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TWO DIMENSIONAL SYSTEMS IN SOLID STATE AND SURFACE PHYSICS: STRONG ELECTRIC AND MAGNETIC FIELDS EFFECTS

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Abstract - Two-dimensional condensed matter systems provide a fertile ground for the study of strong magnetic and electric field effects. We review the present status of studies in three such systems - the electron space-charge layer at the semiconductor surface in a metal-oxide-semiconductor (MOS) system, in heterojunction superlattices, and at the surface of liquid helium. In addition, we present the results of new theoretical calculations on the latter system, which we obtained in collaboration with J.A.C. Gallas.

1. Introduction - During the past decade there has been a burgeoning interest in what is commonly referred to as strong-field problems i.e., problems in which more than one field is playing a leading role in determining the dynamics. For the most part, these problems involve external magnetic and/or electric fields combined with a Coulomb field such that all fields make comparable contributions to the basic forces and energies. It is now conventional to refer to such external fields as strong magnetic and strong electric fields. Thus, we are dealing with situations for which conventional perturbation theory is no longer adequate. Instead one must resort to such methods as numerical integration, variational calculations, semi-classical methods and WKB techniques.

It is of interest to trace the history of strong-field problems particularly as textbooks invariably treat problems in which only one field is dominant, a particular favorite being the linear Zeeman effect in which a weak magnetic field perturbs the atomic Coulomb field. However, in 1939 it was recognized by Schiff and Snyder [1] that for highly-excited states of atoms, it was possible to achieve a strong-field situation. In other words, because the ratio of the Coulomb and quadratic Zeeman magnetic contributions decrease roughly \( n^{-6} \), where \( n \) is the principal quantum number of the atom, it is possible to use laboratory-produced magnetic fields to obtain magnetic and Coulomb forces of comparable strength if one studies electron in high \( n \) orbits. The theoretical analysis of Schiff and Snyder was motivated by the pioneering experimental investigations of Jenkins and Segre [3], who examined sodium and potassium absorption lines for \( n \) values in the range \( n = 10 \) to 35.

Strong-field problems in atomic physics lay dormant for a long time but in the meantime the subject was taken up by solid-state investigators [4-6]. The interest here involved electrons and excitons in semi-conductors. Because of the small effective mass and large dielectric constant associated with semi-conductors, the Coulomb field is relatively weak and the magnetic field relatively strong so that magnetic forces can be as large as Coulomb forces even for the ground state. This is now a mature but still active area of research, dealing with "three-dimensional" systems as distinct from the "two-dimensional" systems discussed below.
The next impetus for such strong-field studies came from an unexpected source viz. astrophysics. In 1967 pulsars were discovered [7] and in 1969 Gold proposed [8] what is now commonly accepted as the correct theory of such objects viz. that they consist of neutron stars with densities of the order of nuclear densities and having magnetic fields of the order of \( 10^{13} \) G i.e., about eight orders of magnitude larger than laboratory-produced fields. It was clear that, for atoms on the surface of such neutron stars, one was dealing with a strong-field problem even for electrons in the ground-state. This gave rise to a flurry of theoretical investigations by many groups [9,10]. Interest in such studies was further heightened by the first observational confirmation of the existence of large magnetic fields on dense bodies viz. the discovery by Kemp [11] in 1970 of a magnetic field \( = 10^8 \) G in the white dwarf star Grw + 70° 8247, and the subsequent discovery of comparable fields in other white dwarfs [12].

In 1969 the pendulum swung back to the realm of atomic physics when Garton and Tomkins [13] carried out their classic investigation of the absorption spectrum in the principal series of Ba I for \( n \) values as high as \( n \sim 75 \), in a magnetic field of \( 2.4 \times 10^5 \) G. The spectra obtained by these authors gave a significant stimulus to theoretical investigators since it was clear that the variational techniques heretofore applied so successfully for ground-state problems were not suitable for highly excited state situations. As a result the use of semi-classical [14] and WKB techniques [15-16] came to the fore. In addition, the scope of the experimental investigations was considerably extended by other groups, involving now the effect of internally generated electric fields [17].

In recent years strong-free effects in solid-state and surface physics have moved to center stage. The effects we refer to occur in so-called "two-dimensional systems" i.e., systems for which electronic motion is quantized in one direction, say \( z \), whereas in the other two directions the electron motion may be free (but we also examine situations in which this two-dimensional freedom may be wiped out by the application of an additional magnetic field). An excellent encyclopedic review of this general area has just appeared [18]. Here we concentrate on the salient features which involve strong magnetic and electric fields. However, under this umbrella, we also wish to include situations in which the temperature is so low that magnetic field energies are greater than thermal energies. This gives rise to many intriguing oscillatory effects in various physical quantities, such as the quantized Hall effect and the Shubnikov-de Haas oscillations of the longitudinal magnetoresistance.

In Sections 2 - 4 we will examine in detail the most commonly studied two-dimensional systems:

(i) The electron space-charge layer at the semiconductor surface in a metal-oxide-semiconductor (MOS) systems. Here the dominant field is the applied electric field which gives rise to the space-charge layer. Coulomb fields come into play via the strong many-body effects of electron-electron interactions. The application of a magnetic field tilted with respect to the semiconductor surface gives rise to combined resonance transitions [19] and the application of a magnetic field perpendicular to the surface gives rise to the now-famous quantized Hall measurements, which were pioneered by Von Klitzing [20] and form the basis of a new resistance standard. Strong magnetic field effects are also of interest in facilitating the formation of a Wigner lattice in a MOS system [21].

(ii) Alternating ultrathin layers of two semiconductors that closely match in lattice constants, referred to as a heterojunction superlattice, give rise to potential wells for electrons and holes so that the electronic quantum states are essentially two-dimensional in nature. The application of a magnetic field can give rise to a quantized Hall effect [22] and also evidence has been
presented for the existence of a highly correlated state such as a charge density wave or a Wigner lattice [23].

(iii) **Electrons at the surface of liquid helium** [24–26] constitute perhaps the "cleanest" two-dimensional system which can be studied. As distinct from the other systems, here we are dealing with a non-degenerate electron gas, for which - in particular - the complicating effects due to many-body interactions are negligible.

In this system we are dealing with a one-dimensional Coulomb potential in the presence of strong electric and magnetic fields. Here the Coulomb field is considerably smaller than hydrogenic fields because the "effective 2" is small due to the small deviation from unity of the dielectric constant of helium. We will survey the experimental and theoretical results already obtained for transitions in this system and we will also comment on the strong evidence for the existence of a Wigner lattice [27].

In addition, we will present the results of new theoretical calculations which were obtained by WKB techniques, in collaboration with J. A. C. Gallis.

2. The Metal-Oxide-Semiconductor (MOS) System — When a semiconductor surface is brought into contact with another substance - usually a metal, insulator, or another semiconductor - a space-charge layer arises on the surface. Such a surface space-charge layer is perhaps the basic key ingredient of the microelectronics industry.

It is generally agreed that the metal-insulator-semiconductor (MIS) system - first proposed by Moll [28] and by Pfann and Garrett [29] in 1959 as a voltage variable capacitor - is the most useful of the plethora of devices which exist. Since by far the most common insulator used is silicon dioxide, most studies and applications make use of the metal-oxide-semiconductor (MOS) system, as depicted in Fig. 1.

The MOS system is of interest for both technological and basic scientific reasons. First of all, the MOS system is the gate structure for most of the insulated-gate field-effect transistors (IGFET) or the metal-oxide-semiconductor field-effect-transistor (MOSFET). Secondly, the MOS system is of interest from the viewpoint of basic physics research because most of the important parameters can be varied simply by "turning a knob" [18,30], e.g., by simply changing the gate voltage (see below) from its threshold value up to \( \approx 10^6 \) V/cm, the charge carrier density can be varied continuously by about many orders of magnitude, \( \left( 10^{15} - 3 \times 10^{19} \right) \text{ cm}^{-3} \). This corresponds to a surface charge-density range of \( 10^{10} - 10^{13} \) cm\(^{-2} \) (the latter value corresponding to the maximum total induced charge beyond which the insulator breaks down). Thus, we have the possibility of studying many-body effects under controlled conditions.

Let us consider the details of a MOS system. To be specific, consider a layer of p-type Si = 10\(^{-2} \) cm thick. By oxidizing the surface of the Si at high temperatures a thin (1,000 - 5,000 Å) insulating layer of Si\(_2\)O\(_2\) is formed. By use of a metal layer of thickness 20–50 Å, called the gate, a gate voltage \( V_g \) is applied. If a positive voltage is applied at the gate then a negative charge is induced at the surface of the Si interfacing with the insulator.

The effect of a positive gate voltage \( (V_g > 0) \) is to bend the conduction and valence bands of the semiconductor downwards and to deplete the majority carriers (holes) in a region, called the depletion layer, which extends into the semiconductor about 10\(^4 \) Å. If \( V_g > 0 \) is large enough such that the conduction band edge \( E_c \) crosses over the Fermi level \( E_F \), the number of minority carriers (electrons) at the interface becomes larger than that of the majority carriers and therefore the surface is inverted. This region is called an inversion layer.
and extends [31] into the semiconductor to about 100 Å. The case in which we are concerned is that of an n-type inversion layer in p-type Si, as shown in Fig. 1.

![Image of MOS system](image1)

2.1 Electric Subbands — In 1957, Schrieffer [32] proposed that the electric field associated with an inversion layer was strong enough to produce a potential well whose width perpendicular to the interface, which we take as the $z$ direction, was small compared to the wavelengths of the carriers. This one-dimensional potential well is responsible for the quantization of the energy levels of the inversion layer electrons. These energy levels are grouped into what are called electric subbands, each corresponding to a quantized level for motion in the $z$ direction and no restrictions on the motion in the $xy$-plane, the plane parallel to the interface (see Fig. 2). This two-dimensional structure of the electron gas was confirmed experimentally in 1966 by Fowler et al. [33] using cyclotron resonance techniques.

The most extensive calculation of energy subband levels $E_n$ has been carried out by Stern and Howard [31] using a self-consistent calculation in which the surface layer charge is used in conjunction with Poisson's equation to determine the profile of the surface potential. The latter in turn is substituted into Schrodinger's equation to give the wave-function from which the surface charge can be determined — and so on. As it turns out a good estimate of the relevant parameters — with an attendant physical insight — may be obtained if we follow Stern [34] and use the triangular-potential approximation i.e., $V(z) = eFz$ for $z > 0$, with an infinite barrier for $z < 0$. The corresponding wave-function is an Airy function, from which it readily follows that [34]
and

\[ E_n = \left( \frac{\hbar^2}{2m^*_1} \right)^{1/3} \left[ \frac{3}{2} \pi eF \left( n + \frac{3}{4} \right) \right]^{2/3}, \]  

(1)

where \( F \) is the field applied normal to the semiconductor voltage by means of the gate voltage, and where \( m^*_1 = 0.9160m \) is the effective mass in the direction normal to the surface, \( m \) denoting the free electron mass. Thus, in atomic units \( (\hbar = m = e = 1) \), so that the unit of energy is 27.18 eV and the unit of length is the Bohr radius \( a_0 \), and hence the atomic unit of electric field is \( F_0 = e/a^2 = 5.142 \times 10^9 \text{ V/cm} \),

\[ E_0 = (0.381)^{-1/3}(9\pi/8)^{2/3} F^{2/3} = 3.2F^{2/3}. \]  

(3)

Thus, for a typical field of \( 10^5 \text{ V/cm} = 1.945 \times 10^{-5} F_0 \), it follows that \( E_0 \approx 63 \text{ meV} \). Similarly, the confinement length is given by

\[ z_{oo} = 2E_0/3F \approx 63a_0. \]  

(4)

Correspondingly, the subband splittings are typically 10-100 meV and so the spectroscopic range of interest is in the infra-red. These energies are, of course, much smaller than the Si band-gap energy of 1.11 eV.

In addition, we note that a typical surface density of \( 10^{12}/\text{cm}^2 \) implies a Fermi energy \( 10 \text{ meV} \) and hence, at low temperatures, the system is a degenerate Fermi gas.

Observation of interband optical transitions is the best means of obtaining information on the subband structure. However, in this system many-body contributions are very significant so that, in general, the resonant energy does not correspond to the appropriate subband energy separation. In particular, there is a depolarization (resonance screening) effect \([35]\) and a local-field correction (exciton-like effect) \([36]\). The former increases the resonance energy whereas the latter decreases it so that overall these effects tend to cancel out \([36]\). Because of the latter and cognizant of the fact that the inclusion of such effects must be treated numerically, - with a corresponding loss in physical insight - we ignore such effects from henceforth.

2.2 Combined subband-Landau level transitions - We will now turn to a consideration of the magnetic field effects. If \( B = B_z \) (i.e. if the magnetic field is applied along the same direction as the electric field) then each subband is further quantized into discrete Landau levels so that the energy becomes \( E_{n,N} = E_n + (N + 1/2)\hbar\omega_c \), where \( \omega_c = (eB/cm^*_1) \) is the cyclotron frequency.

A more interesting scenario is that of "tilted magnetic fields." In particular, Beinvogl and Koch \([19]\) investigated electrons on Si(1,0,0), in the presence of a magnetic field tilted with respect to the sample surface \( (B_x \text{ and } B_z \text{ components}) \), and observed combined resonance transitions because of a coupling of Landau levels and subband states. Surprisingly, they found that the sum of the separations for the \( \Delta N = 1 \) and \( \Delta N = -1 \) transitions is \( (0.7 - 1.6)\hbar\omega_c = 2.3\hbar\omega_c \) i.e. 15% higher than the expected result of \( 2\hbar\omega_c \).
Prior theoretical work on this problem by Ando [37] reached the conclusion that

\[ E_{n,N} = E_n + \Delta E_n + E_N \]  \hspace{1cm} (5)

\[ E_N = (N+\frac{1}{2})\hbar\omega_c, \] \hspace{1cm} (6)

\[ \Delta E_n = \frac{e^2}{2m^*}\hbar^2 y (z_{nn} - (z_{nn})^2). \] \hspace{1cm} (7)

Thus we get the characteristic energy changes of \((\hbar\omega_c)\Delta N\) corresponding to \(\Delta N\) transitions. As shown by Ando [37] this conclusion is not affected by the inclusion of many-body and other effects. The latter affects the difference in the positions of the main \((\Delta N=0)\) and a combined \((\Delta N \neq 0)\) resonance peak but does not affect the difference in the positions of two combined resonance peaks. A basic assumption made by Ando was to treat \(B_y\) as a perturbation so that its influence on the \(z\)-part of the wave function is neglected. Recently Ando [38] carried out a more detailed investigation without this restriction but it is clear (see fig. 9 of ref. 38) that discrepancies between experiment and theory still exist. Ando used an approximation based on the local-density-functional theory, taking into account 5 subbands and 20 Landau levels for the inversion layer.

2.3 Wigner Lattice - We turn now to another phenomenon in which the presence of a strong magnetic field plays a significant role (essentially by localizing the electrons in their quantized orbits). This concerns the very recent evidence for the existence of a highly correlated or crystallized ground state - a Wigner lattice [39] - in Si inversion layers in the extreme quantum limit. This evidence came from infrared measurements of the cyclotron resonance in the two-dimensional electron gas, which revealed a remarkable line narrowing and shift in the resonance frequency to higher values. As emphasized by Wilson et al. [21] "...the strong line narrowing is not a feature of the one-electron theories of Ando...". Qualitatively speaking, the formation of a lattice is facilitated by having low inversion-layer electron concentrations \(n_s\), low temperatures \(T\), and high magnetic fields \(B\). In the experiments of Wilson et al. [21], typical values used were \(n \approx 10^{11}/\text{cm}^2\), \(T = 1.2\ \degree \text{K}\), and \(B\) values of 6.15 T and 7.69 T.

Several investigators [40,41] have pointed out that a strong magnetic field \(B\) can induce Wigner crystallization. In particular, Alastuey and Jancovici [41] have made use of the Wigner - Kirkwood expansion to calculate quantum corrections to the free energy in the fluid and solid phase. In addition, they point out that, if \(B\) is sufficiently large so that the cyclotron radius \(\ell\) is less than the de Broglie wavelength \(\lambda\), then \(\ell\) replaces \(\lambda\) as a characteristic quantum length scale and the motion transverse to the magnetic field has a large effective mass associated with it. As a result, the quantum fluctuations are quenched and a degenerate system such as the electrons in the MOS inversion layer becomes more classical. This leads to the aforementioned conclusion that a strong \(B\) increases the size of the solid-phase domain.

Finally, we speculate that the existing discrepancies between theoretical expectations and the observations of Beinvogl and Koch might be due to the complete neglect of collective effects in the theoretical analysis, since
one-electron theories are the basis of all the existing theoretical investigations of transition energies. As we have previously noted [42], the values of $n_s$ and $B$ in the Wilson et al. experiments are not very different than those used by Beinvogl and Koch [19], but the latter authors used $T = 4.2$ °K. Now Wilson et al. [21] point out that the cyclotron resonance broadens and shifts to lower frequency (i.e. the Wigner lattice starts to disappear) across the temperature range of 5 - 20 °K and that the temperature dependence is apparently independent of the value of electron density, electric field, or magnetic field. The $T$ value of 4.2 °K used by Beinvogl and Koch is thus seen to be on the borderline and perhaps the system itself is borderline between the extremes of a electron gas and a Wigner lattice.

2.4 Magneto-Optical Experiments - Optical experiments can be divided naturally into two distinct classes, viz. interband and intraband effects. The interband transitions involve quantum states in two different energy bands while intraband transitions, or simply free carrier effects, involve only a single energy band. Our attention here will be confined to intraband effects and since the energy band gap [42a] of Si at 0 °K is 1.16 eV (and 1.12 eV at 300 °K), we are therefore restricted to photon energies in the range $10^{-6}$-1 eV which is the mid-infrared, far infrared and microwave regions of the electromagnetic spectrum.

Cyclotron resonance has been the most useful and most frequently used magneto-optical tool since its discovery in the early fifties [43]. If an electromagnetic wave of angular frequency $\omega$ is sent through the system, the electron will oscillate at the frequencies $\omega$ and $\omega_c$ simultaneously. When $\omega$ is adjusted to equal $\omega_c$, resonant absorption occurs and the electron will move in an orbit of increasing radius until it collides after a time $\tau$, called the collision time, with the lattice. This method has been used extensively to obtain information on the effective mass of the inversion layer electrons in MOS systems and particularly its dependence on $T$, $B$, $\omega$ and $n_s$ [18,44].

Another useful tool, which both complements and supplements the cyclotron resonance work is the observation of Faraday rotation, first investigated in 1845 by Faraday [45]. It refers to the rotation of the polarization direction of a linearly polarized electromagnetic wave propagating parallel to the magnetic field direction. Faraday rotation measurements have only recently been carried on the MOS system [46] and they should provide information not only on effective mass values but also on collision times [47], as well as possibly throwing some light on the so-called cyclotron resonance dilemma [48]. In addition, we have recently pointed out the importance of including multiple reflection effects in the inversion layer [49] in the interpretation of Faraday rotation and ellipticity measurements.

2.5 Quantized Hall Effect - Condensed matter systems are often considered to be "dirty" in the sense that their complexity often mitigates against obtaining a very accurate experimental number. A counter example of course is the determination of e/h from the ac Josephson effect. A further example has recently emerged from measurements of the quantized Hall effect [20]. This provides a new method for a precision determination of the fine-structure constant $\alpha$ or, alternatively, if $\alpha$ is assumed to be known from some other experiments (as, for example, measurements of the ac Josephson effect and the gyromagnetic ratio), then we are provided with an improved standard resistance.

In the pioneer quantized Hall experiment, von Klitzing et al. applied a magnetic field of 150 kG normal to the oxide-silicon surface of the MOS system (z direction), at a low temperature of 1.5 °K. Next, an electric field was applied in the x direction giving rise to a longitudinal current $j_x = \sigma_{xx} F_x$ and a Hall current $j_y = \sigma_{yx} F_x$, where $\sigma_{ij}$ denotes the conductivity tensor. Then the gate voltage was varied from 0 to 25 V resulting in an increase in $n_s$ with a
resultant approximate linear increase in the Fermi energy $\approx 6 \, \text{meV}/10^{12}$ electrons $\text{cm}^{-2}$, relative to the lowest electric subband $E_0$. The remarkable result found was that the Hall resistance, $R_H$, say, displayed quantized steps as a function of $V$ (see Fig. 3) and at each step $R_H$ is given in terms of the fundamental constants to an accuracy of about 1 ppm. Specifically,

$$R_H = \frac{h}{e^2 n} \quad (n = 1, 2, \ldots), \quad (8)$$

where $n$ is the number of Landau levels lying below the Fermi energy.

We turn now to an explanation of this result. The key requirements are low temperature and strong magnetic field. Since a temperature of 1.5 K implies a $kT$ value of only $0.13 \, \text{meV}$, the separation between $E_0$ and $E_1$ is such that for $n_s < 6 \times 10^{12} \, \text{cm}^{-2}$ only the $E_0$ level is occupied (electric quantum limit). In the presence of the magnetic field $E_o$ separates into Landau levels and for a B field value of 150 kG and an effective mass of $0.19 \, M_0$ (appropriate to Si) the Landau level separations $\hbar \omega_c$ equals $9.13 \, \text{meV}$.

In general, when $kT < \hbar \omega_c$ one can observe oscillatory behavior, periodic in $B^{-1}$, in various physical quantities as, for example, the Shubnikov-de Haas oscillations in the longitudinal conductivity $\sigma_{xx}$. For the inversion layer, these oscillations can be seen not only in the usual manner by variation of the magnetic field but also by varying the surface concentration, the latter being achieved simply by changing the gate voltage. It turns out that the period is independent of the effective mass but increases when electrons populate the higher subbands, thus providing a method for the determination of the electron density in each subband. It is also of interest to note that these oscillations do not appear when $B$ is applied parallel to the surface, hence providing a
strong confirmation for the two-dimensional nature of the electronic space-charge system.

The gate voltage is now adjusted to set \( E_F \) between two Landau levels so that all the Landau levels below \( E_F \) are completely occupied and those above are empty. As a result \( \sigma_{yx} \) (and hence \( j_x \)) essentially drops to zero because the electrons have no place to scatter into due to the large gap of \( \hbar \omega_c \) between the filled and empty Landau levels. Furthermore, \( \sigma_{yx} = n_s e^2 c/\hbar \) and \( n_s = n g_n \) where \( g_n \) is the degeneracy of the \( n^{th} \) Landau level. However, since \( g_n = eB/\hbar c \) it follows that \( \sigma_{yx} = ne^2/\hbar \) i.e. the Hall conductivity (and hence the Hall resistance) does not depend on any of the MOS parameters but only on fundamental physical constants.

3. Heterojunction Superlattices - Alternating ultrathin layers of two semiconductors that closely match in lattice constants are referred to as a heterojunction superlattice. Studies so far have concentrated on two systems, one being made of GaAs-AlAs or GaAs-Ga\(_{1-x}\)Al\(_x\)As [50] and the other is of InAs-GaSb or In\(_{1-x}\)Ga\(_x\)As-Ga\(_{1-y}\)Sb\(_y\)As [51]. Most investigations have been carried out on the former system and for this reason or remarks will be confined to it. We note that Ga\(_{1-x}\)Al\(_x\)As is a single crystal material in which Al atoms have replaced a fraction \( x \) of the gallium atoms. This three-atom system possesses a higher energy gap and a smaller index of refraction than GaAs.

One can regard the superlattice as essentially "...nothing more than a string of back-to-back single interfaces..." [50]. A selectively doped GaAs - Al Ga As system can result in an energy level diagram depicted in Fig. 4. This comes about from the demand that the Fermi level be constant across the system with the result that electrons move across the interface. This in turn creates a strong internal elective field which causes a significant bending of the bands at the interface. As a result, we see that there is a strong confinement of electrons in a one-dimensional potential well perpendicular to the surface. This region in Fig. 4 is marked "2 DEG" referring to the two-dimensional electron gas, since electronic motion along the two-dimensional interface is unhindered.

![Fig. 4: Energy level diagram for a GaAs - AlGaAs superlattice showing the potential well which gives rise to the two-dimensional electron gas (2 DEG).](image-url)
Thus we have a situation very similar to that which results in an inversion layer in the MOS system, with a corresponding range of studies. However, the density of two-electrons electrons in Ga As cannot be varied easily (in contrast to the MOS system) since the electrons arise from donor impurities placed inside the $\text{Al}_x\text{Ga}_{1-x}\text{As}$ crystal. On the other hand, one advantage of the superlattice systems is the fact that we are dealing with relatively low transverse effective masses viz. $0.07 \text{ m}$ for Ga As and $0.023 \text{ m}$ for In As (to be compared to $0.1905 \text{ m}$ for the MOS system). As a result, strong field regimes start at lower field values. To cite two examples, we note that $\omega_c$ is proportional to $m^{-\frac{1}{2}}$ and that energy levels in an electric field are proportional to $m^{-2}$ (see footnote in ref. 52).

In particular, since the effective mass in Ga As is only about 1/3 of its value in Si, it was possible to carry out quantized Hall measurements in the Ga As - Al Ga As system in a field of 80 kG and at a temperature of 4 K [22] which, of course, makes this heterojunction superlattice a prime candidate as a potential standard of absolute resistance in laboratories throughout the world, particularly since it is significantly cheaper to produce magnetic fields of this value vis-a-vis the 150 kG fields required in MOS systems.

As emphasized by Tsui et al. [22], the quantum regime can be reached with even smaller B values in narrow-gap semiconductors, pointing out as an example the relatively low value of 10 kG required to obtain the quantized Hall effect using the two-dimensional electron system in In Sb. In fact, Tsui et al. [22] have determined $\alpha^{-1}$ to 0.17 ppm, in excellent agreement with the 0.11 ppm value obtained from the gyromagnetic ratio of the proton combined with the Josephson effect.

Finally, we would like to mention that magnetotransport studies of two-dimensional electrons in thin Ga As - Al$_x$Ga$_{1-x}$As multilayers give evidence for the formation of either a Wigner lattice or a charge-density wave [23]. Magnetic fields as high as 210 kG were used and temperatures from 2.4 to 4.2 K.

4. Electrons at the surface of liquid Helium - The study of image-potential-induced surface states for electrons outside of liquid helium has attracted much interest in recent years [24-26]. This system is of interest due to the fact that to a good approximation the electrons form a two-dimensional electron gas and also because of the very large mobility of the electrons along the surface. In addition, it provides information on the nature of the helium liquid-vapor interface and, most significantly, it appears to be the first system in which the existence of a Wigner lattice [40] has been experimentally verified [27].

These very interesting properties originate from the circumstance of the dielectric constant $\varepsilon$ of helium being very slightly greater than 1 ($\varepsilon = 1.0572$), as was first pointed out independently by Cole and Cohen [24] and by Shikin[25]. In the case of a typical dielectric the electrons are attracted to the surface by an electrostatic image force and then become bound to specific atoms or defects. However, in the case of liquid helium, this force, which is proportional to $\varepsilon^{-1}$, is very weak. Also, there is a potential barrier at the surface of 1 eV which prevents the electron penetrating into the surface. As a result the electrons are trapped in a one-dimensional well, giving rise to quantized states in this direction ($x$ say). The potential is given by

$$V(x) = -ze^2/x, \quad (9)$$

where [53] $Z = k_0 (\varepsilon-1)/(\varepsilon+1) = 6.951 \times 10^{-3}$. If we now assume that the barrier is infinite at the origin we are led to a "hydrogenic" spectrum for electrons trapped at the surface. Some interesting characteristics of this spectrum were discussed by Grimes and Brown [54]. In particular, the ground state energy is
0.6 meV and the associated Bohr radius is 114 Å. The theoretical transition energies from the $n = 1$ ground state to the $n = 2$ and $3$ excited states are $119.3$ GHz, and $141.3$ GHz respectively, whereas the experimental values \cite{55} are $125.9$ ±$0.2$ and $148.6$ ±$0.3$ GHz. Also, because the splittings are $5.7$ K and $6.8$ K in temperature units, only the ground state is significantly populated at the temperature $1$ K at which the experiments are performed.

To improve agreement between theory and experiment, and also to provide a more realistic calculation, Cole and Cohen \cite{24} argued that the $x^{-1}$ potential should be truncated near the interface and in fact they assumed a constant potential $-Ze^2/b$ for $0 \leq x \leq b$ (see Fig. 5a), obtaining agreement with experiment for a $1$ eV repulsive barrier and a $b$ value $\approx 10\AA$. However, such a large value of $b$ is inconsistent with the premise of the model that Eq. (9) is reasonable for values of $x$ greater than the inter-atomic spacing in the liquid, which is $3.6$ Å. Next, Grimes et al. \cite{55} proposed a potential (see fig. 5b) $V(x) = 1.0$ eV for $x \leq 0$ and $v(x) = -Ze^2/(x+\beta)$ for $x > 0$, which is equivalent to assuming that the image potential has its origin a small distance inside the liquid. They found agreement with experiment for $\beta = 1.04$ Å. Other calculations \cite{56} have also taken account of the fact that the helium density does not change abruptly from the bulk liquid value to the vapor value.

These studies took on a new dimension by the application of an electric field $F$. As emphasized by Grimes et al. \cite{55}, since the splitting between energy levels is in the mm-wave region it is convenient from an experimental point of view to work with a fixed frequency and to vary the energy separations by means of the electric field. The electric field serves to press the electrons toward the surface with a consequent compression in the wave functions. Grimes et al. analyzed their experimental results using a variational calculation and employing the adjustable parameter $\beta$. More recently, Lambert and Richards \cite{57}, using far-infrared laser spectroscopic techniques, observed transitions at various electric field on the same system. To model their observations they used the one-dimensional Hamiltonian

\begin{equation}
H = \frac{p^2}{2m} - \frac{Ze^2}{x} + eFx,
\end{equation}

where $p$ is the momentum in the $x$ direction, $Z = 6.95 \times 10^{-3}$ and $F$ is the applied electric field. By numerically integrating eq. (10) they were able to show that this model can accurately reproduce the observations.

In a previous work \cite{58} we have studied the dependence of the transition frequencies as a function of the electric field, in the "hydrogenic" model, using two different approaches: a semiclassical \cite{58,59} and a WKB one \cite{58}. The WKB approach is particularly simple to apply since the corresponding quantization rule can be reduced to standard elliptic integrals which, in their turn, can be evaluated even with programmable pocket calculators. The first-order WKB quantization rule corresponding to the Hamiltonian in Eq. (10) is given by (using atomic units)

\begin{equation}
I = (n+\frac{1}{2})\pi,
\end{equation}

where

\begin{equation}
I = \int_0^a \left\{ 2(E + \frac{Ze^2}{x} - Fx) \right\}^{1/2} \, dx
\end{equation}

and where $E$ is the energy. The turning point $a$ is the positive root of the radical of Eq. (12), namely
Fig. 5: Model of the potential seen by electrons at the surface of liquid helium, as proposed by (a) Cole and Cohen (24) and (b) Grimes et al. (55).

Eq. (12) may be then rewritten as

\[-x^2 + \frac{E}{F}x + \frac{Z}{F} = (a-x)(x-b) = 0 \quad . \tag{13}\]

\[I = \left(2F\right)^{1/2} \int_0^a \left\{ \frac{1}{x} \left(-x^2 + \frac{E}{F}x + \frac{Z}{F}\right) \right\}^{1/2} dx\]

\[= \left(2F\right)^{1/2} \int_0^a \left\{ \left(a-x\right)\left(x-b\right) \right\}^{1/2} \frac{x}{\lambda} \right\} \frac{dx}{\lambda} \]

\[= \frac{2}{3} \left[ 2F(a-b) \right]^{1/2} \left\{ (a+b)E(k) - bK(k) \right\} \tag{14}\]

where $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kind, respectively, and $k^2 = a/(a-b)$. A closely related integral is $dE/dn$, the energy spacing between the energy levels. By differentiating Eq. (11) with respect to $n$ and integrating one obtains

\[
\left( \frac{dE}{dn} \right)^{-1} = \frac{1}{\pi} \left[ 2F(a-b) \right]^{1/2} \left\{ (a+b)E(k) + bK(k) \right\} \tag{15}\]
Fig. 6: Plot of transition frequencies vs electric field for the bound electron states at the surface of liquid helium. The crosses are the measured data points of Grimes et al. (55) while the solid curves are the result of a WKB calculation based on the model Hamiltonian given by Eq. (10) in the text.

Fig. 7: Plot of transition frequencies vs electric field for the bound electron states at the surface of liquid helium, corresponding to the first 9 transitions from the ground state recently observed by Lambert and Richards [57]. The curves are the result of a WKB calculation based on the model Hamiltonian given by Eq. (10) in the text. The bottom curve is simply a blow-up of a portion of the top curve.
The elliptic integrals $K(k)$ and $E(k)$ needed in these expressions can be computed very efficiently. From the several algorithms to this end available in the literature [60] we call attention to the ones by Carlson [61], which, as previously mentioned, may easily be implemented in programmable pocket calculators.

Using Eqs. (11), (14) and (15) we calculated the field dependence of the frequency for the $1 \to 2$ and $1 \to 3$ transitions. These results are given in Fig. 6 along with the experimental values of Grimes et al. [55] as read from their Fig. 2. As can be seen from Fig. 6, the WKB results are in good agreement with the variational calculation of Grimes et al. As in the calculation of Grimes et al., the agreement with the measured values degenerates as the field increases. Since Fig. 6 was obtained in the "hydrogenic" model, for which the calculated values are slightly smaller than the measured ones, both curves were normalized to agree at the first measured point.

In Fig. 7 we give the field dependence of the frequency for the first 9 transitions which were recently observed by Lambert and Richards [57]. Our curves were obtained from the WKB approximation, using Eq. (14), and Fig. 7b should be compared with the corresponding curves in Fig. 5 of Ref. 57, obtained through a numerical integration of Eq. (10). The overall agreement is very good if one notes that Fig. 7b has not been rescaled to take the "non-hydrogenic" character into account.

Actually, since we are using an infinite barrier (which is a good approximation since the actual barrier of 1 eV is more than $10^3$ times the typical binding energies), then using an argument due to Langer [62] (see also Miller and Good [63] and Adams and Miller [64]) Eq. (12) for $I$ must be modified (by adding a $-1/8x^2$ term under the radical) for use in Eq. (11) so that in the limit $F = 0$ we obtain the correct quantization rule $E_n \sim n^{-2}$. However, it turns out that this term does not make a significant contribution to the energy spacings except for small $F$ values [65].

We would also like to point out that the WKB approximation provides an easy way to study the field dependence of the transition frequencies for alternative models, e.g. the "8-shifted" model of ref. (55). These topics, however, will be discussed elsewhere [65].

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