Design sensitivity analysis for nonlinear magnetostatic problems by continuum approach

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Résumé. — En utilisant la notion de dérivée matérielle de la mécanique des milieux continus et une méthode de variable adjointe, pour des problèmes magnétiques non linéaires bidimensionnels, la formule de sensibilité est dérivée sous forme d'une intégrale de contour sur la surface de modification. Les coefficients de sensibilité sont numériquement évalués avec les variables d'état et adjointes calculées à partir du logiciel existant d'éléments finis. Pour vérifier cette méthode, le problème d'optimisation de forme d'un quadripôle est décrit

Abstract. — Using the material derivative concept of continuum mechanics and an adjoint variable method, in a 2-dimensional nonlinear magnetostatic system the sensitivity formula is derived in a line integral form along the shape modification interface. The sensitivity coefficients are numerically evaluated from the solutions of state and adjoint variables calculated by the existing standard finite element code. To verify this method, the pole shape design problem of a quadrupole is provided.

Introduction.

Magnetic devices usually consist of an iron, a current coil, an air and permanent magnet and many of them are designed to operate in the saturation region of iron. This paper presents a procedure of exact sensitivity formula derivation for the geometrical design of a nonlinear magnetic system using the material derivative concept of continuum mechanics and an adjoint variable method [1, 2] just as in the linear magnetic system [3, 4]. The derived sensitivity formula is expressed only with respect to the geometrical variation of design interface, the both sides of which can be any two among an iron, a current, an air and a permanent magnet. In the formula in the form of line integral, the integrand is a function of state and adjoint variables. However, since it is almost impossible to obtain the exact state and adjoint variables, its approximate values are calculated using an existing finite element code. With the approximate ones the derived sensitivity formula is numerically evaluated by Gaussian quadrature. This method does not depend on a discretization model and does not require a differentiation of stiffness matrix and forcing vector as is needed in that that is based on a finite element method [5].

Since nonlinearity characteristics in a magnetic system arise usually in the permeability of iron, in this paper we will derive only the sensitivity formula with respect to the interface with different permeability characteristics. By adding the obtained nonlinearity term to the general sensitivity formula of linear magnetic system, the general sensitivity formula in a nonlinear magnetic system is obtained.

Nonlinear magnetostatic system in variational form.

In a nonlinear magnetic system, the variational form of a governing equation for the magnetic vector potential is obtained as follows, by multiplying, by virtual variable $\bar{A}$, both sides of the Ampere's law in the differential form and integrating them over the analysis domain

$$\int_{\Omega} \nu(B^2) B(A)^{\mathrm{T}} B(\bar{A}) \, \mathrm{d}\Omega - \int_{\Omega} J\bar{A} \, \mathrm{d}\Omega = - \int_{\Gamma} \nu(B^2) B_{\mathrm{t}}(A) \bar{A} \, \mathrm{d}\Gamma \quad \text{for all } \bar{A} \in \Phi \tag{1}$$

where ν is reluctivity, $B(\ )$ curl operator to its argument and Φ the space of an admissible state variable. The right-hand side of (1) is zero with Dirichlet and homogeneous Neumann boundary conditions. The variational equation (1) is rewritten with another notation a_{v} and l as

$$a_{\mathrm{v}}(\nu(B^2), A, \bar{A}) = l(\bar{A}) \quad \text{for all} \quad \bar{A} \in \Phi \tag{2}$$

where $\quad a_{\mathrm{v}}(\nu(B^2), A, \bar{A}) = \int_{\Omega} \nu(B^2) B(A)^{\mathrm{T}} B(\bar{A}) \, \mathrm{d}\Omega \quad \text{and} \quad l(\bar{A}) = \int_{\Omega} J\bar{A} \, \mathrm{d}\Omega\,.$

Note that since ν in (1) is a function of flux density, a_{v} is neither linear to A nor symmetrical to A and $\bar{A}$. In (2), a small variation of state variable provides the following relation

$$a_{\mathrm{v}}(\nu_0 + \Delta\nu, A_0 + \Delta A, \bar{A}) = l(\bar{A}) \quad \text{for all} \quad \bar{A} \in \Phi \tag{3}$$

where $\nu(B^2) = \nu_0 + \Delta\nu$ and $A = A_0 + \Delta A$. By linearizing (3) at A_0, it is approximated as

$$\int_{\Omega} (\nu_0 B(\Delta A)^{\mathrm{T}} B(\bar{A}) + 2\,\kappa B(A_0)^{\mathrm{T}} B(\Delta A) B(A_0)^{\mathrm{T}} B(\bar{A})) \, \mathrm{d}\Omega =$$

$$= l(\bar{A}) - a_{\mathrm{v}}(\nu_0, A_0, \bar{A}) \quad \text{for all } \bar{A} \in \Phi \tag{4}$$

where $\kappa = \mathrm{d}\nu/\mathrm{d}B^2$. (4) represents Newton-Raphson algorithm expressed in the variational form. Since the left-hand side of (4) was linearized to ΔA, the left-hand side of (4) is linear to ΔA and $\bar{A}$. Here, we define another bilinear form a_{n} for later use in the next section as

$$a_{\mathrm{n}}(C, \bar{C}) = \int_{\Omega} (\nu B(C)^{\mathrm{T}} B(\bar{C}) + 2\,\kappa B(A)^{\mathrm{T}} B(C) B(A)^{\mathrm{T}} B(\bar{C})) \, \mathrm{d}\Omega \tag{5}$$

where ν, κ and A are given. Note that a_{n} is bilinear and symmetrical to its arguments.

Sensitivity formula derivation in nonlinear magnetostatic system.

The material derivative concept and the material derivative formulas can be referred to [2, 4]. But some relations used in this paper for the sensitivity formula derivation have been rewritten. The point-wise material derivative of the state variable is expressed as

$$\dot{\phi} = \phi' + \nabla\phi^{\mathrm{T}} V(x) \tag{6}$$

where ϕ' is the partial derivative to time t, ∇ the gradient operator and $V(x)$ the velocity field. The point-wise material derivative of the virtual variable can be taken as

$$\dot{\bar{\phi}} = \bar{\phi}' + \nabla\bar{\phi}^{\mathrm{T}} V(x) = 0 . \tag{7}$$

When an integral function F is defined as

$$F = \int_{\Omega_t} f_t(x_t)\, \mathrm{d}\Omega ,$$

its material derivative at Ω is proved [2] to be

$$\dot{F} = \int_{\Omega} f'(x)\, \mathrm{d}\Omega + \int_{\Gamma} f(x)\, V^{\mathrm{T}} n\, \mathrm{d}\Gamma \tag{8}$$

where Γ is the boundary of Ω and n is the outward normal vector on Γ.

To derive the sensitivity formula to interface variation, consider in figure 1 two regions with different permeability characteristics along interface γ. The variational governing equation is expressed as follows with Dirichlet b.c. on Γ^0 and homogeneous Neumann b.c. on Γ^1

$$a_v(\nu, A, \bar{A}) = l(\bar{A}) \quad \text{for all} \quad \bar{A} \in \Phi \tag{9}$$

where

$$a_v(\nu, A, \bar{A}) = \int_{\Omega_1} \nu^*(B^2)\, B(A^*)^{\mathrm{T}} B(\bar{A}^*)\, \mathrm{d}\Omega + \int_{\Omega_2} \nu^{**}(B^2)\, B(A^{**})^{\mathrm{T}} B(\bar{A}^{**})\, \mathrm{d}\Omega ,$$

$$l(\bar{A}) = \int_{\Omega_1} J^* \bar{A}^*\, \mathrm{d}\Omega + \int_{\Omega_2} J^{**} \bar{A}^{**}\, \mathrm{d}\Omega$$

and

$$\Phi = (\bar{A} = (\bar{A}^*, \bar{A}^{**}),\ \bar{A}^* = \bar{A}^{**} \text{ on } \gamma \text{ and } \bar{A}^{**} = 0 \text{ on } \Gamma^0) .$$

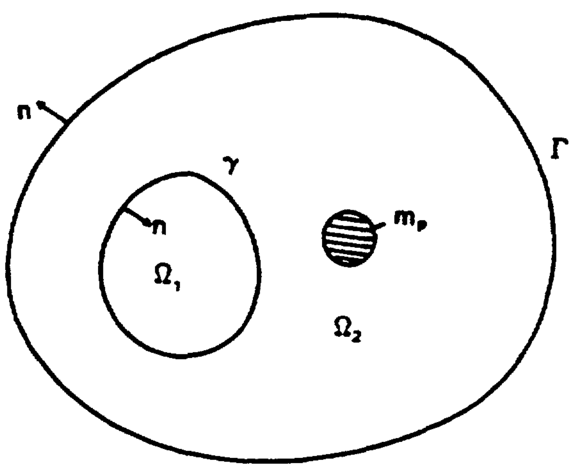

Fig. 1. — Interface problem.

Consider an objective function defined inside a region Ω_2 in figure 1 as

$$F = \int_{\Omega_2} g(B(A^{**}))\, m_p \,\mathrm{d}\Omega \tag{10}$$

where g is a differentiable function and m_p is constant on Ω_p and zero outside of Ω_p.

First, the material derivative of system equation (9) with (6, 7, 8) and $V = 0$ on Γ gives

$$a_n(\dot{A}, \bar{A}) + a'_n(A, \bar{A}) = \dot{l}(\bar{A}) \quad \text{for all} \quad \bar{A} \in \Phi \tag{11}$$

where

$$a'_n(A, \bar{A}) = -\int_{\Omega_1} (\nu^* B(\bar{A}^*)^T B(\nabla A^{*T} V) + 2\,\kappa^* B(A^*)^T B(\bar{A}^*) B(A^*)^T B(\nabla A^{*T} V))\,\mathrm{d}\Omega$$

$$-\int_{\Omega_2} (\nu^{**} B(\bar{A}^{**})^T B(\nabla A^{**T} V) + 2\,\kappa^{**} B(A^{**})^T B(\bar{A}^{**}) B(A^{**})^T B(\nabla A^{**T} V))\,\mathrm{d}\Omega$$

$$-\int_{\Omega_1} \nu^* B(A^*)^T B(\nabla \bar{A}^{*T} V)\,\mathrm{d}\Omega - \int_{\Omega_2} \nu^{**} B(A^{**})^T B(\nabla \bar{A}^{**T} V)\,\mathrm{d}\Omega$$

$$-\int_{\gamma} (\nu^* B(A^*)^T B(\bar{A}^*) - \nu^{**} B(A^{**})^T B(\bar{A}^{**}))\, V^T n \,\mathrm{d}\Gamma \tag{12}$$

and

$$a_n(\dot{A}, \bar{A}) = a_n(\dot{A}^*, \bar{A}) + a_n(\dot{A}^{**}, \bar{A}) . \tag{13}$$

Using the fact $J = 0$ on γ, the right-hand side of (11) is expanded as

$$\dot{l}(\bar{A}) = -\int_{\Omega_1} J^* \,\nabla\bar{A}^{*T}\, V \,\mathrm{d}\Omega - \int_{\Omega_2} J^{**} \,\nabla\bar{A}^{**T}\, V \,\mathrm{d}\Omega . \tag{14}$$

For later use, (11) can be rewritten as

$$a_n(\dot{A}, \bar{A}) = \dot{l}(\bar{A}) - a'_n(A, \bar{A}) \quad \text{for all} \quad \bar{A} \in \Phi . \tag{15}$$

If A and V are given, (15) is a variational equation for $\dot{A}$.

Secondly, the material derivative of the objective function (10) with (6, 7) and $m'_p = 0$ and $m_p = 0$ on γ gives

$$\dot{F} = \int_{\Omega_2} g_B(A^{**})^T B(\dot{A}^{**})\, m_p \,\mathrm{d}\Omega - \int_{\Omega_2} g_B(A^{**})^T B(\nabla A^{**T} V)\, m_p \,\mathrm{d}\Omega . \tag{16}$$

Thirdly, to express (16) in terms of velocity field V, the first integral of (16) must be rewritten. For that, by replacing $\dot{A}$ by a virtual variable $\bar{\lambda}$ and equating it to the bilinear form a_n in λ and $\bar{\lambda}$, an adjoint variable equation is introduced as [2]

$$a_n(\lambda, \bar{\lambda}) = \int_{\Omega_2} g_B(A^{**})^T B(\bar{\lambda}^{**})\, m_p \,\mathrm{d}\Omega \quad \text{for all} \quad \bar{\lambda} \in \Phi \tag{17}$$

where the left-hand side can be expanded using the definition (5).

Next, to express (16) with V instead of $\dot{A}$, we couple (15) and the first integral of (16). For that, (17) is evaluated at $\bar{\lambda} = \dot{A}$ since $\dot{A} \in \Phi$ to give

$$a_n(\lambda, \dot{A}) = \int_{\Omega_2} g_B(A^{**})^T B(\dot{A}^{**}) \, m_p \, d\Omega \,. \tag{18}$$

Similarly, (15) is also evaluated at $\bar{A} = \lambda$ since $\lambda \in \Phi$ to give

$$a_n(\dot{A}, \lambda) = \dot{l}(\lambda) - a_n'(A, \lambda) \,. \tag{19}$$

One sees in (18) and (19) that their left-hand sides are equal from the fact that a_n is symmetrical to its arguments from the definition (5). Thus, we have the following relation

$$\int_{\Omega_2} g_B(A^{**})^T B(\dot{A}^{**}) \, m_p \, d\Omega = \dot{l}(\lambda) - a_n'(A, \lambda) \,. \tag{20}$$

Using (20), (16) is expressed as

$$\dot{F} = \dot{l}(\lambda) - a_n'(A, \lambda) - \int_{\Omega_2} g_B(A^{**})^T B(\nabla A^{**T} V) \, m_p \, d\Omega \tag{21}$$

(21) can be expanded with the relations (12) and (14). But it still has region integrals. To transform them to the surface integrals on γ, we need the following procedure. The adjoint variable equation (17) in the variational form is equivalent to the differential equation

$$\nabla \times P(\lambda^*) = 0 \qquad \text{on} \quad \Omega_1 \tag{22}$$

$$\nabla \times P(\lambda^{**}) = \nabla \times (g_B(A^{**}) \, m_p) \quad \text{on} \quad \Omega_2 \tag{23}$$

with b.c. $\lambda^{**} = 0$ on Γ^0 and $P_t(\lambda^{**}) = 0$ on Γ^1

where

$$P(\lambda) = \nu B(\lambda) + 2\,\kappa B(A)^T B(\lambda) B(A) \,. \tag{24}$$

And the interface condition for λ is $P_t(\lambda^*) = P_t(\lambda^{**})$ and $B_n(\lambda^*) = B_n(\lambda^{**})$ on γ.

In (17), since the geometrical symmetry is also conserved for λ, the same Neumann condition is applied to Γ^1 and since there is no external source for λ, the Dirichlet condition on Γ^0 is zero. And since the source term in (23) is defined inside of Ω_2, continuity to the tangential component of P is satisfied as the interface condition. Using the relation (1), (22) and (23) are transformed to the variational form as

$$\int_{\Omega_1} P(\lambda^*)^T B(\bar{\lambda}^*) \, d\Omega = \int_{\gamma} P_t(\lambda^*) \, \bar{\lambda}^* \, d\Gamma \quad \text{for all} \quad \bar{\lambda}^* \in \Phi \tag{25}$$

$$\int_{\Omega_2} (P(\lambda^{**})^T B(\bar{\lambda}^{**}) - g_B(A^{**})^T B(\bar{\lambda}^{**}) \, m_p) \, d\Omega =$$

$$= - \int_{\gamma} (P_t(\lambda^{**}) \, \bar{\lambda}^{**} - (g_B(A^{**}))_t \, m_p \, \bar{\lambda}^{**}) \, d\Gamma \quad \text{for all} \quad \bar{\lambda}^{**} \in \Phi \,. \tag{26}$$

By substituting for the region integrals of (21) the results of (25) and (26) evaluated at $\bar{\lambda}^* = \nabla A^{*T} V$ and $\bar{\lambda}^{**} = \nabla A^{**T} V$ and ones of (1) evaluated at $\bar{A}^* = \nabla \lambda^{*T} V$ on

Ω_1 and $\bar{A}^{**} = \nabla\lambda^{**\mathrm{T}} V$ on Ω_2, (21) is rewritten as

$$\dot{F} = \int_\gamma (P_t(\lambda^*)\nabla A^{*\mathrm{T}} V - P_t(\lambda^{**})\nabla A^{**\mathrm{T}} V + \nu^* B_t(A^*)\nabla\lambda^{*\mathrm{T}} V - \nu^{**} B_t(A^{**})\nabla\lambda^{**\mathrm{T}} V)\,\mathrm{d}\Gamma + \int_\gamma (\nu^* B(A^*)^{\mathrm{T}} B(\lambda^*) - \nu^{**} B(A^{**})^{\mathrm{T}} B(\lambda^{**}))\, V^{\mathrm{T}} n \,\mathrm{d}\Gamma . \quad (27)$$

Using the interface conditions for A and λ and the definition (24), the sensitivity formula is obtained from (27) as

$$\dot{F} = \int_\gamma \left((\nu^* - \nu^{**}) B(\lambda^*)^{\mathrm{T}} (B(A^{**})) + \frac{2\kappa^*}{\nu^*} B(A^*) B_t(A^*) B_t(A^{**}) \right) V^{\mathrm{T}} n \,\mathrm{d}\Gamma . \quad (28)$$

In many shape design problems of nonlinear magnetostatic system, only one side of interface is nonlinear and the other side is linear. In this case, if we choose Ω_1 as a linear region, κ^* becomes zero and the sensitivity formula is the same as that in linear magnetostatic problems. In the nonlinearity term of (28), only the tangential components contribute to the variation of reluctivity. It is because the normal ones are continuous on the interface.

Numerical evaluation of sensitivity formula.

After the sensitivity formula is obtained, one calculates its numerical value as in the following procedure :

(I) Solve the system equation (2) for the state variable (magnetic vector potential) by the finite element code for nonlinear analysis.
(II) Calculate the adjoint load in (17) with the result of (I).
(III) Solve the adjoint equation (17) for the adjoint variable by the finite element code.
(IV) Calculate the sensitivity coefficients by numerical line integration for the derived sensitivity formula (28).

In (I), one solves the system equation of nonlinear system iteratively. But in (III), since the system matrix of the adjoint equation is the same as the converged one after enough iteration in (I), we do not have to solve it iteratively but we can solve it directly once. In (IV), one evaluates the sensitivity coefficients by a numerical line integration. In the above numerical procedure one can see that the sensitivity calculation can be done externally to a finite element code and so the use of this algorithm will make it possible to do the structured programming for the entire shape optimization procedure.

Numerical example.

The pole shape design problem of a quadrupole is provided to verify this continuum approach in a nonlinear magnetostatic system. An analysis model is shown in figure 2 where the objective is to obtain a linear distribution of flux density on the x-axis by optimizing the pole shape of the quadrupole. The objective function is defined as

$$F = \int_{\Omega_2} \sum_{i=1}^{nn} [(B_y/X_{0i} - S_0)^2\, \delta(X - X_{0i})]\,\mathrm{d}\Omega \quad (29)$$

where nn is the number of measure points, X_{0i} is i-th measure point, S_0 the target slope of flux density and $\delta(x)$ the Dirac-delta function at the original point. The adjoint load in the adjoint variable equation is provided from the material derivative of the objective function (29) and the adjoint variable equation is

$$a_n(\lambda, \bar{\lambda}) = \int_{\Omega_2} 2 \sum_{i=1}^{nn} [(B_y/X_{0i} - S_0)/X_{0i} B_y(\bar{\lambda}) \delta(X - X_{0i})] d\Omega . \tag{30}$$

In this problem, since the air region on the interface can be taken as Ω_1, we use the same sensitivity formula as that of linear magnetostatic problems. The design variables are defined as the y-components of all the node positions on half the pole surface. In an optimization technique, the steepest descent is used as a search direction and the step-size in the line search is determined by the ratio of the objective function value to the norm of the sensitivity vector. Figure 3 shows the convergence history of the objective function with an iteration number. The final shape of the quadrupole compared to that in linear magnetostatic case is shown in figure 4 where the nonlinear result is more projected than the linear one to compensate the saturation effect of iron. At the final shape the flux density distribution is shown in figure 5 where the maximum relative deviation to the target value is 0.06 %. Above results are almost the same as in [5]. The sensitivity formula is exact, but since its numerical values depend on accuracy of numerical analysis, under the same conditions of modelling almost the same results as in [5] are expected.

Conclusion.

By exploiting the variational equation which is not discretized, the exact sensitivity formula is derived and it is expressed as a line integral of a function of state and adjoint variables along the movable interface. Results presented in this paper show the feasibility and numerical efficiency of software implementation of the theoretical design sensitivity analysis with

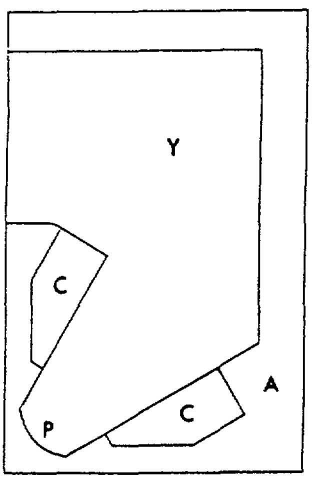

Fig. 2.

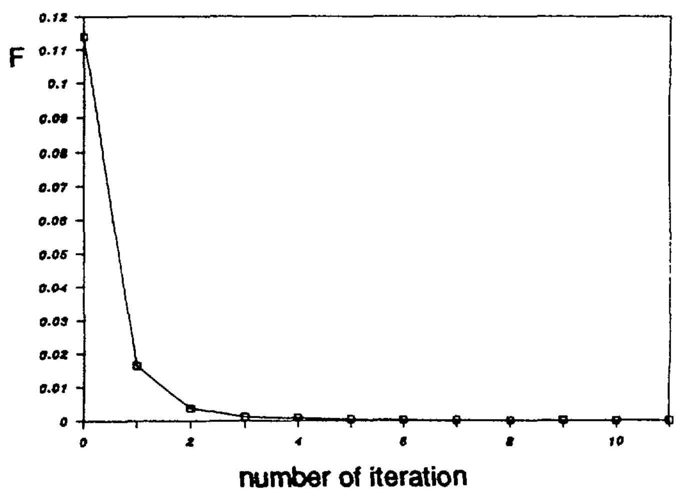

Fig. 3.

Fig. 2. — Analysis model.

Fig. 3. — Objective function with iteration.

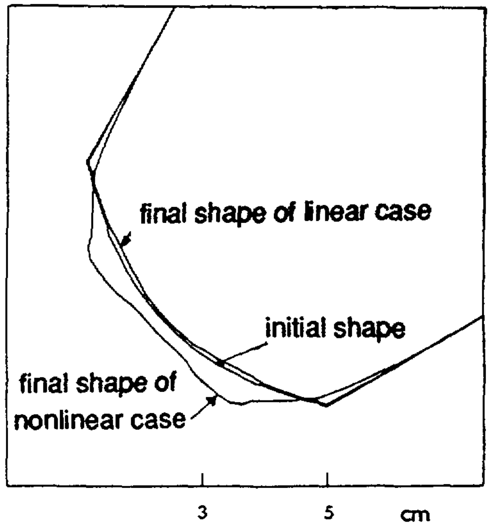

Fig. 4. — Initial and final shape of quadrupole.

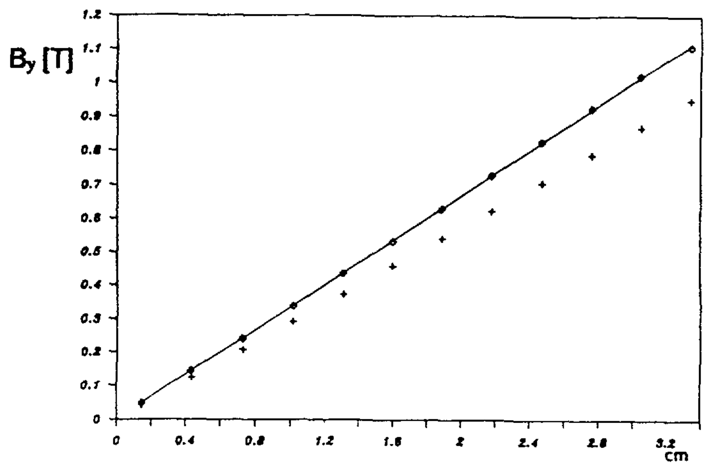

Fig. 5. — Flux density distribution on x-axis : (–) target, (+) initial, (◇) final.

existing finite element code. An important advantage of the method is that one can do sensitivity evaluation outside an existing code and one does not have to embed sensitivity analysis program into an existing finite element code. The variational method and the adjoint variable method used in this paper does not require a differentiation of system matrix and a forcing vector as in that based on finite element discretization [5].

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