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A NOTE ON THE FERROMAGNETIC FARADAY EFFECT AT CENTIMETRE WAVELENGTHS

By F. F. ROBERTS.

Sommaire. — On discute la rotation du plan de polarisation d'une onde électromagnétique transmise à travers une matière ferromagnétique parallèlement à un champ magnétostatique appliqué à l'extérieur. Le mécanisme de cette rotation est différent pour les ondes optiques et les ondes centimétriques et l'on indique une théorie pour l'effet centimétrique. Des mesures à $\lambda \approx 3$ cm pour quelques ferrites de magnésium sont en concordance demi-quantitative avec la théorie. Les écarts sont peut-être dus à la dissipation négligée.

1. **Introduction.** — The Faraday effect to be considered here is the rotation of the plane of polarisation of a plane-polarised wave transmitted through the material on test when a magnetic field is applied parallel to the direction of propagation. The associated Kerr effect is the rotation of the plane of polarisation of a similar wave, reflected nearly normally from one face of the sample, due to an applied magnetic field normal to that face. Much work has been carried out on these effects in the past at optical wavelengths [1]. The Faraday effect has been investigated mainly for diamagnetic and paramagnetic materials, and the Kerr effect for ferromagnetic materials. The most thorough experimental work on the ferromagnetic optical Faraday effect seems to have been that by Cau [2], and a theoretical analysis for this case has been carried out by Hulme [3]. Van Vleck and Hebb [4] and J. Becquerel [5] have discussed the correlation between the Faraday rotation and the magnetisation for paramagnetic salts as functions of temperature and applied field strength.

Cau's measurements confirmed the very large Faraday rotations, about 20 000° : mm at a field strength of 10 000 Oe, found by earlier workers for thin iron films, and also showed that the transmitted wave was elliptically polarised. The rotations were always positive, that is, in the same sense as the positive current flow in the magnet solenoid surrounding the sample. The rotations increased nearly linearly with field strength but showed signs of incipient saturation at fields above 10 000 Oe. The Kerr effect showed a similar behaviour as the field was varied, but the rotation was always negative, and rapidly approached a limiting value of about 0.2° as the film thickness was increased beyond about 50 mμ. The reflected wave was also elliptically polarised to a small extent.

The Kerr effect is relavant in the optical case, because (*a*) it alone has so far been measured for a range of ferrites and ferromagnetic alloys [6], and (*b*) Cau's phenomenological theory [2] has shown that the Faraday and Kerr effects should in general show similar trends with field strength and wavelength except that their algebraic signs should be opposite. Among the metals, Mn — Bi alloy is the only case recorded giving a positive Kerr rotation over part of the visible spectrum, but magnetite, Zn-ferrite and Cu-ferrite all [6] apparently give positive Kerr rotations (and presumably therefore negative Faraday rotations) over a band in the longer optical wavelengths.

The established quantum-mechanical theories [1] of the optical Faraday effect (including Hulme's theory for ferromagnetics [3] show that the Faraday rotation should vary as the square of the frequency of the incident radiation for all frequencies well below the atomic (visible or ultra-violet) absorption frequencies of the material. Molecular (infra-red) resonances do not appear to contribute to the rotation [7]. We should therefore expect, even for materials giving optical rotations as large as Cau measured for Fe, extremely small Faraday rotations at the highest accessible radio frequencies.

In the present Note, measurements of considerable Faraday rotations at a wavelength of 3 cm are reported for certain ferrite materials, and the measurements are shown to be qualitatively consistent with the current theories of gyromagnetic precession resonance in such materials [8].

2. **Technique of Measurement.** — The measurements to be described have been confined to a wavelength of about 3 cm, but further measurements will be carried out in the near future at both longer and shorter wavelengths using the same technique but necessarily different sets of apperatus. The apparatus employed is essentially the waveguide analog of that usual for optical Faraday rotation measurements.

The test signal is obtained from a standard low-power velocity-modulation tube („ Klystron " type) and is passed into rectangular waveguide of internal cross-section 25.4 mm × 12.7 mm, in which is inserted a matched variable attenuator of the resistive vane type. The empty rectangular waveguide uniquely determines the plane of polarisation of the propagating wave, and it is necessary to pass to a circular cross-section to enable the Faraday rotation to take place freely in the succeding sample-filled section of guide. A transition length of waveguide accordingly follows the attenuator and has the effect of smoothly transforming the H_{01}-mode of wave in the rectangular guide into a H_{11}-mode in the 22.2 mm diameter circular guide cross-section at its output. The plane of polarisation is, however, maintained through the transition section.

The sample, in the form of polycrsytalline powder, is pressed between quarter-wave slabs of low-loss dielectric („ Distrene ") in a short length of circular guide, the sample thickness usually being a few millimètres, and this length of guide is arranged centrally along the axis of the magnetic field coil. The free face of each of the quarter-wave slabs may be coated with a uniform layer of colloidal graphite („ Aquadag ") for the purpose of suppressing multiple reflections, a purpose largely achieved in the optical case by tilting the sample face slightly off-normal to the incident radiation.

Two forms of analyser have been used in the cm-wave apparatus : (i) a simple silicon crystal detector mounted transversely across the diameter of a further section of circular waveguide, rotatable relative to the remainder of the system, and backed with a tuning piston, or (ii) a second transition-section of guide transforming the circular H_{11}-mode back into the rectangular H_{01}-mode, followed by a standard matched detector unit in rectangular guide, this transition section and detector being rotatable as a whole relative to the remainder of the apparatus. The second type of analyser is preferred, as only one polarisation component is affected by residual detector mismatch in this type. Apart from this, the use of both types is exactly as in the optical analog, the analyser being rotated to obtain zéro or minimum response for each value of the magnetic field, the angle of rotation being measured relative to the analyser position obtained with zero magnetisation.

3. **Experimental results for certain Ferrites.**

— In the preliminary measurements unexpectedly large rotations, together with large ellipticities, were observed for several materials. On investigation these effects were shown to be largely due to waves multiply-reflected between the wave-guide transition sections. For most of the ferrites examined the Aquadag coatings referred to earlier have been found to reduce these reflections to unimportant proportions. In some cases, however, especially for materials of high effective dielectric permittivity, marked ellipticity remains, almost certainly arising from multiple reflections between the Distrene slabs and entirely within the sample. A set of measurements, in which the effect of spurious reflections are believed to be negligible, are shown graphically on the attached figure 1.

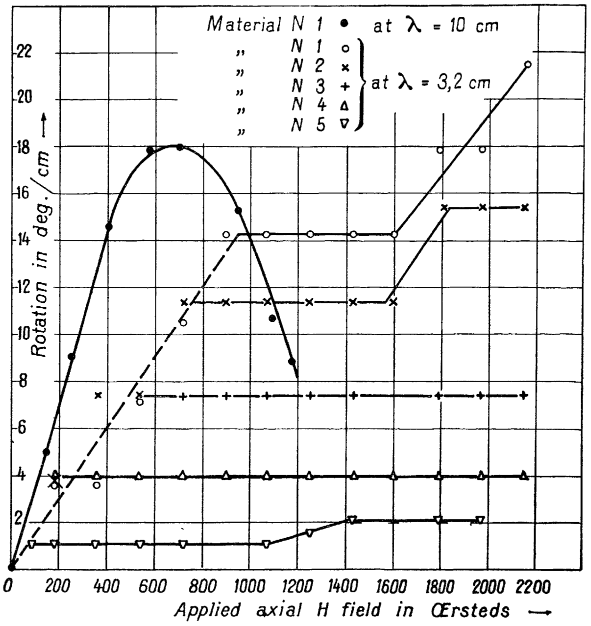

Fig. 1. — Effet Faraday à 9 470 Mc/s à 20°C.

The materials concerned are a series of magnesium ferrites in which the magnesium is progressively replaced by aluminium. The approximate compositions and room temperature saturation magnetisations are given in Table I. These materials were kindly supplied by Dr Welch of the Royal College of Science in London.

TABLE I.

Material (No).	Composition.	$4\pi J_{sat}$. (gauss).
1..........	$MgO\,Fe_2O_3$	895
2..........	$MgO(Al_2O_3)_{0.1}(Fe_2O_3)_{0.9}$	770
3..........	$MgO(Al_2O_3)_{0.2}(Fe_2O_3)_{0.8}$	540
4..........	$MgO(Al_2O_3)_{0.3}(Fe_2O_3)_{0.7}$	325
5..........	$MgO(Al_2O_3)_{0.4}(Fe_2O_3)_{0.6}$	54

The direction of the rotation is positive in all cases, and is seen to rise for applied field strengths greater than the $4\pi J_{sat}$ value, to a constant angle approximately proportionnel to the $4\pi J_{sat}$ value in each case.

4. **An elementary theory of the rotation.**

— A theory of the rotation is implicit in Polder's analysis of the gyromagnetic precession resonance effect in ferromagnetics [8]. This analysis assumes that the applied magnetostatic field is large enough to ensure saturation and neglects all damping effects. It is found that, when the static field is along the direction of propagation of an unbounded linearly-polarised plane wave, the latter splits into positive-and negative circularly-polarised components for which the effective magnetic susceptibilities are given by :

$$\chi_{\pm} = \frac{\dfrac{J_c}{H_c}}{1 \mp \dfrac{\omega}{\omega_c}}, \tag{1}$$

where H_0 and J_0 are the static field and magnetisation and $\omega_c = \gamma H_c$ is the precession frequency of the ionic magnetic tops in H_c (for electron spins $\gamma = 2.8$ Mc : s : Oe). The rotation of the plane of polarisation in an axial distance z is then

$$\left.\begin{aligned} \theta &= \frac{\pi z (n_{-} - n_{+})}{\lambda}, \\ \theta &= \frac{\omega z (n_{-} - n_{-})}{2c}, \end{aligned}\right\} \tag{2}$$

$$n_{\pm}^2 = \varepsilon(1 + 4\pi\chi_{\pm})$$

ε being the dielectric permittivity.

For the sample shape employed, the demagnetising field will be large, so that under the experimental conditions we have $\omega_c \ll \omega$. Furthermore, for the materials considered we may approximate

$$4\pi\gamma J_c \ll \omega$$

so that the rotation may be simplified to :

$$\theta \approx \frac{\omega z}{2c} \frac{4\pi J_c \gamma}{\omega} \sqrt{\varepsilon} = \frac{\omega_J z \sqrt{\varepsilon}}{2c}. \tag{3}$$

where $\omega_J = 4\pi J_c \gamma$.

Thus, within the approximations stated, the rotation should be positive and (i) independent of ferquency; (ii) independent of H_c and (iii) proportional to $4\pi J_c$. The exact analysis predicts interesting behaviour for higher values of H_c, in particular a finite range of H_c over which only the negative circular component is propagated, but damping may prevent this last condition from geing fully realized.

5. **Discussion.** — The predictions (ii) and (iii) of the theory are seen to be substantially confirmed by the observations once saturation is reached in each case, and the sign is also correct. Measurements at other frequencies are still required to test prediction (i). Insertion of the relevant numerical constants in equation (3), however, leads to predicted rotations about twice as great as those observed. The discrepancy is possibly a consequence of the neglected damping. If a damping frequency ω_1 is arbitrarily introduced in such a way that equation (1) is replaced (consistently with Frenkel's equation [9] for the linearly polarised susceptibility) by :

$$\chi_{\pm} = \frac{\dfrac{J_c}{H_c}}{\left\{1 - \left(\dfrac{\omega_1}{\omega_0}\right)^2\right\}^{\frac{1}{2}} \mp \left(\dfrac{\omega}{\omega_0} - \dfrac{j\omega_1}{\omega_0}\right)}. \tag{4}$$

then a value of ω_1 comparable to ω appears to be required to give agreement with the observations. It remains to be seen whether equation (4) will satisfactorily describe the experimental behaviour of the rotation over a wide frequency range.

6. **Conclusion.** — The Faraday rotation technique provides a new and to some extent independent method of investigating the magnetic properties of ferrites and similar materials up to the highest accessible radio frequencies. The degree of correlation between the preliminary test results here reported and the theory outlined, indicate the desirability of extending both measurements and theory to fill in the gap between microwaves and the infra red, taking account in the theory of exchange interaction at the higher microwave frequencies.

7. **Remark.** — Some measurements at $\lambda = 10$ cm have been made since the above report was prepared. The result for material NI has been added to the original graph. The peak of rotation occurs approximately at the H value which gives an absorption peak for a small sample of the same material in a transverse static field in a rectangular cavity.

8. **Acknowledgment.** — This Note is an account of work carried out at the Post Office Research Station, Dollis Hill, and the author desires to thank the Engineer-in-Chief of the Post Office and the Chief Scientist of the Ministry of Supply, for permission to publish the material.

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