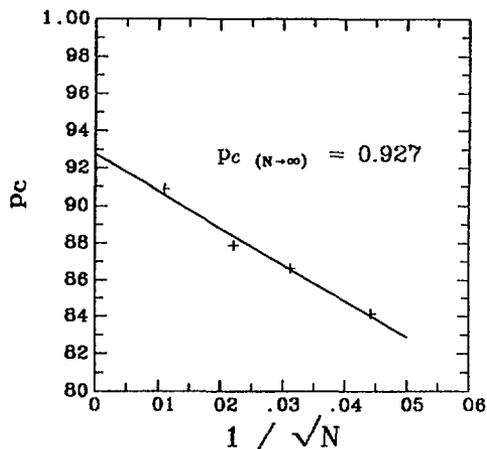


Fig. 4. — The critical value $p_c(N)$ versus $1/\sqrt{N}$. The extrapolation yields a limiting value of $p_c = 0.93$ (i.e. $\lambda_c = 0.85$).

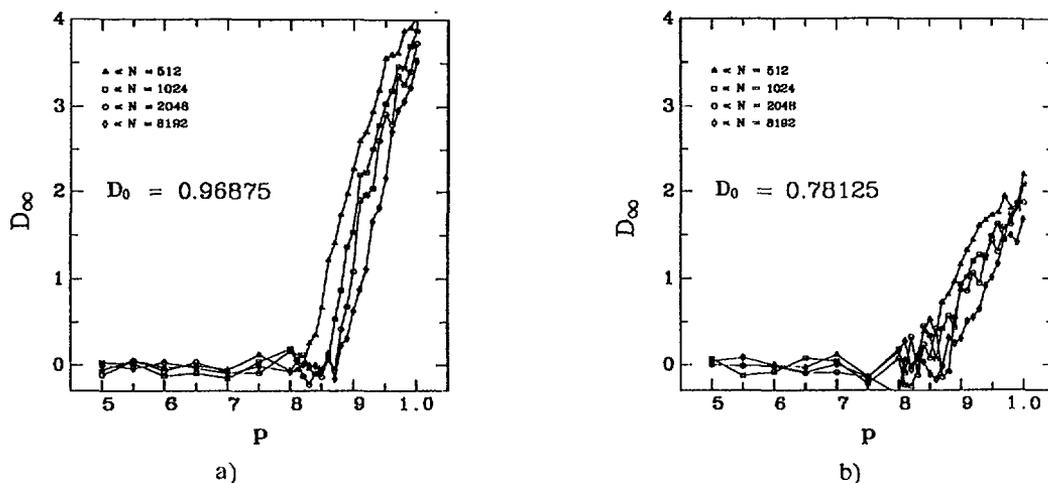


Fig. 5. — Long-time limit D_∞ of the overlap of two configurations with an initial overlap of a) $D_0 = 0.97$ and b) $D_0 = 0.78$ and different p and for four different system sizes. The behavior is similar to that of the remanent magnetization.

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