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Itinerant electron metamagnetism

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1. Introduction. — The metamagnetic transition (MT) from the paramagnetic state to the ferromagnetic state by applying the magnetic field was first discussed by Wohlfarth and Rhodes [1] for the itinerant electron model. They showed that the paramagnetic metals or alloys will show MT, if there is a maximum in the temperature variation of the paramagnetic susceptibility as in Pd, or as a more strict condition, if the coefficient $a_3$ in the expansion of the magnetic energy $E$ given by

$$E = \frac{1}{2} a_1 M^2 + \frac{1}{4} a_3 M^4 - MH,$$

where $H$ is the applied magnetic field, as a power series of the magnetization $M$, is negative. They defined the critical field $H_c$ for MT from the paramagnetic state to the ferromagnetic state by the inflection point in the plot of $E$ against $M$ as a function of $H$.

Recently, using the same definition of $H_c$ as before, Wohlfarth [2] has predicted the value of $H_c$ for Pd, YCo$_2$ and TiBe$_2$ as $1,000$, $170$ and $58$ kOe, respectively. However, experimentally MT has so far never been observed for Pd up to $325$ kOe [3], for YCo$_2$ up to $150$ kOe [4] and $380$ kOe [5] and for TiBe$_2$ up to $190$ kOe [6] and $213$ kOe [7]. In the cases of YCo$_2$ and TiBe$_2$ the highest fields used in experiment are much higher than the values predicted by Wohlfarth [2]. Cyrot and Lavagna [8] have also predicted MT for YCo$_2$ at about $1,000$ kOe, using their calculated density of states. In our estimation from their calculated density of states, MT seems to be impossible in YCo$_2$. Recently, Jarlborg and Freeman [9] have also predicted MT in Pd at $3,220$ kOe from the calculation of its electronic structure under the magnetic field.

In this paper it is shown that the condition for MT, $a_3 > 0$ is not complete and the previous estimations of $H_c$ are not reliable. We should include the higher order terms at least up to the $M^6$ term in the expansion of the magnetic (free) energy $E$ in order to discuss MT, as shown below. It is possible that even if $a_3 < 0$ the second minimum corresponding to the ferromagnetic state in addition to the first minimum corresponding to the induced paramagnetic state in the plot of $E$ against $M$ does not appear even at very high field when the contribution from the higher order terms to $E$ is large. The condition for MT is more strict than $a_3 < 0$. It is well-known that in the case $a_3 < 0$ the paramagnetic susceptibility $\chi$ increases with increasing $H$. Therefore, the increase of $\chi$ with increasing $H$, as observed in Pd [3], YCo$_2$ [4, 5]...

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and TiBe₂ [6, 7], does not necessarily mean the occurrence of MT, although it is one of the necessary conditions.

On the one hand, there are many intermetallic compounds of rare earth metals and transition metals, e.g. RCo₂ [10, 11], where R means the rare earth metals and is Dy, Ho or Er, (Ho, Y)Co₂ [12], etc., which show MT from the paramagnetic state to the ferromagnetic state and also many compounds, e.g. ThCo₂ [13], Y₂Ni₁₋₆ [14], Co(S, Se)₂ [15, 16], (Co, Ni)S₂ [17], etc. show MT from the weak ferromagnetic state to the ferromagnetic state with a larger magnetization.

On the other hand, there are many alloys and rare earth compounds which also show MT from the antiferromagnetic state to the ferromagnetic or ferromagnetic state or from the ferrimagnetic state to the ferromagnetic state and which show the dependence on \( H \) of \( M \) very similar to the one of MT that can be explained by the model of narrow domain wall [18]. In this paper we will discuss only the former case and will no discuss the latter case. There are review articles for the magnetic properties of intermetallic compounds of rare earth metals and transition metals [10, 19-21].

Especially, in some of \( \text{RCo}_{2} \) compounds, ErCo₂, HoCo₂ and DyCo₂ the first-order transition was observed [10, 11, 22] and MT was observed only in a very narrow temperature range, \( 5 \sim 10 \text{ K} \) [10, 11]. The first-order transition, which corresponds to the case \( H_{c} = 0 \) in MT, in these compounds was qualitatively explained by the combined model of the localized magnetic moment on R atoms and itinerant electrons on Co atoms or the so-called s-d model [4], where the expansion of free energy with respect to \( M \) only due to the itinerant electrons was taken into account. Recently, it has been pointed out that the expansion of the magnetic free energy with respect to the total magnetization given by the sum of the magnetization due to the localized moments and that due to the itinerant electrons is very important to discuss the first-order transition and to derive this transition temperature [23]. This expansion of the free energy with respect to the total magnetization is also very important to discuss MT in various kinds of rare earth compounds, as shown below.

In the case of Co(S, Se)₂ [15, 16] and (Co, Ni)S₂ [17], the simple Heisenberg model seems to be appropriate, because the magnetization changes very little at very high field or the high-field susceptibility is very small and the temperature variation of \( \chi \) at high temperatures obeys the Curie-Weiss law. However, all the coefficients except the first coefficient of the \( M² \) term in the expansion of the free energy as a power series of \( M² \) in the molecular field approximation are always positive in the simple Heisenberg model, where the average of the magnetic interaction is proportional to \( M² \), so that we cannot expect MT and the first-order transition in this model. Therefore, in order to explain MT in the Heisenberg model we have to introduce some modifications of the model, e.g. the magnetic interaction with higher powers of \( M \) than \( M² \), the temperature dependence of the molecular field coefficient, the complicated magnetic structures, the distribution of the magnetic moments, etc. In the case of Co(S, Se₁₋₂)₂ compounds the distribution of cobalt atoms with and without magnetic moment has been ascertained in experiment [16] and MT has been discussed in terms of the distribution of the two kinds of Co atoms with and without magnetic moment [24, 25].

In section 2, general conditions for MT based on the thermodynamics are shown without regard to any specific model for MT. In section 3, MT between a simple paramagnetic state and a ferromagnetic state is discussed in the simplest model, that is, the least number of terms in the expansion of the free energy in a power series of \( M² \) and the analytical expressions of the conditions for MT and of the critical fields which show a hysteresis are obtained. In section 4, MT between a weak ferromagnetic and normal ferromagnetic states is discussed in terms of the least number of terms in the expansion of the free energy and the conditions for this kind of MT and the critical fields are obtained. In section 5, from the experimental results of the field dependence of \( M \) for TiBe₂, YCo₂ and Pd the values of the coefficients in the expansion of the free energy with respect to \( M \) are estimated and the conditions for MT are examined. It is concluded that TiBe₂ will not show MT and for Pd and YCo₂ it is very difficult to predict whether they show MT or not from our present knowledge for the dependence on \( H \) of \( M \). Even if it exists it occurs at very high fields higher than 325 kOe for Pd and 380 kOe for YCo₂. In section 6, the reason for the occurrence of the variety of MT in various rare earth compounds and the character of MT are discussed and some conclusions are given:

### 2. General conditions for the metamagnetic transition

The magnetic free energy \( f \) can be given as a function of the temperature \( T \), the volume \( V \), the concentration \( c \), if necessary, and the magnetization \( M \) of a metal, alloy or compound. By the thermodynamic relations the magnetic field \( H(M) \) is given as a function of \( T, V, c \), and \( M \) as

\[
H(M) = \frac{\partial f}{\partial M}.
\]

For the moment we consider the dependence only on \( M \) of \( f \) and \( H(M) \). In the itinerant electron system \( H(M) \) can be written as \[26, 27\] :

\[
H(M) = \Delta \zeta / 2 \mu_B - \alpha M,
\]

where \( \Delta \zeta \) is the so-called exchange splitting, i.e. the difference between the chemical potentials of electrons with up (+) and down (−) spins when the magnetization \( M \) exists and \( \alpha \) is the molecular field coefficient. We can obtain the dependence on \( M \) of \( \Delta \zeta \) from the calculated density of states and actually \( \Delta \zeta \).
shows a very complicated dependence on $M$, as shown schematically before [28] and for ZrZn$_2$ recently [29].

$\Delta \zeta$ should be a monotonically increasing function of $M$, i.e. $\partial \Delta \zeta / \partial M \geq 0$ and it can be expanded when $M$ is small as

$$\Delta \zeta / 2 \mu_B = a_1 M + a_3 M^3 + a_5 M^5 + a_7 M^7 + \cdots,$$

(3)

where the coefficients $a_1, a_3, a_5, a_7$, etc. are in general functions of $T, V$ and $c$ and they can be expressed in terms of the electronic density of states and its derivatives [27]. All coefficients except $a_1$ are possible to become negative. However, we must remember that these coefficients at finite temperature are modified by the effect of spin fluctuations in the itinerant electron system as shown in [27, 30]. Then $H(M)$ is expanded as

$$H(M) = a'_1 M + a_3 M^3 + a_5 M^5 + a_7 M^7 + \cdots,$$

where $a'_1 = a_1 - \alpha = 1/\chi'.

(4)

On the other hand, for a Heisenberg system of $N$ magnetic moments with spin $S$ and Landé factor $g$, $H(M)$ can be written as

$$H(M) = (g\mu_B S)^{-1} B_s^{-1}(M/N\mu_B S) - KM,$$

(5)

where $B_s = 1/k_B T$, $B_s^{-1}$ is the inverse Brillouin function and $K$ is the molecular field coefficient between the localized moments. The right-hand side of equation (5) can also be expanded as a power series of the magnetization $M$ as [23]

$$H(M) = b'_1 M + b_3 M^3 + b_5 M^5 + b_7 M^7 + \cdots,$$

(6)

where

$$b'_1 = b_1 - K = 1/\chi,'$$

and

$$b_1 = 3 \beta^{-1} N^{-1}(g\mu_B)^{-2} S^{-1}(S + 1)^{-1},$$

$$b_3 = \frac{9}{20} \beta^{-1} N^{-3}(g\mu_B)^{-4} \{ (2S + 1)^3 + 1 \} S^{-3}(S + 1)^{-3},$$

$$b_5 = \frac{27}{800} \beta^{-1} N^{-5}(g\mu_B)^{-6} \{ 11(2S + 1)^4 + 32(2S + 1)^2 + 11 \} S^{-5}(S + 1)^{-5},$$

$$b_7 = \frac{81}{560000} \beta^{-1} N^{-7}(g\mu_B)^{-8} \{ 19(2S + 1)^6 + 107(2S + 1)^4 + 107(2S + 1)^2 + 19 \} S^{-7}(S + 1)^{-7}.$$  

(7)

It is seen that all coefficients $b_i$ are proportional to $T$ and positive. In general, $K$ may depend on $T, V, c$ and $M$. In the case of the Heisenberg model all the coefficients $b_i$ are of course modified by the effect of the spin fluctuations in the same way as in the itinerant electron system [27, 30].

For a combined system of the localized magnetic moments and itinerant electrons, such as Pd-Fe and Pd-Co alloys and compounds of rare earth metals and transition metals, $H(M)$ can be expanded as a power series of the total magnetization $M$ as [23]

$$H(M) = c_1 M + c_3 M^3 + c_5 M^5 + c_7 M^7 + \cdots,$$

(8)

where $M$ is the sum of the magnetization of the localized moments and that of the itinerant electrons and the coefficients $c_1, c_3, c_5$ and $c_7$ are given by

$$c_1 = Q_d Q_1 Q^{-1} - J,$$

$$c_3 = (a_3 Q^* + b_3 Q_d^*) Q^{-4},$$

$$c_5 = (a_5 Q_1^* + b_5 Q_d^*) Q^{-6} - 3(a_3 Q_1^* - b_3 Q_d^*) Q^{-7},$$

$$c_7 = (a_7 Q_1^* + b_7 Q_d^*) Q^{-8} - 8(a_3 Q_1^* + b_3 Q_d^*) (a_3 Q_1^* - b_3 Q_d^*) Q^{-9} + 8(a_3 Q_1^* + b_3 Q_d^*) (a_3 Q_1^* - b_3 Q_d^*) Q^{-10} +$$

with

$$Q_d = \chi_d^{-1} + J, \quad Q_1 = \chi_1^{-1} + J$$

and

$$Q = Q_d + Q_1,$$

where $J$ is the molecular field coefficient between the localized moments and itinerant electrons.

Now, we discuss the possibility of MT between the paramagnetic and ferromagnetic states by assuming a proper dependence of $H(M)$ on $M$ for several cases, (for example, $a'_1 \geq 0$, $a_3 < 0$ and $a_5 > 0$ in the itinerant electron model or $c_1 \geq 0$, $c_3 < 0$ and $c_5 > 0$ in the combined model), as shown by schematic curves in figure 1, here and hereafter $\times$ shows an inflection point and a small vertical bar shows the position of a maximum or minimum. This figure gives
of course the variation of $M$ with respect to the external field $H$ as usually observed in experiment. Here and hereafter, we make use of only the expansion of $H(M)$ given by equations (4) for the itinerant electron system and the same discussion can be applied to the combined system of the localized moments and itinerant electrons by replacing equation (4) by equation (8). There is an inflection point at $M = M_0$ and $H = H_0$ in the curve 2, there are a maximum and a minimum in the curves 3, 4, 5 and 6 and there is only a minimum in the curve 7 (e.g. if $a_1 = 0$ or $a_3 > 0$). It is easily seen from figure 1 that in the cases corresponding to the curves 3-6 MT occurs and there are hysteresis in the plot of $M$ against $H$ and in the case of the curve 1 MT does not occur.

The critical condition for the occurrence of MT corresponds to the curve 2 and it is given by the relation which satisfies two equations

$$\frac{\partial H(M)}{\partial M} = 0 , \tag{10}$$

and

$$\frac{\partial^2 H(M)}{\partial M^2} = 0 . \tag{11}$$

And the critical value $M_0$ of $M$ and the corresponding value $H_0$ of the magnetic field $H$ are also obtained.

Analytical expressions of this critical condition obtained from the expansion of $H(M)$ in equation (4) or (8) will be given in § 3.

For each case corresponding to each curve in figure 1 the effect of $H$ is examined in the following way. The magnetic free energy $f$ for any magnetic system under $H$ can be written as

$$f = \int_0^M H(M) dM - HM \tag{12}$$

Besides a constant term. Then, we can get the dependence of $f$ on $H$ by integrating the curves in figure 1 with respect to $M$. The schematic dependences of $f$ on $M$ as a function of $H$ and the schematic dependences of $M$ on $H$ are shown in figures 2a, b, c, d, e and 3a, b, c, d, e, which correspond to the curves 1 and 2 (case I), curve 3 (case II), curve 4 (case III), curve 5 (case IV) and curve 6 (case V) in figure 1, respectively.

In the case I (Figs. 2a and 3a) there is no minimum of $f$ at finite value of $M$ when $H = 0$. With increasing $H$ the paramagnetic state at $M = 0$ when $H = 0$ simply shifts to a larger value of $M$, as there is only one solution $H(M) = H$ in this case. There is no MT and the dependence of $M$ on $H$ is simply given by curve 1 or 2 in figure 1 or 3a. This type of curve 1 in figure 3a has been found experimentally for TiBe$_2$ [6, 7]. In Pd [3] and YCo$_2$ [4, 5] the bend of the curve toward $H$ axis has not yet been observed.

In the case II (Figs. 2b and 3b) $f$ does not show any minimum at finite $M$ when $H = 0$. The cross points between the curve and vertical straight lines $H = constant$ in figures 3 give the inflection point, maximum or minimum in the curve of $f$ against $M$ in figures 2. Therefore, in the case II, MT occurs and the hysteresis appears. The lower and higher critical fields and the critical field in the equilibrium, where the values of $f$ at the induced paramagnetic and ferromagnetic states are equal, are denoted by $H'_c$ and $H''_c$ and $H_c$, respectively, in figures 2b and 3b. The largest value of $M$ corresponding to $H'_c$ in the induced paramagnetic state and the smallest value of $M$ corresponding to $H''_c$ in the ferromagnetic state are denoted by $M'_c$ and $M''_c$, respectively, in figure 3b. Analytical expressions of $H_c$, $H'_c$, $H''_c$, $M'_c$ and $M''_c$ obtained in the simple model are given in § 3. The slope of the curve of $M$ at $M = 0$ in figures 3, gives the paramagnetic susceptibility and that at high $H$ gives the high-field susceptibility. The MT in the case II (Fig. 3b) was observed in experiment for many rare earth compounds, RC$_2$ [10, 11], (Ho, Y)Co$_2$ [12], Co(S, Se)$_2$ [15] and (Co, Ni)S$_2$ [17]. In Co(S, Se)$_2$ the transition seems to be between the weak ferromagnetic and normal ferromagnetic states [16].

In the case III (Figs. 2c and 3c), there is a minimum of $f$ at $M \neq 0$ even when $H = 0$ and the lower critical field $H'_c$ does not exist. In the case IV (Figs. 2d and 3d), the minimum values of $f$ at $M = 0$ and at $M \neq 0$ are equal to zero when $H = 0$ and we have $H'_c = 0$. In the case V (Figs. 2e and 3e), the minimum value of $f$ at $M \neq 0$ is already negative when $H = 0$ and we have only the critical field $H''_c$. The magnetization curves against $H$ in the cases III, IV and V (Figs. 3c, d, e) were observed in experiment for RCo$_2$ [10, 11] and (Ho, Y)Co$_2$ [12].

Now, we discuss MT between the weak ferromagnetic state and the ferromagnetic state with a larger $M$. The relations between $H(M)$ and $M$ for several cases, (for example, in the conditions that $a_1 > 0$, $a_3 < 0$ and $a_7 > 0$ or $c_1 < 0$, $c_3 > 0$, $c_5 < 0$ and

Fig. 1. — Schematic dependences on the magnetization of the magnetic field $H(M)$ for the cases of the metamagnetic transition between the paramagnetic and ferromagnetic states, e.g. for the cases, $a_1 > 0$, $a_3 < 0$ and $a_7 > 0$ in the expansion of $H(M) = a_1 M + a_3 M^3 + a_7 M^7$. Here and hereafter $\times$ shows an inflection point and small vertical bars show a maximum or minimum.
Fig. 2. — Schematic dependences on the magnetization $M$ of the magnetic free energy $f$ under the magnetic field $H$, where the figures a, b, c, d and e correspond to the curves 1 and 2, 3, 4, 5 and 6 in figure 1, respectively, $H_c^e$ and $H_c^r$ are the lower and higher critical fields and $H_c^e$ is the critical field in equilibrium.

Fig. 3. — Schematic dependences on the external magnetic field $H$ of the magnetization $M$, where the figures a, b, c, d and e correspond to the curves 1 and 2, 3, 4, 5 and 6 in figure 1, respectively, $M_0$ and $H_0$ are the magnetization and the magnetic field at the critical condition for the metamagnetic transition and $M_c^e$ and $M_c^r$ are the magnetizations corresponding to the critical fields $H_c^e$ and $H_c^r$, respectively.
c_7 > 0), are shown schematically in figure 4. In this case there is always a minimum of \( H(M) \) at a low value of \( M \). The corresponding dependences of \( M \) on \( H \) are shown in figure 5.

The curve 1 in figures 4 and 5 shows the relation between \( M \) and \( H \) at the critical condition where MT occurs or does not occur. In the cases of the curves 2, 3 and 4 in figures 4 and 5, MT occurs and the hysteresis appears. The critical condition for MT is similarly given as above by the relation which satisfies two equations (10) and (11) and the critical value \( M_0 \) of \( M \) and the corresponding critical value \( H_0 \) of \( H \) are also obtained. Analytical expressions for the critical condition and the critical fields \( H_0', H_0'', H_0' \) and \( H_0'' \) obtained in the simple model are shown in § 4. In the case of the curve 4 in figures 4 and 5 two minimum values of \( f \) are equal and we have \( H_0 = 0 \).

In the case of the curve 5 in figures 4 and 5, the ferromagnetic state with a large \( M \) is always stable. This type of MT was observed in ThCo_5 [13], Y_2Ni_{16} [14] and Co(S, Se)_2 [16].

3. Metamagnetic transition between the paramagnetic and ferromagnetic states. — It is clear from the discussion given in § 2 that at least we should retain three terms in equation (4) or (8) as

\[
H(M) = a_1 M + a_3 M^3 + a_5 M^5, \quad (13)
\]

and we can expect the same dependences on \( M \) of \( H(M) \) as those shown by the curves in figure 1, when \( a_1' > 0, a_3 < 0 \) and \( a_5 > 0 \). We discuss this case below.

The critical condition for MT in the case I (curve 2 in figure 1) is easily obtained from two equations (10) and (11) as

\[
a_1' a_5/a_5^3 = 9/20 \quad (14)
\]

and \( M_0^2 \) and \( H_0^2 \) at the inflection point are given, respectively, by

\[
M_0^2 = -\frac{3}{10} a_3/a_5 \quad (15)
\]

and

\[
H_0^2 = -\frac{54}{3125} a_5^2/a_5^3. \quad (16)
\]

The condition that the critical field \( H_0 \) in equilibrium becomes zero in the case III (curve 5 in figure 1) in § 2 or the condition that the two minimum values of \( f \) at \( M = 0 \) and \( M = M_1 \neq 0 \) are equal was already obtained before [26] and it is written as

\[
a_1' a_5/a_5^3 = 3/16. \quad (17)
\]

The value of \( M_1 \) is given by

\[
M_1^2 = -\frac{3}{4} a_3/a_5. \quad (18)
\]

Therefore, the condition for MT to occur is given from equations (14) and (17) as

\[
\frac{3}{16} < a_1' a_5/a_5^3 < \frac{9}{20} \quad (19)
\]

besides the conditions \( a_1' = a_1 - \alpha = 1/\chi_3 > 0, a_3 < 0 \) and \( a_5 > 0 \). For the combined model of localized moment and itinerant electrons \( a_1', a_3 \) and \( a_5 \) in equation (19) are replaced by \( c_1, c_3 \) and \( c_5 \), respectively.

The critical fields \( H'_e \) and \( H''_e \) and the corresponding critical values \( M'_e \) and \( M''_e \) in figure 3b for the hysteresis of MT can be calculated from equation (10) and are given by, respectively,

\[
H'_e = \frac{2}{5} (2 a_1' + a_3 M'_e^2) M'_e, \quad (20)
\]

\[
H''_e = \frac{2}{5} (2 a_1' + a_3 M''_e^2) M''_e, \quad (21)
\]

\[
M'_e^2 = \frac{1}{10} (-3 a_3 + \sqrt{9 a_5^2 - 20 a_1' a_5})/a_5. \quad (22)
\]
The critical field $H_c$ in equilibrium is calculated approximately from the condition
\[ f(M') - H_c M' = 0 \]
and is given by
\[ H_c \approx \left( \frac{7}{15} a_1' + \frac{3}{20} a_3 M_c^2 \right) M' \].

The definition of the critical field by Wohlfarth [2] is similar to $H_c'$ in this paper, but the expansion of $H_c^\prime$ in equation (21) for the case of small $a_3$ does not coincide with his result. The condition that $H_c'$ becomes zero is obtained from equations (20) and (22) as
\[ a_1' a_3/a_1^2 = \frac{1}{4} \].

and the value of $M_c'^2$ and $M_c'^{n^2}$ are given, respectively, by
\[ M_c'^2 = -\frac{1}{2} a_3/a_5 \]
and
\[ M_c'^{n^2} = -\frac{1}{10} a_3/a_5 \].

The condition that $H_c$ becomes zero is given approximately from equation (24) as
\[ a_1' a_3/a_1^3 = \frac{135}{784} \].

As long as $a_1'$ is positive, $H_c'$ always exists. Therefore, if any ferromagnet shows the first-order transition at $H = 0$ in the case $a_1' > 0$ it should always show the hysteresis like that shown in figure 3d.

4. Metamagnetic transition between the weak ferromagnetic and usual ferromagnetic states.  From the discussion given in § 2, we should retain at least four terms in equation (4) or (8) as
\[ H(M) = a_1' M + a_3 M^3 + a_5 M^5 + a_7 M^7 \],
with
\[ a_1' < 0 \quad a_3 > 0 \quad a_5 < 0 \quad a_7 > 0 \],
in order to describe the curves shown in figure 4.

Similarly as in § 3, the critical conditions for MT in the case shown by the curve 1 in figure 4 is obtained from equations (10) and (11) as
\[ a_1' + \left( 2 a_3 + \frac{5}{3} a_5 M_0^2 \right) M_0^2 = 0 \],
where $M_0'$ is the value of $M$ at the inflection point and is given by
\[ M_0'^2 = \frac{1}{21} \left( -5 a_5 + \sqrt{25 a_1^2 - 63 a_3 a_5 a_7} \right) \]
and the corresponding value of $H_0'$ at the inflection point is given by
\[ H_0' = (a_1' + a_3 M_0'^2 + a_5 M_0'^4 + a_7 M_0'^6) M_0' \].

The condition that the critical field $H_c$ in equilibrium becomes zero in the case given by the curve 4 in figure 4 is obtained from the fact that two minimum values of $f$ at $M_1$ and $M_2$ are equal, where $M_1^2$ and $M_2^2$ are given by the smallest and largest positive roots of the equation
\[ H(M) = a_1' + a_3 M^2 + a_5 M^4 + a_7 M^6 = 0 \],
respectively, and it is given by
\[ a_1' - \frac{1}{9} a_3 a_5/a_7 + \frac{1}{9} (3 a_5 - a_3^2/a_7) (M_1^2 + M_2^2) = 0 \].

Then the condition for MT between the weak ferromagnetic and normal ferromagnetic states can be written from equations (30) and (33) as
\[ \frac{1}{9} a_3 a_5/a_7 - \frac{1}{9} (3 a_5 - a_3^2/a_7) (M_1^2 + M_2^2) < a_1' < -\left( 2 a_3 + \frac{5}{3} a_5 M_0'^2 \right) M_0'^2 \].

This condition can be rewritten approximately as
\[ -\frac{3 a_3}{8 a_5} + \frac{17 a_3}{48} a_7 - \frac{93 a_3}{288} a_7^2 < a_1' < -\frac{3 a_3}{4 a_5} + \frac{10 a_3}{7} a_7 - \frac{500 a_3}{323} a_7^2 \].

The critical fields $H_c'$ and $H_c''$ are defined by $H_c' = H(M_c)$ and $H_c'' = H(M_c')$, respectively, where $H(M)$ is given by equation (29) and $M_c$ and $M_c'$ are the largest and smallest positive roots of the equation $dH(M)/dM = 0$. The critical field $H_c$ in equilibrium is given by the solution of equation
\[ H_c = \left\{ \frac{1}{2} a_1' + \frac{1}{4} a_3 (M_1^2 + M_3^2) + \frac{1}{6} a_5 (M_1^4 + M_1^2 M_3^2 + M_3^4) + \right. \\
\left. + \frac{1}{8} a_7 (M_1^6 + M_1^4 M_3^2 + M_1^2 M_3^4 + M_3^6) \right\} (M_1 + M_3) \].
where $M_1$ and $M_3$ are the smallest and largest positive roots of the equation $H(M) = H$. The analytical expressions in this section are very complicated, because the analytical roots of the cubic equation have complicated expressions, but the numerical calculations of the critical conditions and the critical fields are very easy if the numerical values of all the coefficients $a_1', a_3$, $a_5$ and $a_7$ are given.

5. Comparison with the experimental results. — From the experimental results obtained by Monod et al. [6] on the field dependence of $M$ at 4.2 K up to 180 kOe for TiBe$_2$, which is similar to the curve 1 in figure 3a, the values of $a_1', a_3$ and $a_5$ are estimated as

\begin{align*}
a_1' &= 1.18 \times 10^2 \text{ Oe (mole/emu)}, \\
a_3 &= -4.50 \times 10^{-5} \text{ Oe (mole/emu)}^3 \\
a_5 &= 6.58 \times 10^{-11} \text{ Oe (mole/emu)}^5,
\end{align*}

where the unit of emu/mole for $M$ is used, and we obtain the value of $a_1' a_3/a_5^2 = 3.83$. From the experimental data obtained by Acker et al. [7] similar values of $a_1', a_3$ and $a_5$ are obtained. Therefore, TiBe$_2$ does not satisfy the condition for MT given by equation (19). However, the recent data [31] shows that the relation between $M$ and $H$ cannot be described by a simple expansion like equation (13).

From the recent calculated results of the density of states for TiBe$_2$, the values of $a_3$ and $a_5$ at 0 K can be determined, but we obtain $a_3 > 0$ and $a_5 < 0$ from the calculated results by Jarborg and Freeman [32] and $a_3 \leq 0$ and $a_5 \geq 0$ from the results by de Groot et al. [33].

For YCo$_2$ the values of $a_1'$ and $a_3$ are estimated as

\begin{align*}
a_1' &= 5.88 \times 10^2 \text{ Oe (mole/emu)} \\
a_3 &= -1.53 \times 10^{-4} \text{ Oe (mole/emu)}^3,
\end{align*}

respectively, from the data of Schinkel [5], but the value of $a_5$ cannot be determined, as the field is still not high enough to determine the value of $a_5$. Therefore, it is very difficult to say at present whether YCo$_2$ shows MT or not. If MT would occur, the critical field should be much higher than 325 kOe. However, it seems that the calculated density of states for Pd [35] does not satisfy the condition $a_3 < 0$ and this fact is not favourable to MT in Pd.

6. Discussion and conclusions. — The MT has been found for many intermetallic compounds of the rare earth metals and transition metals, as shown in § 1. The MT is not necessarily sharp [10, 11] and the field dependence of $M$ is very similar to that given by the curve 1 or 2 in figure 3a. However, as the first-order transition has been found in other properties of these compounds at $H = 0$ and therefore MT should occur. The broad transition may be attributed to the effect of defects, distribution of concentration, magnetostriiction or any micro-structure in these compounds.

For the compounds of the heavy rare earth metals and transition metals the combined model of the localized moment on the rare earth atoms and itinerant electrons on transition metals will be appropriate and the expression of $f$ in equations (8) and (9) is useful to discuss MT in these compounds. In order to satisfy the condition of MT given by equation (19), where $a_1', a_3$ and $a_5$ should be replaced by $c_1$, $c_2$ and $c_5$, respectively, the value of $c_3$ or $a_3$ must be largely negative and the values of $c_1$ and $c_3$ must be small. It is easily seen from equation (9) these conditions can be much easily satisfied in the combined model than in the simple itinerant electron model, especially the values of $c_1$ and $c_3$ are reduced in this combined model. Sometimes $c_5$ may become negative at low temperature and in this case we have to take into account the higher order terms than $c_5 M^5$ in the expansion of $f$ in equation (8), as the values of $M$ are always limited.

In the combined model of the localized moments and itinerant electrons the coefficients given by equation (9) always change their sign at high temperature if they are negative at low temperatures. The temperatures where these coefficients change their sign depend on the values of $g$, $S$, $N$, $K$, $J$, $a$ and the shape of the density of states for the itinerant electrons. These facts will be the main reasons for the appearance of the various kinds of MT in the rare earth compounds. In this paper we have discussed only a few simple cases and we can easily extend the present discussions to more complicated MT.

In heavy rare earth-cobalt compounds the value of $c_1$ becomes negative below the temperature $T_w$ where the paramagnetic susceptibility is expected to diverge and which should be just below the first-order transition temperature $T_w$ where $H_c = 0$, and the value of $c_3$ when $a_3 < 0$ becomes positive above a certain temperature higher than $T_w$, as seen from equation (9). Therefore, the range of temperature, where the conditions for MT, i.e. equation (19) for $c_1$, $c_2$ and $c_5$ with $c_1 > 0$, $c_2 < 0$ and $c_3 > 0$, in the combined model are satisfied, is rather narrow and this result is consistent with the experimental results for RCo$_2$ [10, 11]. The detailed comparison between the present theory...
and the experimental results for MT in RCo$_2$ will be published elsewhere.

Moreover, it is seen from equation (9) that $\chi$ can change its sign from the negative value due to $a_3$ at low temperature to the positive values with increasing temperature, even if $a_3$ does not become positive at high temperature, although the change of the sign in $a_3$ was assumed before to explain the first-order transition in RCo$_2$ [4]. As the dependence on $H$ of $M$ in YCo$_2$ does not show any bend toward the $H$-axis at 300 K [4], the value of $a_3$ in YCo$_2$ and also for itinerant electrons in RCo$_2$ will be always negative at least below 300 K.

It is interesting to note that MT in Co(S, Se)$_2$ compounds is rather sharp [15, 16], though there is a random distribution of two kinds of cobalt atoms with and without local moment. This result will be explained in the following way. The effect of the magnetostriction in the magnetizing process or any micro-structure is averaged out by the random distribution of two kinds of cobalt atoms and MT becomes sharp.

It is seen from the discussions in the previous sections the condition $a_3 < 0$ in the simple itinerant electron system, i.e. the paramagnetic susceptibility $\chi$ increases with the increasing field, is not sufficient to show MT. When $a_3 < 0$ at low temperature, $\chi$ at low temperature always increases with increasing temperature and shows a maximum, but even if $\chi$ shows a maximum in its temperature variation $a_3$ is not necessarily negative. Therefore, the existence of a maximum of $\chi$ in its temperature variation is, of course, not a sufficient condition for MT. The range of the values of the coefficients $a_1$, $a_2$ and $a_3$ which satisfy the condition for MT given by equation (19) with $a_1' > 0$, $a_3 < 0$ and $a_3 > 0$ is rather narrow and this will be the reason why there exist no simple materials which show MT, such as the transition metals to which the simple itinerant electron model can be applied.

Although TiBe$_2$, YCo$_2$ and Pd show a maximum in the temperature variation of $\chi$ and $\chi$ increases with increasing $H$, TiBe$_2$ clearly does not satisfy the condition for MT given by equation (19), then it is concluded that TiBe$_2$ will not exhibit MT. For YCo$_2$ and Pd there are not enough data at present to judge whether they exhibit MT, but it seems that these substances are also unlikely to exhibit MT.

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