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To cite this version:

Y. Okabe, K. Niizeki. PHASE TRANSITION OF THE ISING MODEL ON THE TWO-DIMENSIONAL QUASICRYSTALS. Journal de Physique Colloques, 1988, 49 (C8), pp.C8-1387-C8-1388. <10.1051/jphyscol:19888636>. <jpa-00228864>

HAL Id: jpa-00228864
https://hal.archives-ouvertes.fr/jpa-00228864
Submitted on 1 Jan 1988

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PHASE TRANSITION OF THE ISING MODEL ON THE TWO-DIMENSIONAL QUASICRYSTALS

Y. Okabe and K. Niizeki
Department of Physics, Tohoku University, Sendai 980, Japan

Abstract. — The Ising models on the Penrose lattice and its dual lattice are investigated using a Monte Carlo method. The duality relation between the critical temperatures with the ferromagnetic coupling holds for a quasicrystal. The antiferromagnetic system on the dual Penrose lattice shows no long-range order due to a frustration.

1. Introduction

The spin statistics on quasicrystals has been of growing interest recently [1-4]. It is interesting to study the effects of quasiperiodicity and self-similarity on the phase transition and the critical phenomena.

In this paper we study the Ising models on the Penrose lattice and its dual lattice. The coordination number of the Penrose lattice ranges from three to seven (the average is four), whereas that of the dual Penrose lattice is four. The Penrose lattice is 'loose-packed' and there exists a complete symmetry between the systems with ferromagnetic (F) and antiferromagnetic (AF) couplings. On the other hand, the dual Penrose lattice is 'close-packed', and there exists a frustration in the case of AF coupling.

A Monte Carlo method is used to study the F and AF Ising models. We make simulations on the 'periodic' Penrose lattice with a large unit cell, which was systematically constructed by Tsunetsugu et al. [5]. We extend the fast algorithm of multispin coding by Bhanot et al. [6] to the study of quasicrystals. The calculation is fully vectorized by dividing the lattice into appropriate interpenetrating sublattices. With these techniques we can treat the system of a size up to 439,294 sites in a tractable computation time.

2. Ferromagnetic Ising model and duality relation

First we consider the F coupling case. We show the temperature dependence of the magnetization per spin, $\sqrt{\langle m^2 \rangle}$, for the Penrose lattice and the dual lattice in figure 1. The largest lattice has 439,204 sites. Simulations were made for 20,000 Monte Carlo steps per spin (MCS), and the first 4,000 MCS were excluded when talking an average. We clearly see a second-order paramagnetic-ferromagnetic phase transition for both lattices. In order to investigate the critical phenomena we also made longer-time simulations (200,000 MCS) for the temperatures near $T_c$. Using an analysis based on the finite-size scaling, we estimate the critical temperature and critical exponents. For the details, refer to other references [7].

The critical exponents of quasicrystals are found to be the same as those of two-dimensional (2d) regular lattices within the numerical errors. That is, the magnetic and thermal exponents of the regular lattices are $\gamma_H = 15/8$ and $\gamma_T = 1$, respectively.

The estimated critical temperatures for the Penrose lattice and the dual Penrose lattice are given in table I together with those for the square, the diced and Kagomé lattices.

Table I. — Critical temperature of the ferromagnetic Ising model on the 2d lattices. The coordination number $z$ is also shown.

<table>
<thead>
<tr>
<th>lattice</th>
<th>$z$ (average)</th>
<th>$T_c/J$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penrose</td>
<td>3 ~ 7 (4)</td>
<td>2.392 ± 0.004</td>
</tr>
<tr>
<td>dual Penrose</td>
<td>4</td>
<td>2.150 ± 0.006</td>
</tr>
<tr>
<td>square</td>
<td>4</td>
<td>2.269</td>
</tr>
<tr>
<td>diced</td>
<td>3, 6 (4)</td>
<td>2.406</td>
</tr>
<tr>
<td>Kagomé</td>
<td>4</td>
<td>2.143</td>
</tr>
</tbody>
</table>
the Kagomé lattices for comparison. The estimated $T_c$ of the Penrose (dual Penrose) lattice is higher (lower) than that of the square lattice, although the average coordination numbers of these three lattices are the same. A similar situation exists for the dual pair of the diced and the Kagomé lattices [8, 9]; the lattice which has heterogeneity in the coordination number has a higher $T_c$ than the homogeneous lattice with the same coordination number.

A simple relation is known between the partition functions of the F Ising models on a 2d regular lattice and its dual lattice [19]. The duality relation between the critical temperatures, $T_c$ and $T_c^*$, is given by $\sinh (2J/T_c) \sinh (2J/T_c^*) = 1$. From the estimated critical temperatures of the Penrose lattice and its dual lattice, we have $\sinh (2J/T_c) \sinh (2J/T_c^*) = 1.003 \pm 0.005$. Thus, we conclude that the duality relation holds for quasicrystals as well as regular lattices. Actually, this statement is rigorous; the proof of the duality relation in the case of a regular lattice is based only upon a local topological argument, not upon the periodicity in the structure.

3. Frustration in the antiferromagnetic Ising model

Let us consider the AF Ising model on the dual Penrose lattice. Figure 2 gives the temperature dependence of the susceptibility. At low temperatures the susceptibility diverges as $\sim 1/T$, which is the same as the case of the triangular lattice [10]. There exists no long-range order in this model due to a frustration. However, we found a growth of the fluctuation of the sublattice magnetization at low temperatures. Here we consider the decomposition into ten sublattices which are associated with obtuse rhombi with five orientations and acute rhombi with five orientations. This type of the sublattice ordering is similar to the case of the AF Ising model on the triangular lattice [10].

It is interesting to study the present system under a uniform magnetic field because this system is unstable against a small perturbation. We can pursue an analytical method for the ground state properties. The Monte Carlo study to explore a phase diagram is now in progress.