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Minimum Quality Standard and Premium Private Labels

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Résumé: Cet article propose une étude théorique des nouvelles générations de Marques de Distributeurs dans le secteur alimentaire. Nous proposons un modèle original de relation verticale, intégrant l'existence d'un marché intermédiaire de type concurrentiel (marché spot) parallèlement à la mise en place d'une relation contractuelle privilégiée entre une partie des producteurs amont et un distributeur. On montre alors sous quelles conditions les producteurs impliqués et le distributeur vont effectivement mettre en place ce type de démarches. Il apparaît néanmoins qu'à l'inverse des producteurs, les distributeurs et les consommateurs peuvent préférer une amélioration légiférée de la qualité par un renforcement des standards de qualité minimum.

Abstract: This article gives a theoretical analysis of a new type of private labels in the food sector. We propose an original model of vertical relationship between producers and retailers which takes into account two supply sources through (i) a competitive spot market and (ii) supply contracts. We study how the producers and retailers could cooperate to set up these new labels. However it appears that retailers and consumers could prefer the stepping up of minimum quality standards rather than this type of private labels.

Mots clés : Relations verticales, Grande distribution, Sécurité sanitaire, Standard de qualité minimum, Marques de Distributeurs

Key Words : Vertical relationship, Retailer, Food safety, Minimum Quality Standard, Private Labels

Classification JEL: L22, L23, Q13, Q18

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1 Introduction

In the wake of the safety crises of the past ten years in the food sector, supermarket chains have involved themselves in radically new relationships with their upstream suppliers. Even as the public authorities were tightening Minimum Quality Standard (MQS) and creating new control procedures, new labelling strategies were being adopted by certain retailers on the basis of a closer involvement in upstream farming. Initially developed in the meat sector which had been directly impacted by the “mad cow” crisis, these procedures next spread to other sectors, such as fruits and vegetables, fish, cheeses and wines.

The main large scale retailers, whose image had suffered due to this unprecedented crisis in food consumption, sought ways of satisfying consumer expectations for transparency and product safety guarantees. In the United Kingdom, procedures developed in the meat sector by Sainsbury, Marks and Spencer or Tesco are good examples (Fearne, 1998). For France, the Carrefour example can be cited with the development of “Carrefour Quality Chain” supply agreements with producer groups. At the beginning of 2003, Carrefour has already set up over 250 partnership agreements with over 35,000 producers (see Codron et al. (2003) and Mazé (2002) for the European experience).

In the United States, several studies show the increasing market shares of private labels and the development of contracts between retailers and producers in the fresh produce sector (Calvin and Cook, 2001).

One important aspect of these new procedures is that they are directed towards constructing “Premium Private Labels” (PPL). These PPL represent a new generation of private labels set up by the retailers which differ fundamentally from those set up in the 70s. Indeed, PPL place themselves at a higher level of price than that of unbranded products (here referred to as “generic products”) with guarantees for consumers to better take quality, food safety and environmental security into account. They therefore require stricter production specifications for upstream producers and greater involvement of retailers in drawing them up. From then on, product credibility depends on a displayed partnership between retailers and the production sector upstream.

There is a relatively extensive literature on private label as they have taken on considerable importance over the past twenty years in most developed countries (see, for example, Salmon and Cmar (1987), Hoch and Banerji (1993)). Most of these works aim to understand the competitive interaction between private label and producers’ brands. They examine to what degree these procedures create value and how, depending on the case, this created value is shared among the various stakeholders. Mills (1995) offers such an analysis of a producer-retailer relationship in which the producer’s brand (referred to as national brand) and the private label are in competition in the market. This author shows how the private label enhances retailer performance (i) by redirecting sales which previously went to national brand towards private label supplied at a lower gross cost and (ii) by increasing the margins won from the national brand. Other works extend this type of analysis by placing the accent on the differentiation strategy choice and examine the quality and price positioning of the store brands compared to the existing national brands (Bontems et al., 1999; Connor and Peterson, 1992; Slade, 1995; Raju et al., 1995 and Cotteril et al., 2000) propose different econometric methods to estimate market shares and price interactions between private labels and manufacturers’ brands. Hosh (1996), Quelsch and Harding (1996) and Mills (1999)
analyse the counterstrategies the manufacturer is able to adopt in order to reduce the bargaining power of the retailer.

The main part of these works stresses on the power shifting to the retailers and show that the private label are positioned at lower retail price and quality levels than the national brands or like close substitutes to these national brands. However, Dunne and Narasimhan (1999) (see also Dunne, 1999) give the example of Loblaws, the largest Canadian Grocery, which added the new Premium President’s Choice line to its traditional unbranded product. The marketing messages of Loblaws stress the quality of the ingredients and the preparation of President’s Choice. Its olive oil, for example, is claimed to be “harvested from trees planted more than 80 years ago and produced from the first cold pressing of sunripened olives”, that it means higher quality and production costs. It is exactly the same type of marketing messages which are used by the retailers in the meat and fresh produce sectors about the quality, safety and environmental guarantees given by the PPL.

The goal of this paper is to analyze this new generation of private label and to get deeper insight about (i) why retailers set up such procedures, (ii) to what extent suppliers have interest to involve in it, and (iii) what are the consequences for the consumers. To evaluate these new procedures, several elements must be taken into account:

- **The degree of differentiation of the PPL compared to the generic product.** This degree of differentiation is determined by the specification imposed by the retailer. Additional costs are then induced according to the quality level of the product and the reinforcement of controls carried out at all levels of the chain, by both stakeholders and outside control and certification organizations.

- **The retail prices and the consumer willingness to pay.** Both of these criteria depend not only on the degree of differentiation compared to the standard product, but also on communication policies, that is to say, the investment in marketing undertaken to publicize the product and reassure consumers about its characteristics.

- **The buying and selling alternatives available to the suppliers and the retailers.** Depending on the degree of exclusivity of the relationship, each of them can have alternatives for distributing or ordering the differentiated product on other networks. This possibility has two effects: for the producer, threatening to ration the retailer can improve his bargaining power and make it possible for him to capture more of the created value; for the retailer, threatening to set up competition between several competitors all capable of supplying the differentiated product gives him more negotiating power.

To quantify the economic impact of these different elements, we propose an original vertical relationship model which integrates the existence of an intermediary spot market in parallel with a contractual privileged relationship (i.e a PPL) between part of the upstream producers and the retailer. To begin with we consider an arbitrarily large number of upstream producers susceptible of supplying the downstream retailers in the chain. One such retailer decides to set up a higher quality product than that presently offered to consumers (the “generic” quality product) and, hence, gives higher guarantees than the other retailers. This PPL is associated with the implementation of a specification with a subset of producers who develop the higher quality product. Generic quality supply for all retailers goes through the spot market fed by the overall group of producers. This
generic quality is defined by public authorities and corresponds to a MQS. Under these conditions, if we interest ourselves in the impact of the producer-retailer relationship on the nature of the food offer made to the consumers, the strategic decision which is important to study is that of the quality/price positioning of the PPL. This decision is linked to an increase in production costs with quality resulting in a higher end price and a smaller share of the shelf space dedicated to the differentiated product. The PPL operators are, hence, faced with the following alternative: either the qualitative differentiation of the PPL compared to the standard product is low along with a low difference between its production cost and retail price compared to the standard while a large part of the food offer takes advantage of a “little plus” in terms of quality and safety guaranties; or the qualitatively differentiation is strong, resulting in contrast to higher costs and a higher retail price, but for considerable quality and security guaranties on only a small part of the retail shelf space.

We show how the PPL can go beyond the highest quality of the standard susceptible of being set up by the public authorities. At the same time, we show that the interest of the producers can diverge from that of the retailer involved in the PPL. In contrast to the producers, it seems that the retailers and the consumers could, in fact, prefer regulated product improvement through the reinforcement of the MQS. This reinforcement would lead to limiting PPL supply, even to totally eliminating it. In other terms, creation of PPL could, paradoxically, be carried out more in the interest of the producers than in that of the retailers.

Our article is organized in the following manner. In section 2 we describe our model in the absence of the PPL and we solve it to define a reference for the vertical relationship. Section 3 fixes the analytic framework of an PPL by defining the optimal quality and supply for the markets for a fixed MQS. Section 4 is dedicated to the influence this standard has on producer and retailer profits, as well as on the consumer surplus. Section 5 closes the article by referring to possible extensions of the model.

2 Benchmark Modelling

The analytic framework of the initial situation is shown in figure 1. Consider a set of $J$ producers offering an identical product represented by a quality index $k_0 > 0$. The parameter $k_0$ corresponds to a MQS to which all the producers are subjected. In this first phase we suppose that the producers have no possibility of improving quality beyond the level of the standard.

Each producer has an identical production capacity $\alpha = \frac{K}{J}$ (where $K$ is the total capacity of all the producers) and supplies an intermediary market supplying $R$ retailers. Each retailer thus finds himself in the position of a monopoly\(^1\) to supply a “reserved” market of the size $M_r (r = 1, ..., R)$

---

\(^1\)This assumption can be justified by the fact that consumers choose \textit{ex ante} the store according to general features like location, product diversity and general level of prices and quality and not by comparing for each specific product the offers of the different retailers (Chardon and Dumartin, 1998). It means that it is not relevant for our purpose to assume a price competition for each specific product between the different supermarkets chains. Otherwise, since Holton (1957) the theoretical literature dealing with retailing has been taking into account the shopping costs. These shopping costs are important to consider because the consumers buy usually several products in the store. This point also justifies our assumption insofar we limit our analysis to only one product (see Chambolle (2002) for a competitive analysis of multiproduct retailers).
and we pose \( M = \sum_{r=1}^{R} M_r \).

The intermediary market is taken to be a competitive market on which the price \( \omega_0 \) results from the balance between the upstream producers’ offer and the downstream retailers’ demand. The \( k_0 \) quality results in a unit production and certification cost covered by the producers upstream. We designate this unit cost \( c_0 \) and retain the quadratic specification \( c_0 = c k_0^2 \) (\( c \geq 0 \)). Hence, for a fixed price \( \omega_0 \), an upstream producer’s profit \( j \) (\( j = 1, \ldots, J \)) offering quantity \( q \) is written \( B_j(q) = (\omega_0 - c_0)q \). Under these conditions, a producer has only two possibilities: either offer quantity \( \alpha \) if \( \omega_0 \geq c_0 \), or offer nothing if \( \omega_0 < c_0 \). When \( \omega_0 \geq c_0 \) the total quantity offered on the intermediary market is inelastic and is equal to \( K \).

Figure 1 : Producer-retailer relationship in the absence of a PPL.

We consider that the price \( \omega_0 \) is imposed upon each retailer \( r \) but that each of them is free to choose the quantity \( x_r \) he needs depending on the demand of the final market. On this market of the size \( M_r \), consumers are distinguished by a \( \theta \) taste parameter compared to the quality offered and we suppose that the \( \theta \) parameter is uniformly distributed over the \([0, \theta]\) interval. As Mussa and Rosen (1978), we consider that the surplus of a consumer of \( \theta \) type buying a unit of \( k \) quality product at price \( p \) is given by the expression \( S(\theta) = \theta k - p \). In this case, only those consumers allowing \( S(\theta) \geq 0 \) (\( \theta \geq \frac{p}{k} \)) buy the goods. Hence, for the reference situation defined above (i.e in the case where only quality \( k_0 \) is offered at price \( p_0 \)) total demand \( d_r(k_0, p_0) \) for each retailer \( r \) on the end market is written:

\[
d_r(k_0, p_0) = \frac{M_r}{\theta} (\theta - \frac{p_0}{k_0})
\]  

See, for example, Latouche et al. (2000), Dickinson and DeeVon Bailey (2002), Lusk et al. (2003) for assessment of the consumers willingness to pay stronger guarantess in food safety, quality and environmental security after the “mad cow” crisis.
Using the equation (1) one can write the inverse demand function describing the price $p_0(x_r)$ depending on quantity $x_r$ purchased on the intermediary market and resold on the end market:

$$p_0(x_r) = \frac{\theta k_0}{M_r}(M_r - x_r)$$  \hspace{1cm} (2)$$

We thus give the profit of retailer $r$ buying quantity $x_r$ on the intermediary market at price $\omega_0$ and valorizing it at price $p_0(x_r)$ on the end market as $\Pi_r(x_r) = (p_0(x_r) - \omega_0)x_r$. The optimal quantity of $x_r$ placed on the market is therefore given by:

$$x_r = \frac{M_r}{2\theta k_0}(\theta k_0 - \omega_0)$$ \hspace{1cm} (3)$$

Under such conditions, the retailers’ total demand $D(\omega_0)$ on the intermediary market is such that $D(\omega_0) = \sum_{r=1}^{N} x_r$. Accordingly, the price $\omega_0^*$, balancing out the total offer $K$ and the demand $D(\omega_0)$, is given by the following equation (4):

$$\omega_0^* = \frac{\theta k_0(1 - 2\xi)}{c}$$ \hspace{1cm} (4)$$

The parameter $\xi = \frac{K}{M} \left( \xi \leq \frac{1}{2} \right)$ expresses the relationship between the producers’ total offer and the global size of the downstream market (which we call the “relative capacity of the offer” on the market). As shown by equation (4), the intermediary price $\omega_0^*$ shrinks naturally depending on the $\xi$ parameter and grows depending on quality $k_0$ (since the demand on the end market also grows according to $k_0$). What is more, this price $\omega_0^*$ remains superior to the cost $c_0$ as long as the following condition is met:

$$k_0 \leq \bar{k}_0 = \frac{\theta(1 - 2\xi)}{c}$$ \hspace{1cm} (5)$$

Hence, if the $k_0$ standard is not too high and remains inferior to the threshold value $\bar{k}_0$, each producer receives positive profit by offering his total production capacity. Above the $\bar{k}_0$ threshold, quality becomes too expensive to produce and the producers can no longer supply the intermediary market. The $\bar{k}_0$ threshold thus corresponds to a maximum value for fixing the MQS. One can also see that the $\bar{k}_0$ threshold shrinks so directly according to the $\xi$ parameter that an increase in the offer upstream of the production chain limits the possibilities of fixing a minimum quality standard.

Finally, by using (2),(3) and (4), we find that each retailer buys quantity $\bar{x}_r$ at price $\omega_0^*$ and resells it on the end market at price $\bar{p}_0$ defined by:

$$\begin{align*}
\bar{x}_r &= M_r\xi & r &= 1, ..., R \\
\bar{p}_0 &= \theta(1 - \xi)
\end{align*}$$ \hspace{1cm} (6)$$

The profit $\bar{B}_j$ of each producer $j$ ($j = 1, ..., J$) and the profit $\bar{\Pi}_r$ of each retailer $r$ ($r = 1, ..., R$) are, then, written as:

$$\begin{align*}
\bar{B}_j &= \alpha k_0(\theta(1 - 2\xi) - ck_0) & j &= 1, ..., J \\
\bar{\Pi}_r &= M_r\theta k_0\xi^2 & r &= 1, ..., R
\end{align*}$$ \hspace{1cm} (7)$$
Of course the retailer’s $\Pi_r$ profit is a increasing $k_0$ since he does not finance the production and certification costs linked to this level of quality. On the other hand, $\Pi_f$ profit is a concave function, optimized for the $\frac{k_0}{2}$ value, which corresponds to the “ideal” value of the MQS from the producers’ point of view.

Insofar as only the $k_0$ quality is offered on the market, the surplus $W_c(k_0)$ obtained for the consumers, customers of retailer $R$, is written simply:

$$W_c(k_0) = M_R \int_{p_0/k_0}^{p_0} (\theta k_0 - p_0) f(\theta) d\theta = \frac{M_R p_0 \xi^2}{2}$$

Here again the $W_c(k_0)$ surplus is increasing in $k_0$. This result is a direct consequence of the hypothesis we adopted on the form of production costs. Indeed, the linear character of the functional form retained leads to a maladaptation of the quantities placed on the market according to the level of the $k_0$ standard (if not when this standard passes the $k_0$ threshold level). An effect of $p_0$ price increase depending on scarcity is thus eliminated, with the result that consumers always prefer enhanced quality. Hence, a quality standard restrained by reference to a maximum retained value can only be explained by its interest for the upstream producers, never by its interest for retailers or consumers.

3 Premium Private Label modelling

Now we will envisage the creation of a PPL on the market. The analytic framework is shown on figure 2. We consider the existence of a group of $G$ producers upstream of the market. This producer group is represented by a sole entity capable of making strategic production and sales decisions for all the members of the group. Each member’s capacity constraint is identical to the $\alpha$ value defined in the preceding section.

We study the possibility of setting up a partnership (which we call an “integrated chain”) between the producer group and retailer $R$. The goal of this integrated chain is to divert part of the exchanges carried out on the spot market\(^3\) and to constitute a PPL which is differentiated from the MQS $k_0$\(^4\). This PPL is characterized by superior quality valorized by consumers on the end market. More precisely, a PPL corresponds to a partnership between a producer group and retailer $R$ in order to:

1. offer consumers a $k_1$ quality product with $k_1 > k_0$;
2. define the quantities $x_R$ and $y_R$, respectively, which can be sold for the qualities $k_0$ and $k_1$.

\(^3\)Our model is quite similar to the Xia and Sexton’s model (2003) which simultaneously takes into account a contractual relationship between producers and retailers and a spot market. However, these authors do not assume a product differentiation between the contractual relationship and the spot market. The contract does not allow a higher intermediary price than on the spot market. Our model is designed to analyze this possibility.

\(^4\)We show further that if there is no differentiation between $k_0$ and the PPL, we get the same results than with the benchmark.
Hence, as shown in figure 2, the producer group supplies retailer $R$ a $y_R$ part of their production potential in superior quality with the rest, $\alpha G - y_R$, being placed on the spot market. To simplify the analysis, we will work within the hypothetical framework of the group always supplying the spot market, that is to say, the relative number of suppliers involved is sufficiently high. We retain the hypothesis (H1) defined by:

$$(H1) \quad \beta = \frac{\alpha G}{M_R} \geq \xi$$

So, retailer $R$ this time offers two qualities to his clientele. By identifying the $\beta$ consumer taste indifferently between the purchase of the $k_0$ quality product, sold at the $p_0$ price and the purchase of the $k_1$ quality product, sold at the price $p_1$, $\left( \beta = \frac{p_1 - p_0}{k_1 - k_0} \right)$, one obtains the quantities demanded in $k_0$ and $k_1$ qualities sold, respectively, at prices $p_0$ and $p_1$:

$$d_0(k_0, k_1, p_0, p_1) = \frac{M_R}{\beta} \left( \frac{p_1 - p_0}{k_1 - k_0} - \frac{p_0}{k_0} \right)$$  \hspace{1cm} (9)

$$d_1(k_0, k_1, p_0, p_1) = \frac{M_R}{\beta} \left( \beta - \frac{p_1 - p_0}{k_1 - k_0} \right)$$

Using the same methodology as in the preceding section, we inverse the system (9) to obtain prices $p_0(x_R, y_R)$ and $p_1(x_R, y_R)$ in function of quantities $x_R$ and $y_R$ placed on the retailer $R$ market at, respectively, qualities $k_0$ and $k_1$.

$$p_0(x_R, y_R) = \frac{M_R}{\beta} \left( M_R - x_R - y_R \right)$$  \hspace{1cm} (10)

$$p_1(x_R, y_R) = \frac{\beta}{M_R} \left( k_1 M_R - k_0 x_R - k_1 y_R \right)$$
Insofar as the set of parameters for the model is common knowledge, we consider that the producer group and the retailer dimension the $k_1$ superior quality to offer consumers in such a way that quantities $x_R$ and $y_R$ are offered on the end market. Production of the new $k_1$ quality leads to increased unit production and certification costs for the quality, which are measured by $\Delta c = c_1 - c_0$ with $c_1 = ck_1^2$. In practice, covering these extra costs is shared by the producer group and retailer $R$. For this reason, we retain that a $\lambda$ ($0 \leq \lambda \leq 1$) part is paid by the retailer and a $(1 - \lambda)$ part by the producer group. The integrated chain profit is:

$$\Pi(k_1, x_R, y_R) = (p_1(x_R, y_R) - c_1) y_R + (p_0(x_R, y_R) - \omega_0)x_R + (\omega_0 - c_0)(\alpha G - y_R)$$

(11)

The problem is then how to optimize the $\Pi(k_1, x_R, y_R)$ profit in function of the three quality and quantity arguments. Still, it is not sure that production and sales of a $k_1 \neq k_0$ superior quality is really optimal from the joint profit point of view defined above. Trade off for the integrated chain must be carried out specifically in function of the $k_0$ MQS as well as in function of the increased production costs linked to developing the $k_1$ quality. One first obtains the following result:

**Proposition 1**

Posing $\tilde{k}_0 = \frac{\theta}{2\epsilon}$, the integrated chain chooses to produce a $k_1 > k_0$ superior quality if and only if $k_0 < \text{Min}\{k_0, \tilde{k}_0\}$. In this case, the integrated chain chooses to supply the $k_0$ and $k_1$ qualities in non nul quantities if and only if $k_0 > \tilde{k}_0 = \tilde{k}_0(1 - 3\xi)$.

Proposition 1 gives immediate portrayal of the importance fixing the level of the MQS has on the private incentive to develop a superior quality product. Hence, if this standard is too high and exceeds the $\tilde{k}_0$ value, the producer group and retailer will not see any interest in developing the PPL. On the other hand, if the MQS is too low, $(k_0 \leq \tilde{k}_0)$, the integrated chain develops a PPL while eliminating supply of the $k_0$ standard. It must also be taken into account that in the case of strong relative capacity of the offer on the market, $(\xi > \frac{1}{4})$, one obtains $\tilde{k}_0 > \overline{k}_0$ and the PPL is systematically developed.

Under these conditions, and as will later see, the question for the public authorities is to know whether to fix a low standard, favoring private investment and making it possible to segment the market (by the creation of a PPL heavily supplied in the linear) or, on the contrary, to raise the standard in order to minimize the amplitude of PPL development. However, to judge the efficiency of arbitration of public policy, it is first of all important to assess the diversity of the products offered, supply and the pricing practiced for the $k_0$ and $k_1$ qualities. We will carry out this initial analysis by placing ourselves within the framework of hypothesis (H2) defined hereunder:

$$(H2) \quad \tilde{k}_0 \leq k_0 \leq \overline{k}_0$$

The hypothesis $k_0 \leq \overline{k}_0$ is placed in the situation defined by (4) of an effective existence of an intermediary market. The hypothesis $\tilde{k}_0 \leq k_0$ makes it possible to simplify the analysis at the technical level without drawing into question the main results that we want to bring to light.

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5 All the demonstrations are detailed in the appendix.
Within this framework of analysis, the two qualities \( k_0 \) and \( k_1 \) will systematically be supplied and it will be possible for us to obtain a simple analytical expression of optimal PPL quality from the point of view of the integrated chain.\(^6\) We first show an initial result of intermediary price invariance:

**Proposition 2**

Under (H1)-(H2) and if \( k_0 < \bar{k}_0 = \frac{\theta}{2c} \), no matter what the level of quality \( k_1 \) of the PPL, the spot market price remains identical to the market price \( \omega^*_0 = \bar{\omega}k_0(1 - 2\xi) \), obtained in the reference situation. The total quantity of supply for the market of retailer \( R \) is such that \( x_R + y_R = \xi M_R \).

The first consequence of proposition 2 is that if a retailer develops a PPL it will not change the remuneration of producers not engaged by this retailer. Diverting part of the offer on the intermediary market is exactly compensated by a drop in the demand of retailer \( R \) on this market, thus explaining the invariance in the price \( \omega^*_0 \). Hence, a PPL is developed only in the interest of the contracting parties and no externality, positive or negative, on the profit of the other producers (i.e. not part of the group) can be associated with this transformation of vertical relationships. Moreover, the PPL does not lead to any modification in the total quantities of end market supply in comparison with the reference situation defined in the preceding section since retailer \( R \) contents himself with modifying the distribution of this quantity between the two qualities. The optimal level of quality of the PPL for the integrated chain as well as the supply associated with it is given in the following proposition.

**Proposition 3**

Under (H1)-(H2) and if \( k_0 < \bar{k}_0 = \frac{\theta}{2c} \), the quality selected by the integrated chain is written:

\[
k_1^*(k_0) = \frac{\bar{\omega} + ck_0}{3c}
\]

Supply of the \( k_0 \) and \( k_1^*(k_0) \) is obtained by:

\[
\begin{align*}
x_R^* &= \left[ \frac{2ck_0 - (1 - 3\xi)\bar{\omega}}{3\theta} \right] M_R \\
y_R^* &= \left[ \frac{\bar{\omega} - 2ck_0}{3\theta} \right] M_R
\end{align*}
\]

The retail prices on the market of retailer \( R \) are given by:

\[
\begin{align*}
p_0^* &= \bar{\omega}k_0(1 - \xi) \\
p_1^* &= \bar{\omega}k_0(1 - \xi) + \frac{2}{3c}[\bar{\omega} - ck_0(\bar{\omega} + 2ck_0)]
\end{align*}
\]

\(^6\)The assumption (H2) is relevant from an empirical point of view because the PPL never supplies the whole shelf space.
The optimal quality adopted by the integrated chain is increasing in \( k_0 \). In annex we also show that the \( k^*_1(k_0) \) quality can exceed the \( \bar{K}_0 \) value once the \( \xi \) ratio is sufficiently high. Hence, private initiative of the integrated chain makes it possible to reach a quality level for the product which would not have been foreseeable using the MQS. Still, one sees that the differentiation between the quality of the PPL and the quality of the \((k^*_1(k_0) - k_0)\) standard is shrinking in \( k_0 \). Hence, the level of \( k^*_1(k_0) \) approaches that of \( k_0 \) as the standard is raised. In the case of \( \tilde{k}_0 \leq \bar{K}_0 \), the differentiation of the products offered to the clientele of retailer \( R \) drops until it cancels itself out for \( k^*_1(k_0) = \tilde{k}_0 \) (on the opposite, if \( \tilde{k}_0 > \bar{K}_0 \), the qualities \( k_0 \) and \( k^*_1(k_0) \) are systematically developed and supplied). The integrated chain progressively decides to increase supply of the quality standard \( (x^*_k \text{ is growing in } k_0) \) and to limit the quantities attributed to the PPL \( (y^*_k \text{ is shrinking in } k_0) \).

One can, moreover, see that the ratio \( \frac{x^*_k}{y^*_k} \) is growing in \( k_0 \) and in \( \xi \). Hence, the higher the total size \( M \) of the end market (\( \xi \) weak), the more the integrated chain favors supply the PPL at the expense of the supplying the generic product (since the price of supplying the generic product on the intermediary market is higher). On the other hand, the size increase of the \( M_R \) of retailer \( R \) (for example following a merger-acquisition) has no influence on his market segmentation since \( \frac{x^*_k}{y^*_k} \) does not depend on \( M_R \).

Once can see that the price of quality \( k_0 \) remains identical to the price \( \bar{p}_0 \) defined by (6) within the framework of reference (since the price is not a function of the total quantity \( x_R + y_R \) of supply on the market of retailer \( R \). The price of the high quality \( k_1 \) is growing in \( k_0 \) until this parameter reaches a critical size \( \sigma = \frac{7 - 9\xi}{8c} \) (we show that \( \sigma < \tilde{k}_0 \) once \( \xi > \frac{1}{3} \)) then begins shrinking in \( k_0 \) due to the drop in the product differentiation between the two qualities.

The question is now to analyze how the created value is shared among the stakeholders of the chain. To study this point, we placed ourselves within the framework of a negotiation procedure between the producer group and retailer \( R \). This negotiation concerns the \( \omega_1 \) unit remuneration price of the producer group.\(^7\) For a fixed price \( \omega_1 \) the \( B(k_0, \omega_1) \) profits of the producer group and \( \Pi_R(k_0, \omega_1) \) of retailer \( R \) are given by:

\[
B(k_0, \omega_1) = [\omega_1 - c_1 + (1 - \lambda)\Delta c]y^*_R + (\omega^*_0 - c_0)((\alpha G - y^*_R)
\]

\[
\Pi_R(k_0, \omega_1) = [p^*_1 - \omega_1 - (1 - \lambda)\Delta c]y^*_R + [p^*_0 - \omega^*_0]x^*_R
\]

(15)

We suppose that the sharing depends on the bargaining power of each of the participants and that this bargaining power is uniquely represented by the profit each participant obtains in the absence of a PPL. Hence, the status quo of the relationship corresponds to the situation of reference of the preceding section with the respective profits \( \bar{\Pi} = G\bar{\Pi}_j \) and \( \bar{\Pi}_R \) obtained with the help of (7). We thus suppose that the remuneration price \( \omega^*_1 \) of the producers engaged is fixed by the Nash solution by solving the following program:

\[
\max_{\omega_1}(\Pi_R(k_0, \omega_1) - \bar{\Pi}_R)(B(k_0, \omega_1) - \bar{B})
\]

(16)

\(^7\)The price \( \omega_1 \) is a mean to share the value created by the integrated chain in practice. Of course, insofar as we have adopted a “collusive solution concept” (described in Schmalensee, 1987) \textit{ex ante} maximizing the stakeholders profit sum, the transfer could be done by sharing the additional profit with the same results.
One then obtains proposition 4:

**Proposition 4**

The unit remuneration price $\omega^*_1$ of high quality is:

$$\omega^*_1 = \omega^*_0 + \lambda \Delta c + \Gamma(k_0)$$

where $\Gamma(k_0) = \frac{\theta - 2ck_0)^2}{18c}$ is decreasing in $k_0$.

Proposition 4 shows how it is possible to fix a unit remuneration price for the producers engaged on the basis of the price observed on the spot market by reimbursing the added production costs actually paid by the latter. In other words, the decision taken *ex ante* to share the added production costs linked to quality has no effect. In addition there is a producers’ bonus given by $\Gamma(k_0)$.

The value of this bonus is all the lower as the MQS is fixed at a high level, explaining the drop in product differentiation between the MQS and the PPL.

In this section we have thus given a theoretical framework for the creation of a PPL so as to on the one hand define its economic interest for the parties involved and, on the other, to define the value sharing which could be carried out following a policy of optimal maximization of total profits. We have also defined an optimal quality level for the PPL and its supply. Still, it is clear that the set of results depends precisely on the level of the MQS adopted by the public authorities. This point will be studied in the next section by showing the diverging interests that can exist on the choice of this standard from the viewpoints of producers, retailers and consumers.

## 4 Choosing a minimum quality standard

Numerous theoretical works have discussed the interest of public regulations to enhance product quality through a MQS. It is not obvious *a priori* that introducing an imposed minimum level of quality will lead to a higher level of average quality of products offered to consumers. Besanko et al (1988) showed that creating a MQS could result in prices increases as well as in reduced variety of products, thus penalizing a fraction of consumers. Other authors took interest in studying the reduction in the number of companies which could be brought about by the creation of a MQS, by the effects induced by rising costs caused by the quality to produce (Ronen, 1991; Crampes and Hollander, 1995), or by the strategies companies put in place to anticipate the reinforcement of quality standards (Ecchia and Lambertini, 2001; Lutz et al., 2000). In total, the theoretical literature remains quite divided over the usefulness and the effects of MQS. Results thus far obtained remain insufficient for clarifying the debate with which we are here interested. Particularly, they do not take into account the vertical relationship between companies and their suppliers or the negotiation power sharing between downstream and upstream companies. One would suppose that the nature of the vertical relationship has an influence on the sharing of profits.

---

8 We frequently observe in practice supply contracts indexed on spot market prices. One of the goal of such a method is to overcome the difficulties linked to demand hazards. Proposition 4 gives a theoretical expression of this price fixation method.
quality costs between suppliers and the retailer and thus conditions the quality positioning of a downstream company and their answer to the introduction of MQS.

For the example which interests us, in our introduction we looked at the PPL procedures which lead to supplying large portions of the end market with products which are more tightly controlled at the safety level and subjected to more demanding specifications than the products spontaneously offered on the spot market. But are these procedures based on private operator initiative efficient from the public interest point of view? How can the intervention of public authorities, particularly as concerns a MQS, influence them? These are the questions we will be examining in this section.

With reference to public objectives, the standard must be chosen on the based of trade off between the interest of production system and the consumers. As we have already shown, such a minimum quality level has a direct influence over the incentive to develop a PPL and it is not sure that an extremely high level of certification fosters a good quality/price ratio or efficient market segmentation from the point of view of consumer surplus. Moreover, compared to the production system, it might be necessary to distinguish between the interest of upstream producers and that of retailers since the costs of accessing the market supported solely by the producers while the added costs associated with higher quality are partially covered by the retailers. Hence, the following proposition:

**Proposition 5**

The profit of upstream producers $B(k_0, \omega^*_1)$ is maximized on $[k_0, \bar{k}_0]$ with $k^*_0(\beta)$ and we have $k^*_0(\beta) < \min\{\bar{k}_0, \bar{k}_0\}$:

$$k^*_0(\beta) = \frac{\bar{\beta}}{4c}(2 - 9\beta + 3\sqrt{9\beta^2 - 8\beta\xi})$$ (18)

We show that $k^*_0(\beta)$ is increasing in $\beta$ on $[\xi, \frac{1}{2}]$. The profit $\Pi_R(k_0, \omega^*_1)$ of retailer $R$ is maximized on $[k_0, \bar{k}_0]$ with the value $\bar{k}_0$.

The first part of proposition 5 means that the producer group upstream prefers to maintain a moderate level of MQS, leaving the field free for development of a PPL. The ideal level for a standard drops with $M_R$ end market size (because growing in $\beta$). One can moreover see that retailer $R$ would prefer the highest possible level for the standard. Such a standard would have the effect of limiting PPL supply (since $y^*_R$ is decreasing in $\xi$) and would even eliminate the PPL in the case of a low relative capacity level of offer on the market (if $\xi < \frac{1}{4}$).

There is, thus, a divergence of interests between the producer group and retailer $R$ on the fixation of the MQS. From the producer group point of view, it is not necessary to fix too high a standard because it will cause too big an increase in production costs. Moreover, one can easily see that the value $k^*_0(\beta)$ is inferior to the value $\frac{\bar{k}_0}{2}$, which was, from the consumer group point of view, its optimum in the reference situation described in the preceding section. Hence, the possibility of developing a PPL leads the producer group to support a lower MQS than the non engaged producers. On the other hand, retailer $R$ continues to seek the highest possible level of standard which would, implicitly, limit development of the PPL. One must, however, keep in mind
that in this article we consider that the standard is fixed \textit{ex ante} and that the producers and retailers did not participate in fixing the level of \(k_0\). For this reason, negotiating profit sharing in the contractual relationship between the producer group and retailer \(R\) is carried out on the basis of the \(k_0\) standard, meaning it is not foreseeable that all participants are assured of receiving what they consider to be the profit they would earn for the ideal \(k_0\) value. We do, however, in the following proposition, show the necessity, from the integrated chain point of view, of leaving an open field for the development of a PPL.

**Proposition 6**

Under (H1)-(H2) the profit of the integrated chain is maximized on \([k_0, \overline{k}_0]\) with \(k_0^{**}(\beta)\) and we have \(k_0^{**}(\beta) < \text{Min}\{\overline{k}_0, \overline{k}_0\}\):

\[
k_0^{**}(\beta) = \frac{\theta}{8c} (4 - 9\beta + 3\sqrt{9\beta^2 - 16\xi} + 8\xi^2)
\]

We show that \(k_0^{**}(\beta)\) is decreasing in \(\beta\) on \([\xi, \frac{1}{2}]\).

Hence, from the integrated chain point of view it is preferable to obtain a MQS at a sufficiently low level to afterwards justify the creation of a PPL. This optimal level is situated between the \(k_0^{*}(\beta)\) value sought by the producer group and \(k_0\) value sought by retailer \(R\). \(k_0^{**}(\beta)\) is all the lower as the relative production capacity compared to the market size of retailer \(R\) (measured by the parameter \(\beta\)) is high.

We are also in a position to assess the impact of developing a PPL on the consumer surplus of the customers of retailer \(R\). This impact can be measured again in function of the \(k_0\) level of quality standard, either anticipating or not the development of the PPL. Formally, the consumer surplus of retailer \(R\) consumers, with qualities \(k_0\) and \(k_1\) sold at prices \(p_0\) and \(p_1\) at their disposal, is written:

\[
W_c(k_0, k_1, x_R, y_R) = \frac{\theta}{2M_R} [k_0 x_R^2 + k_1 y_R^2 + 2k_0 x_R y_R]
\]

One this obtains the surplus \(W_c(k_0)\) by anticipating the PPL quality \(k_1^*(k_0)\) which can be adopted by the integrated chain:

\[
W_c(k_0) = \begin{cases} 
\frac{M_R \overline{k}_0 \xi^2}{2} + \frac{M_R C^2 (\overline{k}_0 - k_0)^3}{8\theta} & \text{if } k_0 \leq \text{Min}\{\overline{k}_0, \overline{k}_0\} \\
\frac{M_R \overline{k}_0 \xi^2}{2} & \text{if } k_0 \geq \text{Min}\{\overline{k}_0, \overline{k}_0\}
\end{cases}
\]

Formulation (21) makes it possible to check in direct that for all levels of quality \(k_0\), consumers get a superior surplus with development of the PPL. Indeed, in the absence of a PPL, consumers systematically obtain the surplus \(W_c(k_0) = \frac{M_R \overline{k}_0 \xi^2}{2}\) whereas in the case of PPL development (if \(k_0 \leq \text{Min}\{\overline{k}_0, \overline{k}_0\}\)), consumers get a supplementary surplus of \(\frac{M_R C^2 (\overline{k}_0 - k_0)^3}{8\theta}\). This property is, however, relatively intuitive since, with the total quantity offered on the market remaining
constant, dividing part of the production onto a superior level of quality necessarily increases the
collectors surplus.

One can also see that if the MQS is varied, consumers always prefer the highest possible
standard.

**Proposition 7**

No matter what the level of parameter $\xi$, $W_c(k_0)$ is maximized in $k_0 = \overline{k}_0$. However, if $k_0 = \overline{k}_0$, the structure of the offer to consumers varies according to $\xi$ in the following manner:

- if $\xi < 1/4$, the offer is not segmented and only quality $k_0$ is offered to consumers;
- if $\xi > 1/4$, both qualities are offered to consumers, the part of the linear supply in $k_1$ growing with $\xi$.

No matter what the relationship between the size of the market and total production capacity, raising the MQS to the highest possible level is desirable from the consumer point of view. In the context of low relative capacity, and therefore of a high general price level, positioning a MQS at $\overline{k}_0$ is incompatible with the existence of a PPL differentiated from the standard. On the contrary, for higher levels of relative capacity, the differentiation procedures carried out by the retailers remain possible even if the public authorities raise the MQS to $\overline{k}_0$. Hence, the defection of part of the consumers (for example upon the occasion of a food safety crisis) can explain the development of a PPL.

**5 Conclusion**

In the face of the problems posed by recent food safety crises, and, in general, by the worries
about the quality of products offered, the question confronting public authorities is whether, how
and to what degree they must reinforce the quality and product security norms and standards.
Without claiming to fix this critical size of the standard here, which would require developing
more precise technical arguments and an econometric estimation of the model, one can see that
an important element in the debate concerns the costs resulting from such quality standards and
sharing these costs between producers, retailers and consumers. Indeed, the imposed standards
determine the level of production, certification and control costs. Moreover, restoring the consumer
trust implies high investment in communication. The trade off which must be carried out by the
public authorities is, hence, conditioned by the way quality and food security develops, varying
the earnings of each type of actor.

Analysis of the procedures using PPL implemented by large scale retailing shows that the
question of interaction between the level of the MQS and the differentiation strategies put in place
by the private operators must be given top priority attention when public authorities reflect on the

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*Maybe, this result would not be obtained if the assumption concerning the supply inelasticity at the producers
level is modified. Indeed, we assumed that the quality standard increase does not modify the total quantity sold by
the producers. But the production cost increase could lead some producers to be bankrupt, decrease the quantities
and increase the retail prices. In this case, producers and consumers could be penalized by a too high increase of
the MQS.*
question. With this point in mind, the proposed model makes it possible to point out the following facts:

- The interest of differentiation based on a PPL all the stronger for the retailer if the level of the standard is low. The “mad cow” crisis which, at least from the consumer’s point of view, can be interpreted as revealing too low a level of standard, for this reason lead retailers to create PPL. Still, for the retailer, raising the MQS reduces the interest of these PPL as far as supply on the spot market “without constraint” is concerned. The higher the MQS, the higher the required quality level of the PPL; but at the same time, its price increases even as its place in the linear shrinks until it finally disappears when differentiation costs become too high. Stated differently, the more public authorities raise the requirements on the MQS to appease consumer fears, the more differentiation procedures carried out by retailers become difficult to implement.

- Retailer earnings are higher when the standard is high. Indeed, in this case retailers can dispense with setting up contacts and simply acquire standard secured products on the spot market forcing competition among suppliers, that is to say, among the very participants who alone support the increased production costs created by the standard (whereas in the case of a PPL the producer supports higher costs but the retailer covers part of the control costs).

- For the producers, raising the MQS is favorable, but it leads to added costs which he must cover and which, after a certain threshold, reduce their earnings. The producers maximize their profits when the MQS is such that standard product and the PPL co-exist in the shelf space. What is more, the level of standard which maximizes their profits is lower in the presence of a PPL than simply on the spot market alone.

- As far as consumers are concerned, they prefer a high level MQS, without PPL if there is no over capacity in the production regime, with PPL if over capacity is high.

- From the public authorities’ point of view, reasoning in terms of maximization of the global surplus, raising the MQS must be envisage up to an intermediate level between $k^*_0(\beta)$, the value for which the engaged producer profit is maximized, and $\bar{k}_0$, the value for which retailer profit and consumer surplus are maximized. It is, however, important to note that in a wide range of situations the existence of the PPL created by retailers makes it possible to raise the average level of quality above that accessible by the sole intervention of the public authorities.

These results are, however, based on certain hypotheses which should be reconsidered in further research. Hence, the hypothesis of producer offer inelasticity could be dropped (even if, in certain concrete cases, this hypothesis is perfectly valid) since the increase in production costs linked to raising the standard could lead to a drop in product volumes. Similarly, we have supposed that retailers had a local monopoly. Now, this hypothesis, which we justify empirically, probably produces effects which should be identified by examining the impact of real competition between retailers if one wants to do a multiproduct analysis. On this last point concerning competition between retailers, it is clear that marketing policies and communication investments play a key role, making them indissociable from restoring consumer confidence. Hence, the introduction of retailing costs would be necessary in a quantified evaluation of this type of model.
References


Appendix

This appendix presents all the calculations needed to demonstrate the propositions 1 to 7. From the benchmark situation, two other cases can happen: either both qualities are supplied, or only the high quality is supplied.

Let us assume that the integrated chain supplies both qualities. Given \( \omega_0 \) (the spot market price) and the qualities \((k_0, k_1)\), the profit maximization program \((A1)\) of the integrated chain is:

\[
\max_{(x_R,y_R)} \Pi(k_1, x_R, y_R) = (p_1(x_R, y_R) - c_1)y_R + (p_0(x_R, y_R) - \omega_0)x_R + (\omega_0 - c_0)(aG - y_R) \quad (A1)
\]

with:

\[
\begin{align*}
p_0(x_R, y_R) &= \frac{\vartheta k_0}{M_R}(M_R - x_R - y_R) \\
p_1(x_R, y_R) &= \frac{\vartheta}{M_R}(k_1 M_R - k_0 x_R - k_1 y_R)
\end{align*}
\]

First-order conditions relative to partial derivatives with respect to \( x_R \) and \( y_R \) give:

\[
\frac{\partial \Pi}{\partial x_R} = 0 \iff M_R(\vartheta k_0 - \omega_0) - 2\vartheta k_0(x_R + y_R) = 0 \quad (A3)
\]

\[
\frac{\partial \Pi}{\partial y_R} = 0 \iff M_R(\vartheta k_1 - \omega_0 - \varDelta c) - 2\vartheta k_0 x_R - 2\vartheta k_1 y_R = 0
\]

Thus, when the retailer \( R \) chooses to sell both qualities \( k_0 \) et \( k_1 \), the marketed quantities are obtained by solving \((A3)\). It gives:

\[
\begin{align*}
x_R(\omega_0, k_0, k_1) &= \frac{M_R}{2} \left[ k_0 \frac{\varDelta c}{\vartheta} - \omega_0 (k_1 - k_0) \right] = \frac{M_R}{2\vartheta} \left[ c(k_1 + k_0) - \frac{\omega_0}{k_0} \right] \\
y_R(\omega_0, k_0, k_1) &= \frac{M_R}{2} \left[ 1 - \frac{\varDelta c}{\vartheta(k_1 - k_0)} \right] = \frac{M_R}{2\vartheta} \left[ c(k_1 + k_0) \right]
\end{align*}
\]

Otherwise, each retailer \( r = 1, ..., R - 1 \) supplies only \( k_0 \) quality on his market \( M_r \) and the sold quantity is \( x_r(\omega_0) = \frac{M_r}{2} \left[ \vartheta k_0 - \omega_0 \right] \). Then, the total demand on the spot market is:

\[
D(\omega_0) = \sum_{r=1}^{R} x_r(\omega_0) = \frac{k_0 \varDelta c M_R + (k_1 - k_0)(\vartheta k_0 - \omega_0) M - \vartheta k_0 M_R}{2\vartheta k_0(k_1 - k_0)} \quad (A5)
\]

The total offer on the spot market is \( K - y_R(\omega_0, k_0, k_1) \) and the \( \omega_0 \) price is obtained as the solution to the equation \( K - y_R(\omega_0, k_0, k_1) = D(\omega_0) \). Introducing the parameter \( \xi = \frac{K}{M} \) one can obtain the equilibrium spot market price \( \omega_0^* \) (proposition 2):

\[
\omega_0^* = \vartheta k_0(1 - 2\xi) \quad (A6)
\]
Using (A4), we obtain the quantities sold by the retailer \( R \):

\[
\begin{align*}
x_R(\omega^*_0, k_0, k_1) &= \frac{M_R}{2\theta} [c(k_1 + k_0) - \overline{\theta}(1 - 2\xi)] \\
y_R(\omega^*_0, k_0, k_1) &= \frac{M_R}{2\theta} [\overline{\theta} - c(k_1 + k_0)]
\end{align*}
\]  (A7)

The solution given by (A7) is foreseeable as soon as the following conditions are fulfilled:

\[
\begin{align*}
x_R(\omega^*_0, k_0, k_1) > 0 &\iff k_1 > \frac{\overline{\theta}(1 - 2\xi)}{c} - k_0 \\
y_R(\omega^*_0, k_0, k_1) > 0 &\iff k_1 < \frac{\overline{\theta}}{c} - k_0
\end{align*}
\]  (A8)

As the quality \( k_1 \) does not influence the spot market price \( \omega^*_0 \), we directly obtain the optimal quality \( k_1^* \) using the envelop theorem:

\[
\partial \Pi[k_1, x_R(\omega_0, k_0, k_1), y_R(\omega_0, k_0, k_1)] / \partial k_1 = \partial \Pi(k_1, x_R, y_R) / \partial k_1 \bigg|_{(x_R(\omega_0, k_0, k_1), y_R(\omega_0, k_0, k_1))} = \overline{\theta} - \frac{\overline{\theta}}{M_R} y_R(\omega^*_0, k_0, k_1) - 2ck_1 = \overline{\theta} + ck_0 - 3ck_1
\]  (A9)

Thus, when \( k_1^*(k_0) \) is strictly superior to \( k_0 \), we obtain:

\[ k_1^*(k_0) = \frac{\overline{\theta} + ck_0}{3c} \]  (A10)

and:

\[ k_1^*(k_0) > k_0 \iff k_0 < \frac{\overline{\theta}}{2c} \]  (A11)

We check the equilibrium conditions given by (A8):

\[
\begin{align*}
k_1^*(k_0) > \frac{\overline{\theta}(1 - 2\xi)}{c} - k_0 &\iff k_0 > \frac{\overline{\theta}(1 - 3\xi)}{2c} \\
k_1^*(k_0) < \frac{\overline{\theta}}{c} - k_0 &\iff k_0 < \frac{\overline{\theta}}{2c}
\end{align*}
\]  (A12)

The relations (A11) and (A12) demonstrate the propositions 1 to 3. Defining \( \overline{k}_0 = \frac{\overline{\theta}}{2c}, k_0 = \frac{\overline{\theta}(1 - 2\xi)}{c} \) and \( k_0 = \frac{\overline{\theta}(1 - 3\xi)}{2c} \), we obtain:

- If \( k_0 \geq Min\{\overline{k}_0, k_0\} \), the high quality production is not set up by the integrated chain (which ends the demonstration of proposition 1).
- If \( k_0 \leq Min\{0, k_0\} \), the high quality production is set up by the integrated chain but not the low quality production.
• If \( Min\{0, \bar{k}_0\} < k_0 < Min\{\bar{k}_0, 0\} \) both qualities are produced and the high quality \( k^*_1(k_0) \) is given by (A10). The high quality can be greater than the maximum value \( \bar{k}_0 \) of the MQS \( (k^*_1(k_0) > \bar{k}_0) \) as soon as \( k_0 > \rho \) with \( \rho = \frac{6\bar{\sigma}}{c}(\frac{1}{3} - \xi) \). Then, \( \rho > \bar{k}_0 \) if and only if \( \xi < \frac{1}{4} \). Thus if \( \xi < \frac{1}{4} \), we get \( k^*_1(k_0) < \bar{k}_0 \). If \( \frac{1}{4} < \xi < \frac{1}{3} \), we get \( k^*_1(k_0) > \bar{k}_0 \) if and only if \( k_0 > \rho \) and if \( \xi > \frac{1}{3} \) we always get \( k^*_1(k_0) > \bar{k}_0 \).

Otherwise, using (A7), we get the equilibrium quantities sold by the retailer \( R \):

\[
x^*_R(k_0) = x_R(\omega^*_0, k_0, k^*_1(k_0)) = \frac{M_R[2ck_0 - (1 - 3\xi)\bar{\sigma}]}{3\bar{\sigma}}
\]

\[
y^*_R(k_0) = y_R(\omega^*_0, k_0, k^*_1(k_0)) = \frac{M_R(\bar{\sigma} - 2ck_0)}{3\bar{\sigma}}
\]

We check that \( x^*_R(k_0) + y^*_R(k_0) = \xi M_R \).

Then, it is possible to calculate the retail prices using (A2) and (A13):

\[
\begin{align*}
p_0(x^*_R(k_0), y^*_R(k_0)) &= \bar{\sigma}k_0(1 - \xi) \\
p_1(x^*_R(k_0), y^*_R(k_0)) &= \frac{1}{3}[2\bar{\sigma}k_1 + \bar{\sigma}k_0(1 - 3\xi) + 2ck_0(k_1 - k_0)]
\end{align*}
\]

Substituting \( k^*_1(k_0) \) for \( k_1 \) we obtain the equilibrium prices:

\[
\begin{align*}
p^*_0 &= \bar{\sigma}k_0(1 - \xi) \\
p^*_1 &= \bar{\sigma}k_0(1 - \xi) + \frac{2}{9c}[\bar{\sigma}^2 - c\bar{\sigma}(\bar{\sigma} + 2ck_0)]
\end{align*}
\]

We check that \( \frac{\partial p^*_1}{\partial k_0} = \frac{8c}{9}(\sigma - k_0) \) with \( \sigma = \frac{\bar{\sigma}(7 - 9\xi)}{8c} \) and that \( \sigma < \bar{k}_0 \) if and only if \( \xi > \frac{1}{3} \).

The proposition 4 is demonstrated by solving the following program:

\[
Max_{\omega_1} \Phi(\omega_1) = (\Pi_R(k_0, \omega_1) - \bar{\Pi}_R)(B(k_0, \omega_1) - \bar{\sigma})
\]

(A16)

In the case where both qualities are supplied, we have:

\[
B(k_0, \omega_1) = [\omega_1 - c_1 + (1 - \lambda)\Delta c]y^*_R + (\omega^*_0 - c_0)(\alpha G - y^*_R)
\]

\[
\Pi_R(k_0, \omega_1) = [p_1 - \omega_1 - (1 - \lambda)\Delta c]y^*_R + [p_0 - \omega_0]x^*_R
\]

(A17)

As we know that \( \frac{\partial B(k_0, \omega_1)}{\partial \omega_1} = -\frac{\partial \Pi_R(k_0, \omega_1)}{\partial \omega_1} = y_R \) and \( p^*_0 - \omega^*_0 = \bar{\sigma}k_0\xi \), we get the contractual price \( \omega^*_1 \) maximising \( \Phi(\omega_1) \):

\[
\omega^*_1 = \omega_0 + \lambda\Delta c + \frac{1}{2}(p_1 - p_0 - \Delta c)
\]

(A18)

Using (A14), we obtain:
\[
\omega_1^* = \omega_0 + \lambda \Delta c + \frac{1}{4}(k_1 - k_0)[\bar{\theta} - c(k_0 + k_1)]
\]

(A19)

Then we get the premium \( \Gamma(k_0) = \frac{1}{4}(k_1 - k_0)[\bar{\theta} - c(k_0 + k_1)] \) substituting \( k_1 \) by the value given in (A10):

\[
\Gamma(k_0) = \frac{(\bar{\theta} - 2ck_0)^2}{18c}
\]

(A20)

Substituting \( \omega_1^* \) into (A17), we obtain:

\[
B^*(k_0) = B(k_0, \omega_1^*) = MR[\beta k_0(\bar{\theta}(1 - 2\xi) - ck_0) + \frac{(\bar{\theta} - 2ck_0)^3}{54c\bar{\theta}}]
\]

\[
\Pi_R^*(k_0) = \Pi_R(k_0, \omega_1) = MR[\bar{\theta}k_0\xi^2 + \frac{(\bar{\theta} - 2ck_0)^3}{54c\bar{\theta}}]
\]

(A21)

The first-order condition \( \frac{\partial B^*(k_0)}{\partial k_0} = 0 \) has only one root, \( k_0^*(\beta) \), corresponding to a local maximum:

\[
k_0^*(\beta) = \frac{\bar{\theta}}{4c}(2 - 9\beta + 3\sqrt{9\beta^2 - 8\xi})
\]

(A22)

Then we easily check that under (H1), \( k_0^*(\beta) > k_0 \), and otherwise \( k_0^*(\beta) < \frac{k_0}{2} \). Thus the profit of the producer group is always maximum for \( k_0^*(\beta) \) \((k_0^*(\beta) \in [k_0, \min\{\bar{k}_0, \tilde{k}_0\}]\).

For the retailer \( R \), the condition \( \frac{\partial \Pi_R^*(k_0)}{\partial k_0} = 0 \) leads to the following optimal value \( k_0 = \tilde{k}_0(1 + 3\xi) \). As this value is greater than \( \min\{\bar{k}_0, \tilde{k}_0\} \), thus \( \Pi_R^*(k_0) \) is maximised for \( \min\{\bar{k}_0, \tilde{k}_0\} \).

Under (H1) et (H2), we demonstrate the proposition 5 calculating the integrated chain profit with respect to \( k_0 \).

- In the simplest case where only the low quality \( k_0 \) is supplied, we obtain from (A1):

\[
\Pi^*(k_0) = MR\bar{\theta}k_0\xi^2 + MR\beta k_0[\bar{\theta}(1 - 2\xi) - ck_0]
\]

(A23)

Thus:

\[
\frac{\partial \Pi^*(k_0)}{\partial k_0} > 0 \iff k_0 < \eta(\beta) = \frac{\bar{\theta}\xi^2 + \beta(1 - 2\xi)}{2\beta c}
\]

(A24)

Nevertheless, we easily check that under (H1), \( \eta(\beta) < \tilde{k}_0 \). Thus, the profit of the integrated chain is maximised for a value lower than \( \tilde{k}_0 \) which allows the setting up of a PPL.

- When both qualities are supplied, the profit \( \Pi^*(k_0) \) is obtained for \( k_1^*(k_0) \) with the equation \( \Pi^*(k_0) = B^*(k_0) + \Pi_R^*(k_0) \) and (A21):

\[
\Pi^*(k_0) = MR\bar{\theta}k_0\xi^2 + MR\beta k_0[\bar{\theta}(1 - 2\xi) - ck_0] + \frac{MR(\bar{\theta} - 2ck_0)^3}{27c\bar{\theta}}
\]

(A25)

Then, we obtain:
\[
\frac{9\theta}{M_R} \frac{\partial \Pi^*(k_0)}{\partial k_0} = -2(\theta - 2ck_0)^2 + 9\theta(\theta - 2ck_0) + 9\theta^2(\xi - 2\beta)
\]  
(A26)

The first-order condition \( \frac{\partial \Pi^*(k_0)}{\partial k_0} = 0 \) has only one root, \( k^*_0(\beta) \) inside the interval \([k_0, Min\{\tilde{k}_0, \bar{k}_0\}]\). The retailer’s profit is maximised for:

\[
k^*_0(\beta) = \frac{\theta}{8c}[4 - 9\beta + 3\sqrt{9\beta^2 - 16\beta + 8\xi^2}]
\]  
(A27)

To obtain the consumers surplus, we first calculate the surplus when only the quality \( k_0 \) is offered to the consumers:

\[
W_c(k_0) = M_R \int_{p_0/k_0}^\theta (\theta k_0 - p_0)f(\theta)d\theta = \frac{M_Rk_0}{2}(\theta - \frac{p_0}{k_0})^2 = \frac{M_R\theta k_0\xi^2}{2}
\]  
(A28)

In this case, the surplus \( W_c(k_0) \) is increasing in \( k_0 \).

When both qualities \( k_0 \) and \( k_1 \) are sold at the retail prices \( p_0 \) and \( p_1 \) (which depend on the quantities \( x_R \) and \( y_R \) given by (A7), the surplus of consumers buying products in the stores of \( R \), is given by:

\[
W_c(k_0, k_1, x_R, y_R) = M_R \int_{p_0/k_0}^\theta (\theta k_0 - p_0)f(\theta)d\theta + \int_{\theta}^\eta (\theta k_1 - p_1)f(\theta)d\theta
\]  
(A29)

Then we obtain the surplus with respect to the quantities and qualities offered to the consumers:

\[
W_c(k_0, k_1, x_R, y_R) = \frac{\theta}{2M_R}[k_0x^2_R + k_1y^2_R + 2k_0x_Ry_R]
\]  
(A30)

Using (A27), we get an explicit formula of \( W_c(k_0) = W_c(k_0, k^*_1(k_0), x^*_R, y^*_R) \) which is:

\[
W_c(k_0) = \frac{M_R}{16\theta}[12k_0\theta^2 - 2(\theta - 2ck_0)^3] = \frac{MR\theta k_0\xi^2}{2} + \frac{MR^2(\bar{k}_0 - k_0)^3}{8\theta}
\]  
(A31)

Then we get:

\[
[\frac{\partial W_c(k_0)}{\partial k_0} = \frac{M_R}{16\theta}[9\theta^2 - 2(\theta - 2ck_0)^2] > 0] \iff [k_0 > \tau = \frac{\theta}{2c}(1 - \frac{3\sqrt{2}\xi}{2})]
\]  
(A32)

The parameter \( \tau \) corresponds to a local minimum of \( W_c(k_0) \). Thus, the consumers surplus is maximised only for \( Min\{0, k^*_0\} \) or \( \tilde{k}_0 \).

Using (A28), it is possible to determine the consumers surplus when \( k_0 = k^*_0 \):

\[
W_c(k^*_0) = \frac{M_R\theta^2\xi^2(1 - \xi)}{4c}
\]  
(A33)

For the \( \tilde{k}_0 \) value, we have to distinguish two cases: first, if \( \tilde{k}_0 > \tilde{k}_0 \) i.e. if \( \xi \geq \frac{1}{4} \) (in this case, both qualities are supplied); second, if \( \tilde{k}_0 > \tilde{k}_0 \) i.e. if \( \xi < \frac{1}{4} \) (in this case, only the low quality is supplied). From (A28) and (A29), we obtain:
\[ W_c(k_0) = \begin{cases} 
\frac{M_R \theta^2 \xi^2 (1 - 2\xi)}{2c} & \text{si } \xi < \frac{1}{4} \\
\frac{M_R \theta^2 (1 - \xi)^2 (10\xi - 1)}{54c} & \text{si } \xi \geq \frac{1}{4}
\end{cases} \quad (A34) \]

- If \( \xi < \frac{1}{4} \), we have \( W_c(k_0) > W_c(k_0) \).
- If \( \xi \geq \frac{1}{4} \), we have \( W_c(k_0) < W_c(k_0) \) if and only if \( 47\xi^2 - 22\xi + 2 > 0 \), i.e., if \( \xi > \hat{\xi} = \frac{11 + \sqrt{24}}{47} \) and using (A31), we get \( W_c(0) = \frac{M_R \bar{\theta}^2}{64c} \).

0.34. Nevertheless, in this case \( k_0 < 0 \) (because \( \xi > \frac{1}{3} \)) and using (A31), we get \( W_c(0) = \frac{M_R \bar{\theta}^2}{64c} \).

Finally, we obtain \( W_c(k_0) > W_c(k_0) \) if and only if \( \xi \leq \hat{\xi} \simeq 0.34 \).