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Communication in procurement: silence is *not* golden.

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Abstract

We study the effect of cheap talk between bidders on the outcome of a first-price procurement game with N sellers in which bidding is costly. Although no side-payements or commitments are allowed, we show that the game admits a unique family of symmetric equilibria in which sellers use communication to collude on a subset of participants and/or to reveal information about their valuation. Contrary to the conventional wisdom, the buyer's expected revenue and the surplus need not decrease with collusion, and the ex-ante surplus increases with the amount of information revealed in equilibrium. This is because when communication is cheap, bidders cannot directly collude on higher prices. Rather, communication leads to a competition between fewer, but more aggressive bidders, which entails more allocative efficiency and a decrease in the total wasteful entry cost.

JEL Classification: C72; D44; D82; L44.

Keywords: Communication; Procurement; Collusion.

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1 Introduction

Communication between bidders is one of the most important target of competition authorities. Most of them have developed guidelines[1] to help governments improve public procurement by fighting bid rigging, according to which bidders have to vouch for they did not communicate with any competitor regarding prices, methods used to calculate prices, and the intention to submit a bid or not. The reason is the conventional wisdom in industrial organization, according to which communication between bidders in public procurement would 1) discourage competition, 2) increase public spending, and 3) decrease efficiency. In this paper, we show in a first-price procurement game with entry cost that none of these points is true when communication is cheap talk. First, bidders cannot use cheap-talk messages to collude on higher prices. Communication only allows to coordinate on a subset of participants and/or to reveal information about individual valuations. Second, communication need not lead to a higher price for the buyer. Finally, the ex-ante surplus need not decrease with more collusion, and even increases with the amount of information transmitted by bidders in equilibrium.

To illustrate these three points, consider the following (true) story. The announcement in March 2008 that Boeing had lost a $40 billion aircraft contract to Airbus with the United States Air Force (USAF) drew angry protests in the US Congress. Upon review of Boeing's protest, the Government Accountability Office ruled in favor of Boeing and ordered the USAF to recompete the contract. Later, the entire call for aircraft was rescheduled, then cancelled, with a new call decided upon in March 2010. Because of rumors[2] according to which Airbus was going to bid aggressively, the European company was expected to win this time as well. However, Pentagon leaders surprised both competing firms by declaring Boeing's proposal was the "clear winner" in February 2011. How did Boeing win? They underbid on a fixed-price contract by several hundred million dollars. Since the two rival tankers had already satisfied 372 mandatory performance requirements, price determined the outcome. In July 2011, it was revealed that the price was so low that Boeing would take a loss on the deal: projected development costs would exceed the contract cap by $300 million. The reasons behind such

[1] See for instance www.oecd.org/gov/ethics/procurement and http://competitionbureau.gc.ca/eic/site/cb-bc.nsf/eng/00599.html

[2] Mostly spread by journalists, see for instance http://www.forbes.com/sites/beltway/2011/02/28/how-boeing-won-the-tanker-war/

a fiasco are complex. Nonetheless, rumors about Airbus's aggressiveness had apparently no collusive effect per se, since it did not impact the number of participants, and clearly benefited the USAF, in making Boeing bid a very low price.

This cast some doubts on the common idea according to which communication in procurement always hurts the buyer. Despite the complete unanimity on this issue among competition authorities, there has been surprisingly few formal theory on the subject. The aim of this paper is to study the role of cheap-talk pre-play communication between bidders on the equilibrium outcome of a first-price competitive bidding game with entry cost. We consider a buyer who seeks to obtain an object by procuring it via a sealed-bid first-price reverse auction. There are N potential sellers, who hold privately known costs of fulfilling the contract (say, of producing the object). Sellers have the option to pay a fixed and non-recoverable entry cost and bid a price, or to stay out of the competition. The entry cost can be interpreted either as a direct participation cost (travel expenses, participation fees,...), or as a bid preparation cost (time spent and resource allocated to preparing the bid, opportunity cost,...). Finally, if at least one seller participates in the procurement, the contract is awarded to the seller submitting the lowest price, and payoffs are realized. Before the bidding stage occurs a communication stage in which bidders send one round of public messages to each other. The literature on collusion in auctions usually assumes the possibility of commitment and the existence of side payments. On the contrary, we assume cheap-talk communication: messages are costless, unverifiable and non binding.[3]

We find two main results. First, even with this simple communication structure, realistic features of collusive behaviors emerge in equilibrium. Communication allows sellers to enforce "taking-turn" participation strategies, and to reveal some of their private information to each other. Collusion thus emerge in equilibrium as the combined effect of coordination and information sharing. Second, the welfare implications of cheap-talk communication between bidders are not that expected by the conventional wisdom. The buyer's expected revenue and the surplus need not decrease with collusion, and the ex-ante surplus increases with the amount of information revealed in equilibrium. Let us give a more precise description of our results.

[3] Another way to model the effects of announcements is to suppose they are costly. A large literature (starting with Fudenberg and Tirole (1983), Sobel and Takahashi (1983)) analyzes how bargainers can improve their terms of trade by undertaking costly actions.

The game admits a unique family of symmetric Perfect Bayesian Equilibria called (θ, Φ)-*equilibria*, in which bidders use communication for two reasons: 1) to coordinate on a subset of participants, and/or 2) to reveal information about their valuation. After the communication stage, a "cartel" of bidders is selected to participate in the procurement on the basis of the message exchange. Members of the cartel participate if and only if their valuation is below some maximal participating valuation, which depends on the size of the cartel and on the information about each other's valuation revealed by the message exchange. The contract is then awarded to the lowest-valuation bidder, among those who participate. In these equilibria, the way bidders are selected to participate in the procurement only partially depends on their valuation. Equilibrium strategies can then be interpreted in terms of "taking turns", and in this sense capture a relevant aspect of collusive agreements. Because participation is endogenous, sellers have an obvious incentive to look more competitive than they actually are, which makes information revelation not straightforward in equilibrium. It occurs because information a seller may credibly reveal through cheap talk has opposite effects on her expected payoff: "claiming to be weak"[4] increases the probability one's opponents participate, but increases also the level of one's opponents' bids, and then one's probability of making a winning bid. "Claiming to be strong" has the opposite effects. Therefore, any information revelation about one's valuation induces a trade-off between bidding positions and probability of participation. We show that in any equilibrium of the game, strong sellers credibly separate from "very weak" sellers, who do not participate in the procurement, even without any competitor.

In any equilibrium of the game, communication decreases the number of potential bidders either directly, by selecting a cartel of participants, or indirectly, by decreasing the maximal participating valuation. However, because of participation costs, fewer potential bidders need not decrease ex ante the buyer's revenue or the total surplus. Perhaps more surprisingly, the ex-ante total surplus increases with the amount of information transmitted via communication whenever the entry cost is positive. This comes from the fact that when it is cheap talk, communication between bidders does not decrease competition: rather, it leads to a competition between fewer, but more aggressive bidders, which results in a decrease in the total wasteful preparation cost and a better allocative efficiency.

[4] Messages sent in cheap-talk games have no informational content per se, "claiming to be weak" is an intuitive way of understanding "sending the same message in equilibrium as high-cost sellers.

The paper is organized as follows. The related literature is exposed in Section 2. Section 3 presents the procurement game with pre-play communication. In section 4, we prove the existence and the unicity of the family of (θ, Φ)-equilibria among the symmetric Perfect Bayesian Equilibria of the game. We show that communication strictly enlarges the set of equilibrium outcomes, in the sense that there is no symmetric equilibrium of the game without communication in which bidders play the strategies they play in a (θ, Φ)-equilibrium. We also discuss the robustness of (θ, Φ)-equilibria to the assumptions of the model. In section 5, we study the welfare properties of cheap-talk communication for the buyer and the total surplus. Section 6 concludes, and main proofs are gathered in the Appendix.

2 Related literature

There is few formal theory on the role of communication in collusion. Kandori and Matsushima (1998) and Compte (1998) explore the role of communication in repeated games with imperfect monitoring and privately observed signals, in which collusion is hardly sustainable because each player observes a different set of signals about other players' past actions. They assume the possibility for players to communicate at the end of each period, and prove a Folk theorem. Aoyagi (2007) studies collusion in repeated auctions when bidders report their private signals to a center, which then returns instructions to them based on the reported signal profile. He finds conditions under which an equilibrium collusion scheme is fully efficient in the sense that the bidders' payoff is close to what they get when the object is allocated to the highest valuation bidder at the reserve price. The closest work to ours is Campbell (1998), who studies non-committal coordination in second-price auction with entry, where coordination consists in playing sunspot or cheap-talk equilibria. He proves this coordination can yield higher ex-post payoff to all bidders, and that coordination through cheap talk is more efficient than coordination through exogenously generated public information. Miralles (2010) generalizes Campbell's result to more-than-two-bidder, more-than-one-object cases. Finally, some experimental works study how communication promotes collusion in the lab. Brosig et al. (2006) compare different coordination mechanisms in a first-price procurement in how they promote collusive arrangements: unrestricted pre-play communication, ability to restrict bidders' bidding range, and opportunity to implement mutual shareholding. They show that, among the three mechanisms considered, pre-play communication is the one that promotes

collusion the most. Cooper and Kühn (2011) show in the lab that pre-play messages including a credible threat to punish cheating are the most effective valuation of message for improving collusion.

This paper is also related to the literature on communication in competitive bidding games. Matthews and Postlewaite (1987), and Farrell and Gibbons (1987) introduced cheap talk to bargaining games, in which a single buyer and a single seller bargain over an exchange price. Although such coordination is different from that between bidders competing on the same side of a market, they find the existence of equilibria in which players reveal information in the same way they do in (θ, Φ)-equilibria: low-value buyers and high-value sellers are willing to jeopardize continued negotiation so as to improve their bargaining position. The two parties use talk to trade-off bargaining positions against the probability of continued negotiation. More recently, Chakraborty and Harbaugh (2003) study a multi-issue bargaining game in which player A sends a message to player B, who makes a "take-it-or-leave it" offer to player A after hearing the message. They show that while the two sides' interests are directly opposed on each issue, cheap talk can be credible if there is bundling over the two issues. In a different communication setting, Rieck (2010) studies signalling in a first-price auction with two bidders, where one of the two bidders has the option to release a signal about her valuation when she learns it. He shows that a bidder may benefit from the presence of an informative signal about her own valuation, if this signal is not too precise. Gonçalves (2008) studies the existence of a communication equilibrium in a model of a common-value English auction with discrete bidding. Experimental support for the efficiency-enhancing effects of communication is provided by Valley et al. (2002) in a double-auction à la Myerson and Satterthwaite (1983) with pre-play communication.

Finally, this paper belongs to the wide set of papers about cheap talk in games, that followed Crawford and Sobel (1982)'s seminal result. Most papers in this literature assume one sender (the informed player) and one receiver (the decision maker), while we assume that all bidders are both sender and receiver. To the best of our knowledge, the only papers assuming this kind of multiple cheap talk between decision makers are Matthews and Postlewaite (1987) and Farrell and Gibbons (1987) in which the seller and the buyer of a good communicate before bargaining, and Hagenbach and Koessler (2010) in which all participants to a beauty contest à la Keynes can send private messages to each other.

3 A procurement game with pre-play communication

3.1 The procurement game

We consider a standard independent private values procurement game with entry à la Samuelson (1985), shortly called *procurement game* in the rest of the paper. A single buyer seeks to obtain an indivisible object from N possible sellers via a sealed-bid first-price reverse auction, i.e. a procurement. We denote by $\mathcal{N}$ the set of possible sellers. Under the procurement rules, the buyer accepts the lowest bid, provided it is below her value for the object $\rho \geq 0$. Ties are resolved via uniform randomization, and losers of the procurement obtain nothing. Sellers are risk-neutral and incur a privately known cost $t \in [0, 1]$ of supplying the object. To avoid confusion with other costs, the supplying cost of a player will be called her *valuation*. Valuations are generated independently from a common distribution F on [0,1], such that [5] $t \longmapsto \frac{t}{F(t)}$ is non-decreasing. Sellers have the option to pay a participation cost $k \geq 0$ and become *active bidders*, or to opt out and remain *inactive*. We denote by $s \in \mathcal{S} = \{\text{Active}, \text{Inactive}\}$ the status chosen by a seller. The name *bidder* will refer to sellers who paid k and actually participate in the procurement. After observing which of their opponents are active,[6] bidders submit a bid $b \in \mathbb{R}_+$. Finally, inactive sellers have payoff 0, and the buyer gets payoff 0 if no seller participates.

The procurement game has a unique symmetric equilibrium in strictly increasing bidding strategies, described in Samuelson (1985), which we call here the *no-talk equilibrium*. The equilibrium status strategy is a valuation cutoff for which sellers are indifferent between participating and opting out. Above this cutoff, thus called the *maximal participating valuation*, they do not participate and get 0. Let us describe how it is determined in equilibrium. Consider a seller with valuation τ whose all opponents participate if and only if their valuation is below τ. If she participates, she will have the highest valuation among actual bidders. She will thus lose the procurement and get $-k$ except in case she faces no bidder, which will occur if every opponent has a valuation above τ. In this case, she will bid the maximal price and get $\rho - \tau$. Her expected payoff if she participates is then $(\rho - \tau)(1 - F(\tau))^{N-1} - k$. In a

[5] Although not directly related to it, this condition has the flavor of the standard decreasing hazard rate property. It is satisfied by usual distribution functions such as uniform, exponential, and normal distributions.

[6] The implications of assuming that sellers might simultaneously decide about their participation and bid are discussed in section 4.5.

symmetric equilibrium, she must have the same payoff whether she participates or not, which gives the equilibrium entry condition $(\rho - \tau)\,(1 - F(\tau))^{N-1} - k = 0$.

Below the cutoff τ, sellers participate and bid the optimal price in a first-price procurement in which valuations are distributed on $[0, \tau]$.

Proposition 1 (**No-talk equilibrium**). *In the procurement game, there is a unique symmetric equilibrium in strictly increasing strategies in which sellers participate whenever their valuation is below the maximal participating valuation τ defined by*

$$(\rho - \tau)\,\big(1 - F(\tau)\big)^{N-1} - k = 0$$

Whenever a bidder with valuation t participates with $q-1$ other bidders, she bids

$$\widehat{b}(t;\tau,q) = \begin{cases} t + \dfrac{\int_t^\tau \big(F(\tau) - F(v)\big)^{q-1} dv}{\big(F(\tau) - F(t)\big)^{q-1}} & \text{if } q \geq 2 \\ \rho & \text{if } q = 1 \end{cases}$$

Proof. See the proof of Proposition 2. □

3.2 Cheap-talk pre-play communication

We assume that before the procurement game occurs a communication stage in which sellers talk to each other by sending messages chosen in a finite set $\mathcal{M} = \{m_1, \ldots, |\mathcal{M}|\}$. The set of probability distributions on $\mathcal{M}$ is denoted $\Delta(\mathcal{M})$. We consider a simple communication structure: communication is cheap talk (costless, non binding and non verifiable), direct, simultaneous, public, and not repeated. In other words, sellers simultaneously send a unique costless message to every other seller.

The procurement game with pre-play communication we study is thus a four-stage game whose timing is as follows: a) in stage 0, nature chooses a valuation for each seller, b) in stage 1, sellers learn their valuation and send a public message to each other, c) in stage 2, sellers make their status decision based on their valuation and the message exchange, d) in stage 3, bidders make their bid decisions based on their valuation, the message exchange and the status of all players.

Histories and beliefs

Let H denote the set of histories of the game, with $H^k \subseteq H$ the subset of histories of length k. Because all players's actions are public, a seller's information set after history $h \in H^k$ simply consists of all histories $h' \in H^k$ in which she has the same valuation as in h. Sellers start the game with a common belief on each other's valuation and update these beliefs conditional on their information set at each stage. The common belief held by all sellers other than seller i about i's valuation after history h is denoted $\mu_i(h)$, and $\mu(h) := (\mu_i(h))_{i \in \mathcal{N}}$ denotes the common belief profile after h. A belief system $\mu : H \to \Delta([0,1])^N$ specifies a belief profile after any history in H. Finally, we assume correct initial beliefs, that is $\mu(h) = F^N$ after all $h \in H^1$.

Strategies

A seller's strategy has three components: a *message strategy* $m : H^1 \to \Delta(\mathcal{M})$, which determines what message to send as a function of her valuation; a *status strategy* $s : H^2 \to \{\text{Active, Inactive }\}$, which prescribes what status to choose after the message exchange; and a *bid strategy* $b : H^3 \to \mathbb{R}_+$, which determines what to bid according to the message exchange and the participation decisions, after any history in which the seller is active in stage 3.

In the sequel, we will distinguish the message strategy of a seller after $h \in H^1$ and the message actually sent by this seller after $h \in H^2$. The former is denoted $m_i(h) \in \Delta(\mathcal{M})$, and the latter $\mathbf{m}_i(h) \in \mathcal{M}$.

We denote by $\sigma_i = (m_i, s_i, b_i)$ the global strategy of seller i, by (σ_{-i}) the strategy profile of i's opponents, and by $P(h \mid (\sigma_i)_i)$ the probability of history h conditional on the strategy profile $(\sigma_i)_i$.

Equilibrium concept

In this paper, we investigate the existence of symmetric Perfect Bayesian Equilibria (PBE) of the game, defined as strategy profiles $(\sigma_i)_i$ and belief systems $(\mu_i)_i$ such that (1) players use symmetric strategies, (2) players' beliefs result from Bayesian updating of their beliefs according to their opponents' strategy whenever possible, (3) players' strategies are best responses according to their beliefs.

Focusing on symmetric equilibria makes our analysis more restrictive than Campbell's (1998), who studies how pre-play communication may enforce coordination on particular

asymmetric equilibria of a second-price auction. The reason of this restriction is the first-price setting, which makes the analysis of asymmetric equilibria barely tractable.

4 Cheap-talk equilibria

As in any game with strategic communication, the game admits babbling symmetric equilibria, in which players consider cheap talk as meaningless and play the no-talk equilibrium strategies. More interestingly, the game admits a family of non-babbling equilibria, in which players use communication for two reasons: 1) to coordinate on a subset of participants, and 2) to reveal information about their valuation. Such equilibria are characterized by two parameters in a way that will be made precise in the rest of the section: a parameter θ, part of players' message strategy, which represents the amount of information revealed by players through communication, and a parameter Φ, part of players' status strategy, which represents the way players coordinate on a subset of participants. These equilibria will be called (θ, Φ)-*equilibria*. Because the probability to participate in the procurement only partially depends on players' valuation, equilibrium strategies can be interpreted in terms of "taking turns", and capture in this sense a relevant aspect of collusive agreements.

In this section, we define, give sufficient existence conditions, and prove the existence of (θ, Φ)-equilibria. We also prove the unicity of (θ, Φ)-equilibria, in the sense that any symmetric PBE of the game is a (θ, Φ)-equilibrium. We emphasize the role of communication in equilibrium by showing that the mapping from valuations to outcomes in a (θ, Φ)-equilibrium could not occur without communication.

4.1 Definition of (θ, Φ)-equilibria

Let us call any collection of players a *cartel*, and denote by $\mathcal{C}$ the set of possible cartels. The construction of a (θ, Φ)-equilibrium works as follows. Given some cartel $c \in \mathcal{C}$, one can construct a PBE of the game without communication in which members of c are the unique *potential bidders* in equilibrium. In the off-path event a player outside the cartel participates, members of the cartel believe that the maximal participating valuation in the group of bidders is $\widetilde{\tau}$. The equilibrium bid conditional to the out-of-equilibrium subjective $\widetilde{\tau}$ gives a negative payoff to the deviator and no incentive to participate. Communication prior

to the procurement game is used by bidders to coordinate on some cartel, and, when it is informative, influences the status and bid strategies of members of the cartel.

The typical strategy played in a (θ, Φ)-equilibrium, called strategy (θ, M, Φ), is characterized by a cutoff $\theta \in [0,1]$, a subset of messages $M \subseteq \mathcal{M}$, and a function $\Phi : \mathcal{M}^N \to \mathcal{C}$ which associates a cartel of potential bidders to each message profile. According to this strategy, a seller whose valuation is below θ sends any message in M with equal probability, and sends a message in $\mathcal{M} \setminus M$ otherwise. In other words, sellers below θ credibly separate from sellers above θ, but sellers whose valuation is below θ do not separate from each other. To the message profile $(\mathbf{m}_i(h))_i$ is associated a cartel $\Phi(h)$. Sellers outside $\Phi(h)$ do not participate in the procurement, and potential bidders in $\Phi(h)$ participate if their valuation is below the maximal participating valuation after h, which depends on the number of potential bidders and on the information revealed by the message exchange. In the last stage of the game, bidders play optimally given the subset of actual bidders and their beliefs about their opponents' valuation.

Let $\mathcal{U}_M$ denote the discrete uniform distribution on M. The strategy (θ, M, Φ) is formally defined as follows.

Definition 1 (**Strategy** $(\theta, \mathbf{M}, \mathbf{\Phi})$)**.** Let $\theta \in [0,1]$, $M \subseteq \mathcal{M}$, and $\Phi : \mathcal{M}^N \to \mathcal{C}$. The strategy (θ, M, Φ) prescribes to play the following message, status, and bid strategies to player i:

- After $h \in H^1$, $m(h) \begin{cases} = \mathcal{U}_M \text{ if } t_i \leq \theta \\ \in \Delta(\mathcal{M} \setminus M) \text{ otherwise} \end{cases}$

- After $h \in H^2$, $s(h) =$Active if and only if $i \in \Phi(h)$ and $t_i \leq \tau(h)$, with $\tau(h)$ solution of

$$\big(\rho - \tau(h)\big)\left(1 - \frac{F(\tau(h))}{F(\theta)}\right)^{|\Phi(h)|-1} - k = 0$$

- After $h \in H^3$ in which i is active, $b(h) = \widehat{b}(t_i; \widetilde{\tau}(h), q(h))$, where $q(h)$ is the number of actual bidders and $\widetilde{\tau}(h)$ the subjective maximal participating valuation in this group of bidders after history h.

The message support M does not affect status and bid outcomes when sellers play (θ, M, Φ). We thus call (θ, Φ)-equilibrium any equilibrium in which sellers all play the same strategy (θ, M, Φ) with $M \subseteq \mathcal{M}$.

4.2 Existence of (θ, Φ)-equilibria

We first give three conditions on θ and Φ that are necessary and sufficient for (θ, M, Φ) to be an equilibrium strategy. Then we show the existence of (θ, Φ)-equilibria by exhibiting strategies (θ, M, Φ) which satisfy these conditions.

The first condition guarantees that randomizing over M is a best-response to (θ, M, Φ). Given a selection function, any message strategy induces a probability distribution on the set of entry coalitions. Sellers obviously hold different preferences on the set of cartels, and may not be indifferent between all messages in M. The selection function Φ must therefore have the property that all messages induce the same probability distribution on $\mathcal{C}$. With such *strategy-proof* selection functions, players cannot improve their chances of being selected in a cartel through their choice of message.

Definition 2 (**Strategy proofness**). The selection function $\Phi : \mathcal{M}^N \to \mathcal{C}$ is strategy-proof on $M \subseteq \mathcal{M}$ for player i if for all $c \in \mathcal{C}$, the probability that Φ selects c does not depend on i's message in M when i's opponents randomize over M.

The second condition is necessary for sellers to reveal whether their valuation is below or above θ. It states that θ must be above $\rho - k$, namely the valuation above which opting out is a dominant strategy. In other words, communication never transmits more information than the identity of some *out-bidders*, who never participate in the procurement. Indeed, low valuation sellers are better off when they can be distinguished from out-bidders. The condition $\theta \geq \rho - k$ guarantees that sellers who send a message in $\mathcal{M} \setminus M$ are actually out-bidders. Furthermore, it is necessary for *all* sellers with valuation below θ to send a message in M. If θ was strictly smaller than $\rho - k$, sellers in $[\theta, \rho - k]$ would strictly prefer participating alone rather than not participating, and would therefore deviate from (θ, M, Φ) in histories in which all sellers would send a message in $\mathcal{M} \setminus M$. This equilibrium condition implies that θ represents the informativeness of communication in a (θ, Φ)-equilibrium. The lowest $\theta \in [\rho - k, 1]$, the more out-bidders are identified by communication, and thus removed from the set of potential bidders by their opponents.

The third condition is necessary to enforce coordination. Sellers outside the cartel of potential bidders must have incentives not to enter. These incentives are maintained by out-of-equilibrium beliefs which work as follows. When at least one seller participates while she

is not supposed to do so, bidders believe that all valuations are below a *subjective* maximal participating valuation $\widetilde{\tau}$, which is defined so that even the zero-valuation seller is better-off opting out. It is not possible to find such out-of-equilibrium beliefs if the participation cost is too small. Indeed, the deviator will receive at least ρ if she is the sole bidder, which occurs at least with probability $\left(1 - \frac{F(\rho-k)}{F(\theta)}\right)$, i.e. the probability that a unique potential bidder does not participate. If the participation cost is smaller than her worst-case-scenario payoff $\rho\left(1 - \frac{F(\rho-k)}{F(\theta)}\right)$, no out-of equilibrium belief could give her incentives not to enter.

Proposition 2 (**Equilibrium conditions**). *Consider the strategy profile $\sigma = ((\theta, M, \Phi))_i$ and the belief system μ in which after any history $h \in H^3$ such that $P(h \mid \sigma) = 0$, sellers commonly believe that bidders' valuation is smaller than $\widetilde{\tau}(h)$. There exists $\widetilde{\tau} : H \to [0,1]$ such that (σ, μ) is a symmetric PBE of the game if and only if*

(i) Φ *is strategy-proof on* M *for all sellers;*

(ii) $\theta \geq \rho - k$*;*

(iii) $k > \rho\left(1 - \dfrac{F(\rho-k)}{F(\theta)}\right)$.

We now show that there exist at least as many (θ, Φ)-equilibria as there are subsets of cartels in $\mathcal{C}$. To do so, for any $C \subseteq \mathcal{C}$, we construct a parameterized strategy (θ, M_C, Φ_C) such that Φ_C is strategy-proof on M_C for all sellers. Consider some subset $C \subseteq \mathcal{C}$ containing ν_C cartels and let us number the cartels in C so that $C = \{c_1, \ldots, c_{\nu_C}\}$.

The message support of the strategy consists of the first ν_C messages in M, namely $M_C := \{m_1, \ldots, m_{\nu_C}\}$.

The definition of Φ_C requires an adjacent function $\widetilde{\Phi}_C : \mathcal{M}^N \to 2^{\mathcal{N}}$, which associates a subset of sellers, possibly empty, to any message profile. To fix the ideas, imagine that $\widetilde{\Phi}_C$ works in two steps. First, it picks a cartel in C on the basis of the messages sent by sellers 1 and 2. Then, it selects from this particular cartel the subset of sellers who sent a message in M_C.

Formally, consider the following $|\mathcal{M}| \times |\mathcal{M}|$-matrix

$$A := \begin{pmatrix} c_1 & \dots & c_{\nu_C - 1} & c_{\nu_C} & c_1 & \dots & c_{\nu_C} & c_1 & \dots & c_x \\ c_2 & \dots & c_{\nu_C} & c_1 & \dots & \dots & \dots & \dots & \dots & c_{x+1} \\ \vdots & \dots & \dots & \dots & \dots & \dots & \vdots & \vdots & \vdots & \vdots \\ c_{\nu_c} & c_1 & c_2 & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \vdots & \vdots & & & & & & & & \vdots \\ c_x & c_{x+1} & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{pmatrix}$$

The first line of A is obtained by replicating y times the vector $(c_1, \dots, c_{\nu_C})$ and one time the vector $(c_1, \dots, c_x)$, where $y := \left\lfloor \frac{|\mathcal{M}|}{\nu_C} \right\rfloor$ and $x := \left\{ \frac{|\mathcal{M}|}{\nu_C} \right\}$ are respectively the integer and the fractional part of $\frac{|\mathcal{M}|}{\nu_C}$. The other lines are regular permutation of the first line: $\forall\, p \in \{2, |\mathcal{M}|\}$, $A_{p,q} = A_{p-1,q+1}$ for $q < |\mathcal{M}$, and $A_{p,|\mathcal{M}|} = A_{p-1,1}$.

With $\mathbf{M}_{p,q}$ standing for the subset of message profiles in which sellers 1 and 2 respectively send m_p and m_q, the adjacent function $\widetilde{\Phi}_C$ is defined by:

$$\forall\, \mathbf{m} \in \mathbf{M}_{p,q},\ \widetilde{\Phi}_C(\mathbf{m}) = A_{p,q} \cap \left\{ i \ \mid \mathbf{m}_i \in M_C \right\}$$

If the cartel $A_{p,q}$ happens to contain only sellers whose valuation is above θ, $\widetilde{\Phi}_C(\mathbf{m})$ is the empty set. For (θ, M_C, Φ_C) to be well-defined, we thus impose that in that case, all sellers who sent a message in M_C are potential bidders. The selection function Φ_C is then defined as follows:

$$\Phi_C(\mathbf{m}) = \begin{cases} \widetilde{\Phi}_C(\mathbf{m}) \text{ if } \widetilde{\Phi}_C(\mathbf{m}) \neq \{\emptyset\} \\ \left\{ i \ \mid \mathbf{m}_i \in M_C \right\} \text{ otherwise} \end{cases}$$

By construction, such a selection function is strategy-proof on M_C for all players. Indeed, the probability distribution on C induced by seller 1's message is given by the lines of A. If all other players randomly choose a message in M_C whenever their valuation is below θ, all subsets of profiles $\mathbf{M}_{p,q}$ have the same probability conditional on $\mathbf{m}_1 = m_p$. Since any cartel c appears the same number of times in any line of A, seller 1 cannot influence the distribution probability on C with her choice of message. The same argument applies for seller 2. Other

sellers have no influence on the probability that the message profile $\mathbf{m}$ belongs to any subset $\mathbf{M}_{p,q}$, and thus on the distribution probability on C.

Claim 1. *For all $C \subseteq \mathcal{C}$, Φ_C is strategy-proof on M_C for all players.*

The corollary of Claim 1 is that (θ, M_C, Φ_C) is an equilibrium strategy, provided that θ and k satisfy the conditions of Proposition 2.

When the set of possible cartels only contains the biggest coalition ($C = \{\mathcal{N}\}$), communication is used in equilibrium only to reveal information. The $(\theta, \Phi_{\{\mathcal{N}\}})$-equilibrium is then the first-price analogous of Campbell (1998)'s *I'm-Out equilibrium*, whose existence is proved in a second-price procurement. In this equilibrium, sellers with valuation above θ send a message $\overline{m}$ while other send a message $\underline{m} \neq \overline{m}$. Then all sellers are implicitly selected to participate, and the contract is awarded to the lowest-valuation bidder, among those who participate. The difference between an I'm-Out- and a $(\theta, \Phi_{\{\mathcal{N}\}})$- equilibrium is that in the second-price setting, communication will affect only whether sellers bid, not what they bid, whereas the value of θ will also affect bid outcomes in a $(\theta, \Phi_{\{\mathcal{N}\}})$-equilibrium.

4.3 Unicity

There exist infinitely many (θ, Φ)-equilibria: the threshold θ may vary from $\rho - k$ to 1, and there are at least as many equilibrium status strategies as there are subsets of cartels in $\mathcal{N}$. However, the (θ, Φ)-equilibria family is the unique family of symmetric PBE of the game in the following sense.

Proposition 3 (Unicity). *Any symmetric PBE of the game is a (θ, Φ)-equilibrium with $\theta \geq \rho - k$.*

This proposition entails that any status equilibrium strategy can be expressed in terms of a selection function Φ. Since any probability distribution on the set of potential bidders can be induced by a selection function, this is not surprising. What is less intuitive is that in any symmetric equilibrium of the game, players reveal information in this simple way: they announce whether their valuation is below or above some threshold $\theta \in [\rho - k, 1]$. Why couldn't

players reveal more information in equilibrium? Consider some seller with valuation $\bar{t} < \rho - k$, who credibly informs her opponents that her valuation is below $\bar{t}$. This will decrease the level of equilibrium bids and, consequently, 1) her opponents' probability of participation, which is a good thing, and 2) her own probability of winning, which is not. Informing her opponents that her valuation is above $\bar{t}$ will have exactly opposite effects. This is the trade-off between probability of participation and bidding position. However, because the probability of ties is zero,[7] the trade-off turns out to be degenerated in equilibrium: if the seller $\bar{t}$ claims to have a valuation below $\bar{t}$, her probability of winning decreases to zero whenever she has a competitor, since her bid will be the maximal one. However, if she claims to have a valuation above $\bar{t}$, then she is sure to win the procurement, even if she has a competitor. A possible interpretation is the following. In any standard auction with private values and no participation cost, bidders' profits can be seen as information rents. Therefore, any information revelation about one's valuation could only decrease payoff. This is why two potential bidders have no interest in being distinguished from one another. However, any potential bidder wants to be perceived as a potential bidder, even if players hold the off-path (correct) belief that any player who enters has a valuation below $\rho - k$ when she deviates.

4.4 Role of communication

The role played by communication in a (θ, Φ)-equilibrium is twofold. Talk may be used by players to collude on a subset of participants (whenever $\Phi \neq \Phi_{\{\mathcal{N}\}}$) and/or to reveal information about their valuation (whenever $\theta < 1$). The next proposition shows that this role is not trivial, in the sense that communication strictly enlarges the set of equilibrium of the procurement game.

Proposition 4 (Role of communication). *If $k > 0$, the equilibrium mapping from profile of valuations to outcomes in a (θ, Φ)-equilibrium differs from that of the no-talk equilibrium whenever $\theta < 1$ or $\Phi \neq \Phi_{\{\mathcal{N}\}}$.*

Indeed, if $\Phi \neq \Phi_{\{\mathcal{N}\}}$, talk achieves some correlation of participation decisions. In the no-talk equilibrium, there is a deterministic mapping from valuations to status decisions. On

[7] This is because we assume a continuous set of bids. In a discrete auction framework, more information could be revealed in equilibrium.

the contrary, in any (θ, Φ)-equilibrium with $\Phi \neq \Phi_{\{\mathcal{N}\}}$, the mapping from valuations to status decisions is random: if a player is not selected in a cartel, her status decision will not depend on her valuation at all, which cannot occur in a Nash equilibrium of the game without talk. This may occur in a public correlated equilibrium. As a matter of fact, a strategy-proof selection function exactly works as a public randomization device. Since players mix over a set of possible messages, the selection function is a random variable whose realization prescribes whether to enter or not. However, whenever $\theta < 1$, players reveal information through communication, and the mapping from valuations to outcomes (status and bid decisions) could not occur in a correlated equilibrium with a public randomization device (the ex-ante probability to participate increases with θ). Therefore, because of information sharing, cheap talk does more than replicating a public randomization device.

4.5 Robustness

The assumption that bidders observe the status of their opponents prior to bidding is crucial for the existence of (θ, Φ)-equilibria. First, it is necessary for information to be revealed in equilibrium. Consider a seller, say i, whose valuation is below but close to $\rho - k$. Without status observation, i can use communication to "fool" her opponents in the following way. She sends a message in $\mathcal{M} \setminus M$, and does not participate unless the selection function selects only one potential bidder, say j. Thinking she has no opponent, j participates and bids ρ. Knowing that, i participates and wins the contract by bidding $\rho - \varepsilon$. This assumption is also necessary for coalitions to be sustainable in equilibrium. If sellers do not observe participants before bidding, they obviously have no way to "punish" deviators. Consequently, without status observation, communication would have no impact on the equilibrium outcome.

Claim 2 (**Robustness to observation**). *If sellers do not observe each other's status before bidding, the only symmetric equilibrium of the game is the no-talk equilibrium.*

Keeping participation secret from bidders could then be a way to prevent collusion. This is consistent with the competition authorities' recommendation according to which sellers should not be allowed to communicate about their intention to participate. Though, this solution is practically hard to implement in the case of public procurements, for which bid preparation may last several years.

The presence of entry costs is also crucial for the existence of (θ, Φ)-equilibria. At a first glance, it is a sufficient motivation for bidders to coordinate, since this is why losers of the procurement strictly wish ex post they had not participated. It turns out to be also necessary for coordination. Indeed, collusion is enforced by particular out-of-equilibrium beliefs. Without participation costs, no out-of-equilibrium beliefs could ever give incentives to the zero-valuation seller to stay out.

Participation costs are not necessary for information revelation, which only requires the existence of out-bidders, that is $\rho - k < 1$. Nonetheless, maximal participating valuations, and, consequently, equilibrium bids, depend on θ only if $k > 0$. Without participation costs, information transmission is thus irrelevant to equilibrium behaviors.

Claim 3 (**Robustness to positive participation costs**). *If $k = 0$, any equilibrium is strategically equivalent to the no-talk equilibrium.*

5 Welfare issues

In a (θ, Φ)-equilibrium, communication decreases the number of potential bidders either directly, by selecting a subset of participants, or indirectly, by decreasing the maximal participating valuation. In a standard private values procurement, the effect of a decrease in the number of potential participants is straightforward: both the revenue and the social welfare will decrease in expectation. Any form of collusion would then decrease the revenue (to zero if there is only one monopolist), and the social welfare (through the loss of allocative efficiency). These classical results do not hold when bidders face participation costs. In this case, Menezes and Monteiro (2000) show that there is a potential trade-off between the number of potential participants and the expected revenue, and Samuelson (1985) that the social welfare need not increase with the number of potential bidders.

In this section, we show that the impact of strategic communication on the buyer's expected revenue and the expected social surplus is not that expected by the conventional wisdom. In line with Menezes and Monteiro (2002) and Samuelson (1985), we show that more collusion, in the sense of a more *selective* selection function, need not decrease the revenue

and the social welfare in equilibrium. Furthermore, conditional on a given number of participants, the expected revenue increases with the informativeness of equilibrium. Finally, the ex-ante total surplus increases with the amount of information transmitted by communication in equilibrium. This emphasizes that the number of competitors does not entirely characterize competition. Indeed, more information leads to a competition between fewer, but more aggressive bidders, which entails a decrease in the total wasteful participation cost and a better allocative efficiency.

5.1 Communication need not decrease the expected revenue

Let us first compute the expected revenue in a (θ, Φ)-equilibrium. Suppose that a cartel of n potential bidders has been selected by Φ. The buyer's ex-post revenue is her valuation minus the winning bid. If no seller participates, the revenue is 0. If one seller participates, she bids the maximal price and the revenue is also 0. If $q \in \{2, n\}$ sellers participate, the expected winning bid is the expected value of the second smallest valuation among q bidders, given that their valuation is smaller than the maximal participating valuation τ. The expected revenue conditional on $q \geq 2$ bidders is then

$$R_\theta(q, n) := \int_0^\tau (\rho - t) dG_{(2)}(t)$$

where $dG_{(2)}(t) = q(q-1)\frac{f(t)}{F(\tau)}\frac{F(t)}{F(\tau)}\left(1 - \frac{F(t)}{F(\tau)}\right)^{q-2} dt$ is the probability density of the second smallest valuation among these bidders.

The expected revenue conditional on n potential bidders is the expectation of $R(q, n)$, i.e.

$$R_\theta(n) := \sum_{q=2}^{n} C_n^q \left(\frac{F(\tau)}{F(\theta)}\right)^q \left(1 - \frac{F(\tau)}{F(\theta)}\right)^{n-q} R_\theta(q, n)$$

The event "Φ selects a cartel of size n" can be fictitiously described as the event "a cartel of size x is picked and $n \leq x$ members of this cartel send a message in M". With $g_\Phi(x)$ standing for the probability that a cartel of size x is picked, the probability of n potential bidders is $\sum_{x=n}^{N} C_x^n F(\theta)^n \big(1 - F(\theta)\big)^{x-n} g_\Phi(x)$.

The expected revenue in the (θ, Φ)-equilibrium is then

$$R_{(\theta,\Phi)} := \sum_{x=2}^{N} g_\Phi(x) \left[\sum_{n=2}^{x} C_x^n F(\theta)^n (1 - F(\theta))^{x-n} R_\theta(n)\right]$$

We first emphasize that the expected revenue need not decrease with "more collusion". We prove it by comparing expected revenues for more or less selective selection functions in the following sense. Let Φ^l denote the selection function that selects only cartels of size l. The probability that a cartel of size x is selected by Φ^l is then 0 if $x \neq l$ and 1 if $x = l$. We shall say that Φ^l is more selective than $\Phi^{l'}$ if and only if $l < l'$.

Claim 4. *If $k > 0$, the buyer's expected revenue need not decrease with collusion.*

Indeed, consider the uniform setting with $\theta = 1$. The expected revenue is $R_{(1,\Phi^l)} = R_1(l)$. At a first glance, the buyer should always benefit from more sellers. Indeed, if $\rho = 1$ and $k \in]0,1[$, $R_1(l)$ increases with l. If, however, $\rho = 10$ and $k = 0.5$, numerical computations give $E[R_1(2)] = 8.3639$, $E_1[R(3)] = 8.2438$, $E[R_1(4)] = 8.18$, etc. The intuition is the same as in Menezes and Monteiro (2000). Suppose the number of potential bidders increases. By the equilibrium entry condition, the maximal participating valuation decreases. This has two effects on the expected revenue: a negative one, through the decrease in the probability that at least two bidders participate, and a positive one, through the decrease in equilibrium bids. When ρ is high enough, the probability that at least two bidders participate is so high that the overall effect is positive.

We cannot conclude on the general effect an increase of θ might have on the expected revenue. However, conditional on a given number of participants, the expected revenue increases with the informativeness of equilibrium.

Proposition 5. *Conditional on $q \geq 2$ participants, the buyer's expected revenue decreases with θ if and only if $k > 0$.*

In other words, as long as the information transmitted by bidders do not impact ex-post status decisions, the buyer benefits from the informativeness of communication.

5.2 Communication increases the expected surplus

Let us now compute the expected surplus of the procurement in a (θ, Φ)-equilibrium. Consider an history in which Φ selected n potential bidders with maximal participating valuation τ. The ex-post surplus depends on $t_{(1)}$, the smallest valuation among n valuations distributed on $[0, \theta]$. Since all transfers between firms and the buyer can be ignored, the expected social

surplus conditional on n is 0 if no seller participates, which happens if and only if $t_{(1)} > \tau$, and is $\rho - t_{(1)} - \sum_{t \leq \tau} k$ if at least one bidder participates, that is if $t_{(1)} \leq \tau$. The expected surplus conditional on n potential bidders writes then:

$$W_\theta(n) := \int_0^\tau (\rho - t) dG_{(1)}(t) - n \frac{F(\tau)}{F(\theta)} k$$

where $G_{(1)}(t) = 1 - \left(1 - \frac{F(t)}{F(\theta)}\right)^n$ is the distribution function of the smallest of n values distributed on $[0, \theta]$ according to $\frac{F(t)}{F(\theta)}$. With the same notation, the expected surplus is

$$W_{(\theta,\Phi)} := \sum_{x=1}^{N} g_\Phi(x) \sum_{n=1}^{x} C_x^n F(\theta)^n (1 - F(\theta))^{x-n} W_\theta(n)$$

We first emphasize that a more aggressive coordination may not hurt the expected surplus.

Claim 5. *If $k > 0$, the expected surplus need not decrease with collusion.*

As for the expected revenue, we prove it by comparing the expected surplus associated to different selection functions Φ^l, in the uniform setting with $\theta = 1$. The expected social surplus is $W_{(1,\Phi^l)} = W_1(l)$. If $\rho = 1$ and $k < 1$, $W_1(l)$ increases with l. If, however, $\rho = 3$ and $k = 0.5$, numerical computations show that $W_1(l)$ is not increasing with l: $W_1(2) = 1.7828$, $W_1(3) = 1.712$, $W_1(4) = 1.681$, $W_1(5) = 1.664$, etc. The intuition is the following. Increasing the number of potential bidders has two opposite effects on the expected surplus. On the positive side, it decreases the value of the smallest valuation, which improves the allocative efficiency of the procurement. On the negative side it increases the total amount of participation costs paid by bidders. If the buyer's valuation is small, sellers' valuations are distributed on a relatively large interval, and the potential benefits of an increase in the number of potential bidders in terms of allocative efficiency are larger than the increase in the total participation costs. If, however, the buyer's valuation is relatively so large that sellers are nearly identical, increasing the number of bidders necessarily conveys little benefit in terms of allocative efficiency. Simultaneously, it brings along the polar risks of inducing too many bidders (and then too much wasteful preparation cost) or no bidder at all, if the maximal participating valuation decreases so much that no seller participates. In this case, the overall effect of a raise in the number of potential bidders is negative.

Furthermore, with positive participation costs, more information transmission always increases the expected surplus.

Proposition 6 (**Efficiency-enhancing effect of information**). *The expected surplus in a* (θ, Φ)*-equilibrium decreases with* θ *if and only if* $k > 0$.

Since transfers are ignored, the number of actual bidders does not matter in the expected surplus, which only depends (negatively) on the expected smallest valuation and on the total participation costs. If there is no participation cost ($k = 0$), variations of θ have no impact on status strategies, and then do not affect the surplus. If the participation cost is positive however, the maximal participating valuation increases with θ. A decrease in the informativeness of equilibrium increases the expected number of potential participants and the expected value of the smallest valuation.

6 Concluding remarks

This paper studies the impact of cheap-talk communication between sellers in a procurement game with entry. We show that in any equilibrium of the game, sellers use communication to coordinate on a subset of participants and to reveal information about their valuation. The main results of the paper are that 1) the only possible information revelation separates sellers for which opting out is dominant from other sellers, 2) cheap talk between sellers may benefit to the buyer, and 3) the expected surplus increases with the amount of information transmitted in equilibrium.

The participation cost plays an important role in these results. Without it, cheap talk has no impact on equilibrium status and bid strategies, even though sellers' message may reveal information about their valuation. Therefore, the efficiency-enhancing effect of cheap talk crucially depends on the existence of participation costs. Furthermore, the participation cost gives a flavor of costly signalling to our cheap-talk setting. Indeed, sellers learn information about their opponents' valuation from the observation of their participation decision, which costs k: participation is the real and costly signal about sellers' valuation.

We do not think theses results advocate authorizing communication between sellers in public procurements, though. Clearly, if communication is not cheap, sellers can either collude

on higher prices, which indeed increases public spending, or on bid rotation schemes which are generally inefficient. Rather, we think these results emphasize how different can be the outcome implications of cheap-talk and binding communication.

Finally, sellers communicate in this model with one unique round of public and simultaneous messages. Even though this simple communication setting is enough for realistic features of collusive behaviors to appear in equilibrium, it is legitimate to wonder whether adding a mediator would enlarge the set of equilibria. The presence of a mediator in this particular collusion setting could be deemed not realistic: one may wonder who she/he could be in reality. However, because cheap-talk equilibria with general message structures converge to communication equilibria, investigating this setting would give an approximate idea of the effect of pre-play communication with more complex structures (several rounds of messages, private messages, ...). This possible research direction is left for further investigations.

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Appendix

Proof of Proposition 2. Let us show that (θ, M, Φ) is a mutual best-response under conditions (i) to (iii). Consider some seller i whose all opponents play (θ, M, Φ).

Bid best-response In stage 3, active bidders after history h consider the group of their opponents and their beliefs about their opponents. Whether h is plausible or not conditional on σ, players commonly believe that their opponents' valuation is below some value $\widehat{\tau}(h)$. Suppose that i is active after h and faces $q(h)-1$ other bidders. If $q(h)-1=0$, i bids the maximal price ρ. If $q(h)-1>0$, she bids the optimal price in a first-price procurement with $q(h)$ bidders whose valuation is distributed on $[0, \widehat{\tau}(h)]$. Suppose that i's opponents all follow the same increasing and C^1 bid strategy $b_j(.)$. Dropping the notation h, bidder i's optimal bid is the solution of:

$$\max_{b\in[t,\rho]}(b-t)\left(1-\frac{F(b_j^{-1}(b))}{F(\widehat{\tau})}\right)^{q-1}$$

The first-order condition to this problem is

$$1-\frac{F(b_j^{-1}(b))}{F(\widehat{\tau})}-(b-t)(q-1)\frac{f(b_j^{-1}(b))}{F(\widehat{\tau})b_j'(b_j^{-1}(b))}=0$$

In a symmetric equilibrium, $b_j^{-1}(b(t))=t$, so the FOC rewrites

$$b(q-1)f(t)-b'(F(\widehat{\tau})-F(t))=t(q-1)f(t)$$

Integrating between t and $\widehat{\tau}$ and rearranging gives

$$\widehat{b}(t;\widehat{\tau},q)=t+\frac{\int_t^{\widehat{\tau}}(F(\tau)-F(v))^{q-1}dv}{(F(\widehat{\tau})-F(t))^{q-1}}$$

The expected equilibrium payoff to bidder i is

$$\widehat{V}(t;\widehat{\tau},q):=(\widehat{b}(t;\widehat{\tau},q)-t)\left(1-\frac{F(t)}{F(\widehat{\tau})}\right)^{q-1}-k=\int_t^{\widehat{\tau}}\left(1-\frac{F(v)}{F(\widehat{\tau})}\right)^{q-1}dv-k$$

Status best-response Consider some history $h\in H^2$ in which $i\notin\Phi(h)$. Let us show that we can define out-of-equilibrium beliefs $\widetilde{\tau}:h\to[0,1]$ such that i has no incentive to enter, even when her valuation is zero.

Sellers $j\neq i$ outside the cartel $\Phi(h)$ will not participate since they follow σ. If i participates, she may thus face $n(h)$ opponents, where $n(h)$ stands for the number of potential bidders in $\Phi(h)$. These potential opponents will participate if their valuation is below

the equilibrium threshold $\tau(h)$. Let us now drop the notation h. For $q \in \{1, \ldots, n\}$, let E_q denote the event "q among n potential bidders participate". The probability of E_q is $P(E_q) = C_n^q \left(\frac{F(\tau)}{F(\theta)}\right)^q \left(1 - \frac{F(\tau)}{F(\theta)}\right)^{n-q}$.

In the event E_0, she gets $\rho - t - k$. In the event E_q with $q \in \{1, n\}$, she gets $\widehat{V}(t; \widetilde{\tau}(q), q)$, where $\widetilde{\tau}(q)$ is the out-of-equilibrium maximal valuation. Her expected payoff if she participates is then

$$D(t) := (\rho - t - k)P(E_0) + \sum_{q=1}^{n} \widehat{V}(t; \widetilde{\tau}(q), q)P(E_q)$$

Let us show that there exists $\widetilde{\tau}$ so that i is better-off opting out even when her valuation is 0, namely such that

$$D(0) = 0$$

A simple way to do so is to chose $\widetilde{\tau}(q)$ so that $\widehat{V}(0; \widetilde{\tau}(q), q) = \overline{v} - k \; \forall \; q$, with $\overline{v} > 0$. The deviator's payoff rewrites

$$D(0) = \rho \left(1 - \frac{F(\tau)}{F(\theta)}\right)^n + \overline{v} \left(1 - \left(1 - \frac{F(\tau)}{F(\theta)}\right)^n\right) - k$$

We chose $\overline{v}$ such that $D(0) = 0$, namely

$$\overline{v} = \left[k - \rho \left(1 - \frac{F(\tau)}{F(\theta)}\right)^n\right] \left[\left(1 - \left(1 - \frac{F(\tau)}{F(\theta)}\right)^n\right)\right]^{-1}$$

The condition (iii) $k > \rho \left(1 - \frac{F(\rho - k)}{F(\theta)}\right)$ together with the assumption that $\frac{t}{F(t)}$ is non-decreasing is necessary for $\overline{v}$ to be positive. Indeed, $\overline{v} > 0$ if and only if $k - \rho \left(1 - \frac{F(\tau)}{F(\theta)}\right)^n > 0$. Replacing k by the equilibrium value of τ, the condition becomes $\rho \frac{F(\tau)}{F(\theta)} - \tau > 0$. For any value of n, $\tau \leq \rho - k$. The assumption $\frac{t}{F(t)}$ non-decreasing implies that $\rho \frac{F(\tau)}{F(\theta)} - \tau > \rho \frac{F(\rho - k)}{F(\theta)} - (\rho - k)$.

Consider now some history $h \in H^2$ in which $i \in \Phi(h)$. The payoffs to bidder i from not participating (0) and participating and loosing the procurement $(-k)$ do not depend on her valuation. Her payoff conditional on submitting a winning bid $b - t$ decreases with her valuation. Therefore, if it is a best response for seller i to participate when her valuation is t, it still is when her valuation is $t' < t$. It follows that there exists some threshold τ_i such that i participates if and only if her valuation is below τ_i. Let us show that this threshold is the same as that of her opponents when all follow the strategy (θ, M, Φ). Suppose that the maximal participating valuation of all sellers is τ. If $\tau_i > \tau$, i will lose the auction with

probability 1 for all valuations in $]\tau, \tau_i]$, so τ_i cannot be the maximal participating valuation. Furthermore, by definition of the maximal participating valuation, it is profitable to enter for any $t < \tau$, so $\tau_i \geq \tau$. It follows that $\tau_i = \tau$. The equilibrium value of τ is the one that makes a seller indifferent between participating and opting out. If i participates with a valuation τ, her expected profit is zero unless nobody else participates, which occurs with probability $\left(1 - \frac{F(\tau)}{F(\theta)}\right)^{n-1}$. In the latter case, she gets $\rho - \tau$. The equilibrium entry condition is then

$$(\rho - \tau)\left(1 - \frac{F(\tau)}{F(\theta)}\right)^{n-1} - k = 0$$

Message best-response If i's valuation is above θ, the condition $\theta \geq \rho - k$ implies that opting out is dominant for i. Therefore, she is indifferent between all messages in $\mathcal{M}$. Suppose now that i's valuation is below θ. We first show that i is better off sending a message in M rather than in $\mathcal{M} \setminus M$.

If she sends a message in $\mathcal{M} \setminus M$, then according to σ she is not supposed to participate in stage 2. If she stays out, she gets 0. If she enters, she will get the out-of-equilibrium payoff $D(t)$, which is negative. If she sends a message in M, she will get 0 if she stays out, will get $D(t)$ if she enters while Φ did not pick her, and a strictly positive payoff if she enters while Φ picked her and her valuation is below the maximal participating valuation. Therefore, her expected payoff is the same or strictly higher when she sends a message in M rather than in $\mathcal{M} \setminus M$.

Finally, let us show that i is indifferent between all messages in M when her valuation lies in $[0, \theta]$. By definition of the strategy-proofness of Φ, if all sellers randomize over M, the probability distribution over the set of cartels does not depend on i's message. It implies that i is also willing to randomize over all messages in M.

□

Proof of Claim 1. Let σ_C stands for the strategy (θ, M_C, Φ_C). Recall that a seller playing σ_C sends a message in $\mathcal{M} \setminus M_C$ when her valuation is above θ, and sends any message in M_C with probability $\frac{1}{\nu_C}$ otherwise. Fix some seller i whose opponents play σ_C, and let us show that for all $c \in \mathcal{C}$, the probability that Φ_C selects c does not depend on i's message in M_C. If $c \notin C$, the probability that Φ_C selects c is 0 by construction. If $c \in C$, the probability that c

is selected only depends on the first two sellers messages. Consequently, sellers $i \geq 3$ cannot manipulate the result of the selection function.

Focus now on seller 1, and let us compute the probability for a cartel $c \in C$ to be selected conditional on player 1's message:

$$\begin{aligned} P(c \mid \mathbf{m}^1 = m_p, (\sigma_C)_j) &= \sum_{q \text{ s.t. } X_{p,q}=c} P(\mathcal{M}_{p,q} \mid (\sigma_C)_j) \\ &= \tfrac{1}{\nu_C} F(\theta) + (N(c,p) - 1)(1 - F(\theta)) \end{aligned}$$

with $N(c,p)$ the number elements equal to c in the p-th line of X. Yet by construction, $N(c,p) = N(c,p')$ for all p, p'. Therefore, $P(c \mid \mathbf{m}^1 = m_p, (\sigma_C)_j) = P(c \mid \mathbf{m}^1 = m_{p'}, (\sigma_C)_j)$ for all p, p'.

The same argument applies for player 2. □

Proof of Proposition 3. 1) Let us first show that the message strategy used in a (θ, Φ)-equilibrium is the only possible equilibrium strategy. To do so, we show that sellers cannot credibly reveal whether their valuation is below or above some parameter θ if $\theta < \rho - k$. To avoid heavy notation, we present a proof for 2 sellers, which can be easily extended to the N-sellers case.

Imagine an equilibrium in which the message strategy is as follows: sellers send $\underline{m}$ when their valuation is below θ, send $\overline{m} \neq \underline{m}$ otherwise, and are indifferent between the two messages when their valuation is θ, with $\theta < \rho - k$. Consider one of the two sellers, say i_θ, whose valuation is θ, and let us compute her expected payoff with message $\underline{m}$.

- If j also sends $\underline{m}$, the two players will have symmetric beliefs. They will thus have a common maximal participating valuation $\tau < \theta$ defined by $(\rho - \tau)\left(1 - \frac{F(\tau)}{F(\theta)}\right) - k = 0$. Then i_θ does not participate and gets 0.

- If j sends $\overline{m}$, players have asymmetric beliefs. Suppose that j has a maximal participating valuation denoted $\tau_j \in [\theta, \rho - k]$. If i_θ does not participate, she gets 0. If she participates, then she would do for any valuation $t \leq \theta$. Seller j's expected payoff is then $(b_j(t) - t)P(b_j(t) < b_i) - k$. j's maximal participating valuation must then satisfy $(b_j(\tau_j) - \tau_j)P(b_j(\tau_j) < b_i) - k$. Yet $b_j(\tau_j)$ must be equal to i's maximal bid, so

$P(b_j(\tau_j) < b_i) = 0$. Therefore, there can be no maximal participating valuation $\tau_j > \theta$, and j does not participate if i_θ participates.

If i_θ does not participate when she sends $\underline{m}$ and j sends $\overline{m}$, i_θ never participates when she sends $\underline{m}$ and thus cannot be indifferent between $\underline{m}$ and $\overline{m}$. If i_θ participates in this configuration, i_θ's payoff when she sends $\underline{m}$ is 0 if j sends $\underline{m}$ and is $\rho - \theta - k$ if j sends $\overline{m}$, which gives her the expected payoff

$$V^{i_\theta}(\underline{m}) = (\rho - \theta - k)\big(1 - F(\theta)\big)$$

Let us now compute i_θ's expected payoff with message $\overline{m}$.

- If j sends $\underline{m}$, i_θ does not participate (as we showed for player j in the same situation).
- If j sends $\overline{m}$, players have symmetric beliefs. They have a common maximal participating valuation $\tau' \in [\theta, \rho - k]$ defined by $(\rho - \tau')\left(\frac{1-F(\tau')}{1-F(\theta)}\right) - k = 0$. Even if j participates, i_θ has a probability 1 of making the lowest bid.

i_θ's payoff when she sends $\overline{m}$ is then 0 if j sends $\underline{m}$ and $(\rho-\theta)\left(\frac{1-F(\tau')}{1-F(\theta)}\right)+(b_i(\theta)-\theta)\left(\frac{F(\tau')-F(\theta)}{1-F(\theta)}\right)-k$ if j sends $\overline{m}$. Her expected payoff is then

$$V^{i_\theta}(\overline{m}) = (\rho - \theta)\left(1 - F(\tau')\right) + (b_i(\theta) - \theta)\left(F(\tau') - F(\theta)\right) - k(1 - F(\theta))$$

Since $b_i(\theta) < \rho$, $V^{i_\theta}(\overline{m}) < (\rho - \theta - k)(1 - F(\theta)) = V^{i_\theta}(\underline{m})$.

Therefore, either i_θ never participate when she sends $\underline{m}$ and thus strictly prefers $\overline{m}$ to $\underline{m}$, or i_θ participates when she sends $\underline{m}$ and j sends $\overline{m}$, in which case she strictly prefers $\underline{m}$ to $\overline{m}$. In both cases, i_θ cannot be indifferent between the two messages.

2) Let us now show that any (symmetric) equilibrium status strategy can be written with a selection function as follows: $s_i(h)$ =Active if and only if $\{i \in \Phi(h) \text{ and } t_i \leq \tau(h)\}$, with $\tau(h)$ defined by the equilibrium entry condition.

We know that, in any equilibrium, players' status strategy are cutoff strategies of the form $s_i(h)$=Active if and only if $t_i \leq \tau_i(h)$ (see the proof of Proposition 2). It follows that $s_i(h)$ =Inactive if and only if $\{\tau_i(h) = 0 \text{ or } t_i > \tau_i(h) > 0\}$.

Let $H_i \subseteq \mathcal{H}$ the subset of histories after which $\tau_i(h) > 0$. For each history h, define $C(h)$ the set of sellers (possibly empty) who participate with a positive probability after h:

$C(h) = \{i \in \mathcal{N} \mid h \in H_i\}$. In any symmetric equilibrium, players will have common beliefs after $h \in H^2$. Then as we showed in the proof of Proposition 2, all players in $C(h)$ must have the same maximal participating valuation $\tau(h)$, defined by the equilibrium entry condition.

Therefore, there exist a function $\Phi : \mathcal{M}^N \to \mathcal{C}$ defined by $\Phi(h) = C(h)$ such that $s_i(h)$ =Inactive if and only if $\{i \notin \Phi(h)$ or $i \in \Phi(h)$ and $t_i > \tau(h)\}$.

□

Proof of Proposition 4. • In any Nash equilibrium of the game without communication, players make their status decision on the basis of their prior belief only. There is thus a deterministic mapping from valuations to status decisions. In a (θ, Φ)-equilibrium however, a player with valuation t may or may not participate ex post, according to which cartel is selected after the message exchange. Since players mix over the set of possible messages, the selection of a cartel is the occurrence of a random event independent of players' valuation.

• Let us show that conditional on at least two participants in the procurement, the equilibrium bid in a (θ, Φ)-equilibrium strictly increases with θ. Recall that the equilibrium bid in a group of q bidders whose maximal participating valuation is τ is

$$\widehat{b}(t;\tau,q) = t + \frac{\int_t^\tau \big(F(\tau) - F(v)\big)^{q-1}}{\big(F(\tau) - F(t)\big)^{q-1}} dv$$

The maximal participating valuation and the equilibrium bid are continuous in θ on $[\rho - k, 1]$. Let us differentiate $\widehat{b}(t;\tau,q)$ with respect to θ. With τ' standing for $\dfrac{\partial \tau}{\partial \theta}$,

$$\frac{\partial \widehat{b}}{\partial \theta} = \frac{f(\tau)\tau'(q-1)(F(\tau) - F(t))^{q-2}}{(F(\tau) - F(t))^{2(q-1)}} \left[(F(\tau) - F(t)) \int_t^\tau (F(\tau) - F(v))^{q-2} dv - \int_t^\tau (F(\tau) - F(v))^{q-1} dv \right]$$

For all $v \geq t$, $F(\tau) - F(t) \geq F(\tau) - F(v)$, so the term into brackets is positive and $\dfrac{\partial \widehat{b}}{\partial \theta}$ signs like τ'. Differentiating the equilibrium entry condition with respect to θ gives:

$$-\tau'\big[F(\theta) - F(\tau) + (\rho - \tau)(n-1)f(\tau)\big] + (\rho - \tau)(n-1)F(\tau)\frac{f(\theta)}{F(\theta)} = 0$$

It follows that $\tau' > 0$ and $\dfrac{\partial \widehat{b}}{\partial \theta} > 0$.

The case $\theta = 1$ corresponds to the equilibrium of the game without communication. Therefore, for any $\theta < 1$, $\widehat{b}(t;\tau,q)$ cannot be an equilibrium of the game without communication.

□

Proof of Claim 2. Suppose that players do not observe each other's status before bidding.

1) Let us first show that in this case, there can be no information revelation in equilibrium. Consider a bidder i whose valuation is below but close to the maximal participating valuation $\rho - k$. Suppose that her opponents follow some strategy (θ, M, Φ) with $\theta \geq \rho - k$. If i sends a message in M and participates, then the only situation in which she can make a positive profit is when all her opponents send a message in $\mathcal{M} \setminus M$. In this case, i is indifferent between messages in M and in $\mathcal{M} \setminus M$. If i sends a message in $\mathcal{M} \setminus M$ however, she can also make a positive payoff in the situation in which there is only one potential bidder: this opponent will participate and bid ρ with probability one (since she is sure to be the sole bidder), and i can profitabily win the auction by participating and bidding $\rho - \varepsilon$. Therefore, it is dominant for players with valuation in $]\rho - k - \varepsilon, \rho - k]$ to deviate from (θ, M, Φ).

2) Second, without status observation, there can be no collusion in equilibrium. Indeed, the construction of a (θ, Φ)-equilibrium rests on the construction of a PBE of the game without communication in which players who are not selected in a cartel have no incentives to enter. If players do not observe each other's participation before bidding, they trivially do not observe whether players deviate from (θ, M, Φ), and deviators cannot be "punished" by out-of-equilibrium beliefs.

Consequently, the only equilibrium of the game is the $(1, \Phi_{\{\mathcal{N}\}})$-equilibrium, namely the no-talk equilibrium.

□

Proof of Claim 3. Suppose that $k = 0$. First, let us show that communication cannot enforce collusion. Recall that in a (θ, Φ)-equilibrium, coordination works so that mixed message strategies induce a random selection of the sellers into an entry coalition, and that the incentives for the outsiders not to enter are maintained by particular out-of-equilibrium beliefs. If there is no participation cost, no out-of-equilibrium beliefs could give incentives to the zero-valuation seller to stay out.

Indeed, consider an history h in which n potential bidders are selected by Φ. Recall that the expected payoff to a seller outside the cartel who considers participating is

$$D(t) = (\rho - t)P(E_0) + \sum_{q=1}^{n} \widehat{V}(t; \widetilde{\tau}(q), q)P(E_q) - k$$

where E_q is the event "q among the n potential bidders participate" and $\widetilde{\tau}(q)$ the out-of-equilibrium maximal participating valuation. If $k = 0$, no value of $\widetilde{\tau}$ could ever make $D(t)$ negative.

Second, $k = 0$ does not preclude information transmission in equilibrium: if $\rho < 1$, $(\theta, \Phi_{\mathcal{N}})$-equilibria are informative whenever $\theta \in [\rho, 1]$. But the maximal participating valuation (ρ) and the equilibrium bid ($\widehat{b}(t; \rho, q)$) would not depend on θ. All $(\theta, \Phi_{\mathcal{N}})$-equilibria are then strategically equivalent in terms of procurement behaviors. □

Proof of Proposition 5. Fix some (θ, Φ)-equilibrium. The expected revenue of the buyer is

$$R_{(\theta,\Phi)} := \sum_{x=2}^{N} g_\Phi(x) \left[\sum_{n=2}^{x} C_x^n F(\theta)^n (1 - F(\theta))^{x-n} R(n) \right]$$

with

$$R(n) = \sum_{q=2}^{n} C_n^q \left(\frac{F(\tau)}{F(\theta)} \right)^q \left(1 - \frac{F(\tau)}{F(\theta)} \right)^{n-q} \int_0^\tau (\rho - t) q(q-1) \frac{f(t)}{F(\tau)} \frac{F(t)}{F(\tau)} \left(1 - \frac{F(t)}{F(\tau)} \right)^{q-2} dt$$

If $k = 0$, the maximal participating valuation is $\tau = \rho$, regardless of θ. Thus the expected revenue $R(q, \tau)$ does not depend on θ. Suppose that $k > 0$. Differentiating $R(q, \tau)$ with respect to τ gives:

$$\frac{\partial}{\partial \tau} R(q, \tau) = \frac{f(\tau)}{F(\tau)} q(q-1) \int_0^\tau (\rho - t) \frac{f(t)}{F(\tau)} \frac{F(t)}{F(\tau)} \left(1 - \frac{F(t)}{F(\tau)} \right)^{q-3} \left(q \frac{F(t)}{F(\tau)} - 2 \right) dt$$

With $x(t) := \frac{F(t)}{F(\tau)}$, the former expression rewrites:

$$\frac{\partial}{\partial \tau} R(q, \tau) = \frac{f(\tau)}{F(\tau)} q(q-1) \int_0^\tau (\rho - t) x'(t) \big(q x(t) - 2 \big) (1 - x(t))^{q-3} dt$$

A primitive of $x'(t)\big(qx(t) - 2\big)(1 - x(t))^{q-3}$ is $-x(t)^2\big(1 - x(t)\big)^{q-2}$. Integrating by parts boils down to:

$$\frac{\partial}{\partial \tau} R(q, \tau) = -\frac{f(\tau)}{F(\tau)} q(q-1) \int_0^\tau x(t)^2 \big(1 - x(t)\big)^{q-2} dt$$

which is negative. Since $\frac{\partial}{\partial\theta}\tau > 0$, an increase in θ entails a decrease in the buyer's expected revenue, as long as the number of actual participants is not impacted.

□

Proof of Claim 5. As for the buyer's expected revenue, we prove the claim by comparing the expected welfare for selection functions Φ^l, in the case where $\theta = 1$ and valuations are uniformly distributed. Here again, it is straightforward to show that in the uniform setting, the expected social surplus in the $(1, \Phi^l)$-equilibrium is $W_{(1,\Phi^l)} = W_1(l)$.

- If $\rho = 1$ and $k < 1$, $W_1(l)$ increases with l. Indeed, in this case, $\mu(1, l) = 1 - k^{1/l}$ and

$$W_1(l) = \frac{l}{l+1} - lk + k^{1+1/l}\frac{l^2}{l+1}$$

Consider that l is continuous, and let us show that $\frac{\partial}{\partial l}W_1(l) > 0$.

$$\frac{\partial}{\partial l}W_1(l) = -k + k^{1+\frac{1}{l}}\left[\frac{l^2+2l}{(l+1)^2} - \frac{\ln k}{l+1}\right] + \frac{1}{(l+1)^2}$$

Let $g(k)$ stand for $\frac{\partial}{\partial l}W_1(l)$, and let us show that $g(k) \geq 0$ for all $k \leq 1$. The first two derivatives of g are $g'(k) = -1 + k^{\frac{1}{l}}\left[\frac{l}{l+1} - \frac{\ln(k)}{l}\right]$ and $g''(k) = \frac{k^{\frac{1}{l}-1}}{l}\left[\frac{l^2-l-1}{l(l+1)} - \frac{\ln(k)}{l}\right]$. Since $g''(k) > 0$ and $g'(1) = \frac{l}{l+1} - 1 < 0$, $g'(k) < 0$ for all $k < 1$. Since in addition, $g(1) = 0$, $g(k) > 0$ for all $k \leq 1$.

- If $\rho = 3$ and $k = 0.5$, $W_1(l)$ is not increasing in l. Numerical computations show that $W_1(2) = 1.7828$, $W_1(3) = 1.712$, $W_1(4) = 1.681$, $W_1(5) = 1.664$, etc...

□

Proof of Proposition 6. The second implication is proved as follows. Suppose that $k = 0$. For any number of potential bidders, the maximal participating valuation is ρ and does not depend on θ. In other words, any seller whose valuation is below the maximal price participates, regardless of the message strategy cutoff θ and the number of sellers sending each message. Therefore, the expected surplus of the procurement does not depend on θ.

The first implication is proved by showing that if $k > 0$, $W_{(\theta,\Phi)}$ decreases with θ on $[\rho-k, 1]$. We will use the notation τ_n for the maximal participating valuation in a group of n potential bidders, and the notation $Z(n, \theta) := F(\theta)^n W_\theta(n)$.

Deriving $W_{(\theta,\Phi)}$ with respect to θ gives:

$$\frac{\partial}{\partial\theta}W_{(\theta,\Phi)} = \sum_{x=1}^{N} g_\Phi(x)\left[Z'_\theta(x,\theta) + \sum_{n=1}^{x-1} C_x^n (1-F(\theta))^{x-n-1}[(1-F(\theta))Z'_\theta(n,\theta) - f(\theta)(x-n)Z(n,\theta)]\right]$$

Let us remark that for $n \geq 2$,

$$\begin{cases} Z'_\theta(n,\theta) = (n-1)f(\theta)g(n,\tau_n) \\ \text{and} \\ Z(n-1,\theta) = g(n,\tau_{n-1}) \end{cases}$$

with $g(n,x) = n\int_0^x (\rho - t)f(t)(F(\theta)-F(t))^{n-2}dt - nkF(x)F(\theta)^{n-2}$.

Furthermore, for $\theta > \rho - k$, $Z'_\theta(1,\theta) = 0$. Therefore, the expression $\frac{\partial}{\partial\theta}W_{(\theta,\Phi)}$ can be arranged so that

$$\frac{\partial}{\partial\theta}W_{(\theta,\Phi)} = \sum_{x=1}^{N} g_\Phi(x)\sum_{n=2}^{x}\big(1-F(\theta)\big)^{x-n} f(\theta)(n-1)nC_x^n\big[g(n,\tau_n) - g(n,\tau_{n-1})\big]$$

The difference $g(n,\tau_n) - g(n,\tau_{n-1}))$ is equal to

$$kF(\theta)^{n-2}\big(F(\tau_{n-1}) - F(\tau_n)\big) - \int_{\tau_n}^{\tau_{n-1}} (\rho - t)f(t)\big(F(\theta)-F(t)\big)^{n-2}dt \tag{1}$$

Let us sign the expression (1). For $t < \tau_{n-1}$, $(\rho - t)f(t)\big(F(\theta)-F(t)\big)^{n-2}$ is larger than $(\rho - \tau_{n-1})f(t)\big(F(\theta)-F(\tau_{n-1})\big)^{n-2}$. Therefore,

$$(1) < \big(F(\tau_{n-1}) - F(\tau_n)\big)\left[-(\rho-\tau_{n-1})\big(F(\theta)-F(\tau_{n-1})\big)^{n-2} + kF(\theta)^{n-2}\right]$$

Yet by the equilibrium entry condition,

$$-(\rho-\tau_{n-1})\left(F(\theta)-F(\tau_{n-1})\right)^{n-2} + kF(\theta)^{n-2} = 0$$

Consequently, $\dfrac{\partial}{\partial\theta}W_{(\theta,\Phi)} < 0$. □