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THE KAPITZA RESISTANCE
AND HEAT TRANSFER AT LOW TEMPERATURES

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Résumé. — Dans cet article nous rappelons quelques problèmes importants de transfert de chaleur à des températures voisines du Kelvin. Nous traitons plus particulièrement de la résistance de contact thermique entre solides et de la résistance thermique entre solides et hélio superfluid (résistance de Kapitza). Nous faisons une comparaison entre les résultats expérimentaux et les modèles théoriques existants, en particulier le modèle de rayonnement de phonons, limite supérieure de transfert de chaleur.

Abstract. — A brief review is given of some important heat transfer problems at 1 °K and below. Included are discussions of the thermal contact resistance between solid-solid and solid-liquid helium II boundaries (Kapitza resistance). The results are compared to available theoretical models, with particular interest placed on the upper limit, the phonon radiation model.

I. Introduction. — There have been three comprehensive reviews of the Kapitza resistance during the last year or so [1-3]. As many of the theoretical and experimental aspects have been treated at length in these sources, what I propose to do here is to try to view the problem in a slightly more general sense (including for example solid — solid resistances) and to give a personal view point on the likely heat transfer mechanisms in the various cases considered.

At the same time I have been unable to resist the usual temptation to dwell on some aspects of the subject which particularly interest me. I have limited the discussion to the high temperature range (1-2 °K) the work below 1 °K being found almost entirely in the publications of Wheatley and collaborators and more recently of Zinovyeva (this colloque).

We start with the acoustic mismatch formulation of the thermal contact resistance between insulating bodies first proposed by Khalatnikov [4] for a solid in contact with liquid helium II. This was later generalised and presented in a simpler form by Little [5] to cover the case of any two solids in contact. We then compare this result with selected experimental data on solid — solid contacts, considering particularly the case of metal films on insulating substrates and the metal — superconductor contact. Some technical aspects are mentioned briefly.

In section III we compare the acoustic mismatch predictions with the experimental conductances of solid-helium II interfaces. The agreement is not good, the observed conductance being generally very much greater than that calculated. We then consider ways in which the agreement might be improved (e. g. dense helium layer, limit of very low temperatures) and other possible contributions to the heat transfer (e. g. the electrons in a metal). The conclusion of this section is that the acoustic mismatch model doesn’t seem to fit the facts and that it should be regarded as a lower limit for the conductance of a perfect surface.

The failure of the acoustic mismatch model leads us to examine the upper limit for phonon transfer in section IV. This limit has often been considered for solid — solid contacts, but the first detailed treatment for the helium problem was given recently by Snyder [3]. It has been noted frequently that the acoustic mismatch model fails worst for solids with a high \( \theta_0 \). The interesting observation made by Snyder is that very good agreement is observed for this case with the phonon radiation limit, the maximum possible rate of energy transfer (i. e. 100 % phonon transmission). A more explicit comparison of the data with the two limits than that made by Snyder is made in this section.

I conclude in section V with a subjective assessment of the likely lines along which the problem may be resolved.

II. The acoustic mismatch model. — In this model, following Little [5] we consider the heat transfer between insulators connected by a perfect interface. Application of the Knudsen cosine law and the Planck distribution gives in the Debye limit for the
net heat flow between one body at temperature $T_1$ and the other at $T_2 = T_1 - \Delta T$

$$\dot{Q} = 2 \times 10^{10} \frac{I T_1^3 \Delta T}{c_1^2} \text{ watt units} \quad (1)$$

where

$$\Delta T = h_{am} \Delta T \quad \text{for} \quad \Delta T \ll T \quad (2)$$

and the transmission coefficient

$$\tau_1(\theta_1) = \tau_2(\theta_2) = \frac{4 \rho_2 c_2 \cos \theta_2}{\rho_1 c_1 \cos \theta_1} = \frac{4 \rho_1 c_1}{\rho_2 c_2} \left( \frac{\rho_2 c_2}{\rho_1 c_1} + \cos \theta_2 \right)^2 \quad (3)$$

for normal incidence and $\rho_2 c_2 \geq \rho_1 c_1$.

Thus the conductance is determined by considering all phonons and incidence angles to determine the total energy flux arriving at the interface from each side. This quantity is then multiplied by a transmission coefficient, determined classically by the mismatch of the acoustic impedances of the two media. There is an obvious analogy to electrical circuitry equations, and indeed Brekhovskikh [6] investigated the electrical and acoustic transmission coefficient for a number of cases using a simple general approach. This has been applied to the Kapitza problem (e.g. Andreev [7]) yielding a result similar to that given here.

The simple result obtained by Little quoted above was obtained for media with the rigidity modulus $\mu = 0$ so that only longitudinal modes were considered. Extension of the model to include transverse modes is simple in principle by replacing

$$\Gamma \rightarrow \frac{\Gamma_1}{c_1^2} + \frac{2 \Gamma_2}{c_2^2} \quad (5)$$

but the calculation of $\Gamma_1$ becomes more complicated.

Machine calculations of $\Gamma_1$, $\Gamma_2$ and $\Gamma_3$ are given by Little for various values of $\rho$ and $c$. $\Gamma_1$ varies from 0 to 0.5 and is nearer the upper limit for a contact between ordinary solids, which are generally quite well matched. Going to the limit of perfect transmission ($\alpha = 1$) with $\rho_1 = \rho_2$ and $c_1 = c_2$, Little showed that (1) reduces to the ordinary formula $K = \frac{3}{2} C_r c_1$ of a homogeneous solid, as it must.

We now turn to a comparison of Little's formulation with the experimental data. A list of results (by no means complete) suitable for comparison is given in table I, together with the predictions of Little's model calculated by the authors in question. With the exception of copper-solid helium, the black body conductances ($\alpha = 1$) are slightly larger than $h_{am}$ and have not been given so as to avoid confusion due to the already large spread in the data, and several large uncertainties in the calculated values. In some cases $h_{obs} > h_{am}$, so if anything, the black body model would do slightly better, a conclusion reinforced by the observations of Von Gutfield et al. [8] given in table II, for the thermal time constants of bolometric films on insulating substrates. For such applications, and for low temperature joints between such materials as Cu, Sn, Pb, In, etc, in general, it would seem to be more simple and more accurate to calculate conductances of perfect junctions using the blackbody limit than to use anything more complicated.

Cu-solid helium is an interesting special case. Experimentally the contact should be intimate and this, coupled with the high thermal conductivity of the materials in question, means that the results should be more reliable than those for the other more ordinary junctions given in table I. From a theoretical standpoint the mismatch is much greater than usual so that a useful comparison with the theory is possible. We have given the Kapitza (acoustic mismatch) value for Cu-solid He as a lower limit modifying the Khalatnikov formula (6) for the appropriate density and sound velocity of the helium, while $h_{am}$ obtained

---

Table I

<table>
<thead>
<tr>
<th>Solids</th>
<th>$(\Theta_0)_1$</th>
<th>$(\Theta_0)_2$</th>
<th>$\dot{Q}_{obs}$</th>
<th>$\dot{Q}_{am}$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)-(2)</td>
<td>°K</td>
<td>°K</td>
<td>watt units</td>
<td>watt units</td>
<td></td>
</tr>
<tr>
<td>Cu-Pb (Sandwich)</td>
<td>343</td>
<td>100</td>
<td>1</td>
<td>1.4-2.9</td>
<td>CC(64) : 9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cu-Pb</td>
<td>343</td>
<td>100</td>
<td>1.5</td>
<td>2.4</td>
<td>BD(66) : 10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cu-Sn</td>
<td>343</td>
<td>195</td>
<td>5</td>
<td>3.1</td>
<td>BD(66) : 10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Al$_2$O$_3$-In</td>
<td>1000</td>
<td>111</td>
<td>0.25</td>
<td>0.18-0.3</td>
<td>ND(64) : 11</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cu-solid helium</td>
<td>343</td>
<td>43.5</td>
<td>0.25</td>
<td>0.05</td>
<td>M-D(64) : 12</td>
</tr>
</tbody>
</table>
using Snyder’s figures gives $h^{pr} \sim 3$ as the upper limit. It is perhaps significant that neither limit is observed, a point we shall return to later.

As most of the solids considered so far have been metals we must now consider the effect of the conduction electrons. For a metal-insulator or metal-superconductor joint at $T \ll T_c$ there is no convincing evidence of the existence of the transmission of an electronic heat current. There is the possibility of an electron-surface wave interaction in some cases. We show in section III that this process seems to be negligible in the metal-helium II problem and in view of the reasonable agreement of the results considered thus far with the black body or acoustic mismatch limits, we don’t feel this question worth pursuing here. However, one interesting exception was noted by Challis and the author [9] for a Cu-Pb sandwich. The behaviour as a function of magnetic field is shown in figure 1. The results are consistent with severe straining of the Pb platelet (presumably by differential contraction) and hence removal of the lattice component and heat transport by the electronic component seems to be the only reasonable explanation. Such a device obviously would be useful as a thermal switch ($K_{el}/K_{el} > K_{sol}/K_{sol}$) and indeed the performance indicated by figure 1 is very promising. At sufficiently low temperatures the lattice should again take over, reducing the switching ratio as a function of $T$, but still leaving us with a better switch than that provided by a simple lead wire.

Finally, there is also in principle a thermal resistance between normal metals in contact due to the difference in electronic parameters. The data, some of which is available from the normal state data in the references cited in table I, indicates that this resistance is virtually insignificant and in one case (Anderson et al., reference 13) for Cu-Cu joints was almost unmeasurable. For technical purposes the summation of appropriate bulk thermal resistances seems generally quite adequate for this type of contact.

III. The Kapitza resistance between solids and helium II. — Khalatnikov (1952) has given a detailed calculation of the heat flow between solids and liquid helium II based essentially on the acoustic mismatch idea. The result, including the contribution by surface waves, is:

$$h_{el} = \frac{16 \pi^3 k_n^2 p c F(c_l/c_s) T^3}{15 k^3 \rho_s c_s^2} \tag{6}$$

$$= \frac{1.37 \times 10^{14} F T^3}{\rho_s c_s^2} \text{ watt units} \tag{7}$$

where $\rho$, $c_s$, $c_l$, the solid density and sound velocity and $\rho_s$, $c_s$, $c_l$, the solid density and longitudinal and transverse sound velocities and $F(c_l/c_s)$ is a numerical factor $\approx 1.5$ which is very weakly dependent on the solid considered. This is again an interesting special case as the acoustic mismatch is usually extremely large, the liquid temperature is quite constant (we thus avoid temperature extrapolation problems in the liquid) and the contact is very intimate for small heat flows. Detailed comparisons between the Khalatnikov model and experiment have been given by many

---

**Table II**

*Observed and calculated thermal time constants of bolometric films on insulating substrates (after reference 8)*

<table>
<thead>
<tr>
<th>Film</th>
<th>Thickness (Å)</th>
<th>Substrate</th>
<th>$\tau$ (Exptl., normalized to 1 000 Å) (ns)</th>
<th>$\tau_{bb}$ (ns)</th>
<th>$\tau_{am}$ (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>In (94 %)-Sn (6 %)</td>
<td>910</td>
<td>z-quartz</td>
<td>9 ± 2</td>
<td>7.5</td>
<td>8-10</td>
</tr>
<tr>
<td>In (94 %)-Sn (6 %)</td>
<td>1 000</td>
<td>x-sapphire</td>
<td>16 ± 2</td>
<td>18</td>
<td>22-29</td>
</tr>
<tr>
<td>In (94 %)-Sn (6 %)</td>
<td>1 200</td>
<td>z-sapphire</td>
<td>17 ± 2</td>
<td>18</td>
<td>22-29</td>
</tr>
<tr>
<td>In (94 %)-Sn (6 %)</td>
<td>1 800</td>
<td>z-sapphire</td>
<td>14 ± 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pb (98.5 %)-Bi (1.5 %)</td>
<td>1 400</td>
<td>z-quartz</td>
<td>33 ± 4</td>
<td>12</td>
<td>12-14</td>
</tr>
<tr>
<td>Pb (98.5 %)-Bi (1.5 %)</td>
<td>2 000</td>
<td>x-sapphire</td>
<td>50 ± 4</td>
<td>29</td>
<td>33-39</td>
</tr>
</tbody>
</table>

---

**Fig. 1.** — The thermal resistance of a Cu-Pb-Cu sandwich as a function of field [9] showing a limiting switching ratio of 80 at 1.35 K. $K_{el}$ in the bulk lead. Our experimental knowledge of the physical state of the Pb and of the joint at 1 K does not permit a more detailed analysis, but this...
authors (see e.g. [1-3]) and for the moment we can sum up the present state of affairs by making the following remarks:

(a) The observed conductance is usually an order of magnitude or more greater than the Khalatnikov value. A rule of thumb experimental value for a typical solid is $h_K = 0.1 T^{+3}$ watt units. A comparison between theory and experiment at 1.9 °K is given in table III.

### Table III

<table>
<thead>
<tr>
<th>Solid</th>
<th>$\theta_0$</th>
<th>$h_K^e$</th>
<th>$h_K^{ob}$</th>
<th>$h_K^m$</th>
<th>$h_K^e$</th>
<th>$h_K^{ob}$</th>
<th>$h_K^m$</th>
<th>Reference</th>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Metals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hg</td>
<td>72</td>
<td>44</td>
<td>3.0</td>
<td>0.46</td>
<td>14.5</td>
<td>6.5</td>
<td>NPW(67) : 33</td>
<td></td>
</tr>
<tr>
<td>Pb</td>
<td>100</td>
<td>19</td>
<td>3.2</td>
<td>0.17</td>
<td>6</td>
<td>19</td>
<td>CC(65) : 24</td>
<td></td>
</tr>
<tr>
<td>In</td>
<td>111</td>
<td>17.1</td>
<td>1.1</td>
<td>0.22</td>
<td>15.5</td>
<td>5</td>
<td>ND(64) : 11</td>
<td></td>
</tr>
<tr>
<td>Au</td>
<td>162</td>
<td>10.5</td>
<td>0.88</td>
<td>0.041</td>
<td>12</td>
<td>21.5</td>
<td>JL(63) : 17</td>
<td></td>
</tr>
<tr>
<td>Sn</td>
<td>195</td>
<td>5.4</td>
<td>1.25</td>
<td>0.039</td>
<td>4</td>
<td>32</td>
<td>GB(62) : 23</td>
<td></td>
</tr>
<tr>
<td>Pt</td>
<td>221</td>
<td>6.2</td>
<td>1</td>
<td>0.016</td>
<td>6</td>
<td>6</td>
<td>L(68) : 34</td>
<td></td>
</tr>
<tr>
<td>Ag</td>
<td>226</td>
<td>5.5</td>
<td>0.6</td>
<td>0.027</td>
<td>9</td>
<td>22</td>
<td>CF(65) : 35</td>
<td></td>
</tr>
<tr>
<td>Cu</td>
<td>343</td>
<td>3.0</td>
<td>0.75</td>
<td>0.014</td>
<td>4</td>
<td>54</td>
<td>JL(63) : 17</td>
<td></td>
</tr>
<tr>
<td>Ni</td>
<td>440</td>
<td>1.9</td>
<td>0.4</td>
<td>0.007</td>
<td>5</td>
<td>57</td>
<td>KW-Y(62) : 36</td>
<td></td>
</tr>
<tr>
<td>W</td>
<td>405</td>
<td>1.8</td>
<td>0.25</td>
<td>0.003</td>
<td>7</td>
<td>84</td>
<td>JL(63) : 17</td>
<td></td>
</tr>
<tr>
<td>Al</td>
<td>433</td>
<td>1.5</td>
<td>0.76</td>
<td>0.002</td>
<td>2</td>
<td>48</td>
<td>CS(70) : 26</td>
<td></td>
</tr>
<tr>
<td>Non-metals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KCl</td>
<td>230</td>
<td>2.2</td>
<td>0.69</td>
<td>0.038</td>
<td>3</td>
<td>18</td>
<td>J(64) : 37</td>
<td></td>
</tr>
<tr>
<td>SiO$_2$(Q)</td>
<td>290</td>
<td>1.9</td>
<td>0.57</td>
<td>0.023</td>
<td>3.5</td>
<td>25</td>
<td>KW-Y(62) : 36</td>
<td></td>
</tr>
<tr>
<td>Si</td>
<td>636</td>
<td>0.64</td>
<td>0.42</td>
<td>0.005</td>
<td>1.5</td>
<td>90</td>
<td>JL(63) : 17</td>
<td></td>
</tr>
<tr>
<td>LiF</td>
<td>750</td>
<td>0.51</td>
<td>0.45</td>
<td>0.003</td>
<td>1</td>
<td>150</td>
<td>JL(63) : 17</td>
<td></td>
</tr>
<tr>
<td>Al$_2$O$_3$</td>
<td>1000</td>
<td>0.15</td>
<td>0.16</td>
<td>0.00035</td>
<td>1</td>
<td>450</td>
<td>GB(62) : 23</td>
<td></td>
</tr>
</tbody>
</table>

(b) The predicted temperature dependence is $T^3$ which in practice is almost never observed over a wide temperature range (see however Zinovyeva, this colloque and reference [14]). Typical exponents vary from $n = 2.5$ to 4.5.

(c) $R_K$ should vary as $\theta_0^{-3}$ by the Khalatnikov model; the experimental scatter is very great but a linear variation seems to fit better [15].

(d) The observed dependence of $h_K$ on the liquid pressure is very small and much less than that predicted by the Khalatnikov model, except in the low temperature limit [14].

Thus there seems to be fundamental disagreement between the Khalatnikov model and experiment. However the model may be partly saved by remarks concerning observations (b) and (d) above. An apparent result of Wheatley’s work in the very low temperature range is that the agreement with Khalatnikov improves as $T$ approaches 0 °K. For example for Cu-HeII, at 0.8 °K $h_K^{ob}$ is more than a factor 30 greater than $h_K^m$, while it is only about four times greater at 0.08 °K. There is also a progressive improvement of agreement of the $T$ dependence with lowering temperature. This tendency is pleasing, as barring other possible mechanisms of heat transfer we would hope for a better agreement with the Khalatnikov model for $T \to 0$, in the long wavelength limit.

The high temperature end ($T \sim 2$ °K) is complicated by the possibility of direct phonon or roton emission, and the much shorter phonon wavelengths. Again, however, the model is partly saved by the calculation of Challis and Wilks [16] which showed that the dense layer of helium of several atomic layers thickness near the wall may act as a matching unit for phonon transmission. A simple model calculation for copper gave an $h_K^{ob}$ increased by a factor of about 4, and $T^3$ becomes $\sim T^{+4}$ near 2 °K. It seems unlikely that the model as it stands could be extended to obtain accord with experiment, but certainly a more refined calculation would be interesting. The model has been applied to a number of other substances [17] but has never been able to give anything like quantitative agreement.

As most of the measurements have been on metals we are again confronted with the question of an electronic contribution to the heat flow. Little [18] and Andreev [7] both considered this question in detail.
In terms of the acoustic mismatch model for an ideal surface, all phonons incident from the liquid are totally reflected, except those incident from a narrow critical cone centred around normal incidence. As in the optical counterpart, total reflection implies the setting up of an inhomogeneous plane wave in the surface layer. Phonons incident near the edge of the critical cone set up an inhomogeneous wave with velocity \( c_s = c_l \), the velocity of Rayleigh waves of the free surface. For phonons incident from other angles the corresponding surface wave velocity is \( c_s = c_l / \sin \theta \). This physical picture corresponds to a resonance phenomenon, and the mathematics of the problem shows that the resonance for phonons incident near the critical cone is extremely sharp, as seen in figure 2 (c) (for \( \nu_l / \nu = 77 \) and \( c_l / c_s = 0.2 \)). If the resulting surface wave is not scattered energy is stored in it, and the reflection coefficient of phonons in the liquid is as shown in figure 2 (a). If the surface wave is scattered, we then have the possibility of transferring energy by a mechanism acting in parallel with phonon transmission, thereby increasing the conductance. For Pb this mechanism could augment the Khalatnikov heat flow by a factor of nearly 2.5 from that predicted by phonon transmission only.

Little [19] suggested that for a perfect surface the surface wave is uncoupled. While this seems unlikely to be the case for a real surface (see however reference [20]), if we take a superconductor at \( T \ll T_c \) and suppose the surface wave uncoupled, then the conductance should increase when the specimen is driven normal by a magnetic field, due to scattering of the surface wave by the conduction electrons. This is the process calculated by Little and Andreev; however even for a free electron metal they come to quite different conclusions, Little that \( h_{\text{Kap}} \) is negligible and Andreev that \( h_{\text{Kap}} \sim h_{\text{env}} \). Challis and the author considered this question in some detail [21] showing that in fact Little's and Andreev's calculations correspond to different limits of the same model. For a free electron metal we expect the Little limit to hold for \( T \sim 1 \) oK and the Andreev limit for \( T \ll 10^{-2} \) oK.

Historically Challis [22] found an increase of \( \sim 3 \) in \( h_{\text{K}} \) at 1.3 oK for Pb driven normal by a field; this result was confirmed by numerous authors [10, 23, 24].

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**Fig. 2.** — The reflection coefficient at a helium solid interface for a solid with \( \rho_l / \rho = 77 \) and \( c_l / c_s = 0.2 \) where \( A_h, A_r \) are incident and reflected amplitudes respectively in the fluid and \( A_l, A_t \) are transmitted longitudinal and transverse amplitudes in the solid. The cases shown are: (a) No surface wave absorption, scale enlarged. (b) Surface wave absorption, scale enlarged. (c) No surface wave absorption, to scale.

**Fig. 3.** — The Kapitza resistance of Pb in a magnetic field at \( T \sim 1.3 \) oK. (a) IV B; specimen cut in liquid helium, annealed 1 day at 77 oK [25]; (b) IV D; specimen cut in liquid helium, annealed 1 week at 300 oK [25]; (c) 3; specimen electropolished and argon ion bombarded [24].
and taken to be evidence of the existence of $h_{\text{elec}}$. More recently, however, several experiments have indicated that this is not the case. Experiments by the author [25] shown in part in figure 3 on Pb surfaces cut in the helium bath showed that $h_N$ could be increased by a factor of up to 25 on application of a field at 1.2 °K. As $h_N \sim$ constant and has about its usual value, and $h_N/h_b \sim 3$ after a room temperature anneal, the observed behaviour does not seem to be a question of the "true" Kapitza resistance. As proposed in [25], the results are consistent with the introduction of a bulk thermal resistance in the superconducting state only, due to damage in a thick surface region on cutting. At the same time, Challis and Sherlock published preliminary results on Kapitza conductance measurements on Pb foils by second sound transmission [26] and found $h_b = h_N$ to 10% or less. They proposed that surface strain in the steady state measurements due to differential contraction between specimen and specimen holder might well account for the values of $h_{N}/h_b$ between one and three which are usually observed. Work is being continued on this problem, our present view being that the above results strongly suggest that the strain explanation is correct, and that $h_{\text{elec}}$ is extremely small, at least near 1 °K.

Several preliminary results on the Kapitza resistance of Pb-2 % In and Pb-20 % In at 25 suggest that $h_{\text{elec}}$ is also negligible in the dirty metal [7]. Due to the large bulk thermal resistivity the correction to the measured resistance is large, and field variations observed in the raw data can be quite happily accounted for by the observed variation of this bulk resistivity.

Another possibility of an electronic contribution to the Kapitza conductance was proposed by F. Bloch [28]. He looked for an effective heat transfer mechanism valid even for a rigid metal and consequently considered a possible electron tunnel effect by means of the leakage of the electronic wave function into the helium. A simple perturbation treatment gave

$$h_{\text{tunnel}}/h_b \sim 5 T^2.$$ 

However, a layer of dirt or even the dense helium layer could possibly suppress this effect. Three attempts to observe it in Cu [17], Pt [23] and Pb [25] all lead to negative results.

Many other systems of great interest have recently been considered, for example CMN-Hex [29] CeE8-HeII [30], Cu-He gas [31]. The most recent investigations of the CMN question [32] suggests a confirmation of the speculation [29] of heat exchange by a direct magnetic interaction, which will undoubtedly lead to some exciting future prospects. As most of these special cases are still at an embryo stage, we shall not deal with them here. We complete our account by a last look at the phonon transmission model. We can summarise this section by saying that even with a surface wave contribution (with or without electrons) and a dense layer, the Khalatnikov model drastically underestimates the heat flow in the 1 °K range. We are thus prompted to look at the phonon radiation limit in hopes of a better agreement.

IV. Phonon radiation limit. — To calculate this limit we suppose a perfect energy transfer mechanism at the interface. Then for phonon transport, the maximum heat flow is limited by the rate at which energy can be brought up to (and taken away from) the interface from each side. As mentioned before, this limit can be obtained from the acoustic mismatch model by putting $\alpha = 1$.

It seems however that we must keep the critical cone in the helium so as to have zero net heat flow at thermal equilibrium.

Snyder has found the same result by considering a slice about one mean free path wide in one of the media. The heat flow across a plane for a medium of energy density $U$ is

$$\dot{Q} = \frac{1}{4} U c.$$  \hspace{1cm} (8)

Then the net heat current across the slice with one side at $T$ and the other at $T + \Delta T$

$$\dot{Q} = \dot{Q} - \dot{Q} = h^{pr} \Delta T \quad \Delta T \ll T$$

gives

$$h^{pr} = \frac{1}{4} c, \tilde{c}$$

$$= \frac{2 \pi^4 k^2}{5 h^2 \theta_D^2} \left( \frac{3 n}{4 \pi} \right)^{2/3} T^3$$

$$= 1.9 \times 10^5 \left( \frac{\rho M}{\theta_D} \right)^{2/3} \frac{T^3}{\theta_D^2}$$ \hspace{1cm} (9)

the result quoted by Snyder [3].

The model is thus analogous to what has been called the black body model [38] for solid-solid contact. As Snyder points out, the result (9) is determined essentially by the properties of the solid.

We have taken table III from Snyder and using her values of $\theta_D$ calculated the Khalatnikov limit as well; the ratios of each conductance to the experimental values are also given in table III, so that small numbers correspond to agreement and large to disagreement, in the appropriate sense.

The most striking fact about the table is seen by looking at the non-metals. We see very clearly that as $\theta_D$ increases the disagreement with Khalatnikov is enhanced while there is an equally dramatic improvement between experiment and the phonon radiation model. In the limiting cases of LiF and Al2O3 it thus appears that all of the phonons are transmitted instead of the now very small fraction permitted by acoustic mismatch. The fact is all the more remarkable in that it is just in this limit that we might expect acoustic mismatch to work best. The same tendency is seen among the metals; while there is more data the
situation is less clear due to a lack of very high $\theta_D$ materials and due to some apparently random behaviour in the lowish $\theta_D$ end. The rather singular behaviour of Pb is somewhat confusing, as it is one of the most studied materials of this group.

Thus in the high $\theta_D$ limit at 1 °K we could say that there appears to be no Kapitza resistance as is usually understood by the term. The maximum possible amount of energy is transferred to the helium, this being however relatively small due to the small number of phonons excited at this temperature. As we increase the number of phonons (by lowering the Debye temperature) a resistive mechanism seems to appear, which however, is much less severe than that proposed by Khalatnikov. In the author’s opinion this observation is of great importance in that at last we can pose a precise question which should be tractable theoretically. It would obviously be desirable to extend the comparison down to lower temperatures when more results become available, both from the point of view of having longer phonon mean free paths and being far away from the $\lambda$ anomaly.

V. Conclusion. — Regarding solid-solid contacts, in most cases the acoustic mismatch and black body models give not too different results and both are not too different from the observed values. The interpretation is complicated by the fact that solids are generally well matched acoustically and the experiments are difficult to evaluate due to inherent uncertainties regarding the physical nature of the junction.

The case of limiting mismatch, that of copper-solid helium fits neither limit well, although agreement with Khalatnikov model does not hold, at least in the high temperature limit. The disagreement becomes progressively worse in the high $\theta_D$ limit, especially for the non metals. The phonon radiation limit shows the opposite trend, and seems to work very well for high $\theta_D$ materials, although the absolute conductance is very small.

On the theoretical side we feel that the problem would be much better understood by considering models (hopefully still simple!) which correspond more closely in the relevant details to the real state of a solid surface in helium. For example calculations elaborating on the presence of a dense helium layer and of surface states, and calculations treating the coming into equilibrium of phonons and electrons near a uniformly rough surface would be of real value.

On the experimental side there seems to be a need for three conditions before quantitative comparison between different specimens and with the theory can be sought:

(a) Better methods of evaluating the true surface temperature or by-passing the problem to some extent by the use of techniques such as those of Challis and collaborators (second sound transmission).

(b) The obtaining of better surfaces and in any case a more precise definition of the physical and chemical state of the surface under study.

(c) Assuming (a) and (b), more data on the same system over the whole temperature range.

It is the author’s opinion that the apparently large variations in Kapitza resistance measurements are largely due to neglect of one or all of these factors, and comparison with any model seems difficult without satisfying condition (c).

Acknowledgements. — The author is grateful to Dr. L. J. Challis and Prof. A. Lacaze for many helpful discussions. He is also grateful to Dr. F. W. Sheard for providing figure 2 (c) and to B. Hebral for his permission to mention unpublished results.

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