MOMENTUM DISTRIBUTIONS IN DEFORMED NUCLEI

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MOMENTUM DISTRIBUTIONS IN DEFORMED NUCLEI[1]

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Abstract. We discuss some distinguishing features of momentum distributions in deformed nuclei.

Quasielastic electron scattering provides a very powerful tool to investigate momentum distributions of protons bound in nuclei[1]. Untill now the experimental information available has been analyzed in terms of the independent particle model assuming nuclei to be spherically symmetric, in which case the spectral function takes the simple form

$$S(E,\vec{p}) = \sum_{\alpha} N_{\alpha} \quad n_{\alpha}(p)\ \delta(E-E_{\alpha}) \qquad (1)$$

where α stands for the quantum numbers of the single particle orbital (nlj); and $n_{\alpha}(p)$, E_{α}, N_{α} are, respectively, the momentum distribution (normalized to 1), the binding energy and the occupation number of said orbital.

It is well known that many nuclei adquire a deformed axially symmetric shape in their ground state. For these nuclei the ground state charge density in the intrinsic frame is given by

$$\rho(\vec{r}) = \sum \quad \rho_{\lambda}(r)\ P_{\lambda}(\theta_r)$$

similarly the momentum distribution of the single particle orbitals will have the form

$$n(\vec{p}) = \sum_{\lambda} n_{\lambda}(p)\ P_{\lambda}(\theta_p).$$

While the effects of nuclear deformation in elastic and inelastic scattering have been extensively discussed[2], those in quasi-elastic

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electron scattering have, to our knowledge, never before been considered.

For the process A(e,e'p')B the dependence of the nuclear structure can be factorized out (in plane wave impulse aproximation (PWIA)) in the spectral function[1]

$$S(E,\vec{p}) = \frac{1}{2J_A+1} \sum_B \delta(E-\varepsilon_B+\varepsilon_A^o) \sum_{\ell j} |\langle\Psi_B J_B || a_{\ell j}(P) || \Psi_A J_A\rangle|^2 \qquad (2)$$

where $\Psi_A^o J_A$, represents the wave function of the ground state of the target, with angular momentum J_A and total binding energy ε_A^o, and $\Psi_B J_B$ the wave function of the residual nucleus B, with binding energy ε_B. In this paper we discuss the form of the spectral function for axially symmetric deformed nuclei. To simplify the discussion we use the Bohr-Mottelson[3] factorization approximation and Nilsson model[4] single particle wave functions.

For axially symmetric deformed nuclei we write the wave function in the laboratory system in terms of the relative orientation of the body fixed system and the intrinsic wave function. For a given band, K_A, in the A nucleus we write (same conventions and notations as those in ref. (5) are used here),

$$|\Psi_A J_A M_A\rangle \equiv |J_A K_A M_A\rangle = \left[\frac{2J_A+1}{16\pi^2(1+\delta_{K_A,o})}\right]^{\frac{1}{2}} \left[D^{+J_A}_{K_A M_A}\Phi_{K_A} + (-1)^{J_A-K_A} D^{+J_A}_{-K_A M_A}\Phi_{\overline{K}_A}\right] \qquad (3)$$

and similarly for the states of the residual nucleus B. With this approximation the spectral function takes the form

$$S(E,\vec{p}) = \frac{1}{2J_A+1} \sum_B \delta(E-\varepsilon_B+\varepsilon_A^o) \sum_{\ell j} |\langle J_B K_B || a_{\ell j}(P) || J_A K_A\rangle|^2 \qquad (4)$$

where, in particular, if the target A is even-even, we have for the transition from the ground state ($J_A=K_A=V_A=0$) to a one quasiparticle state ($V_B=1, K_B$) in the residual nucleus

$$|\langle J_B K_B || a_{\ell j}(P) || J_A K_A\rangle|^2 = 2v^2_{K_B} |\sum_n C^{K_B}_{n\ell j} R_{n\ell j}(P)|^2 \delta_{j,J_B}$$
$$= v^2_{K_B} n_{J_B K_B}(P) \qquad (5)$$

where $v^2_{K_B}$ is the probability for finding the (deformed) single particle (s.p.) state K_B occupied in the ground state of the target nucleus, and $C^{K_B}_{n\ell j}$ are the amplitudes of the s.p. state K_B in the spherical basis

$$|K_B\rangle = \sum_{n\ell j} C_{n\ell j}^{K_B} |n\ell j\ K_B\rangle \tag{6}$$

Hence, one can see that the spectral functions in the deformed and spherical cases are quite different. First of all, the single particle binding energies E_α appearing in eq. (1) are replaced in eq. (4) by

$$\varepsilon_B - \varepsilon_A^o = E_{J_B K_B}^{rot} + E_{K_B} \tag{7}$$

where $E_{J_B K_B}^{rot}$ $(= \frac{1}{2\mathcal{J}_{K_B}}(J_B(J_B+1)-K_B^2+\delta_{K_{B,1/2}}(-1)^{J_B+1/2}\ a_{K_B}(J_B+\frac{1}{2})))$ is the rotational energy and E_{K_B} the quasiparticle energy. Since this spectrum is in general much more dense than that corresponding to the s.p. energies E_α, for a given range of missing energies E the spectral function will have many more and closer peaks than in the spherical case. With the experimental resolution available at present, the rotational levels in a given K band may not in general be resolved, several peaks thus resulting in a single broader one. On the other hand for a narrow range of missing energies in the neighbourhood of a given quasiparticle (or hole) energy the momentum distribution in the deformed case will in general be quite different. While in the spherical case the momentum distribution will be that of a single orbital n_α, in the deformed case it will be a linear combination of many orbitals.

To illustrate this point we consider in what follows the momentum distributions of the last occupied proton states in $^{28}S_i$, using the Nilsson model and neglecting pairing correlations. In the Nilsson model, with deformation parameter in the range $-.2 \leq \delta \lesssim .2$, the last occupied states in $^{28}S_i$ are the lower $K^\pi = \frac{1}{2}^+, \frac{3}{2}^+, 5/2^+$ in the N=2 shell. For $\delta=0$ (spherical case) the three states are degenerate into the $1d_{5/2}$ orbital. For $0<|\delta| \lesssim .2$ the separation in energy of these states among themselves is less than or of the order of $.1\hbar\omega_o$, while the separation in energy with respect to remaining occupied states is of the order of $1\hbar\omega_o$. Therefore if we are interested in the momentum distribution, n(p), obtained by integration of the spectral function over a narrow energy range ($\sim$ 1 MeV) around the minimal removal energy, we can restrict our attention to just these three K s.p. states. Moreover taking into account the spectrum of $^{27}A_\ell$ (6) one sees that only the lowest $J_B K_B = \frac{5}{2}\frac{5}{2}^+, \frac{1}{2}\frac{1}{2}^+$, states will contribute in this energy range. In fig. 1 we show the momentum distribution, $n(p)=n_{5/2\ 5/2^+}+n_{1/2\ 1/2}$ (in units of $(b/\sqrt{\pi})^3$) as a

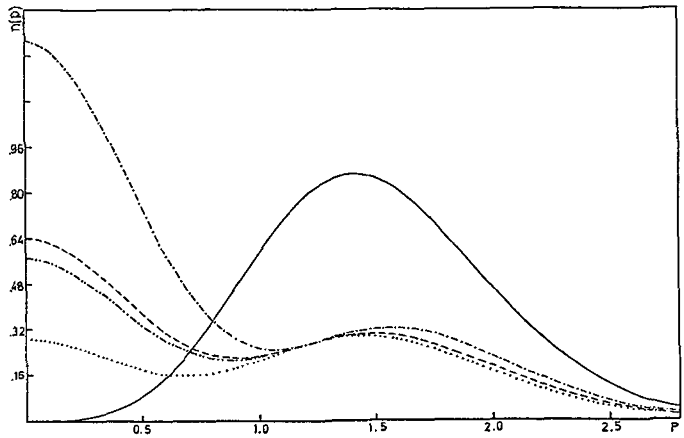

Fig. 1 - Momentum distribution for δ=-.2(-.-), δ=-.1(---), δ=0(——), δ=.1(....), δ=.2(-..-)

function of P (in units of b^{-1}), for different values of the deformation parameter. A marked dependence on deformation is observed. For δ=0 there is only a bump at $P=\sqrt{2}/b$, as $|\delta|$ increases this bump is reduced and displaced to somewhat larger p values, and other bump at p=0 appears whose strength increases as $|\delta|$ increases. In fig. 2 we compare our result for the deformed model with δ=.1 to the experimental data and fit given in ref. (7) (see also ref. (1)).

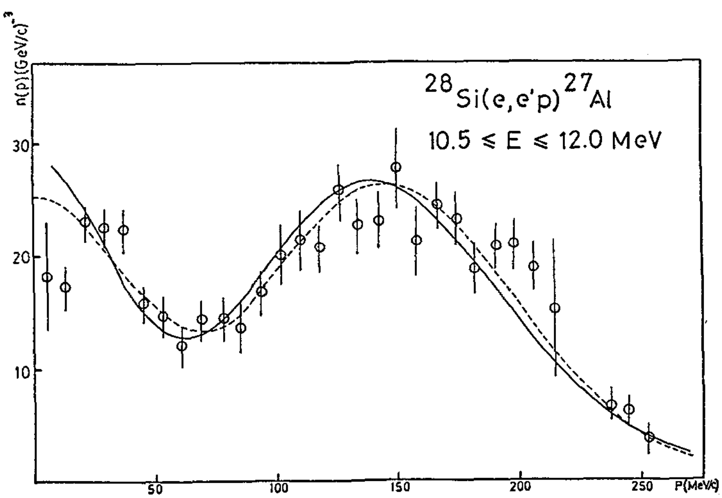

Fig. 2 - Comparison of deformed model result for δ=.1(---) to the experimental data and fit[7].

The fit in ref.(7) was obtained by using eq.(1), with $N_{1d_{5/2}} \approx 5.5$ and $N_{2s_{1/2}} = .4$, in distorted wave Born approximation (DWIA) with wave functions obtained from a Woods-Saxon potential. Our result shown by the dashed line in fig. 2 contains a renormalization factor $\eta = .5$ to take into account the overall reduction in going from PWIA to DWIA[7]. Also the b value used (b = 1.97 fm) was chosen large to take into account the displacement to lower momenta in going from PWIA to DWIA[7]. Taking into account these two considerations, the deformed model gives a very good agreement with the experimental data.

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