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NEUTRON INTERFEROMETRY AND ITS RELATION TO FUNDAMENTAL PHYSICS

H. Rauch

Atominstitut der Oesterreichischen Universitaeten, A-1020 Wien, Austria

RESUME - Différentes lois fondamentales de la mécanique quantique ont été testées à l'aide d'un interféromètre à neutrons, dans lequel des ondes de matière cohérentes complètement séparées peuvent être influencées d'une manière indépendante. La vérification explicite de la symétrie de $4\pi$ des fonctions d'onde spinorielles, de la loi de superposition des spins - tant statique que dépendante du temps - et celle de phénomènes variés liés à la cohérence des ondes de matière sont autant d'exemples caractéristiques.

ABSTRACT - Various fundamental laws of quantum mechanics have been tested by neutron interferometry, where widely separated coherent matter waves can be influenced separately. The explicit verification of the $4\pi$ symmetry of spinor wave functions, of the static and time dependent spin superposition law and of various coherence phenomena of matter waves are characteristic examples.

I - INTRODUCTION

Neutron interferometry most clearly demonstrates the wave properties of the neutron, which exist besides its particle properties mainly discussed during this workshop. The advent of perfect crystal neutron interferometry /1,2/ strongly motivated related measurements. Within this macroscopic quantum device the neutron beam becomes split into two widely separated coherent beams which can be influenced separately. Related measurements dealt with the verification of the $4\pi$-symmetry of spinor wave functions /3-5/, with the influence of gravitational forces /6-8/ and with the coherence properties of matter waves /9,10/. The results of the first period of these measurements have been summarized in the proceedings of an ILL workshop held in 1978 /11/ and more recent results have been reviewed in various papers /12-14/.

In this contribution only very recent results obtained mainly with the joint Dortmund-Grenoble-Wien interferometer set-up at the ILL will be discussed. The use of polarized neutrons and of a vibrating interferometer permit new basic experiments concerning spin-superposition /15,16/ and coherence phenomena in noninertial systems /17/.

The features of the perfect crystal interferometer are described by the dynamical diffraction theory which gives the solution of the Schrödinger-equation for a strictly periodical potential /18,19/. A monolithic cut crystal (Fig. 1) guarantees the coherence of the spacial lattice arrangement over macroscopic distances. A detailed theoretical treatment of such a system has been given in /20,21/ and only a few important results are needed here. The wave function behind the interferometer is composed of wave functions coming from beam path I and II. For a perfect crystal in ideal geometry and under the absence of any disturbing interaction the following relations exist
Any kind of interaction causes at least a phase shift \( (\psi_0^{II} \rightarrow \psi_0 \exp(i\phi)) \), which gives a pronounced modulation of the beam:

\[
I_0 \propto |\psi_0^{I} + \psi_0^{II}|^2
\]

\[
I_H \propto |\psi_H^{I} + \psi_H^{II}|^2
\]

\[
I_0 + I_H = \text{const}
\]

\[
\psi_0^{I} = \psi_0^{II} = \psi_0
\]

(1)

Fig. 1: Sketch of the perfect crystal interferometer

The phase shift for nuclear interaction reads as

\[
\chi = -(1-n)kD = -\lambda N b_c D
\]

(3)

and for magnetic interaction

\[
\alpha = \frac{\hbar}{2} \text{ with } \hbar = \frac{2\mu}{\hbar} \int B dt \approx \frac{2\mu}{\hbar} \int B ds
\]

(4)

where \( n \) is the index of refraction, \( k = 2\pi/\lambda \) is the wave number of neutrons having a wave length \( \lambda \), \( D \) is the thickness of the sample, \( N \) is the particle density, \( b_c \) the coherent scattering length, \( \tilde{\sigma} \) denotes the Pauli spin matrices, \( \mu \) the magnetic moment and \( v \) the velocity of the neutrons. \( \alpha \) formally describes the Larmor precession angle around the magnetic field \( B \) along the neutron path \( ds \).

Various imperfections of the apparatus, of the phase shifter and of the beam itself can reduce the calculated beam modulation. This may be accounted by a visibility function describing the mutual degree of coherence

\[
\gamma = \frac{2|\psi_0^{I} \psi_0^{II}|}{|\psi_0^{I}|^2 + |\psi_0^{II}|^2}
\]

(5)

which can be determined from the intensities having both beam path open and alternatively closed beam paths. In the following sections only the coherent part of the intensity will be discussed.

II - MEASUREMENT OF THE TRANSVERSAL COHERENCE

The longitudinal coherence length \( \Delta_C = \lambda^2/\delta \lambda \) is defined in the same sense than in light optics and is directly related to the wave length spread of the beam. In coherence experiments it causes a loss of contrast at high order, which has been verified experimentally /9,10/.
In perfect crystal neutron interferometry the transversal coherence length can be much larger than the longitudinal one because it is caused by the mutual and multiple interference of the wave fields excited within the perfect crystal slab. This defines the wave function within the Borrmann fan whose width is given as $\Delta^t_C = 2t \cdot \tan \Theta_B$. Within this fan the wave function has a complicated amplitude and phase structure according to the Pendellösung phenomena in dynamical diffraction theory /18,19/. This feature manifests itself in a very narrow central peak of multiple Laue rocking curves /22,23/. The shape of this central peak has been recently calculated for the triple Laue rocking curve /24/

$$I_{c.p.} \propto \frac{3\pi}{16} \left[ 2 \frac{J_0(2\Delta y)}{(2\Delta y)^2} + \frac{J_0(4\Delta y)}{(4\Delta y)^2} \right]$$

and whose half width is

$$\Delta = \frac{2.1}{A} = \frac{2.1 \cdot k \cos \Theta_B}{2\pi b_c N t}$$

which is in the order of 0.001 sec of arc. Inserting macroscopic slits (Fig. 2) a broadening of the central peak can be observed, which allows the determination of the transversal coherence lengths, which was in this case $\Delta^t_C \approx 6.5 \text{ mm} /25/$. The high angular resolution was achieved by using the small deflection at prisms within the beams. Similar conclusions about the transversal coherence length can be drawn from the results of two plate interferometry obtained by the M.I.T. group /26/.

III - SPIN SUPERPOSITION

With the use of polarized incident neutrons inversion of the polarization direction in one coherent beam and superposition of these beams at the third crystal plate permits the measurement of the quantum mechanical spin superposition law on a macroscopic scale. When nuclear ($\chi$) and magnetic ($\alpha$) phase shifts are applied simultaneously the wave function propagates as /27,28/.
\[ \psi(x, \alpha) = e^{iX} e^{-i\frac{\alpha \pi}{2}} \psi(0,0) \]  
which reduces for the situation discussed above and shown in Fig. 3 to

\[ \psi(x, \alpha) = e^{iX} e^{-i\frac{\alpha y \pi}{2}} |\uparrow\rangle = -i\sigma_y e^{iX} |\uparrow\rangle = e^{iX} |\uparrow\rangle \]  

Together with the undisturbed wave function of beam I one obtains for the wave functions behind the interferometer

\[ \psi = |\uparrow\rangle + e^{iX} |\downarrow\rangle = \psi_0(1 + U) \]  

which gives a final polarization perpendicular to both polarization states before superposition

\[ \hat{p} = \frac{\psi_0^+(1 + U)^+ \sigma (1 + U)\psi_0}{I_0} = \hat{x}\sin \chi - \hat{y}\cos \chi \]  

Fig. 3: Experimental arrangement and characteristic results of the spin superposition experiment. When a static \( \pi/2 \) spin turn device in the outgoing beam is activated to rotate the \( y \)-polarization to the \( z \)-direction which is the analyzer direction the polarization modulation according to equation (11) can be observed. An additional Larmor precession occurs when the magnetic guide field \( B_0 \) acts along the beam path. This phenomenon is closely related to the slight difference of the \( k \)-vectors of the beams behind the spin reversal system. Therefore this experiment demonstrates not only the quantum mechanical spin superposition behavior but also that waves having different \( k \)-vectors can coherently overlap when their \( k \)-vector difference exactly fits the condition for Larmor precession (\( \Delta k = \frac{\pi m B_0}{\hbar^2} \)).

IV - TIME DEPENDENT SPINOR SUPERPOSITION

When a resonance spin flipper is used instead of the static flipper shown in Fig. 3 the physical situation changes drastically. Now a time dependent interaction exists and therefore the time dependent Schrödinger equation has to be used to describe this phenomenon. In this case the spin reversal is accompanied with an exchange of a
photon having an energy $h \omega_{r} = 2|\mu|B_0$ between the neutron and the resonance flipper /30,31/. This changes the total and potential energy but not the kinetic energy and therefore not the k-vector of the neutrons. The wave function behind the resonance flipper reads as /32/

$$\psi(\chi,\omega_{r}t) = e^{i\vec{k}\cdot\vec{r}} e^{i\chi} e^{-i(\omega_{r}t)} |\uparrow>$$

and the superposition with the undisturbed wave function of beam I yields again a final polarization in the (xy)-plane which rotates in this plane in phase with the flipper field

$$\vec{P} = \begin{pmatrix} \cos (\chi - \omega_{r}t) \\ \sin (\chi - \omega_{r}t) \\ 0 \end{pmatrix}$$

This time dependent rotation can be detected by a stroboscopic registration of the neutrons synchronized with the phase of the flipper field.

Fig. 4: Experimental arrangement and characteristical results of the time dependent spinor superposition experiment /16/.

The parameters of the experiment (Fig. 4) have been: guide field $B_0 = 190$ G, resonance frequency $\omega_{r}/2\pi = 55.4$ kHz, length of the flipper coil 1 cm, interval between registered time channels 9.03 ms, widths of time channels 4.51 ms, distance to the detector 25 cm which avoids frame overlap for a neutron beam having a mean wave length $\lambda_0 = 1.835$ Å and a spread of $\delta\lambda/\lambda_0 = 0.015$.

The experimental results also agree with the theoretical predictions and demonstrate that coherence can even be preserved when a real energy exchange occurs /16/. One might argue that besides the interference pattern the beam path can be detected by observing the added or missing photon of the resonance circuit or by measuring the change of the kinetic energy of the neutron behind the guide field where the different total energy is due to the longitudinal Stern-Gerlach effect transformed to a change of the kinetic energy of the neutrons /33/. But we note that the detection of a single photon transition simultaneously with the interference pattern is forbidden by the number phase uncertainty relation

$$\Delta N \Delta \Phi \geq 1/2$$
since for the observation of the interference pattern the phase information must be at least half a period. This argumentation holds even when more elaborated formulations of the number-phase uncertainty relations are used, which show that coherent states need not necessarily minimize this product /34,35/.

One now might try to get the information about the path chosen by the neutron by measuring the change of the kinetic energy of these neutrons which passed through the resonance flipper. The related difference in the velocity of the neutrons is \( \Delta v_{rf} = \mu B_0 / m v /35/ \). It can be measured by a time-of-flight system placed at a distance \( \Delta l \) beyond the guide field region when the width of the incident beam is smaller than this quantity \( (\Delta v < \Delta v_{rf}) \). In order to accumulate the neutrons with the correct polarization phase in the correct time channels their widths have to fulfill the condition \( \Delta t > \Delta t_{/2} / \Delta v \), whereas these time channels have to be \( \Delta t < 2\pi /\omega_{rf} \) to obtain the interference pattern. This runs into conflict with the momentum-position uncertainty relation \( \Delta k \Delta l > 1/2 \).

V - PHASE ECHO SYSTEMS

At a workshop at ILL held in 1979 we demonstrated that the neutron interferometer makes phase echo systems feasible /36/. This technique is closely related to spin echo systems /37,38/. A dephasing caused by the dispersive action of a nuclear (\( x \)) and/or a magnetic (\( \alpha \)) phase shifter can be recovered by another phase shifter having an opposite action on the phase. For a proper treatment one can consider the interferometer and the phase shifter as being perfect (equations (1) and (2)) but one has to include the imperfection of the neutron beam. This can be done by describing it in the form of a wave packet

\[
\psi(\mathbf{r},t,\mathbf{k}_0) = (2\pi)^{-3/2} \int d\mathbf{k} A(\mathbf{k},\mathbf{k}_0) e^{i(\mathbf{k} \cdot \mathbf{r} - \omega(k) t)}
\]

which can be separated for the various directions. For a stationary situation and a Gaussian shaped packets centered around \( \mathbf{k}_0 \) it can be written as

\[
\psi(x,k_0) = (2\pi)^{-3/4} \delta k^{-1/2} \int d\mathbf{k} e^{ikx} e^{-[(k-k_0)/4\delta k]^2}
\]

where \( \delta k \) is the standard deviation of the \( k \)-distribution of the beam, which can be obtained independently from a measurement of the momentum distribution of the beam. According to the structure of equations (1) and (2) the result for the intensity or polarization behind the interferometer is the same when using the wave packet representations for \( \psi \) or by averaging the plane wave result by the momentum distribution function /27/

\[
I_0 = \int dk [A(k,k_0)]^2 [1 + \cos x(k) \cos \alpha(k)/2 + \hat{P}_i \sin x(k) \sin \alpha(k)/2]
\]

\[
\hat{P}_0 = \int dk [A(k,k_0)]^2 \hat{B} \sin x(k) \sin \alpha(k)/2 + \hat{B} \hat{P} \sin^2 \alpha(k)/2 + \hat{P}_i [\cos^2 \alpha(k)/2 + \cos \alpha(k)/2 \cos x(k)]/I.
\]

For the simple case \( \alpha(k) = 0 \) and unpolarized incident neutrons \( P_i = 0 \) we obtain

\[
I_0 = [1 + e^{-2\pi(D/D_k)(\delta k/k_0)}]^2 \cos 2\pi D/D_k = [1 + e^{-2\pi(D/D_k)(\delta k/k_0)}]^2 \cos x_0 = [1 + D_0 \cos 2\pi D/D_k]
\]
which represents a reduction of the visibility at high orders $m = D/D_k = D N b_c/k$ which has been verified by experiments /9,10/ (Fig. 5). Analogous to optics the longitudinal coherence length is defined from this reduction of the visibility function or from the momentum distribution function $\Delta k = \lambda^2/\delta \lambda = 2\pi/\delta k$.

The dephasing effect or the visibility function $D\phi$ only depends on the net phase shift between the two beams at the place of superposition. Therefore it may happen that the wave trains at certain positions are longitudinally wider separated than their coherence length. The complete contrast can be recovered when by means of a second phase shifter the spin echo condition $x_1 + x_2 = x_{\text{net}} = 0$ is fulfilled. Therefore a vanishing interference contrast does not necessarily indicate that the beams have lost their coherence. This becomes even more evident when a four or five plate interferometer is considered (Fig. 6). In this case no wave train overlap may exist at the third plate but an interference depending on both phase shifts can be recovered afterwards. Recent calculations on this subject /39/ yield for the situation shown in Fig. 6 the intensity behind the fifth plate

$$I_0 = [1 + 8R^2 \cos \frac{x_1}{z} \cos \frac{x_2}{z} \left(\cos \frac{x_1 + x_2}{z} + 2R^2 \cos \frac{x_1}{z} \cos \frac{x_2}{z}\right)]$$

where
\[ R = \frac{\sin^2 A \sqrt{1 + y^2}}{1 + y^2} \]

is the Laue-case perfect crystal reflectivity /18,19/.

Returning to the conventional triple Laue-case interferometer more complicated phase echo conditions can be discussed. In the case of simultaneous nuclear and magnetic phase shifts equation (17) can be written for unpolarized incident neutrons as

\[ I_0 = \left[ 1 + \frac{1}{2} D_{p+} \cos \left( \chi_0 + \frac{\alpha_0}{2} \right) + D_{p-} \cos \left( \chi_0 - \frac{\alpha_0}{2} \right) \right] \]

\[ D_{p\pm} = \exp\left[ -\left( \chi_0 \pm \frac{\alpha_0}{2} \right)^2 \left( \delta k/k_0 \right)^2 \right] \] (20)

which shows that a partial compensation of nuclear and magnetic phase shifts can be achieved and a partial recovery of the interference signal can be observed (Fig. 7).

Using polarized neutrons a complete phase echo system can be achieved even in the case of simultaneous nuclear and magnetic phase shift. A different situation exists when a complete or partial spin reversal occurs and when the magnetic guide field extends to the place of superposition (chapter III). Thus the medium remains phase dispersive behind the spin turn device and the k-vectors of the two beams become different due to the Zeman splitting \( k_{0\pm} = k_0 \left( 1 \pm \mu B_0/2E \right) \) /31/. Therefore in the spin superposition experiment of chapter III not only \(|+z>\) and \(|-z>\) states are superposed but also slightly different wave packets exist at the place of superposition

\[ \psi_0 = \psi_0^I + \psi_0^{II} = \frac{1}{2} A(k_+,k_{0+}) |+z> + \]

\[ + \frac{1}{2} A(k_-,k_{0-}) e^{i(\chi - \alpha/2)} |-z> \] (21)

An influence of the shift of the wave packets on the interference pattern appear only when the Zeman splitting becomes comparable to \( \delta k \).

Pulsed beams or a time dependent interaction are described by the time dependent Schrödinger equation and a spacial spreading of the wave packet occurs (e.g. /40/).
which is known in any time-of-flight experiment. The spreading phenomenon also appears in the case of a time dependent interaction, which causes a certain phase mark on the wave function. Additionally, energy can be exchanged between the neutron and the apparatus which can separate the packets in momentum and for ordinary space, but coherence phenomena can still be preserved and can be recovered afterwards by applying an opposite interaction. The spreading of the wave packet is analogous to time-of-flight experiments. The phase focussing conditions read as in the stationary case because the spacial shift of the wave packets \((\Delta x = (D/D_k)\lambda)\) is usually smaller than \(\delta x(t)\). Only in hypothetical cases where the condition

\[
\Delta x < \frac{\lambda^2}{\delta x(0)}
\]

is fulfilled, the wave packets become separated in space but they remain coherent. The constraint of equation (23) has been extracted from equation (22) and the uncertainty relation \(\delta x(0)\delta k \leq \hbar/2\). When it is fulfilled the individual packets have information about the beam path but no interference pattern appears. Additional aspects arise when a pulsed beam crosses a time dependent interaction region which changes its properties considerably during the passage of the wave train. In such cases the shape of the wave packet can change drastically /41/.

VI - CONCLUDING REMARKS

Various additional properties of the wave function describing the neutron of a certain beam become observable by neutron interferometry. All observed phenomena belong to the self-interference regime because the time-of-flight of the neutrons through the interferometer is orders of magnitudes smaller than the mean interval of two neutrons. Thus the statement is allowed usually that the next neutron is not yet born when a certain neutron passes through the interferometer. But all neutron have a common history within certain constraints determined by the apparatus and determining the coherence properties. The question whether the wave function describes a single neutron or a beam has to be answered such that it describes a single neutron out of a certain beam. Therefore the wave packet representation contains parameters of the particle and of the beam which are defined by the constraints of the experimental arrangement.

The experimental results manifest that every neutron has at the place of superposition information about the physical situation in both possible beam paths which uniquely determines its future. There exist situations where no interference pattern can be observed but the coherence properties are still preserved and interference phenomena can be recovered by applying phase echo methods. Even a finite change of the energy need not destroy the coherence and need not necessarily be a measuring process. Coherence is preserved as long as the wave function is known and incoherence appears as a lack of information about it. All experimental results are in complete agreement with the formulation of quantum mechanics and should provide help towards a deeper understanding of this fundamental theory.

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