Economically rational expectations theory: evidence from the WTI oil price survey data

Georges Prat, Remzi Uctum

Economically rational expectations theory: evidence from the WTI oil price survey data

Georges PRAT   and   Remzi UCTUM[(*)]
CNRS / University of Paris-X Nanterre (France)

Janvier 2006

Abstract – In the light of the economically rational expectation theory, this article shows how an expert chooses an optimal oil price forecast function given that information is costly. In this framework we propose an expectational process which nests all processes considered in the literature. By aggregating individual processes, it is shown that the overall expectational process may result from individual mixing effects and/or group heterogeneity effects. Using Consensus Forecast survey data, for three and twelve month horizons, we find that the rational expectation hypothesis is rejected and that none of the traditional extrapolative, regressive and adaptive processes and macroeconomic fundamentals is relevant. We show however, that a combination of the three traditional processes explains satisfactorily oil price expectations, which appear to exert a stabilizing strength in the oil market.

KEYWORDS: expectations formation, oil price

CLASSIFICATION: D84, G14, Q43

[(*)] **Corresponding author. MODEM, Université Paris-X Nanterre, 200 avenue de la République, 92000 Nanterre, France. Tel: +33 1 40977848. Fax: +33 1 40975968. E-mail:** uctum@u-paris10.fr**.**

Modeling price expectations in the oil market: evidence from survey data

1. Introduction

Oil price shocks are among the main impulse shocks introduced in macroeconomic models to study the time path describing adjustment towards equilibrium (Garratt and Hall, 1997). Generally, these shocks (inventory shocks, credibility of OPEC's intervention in a target zone model, OPEC's pricing arrangements, conflicts, drilling field discoveries…) are introduced exogenously in these models. In turn, by altering supply and demand on different markets in the economy (Wirl, 1991), they produce feedback effects on oil prices because of changes in the fundamentals (Hawdon, 1989; Walls, 1987) but also because of changes in price expectations (Hawdon, 1987, 1989; Rauscher, 1988; Hammoudeh and Madan, 1995). Hence, a complete understanding of the oil price movements requires a better understanding of how price expectations are formed. However, the processes that govern these expectations remain widely unknown. Indeed, literature deals with price expectations implicitly, that is, by focusing on oil future demand prospects (Stevens, 1987) or by supposing that oil supply is based on a rational expectation scheme (Walls, 1987). Furthermore, studies concerning rational expectation hypothesis are not conclusive regarding this hypothesis. Moosa and Al-Loughani (1994) find that futures prices on the WTI appear to be inefficient predictors of spot prices, and that the time-varying risk-premium hypothesis tested using a GARCH-M framework in not fully capable of explaining this result, leaving the question of whether or not expectations are rational unsolved.

Using private WTI oil price expectations revealed by Consensus Forecasts survey data, this article aims to shed light on these issues.[1] The *economically rational expectations* framework introduced by Feige and Pearce (1976) is appropriate to study the formation of expectations to the extent that it is based on a cost-and-advantage analysis of information. In this framework, agents collect and use information until equality is reached between the unit cost of information and the marginal gain due to the use of this information. This provides a general theoretical basis for all processes from the naive process to the Muthian rationality through the traditional extrapolative, regressive and adaptive processes and any combination of them. This article sets out to show which of these *a priori* possible processes are validated by empirical data.

The paper is organized as follows. Section 2 examines the microfoundations of expectational processes and discusses the aggregation conditions of individual expectations. Section 3 explores which process is effectively validated by survey data. Section 4 provides concluding remarks.

2. The economic rationality of expectational behavior

---

[1] Numerous studies based on survey data have attempted to model expected prices in various markets (exchange rates, stock prices, commodity prices, interest rates), but no such literature is devoted to oil price expectations.

Despite an extensive literature on the microeconomic foundations of the rational expectation hypothesis (REH), studies considering a general form of rationality to explain the formation of expectations, where the REH appears to be a limit case, are surprisingly scant. To partially fill this gap in the literature we devote this section to this issue.

2.1. The case of costless information and unlimited cognitive ability

Here, the dynamics function is observed without bias and thus expectations are homogenous and rational. For different representations of the variable to be forecasted, say the logarithm of the oil price p_t, the optimal forecast models can be determined in the sense that they provide the minimum forecast error variance. Consider the following simple examples:

(i)   Assume that the dynamics of the change in p_t is a series of random observed shocks which are common knowledge :

$$p_t - p_{t-1} = \xi_t - (1 - \beta)\xi_{t-1}, \tag{1}$$

where ξ_t is a white noise with zero mean and constant variance σ_ξ^2 and represents the effect on the endogenous variable of qualitative or quantitative announcements or events that are observed but unpredictable. Solve for ξ_t using the lag operator on the one hand, and take expectations at t-1 on equation (1) on the other hand. Writing the former relationship one period backward and reporting it into the latter, one can verify that the expectations follow the adaptive process (Muth, 1960):

$$E_t p_{t+1} - p_t = (1 - \beta)(E_{t-1} p_t - p_t) \tag{2}$$

Furthermore, from equation (1) and from the adaptive process write both the shock ξ_t and the forecast $E_t p_{t+1}$ conditionally on the current and past observations of p_t. Then it can be easily shown that $E_t p_{t+1} - p_{t+1} = -\xi_{t+1}$. Therefore the adaptive process is optimum since the forecast error variance is minimum (equal to σ_ξ^2).

(ii)   Assume that the change in p_t has an autoregressive representation of the form:

$$p_t - p_{t-1} = \gamma(p_{t-1} - p_{t-2}) + \eta_t \tag{3}$$

where η_t is a white noise with zero mean and constant variance σ_η^2. It follows that the optimal expectational process is the extrapolative process (Baillie and MacMahon, 1992):

$$E_t p_{t+1} - p_t = \gamma(p_t - p_{t-1}) \tag{4}$$

since this process provides the minimum forecast error variance σ_η^2.

(iii)   Assume that the change in p_t exhibits a mean-reversion dynamics of the form:

$$p_t - p_{t-1} = \mu(\bar{p} - p_{t-1}) + \upsilon_t, \quad 0 < \mu < 1 \tag{5}$$

where $\bar{p}$ is some target value and υ_t an unobservable white noise with constant variance σ_υ^2. It therefore follows that the optimal expectational process is the regressive process (Holden, Peel and Thomson, 1995):

$$E_t p_{t+1} - p_t = \mu(\bar{p} - p_t) \tag{6}$$

since this process provides the minimum forecast error variance σ_υ^2.

(iv) Similarly, if the change in p_t is represented by lagged macroeconomic factors, the expected change in oil price is given by actual and lagged macroeconomic variables.

Of course one can consider a complex dynamics defined as some combination of the preceding basic dynamics and it can be easily shown that the optimal expectational process is then a mixed process. In this case, expectations are still homogenous and rational in the Muthian sense.

2.2. The case of costly information and limited cognitive ability

Contrary to the homogeneity and rationality of expectations assumed in 2.1, the empirical results based on survey data on oil price expectations suggest that expectations are heterogeneous and biased. In accordance with the economically rational expectation hypothesis, we relax the assumption that agents use the true dynamics function and show that this framework is consistent with heterogeneous and biased expectations.

2.2.1. Microeconomic behavior of forecasters

The optimizing behavior of an economically rational forecaster can be represented as follows. Assume that the forecaster j bases their forecast on three independent types of information: history of observed and expected values of oil prices (type z), other macroeconomic information that can be used to forecast oil prices (type v) and stochastic observable oil shocks (type φ). Assume further that the squared forecast error is a function of the amounts $H_{z,t}^j$, $H_{v,t}^j$ and $H_{\varphi,t}^j$ of information of type z, v and φ used at the time the forecast was generated, such that:

$$(E_t^j p_{t+1} - p_{t+1})^2 = f(H_{z,t}^j, H_{v,t}^j, H_{\varphi,t}^j) \qquad f_{H_i}' < 0, f_{H_i}'' > 0 \quad i = z, v, \varphi$$

The signs of the first and second derivatives represent the assumption that the marginal efficiency of the amount of information in terms of the forecast accuracy decreases. Let the unit price of collecting (purchase price of the information, time spent to get the information) and processing (cost of analyzing the information) each type of information for agent j be $c_{z,t}^j$, $c_{v,t}^j$ and $c_{\varphi,t}^j$ respectively. The total cost supported by the agent is then:

$$C_t^j = \pi_t^j f(H_{z,t}^j, H_{v,t}^j, H_{\varphi,t}^j) + c_{z,t}^j H_{z,t}^j + c_{v,t}^j H_{v,t}^j + c_{\varphi,t}^j H_{\varphi,t}^j$$

where $\pi_t^j > 0$ represents the aversion of agent j of misestimating future oil prices and $\pi_t^j f(.)$ is a loss function.

The solutions of the minimization of this total cost yields to the equilibrium conditions of the forecaster:

$$c_{i,t}^j = -\pi_t^j \, f'_{H_{i,t}^{j*}} \quad (i = z, v, \varphi) \qquad (7)$$

where the optimal values $H_{z,t}^{j*}$, $H_{v,t}^{j*}$ and $H_{\varphi,t}^{j*}$ are chosen by the forecaster such that the marginal gain, i.e. the marginal decrease in the loss function, due to the decrease in the forecast error equals the unit cost.[2] For an agent j, a given time t and a given type of information, Figure 1 illustrates the relationship between the quantity of information, its unit cost and the marginal gain, the amounts of the other types of information assumed to be given. In the upper quadrant, the bisecting line represents the equilibrium locus corresponding to (7), while in the lower quadrant the effect of the amount of information on the marginal gain is sketched. For a given unit price c_1, the point D corresponds to an initial sub-optimal situation in which the marginal gain is less than c_1 and the information used is H_D. By diminishing the amount of information used, i.e. as H_D reduces to H_1^*, the agent increases the marginal gain up to the optimum level N. At this point, the situation D has converged towards the equilibrium point E_1. Consider next a new cost $c_2 < c_1$ that moves the equilibrium point E_1 down to E_2. It is clear that at this new point, the optimal quantity of information has increased from H_1^* to H_2^*.[3] The maximum unit price $c_{\max}$ is the limit price such that for all prices exceeding or equal to $c_{\max}$, the optimal amount of information chosen is zero. Such a situation may be obtained for a lower value of $c_{\max}$ if the marginal gain line is flatter with respect to H such that it crosses the marginal gain axis at M' instead of M: this corresponds to a lower sensitivity of the loss function to new information. The case of $H^* = 0$ may particularly be viewed as a noise trader's behavior for whom all unit cost is higher than their $c_{\max}$ and whose forecast is purely stochastic. All these cases show why it may be rational for an agent not to use all available information. Conversely to the case $H^* = 0$, the point $H_{\max}$ refers to the maximum amount of costless information the agent is able to collect and process in the limits of their cognitive capacity. The part of the H axis below $H_{\max}$ represents the inefficiency locus of information, i.e. some kind of "information trap": beyond this threshold, the agent cannot reduce their squared forecast error. In the case the cognitive capacity of the agent is unlimited and the cost supported is zero, $H_{\max}$ corresponds to all the relevant information, the agent thus forms rational expectations in the Muthian sense and the squared error is the lowest possible. For a higher level of π_t^j, the marginal gain line shifts to the right. The agent who has a higher aversion to misestimating future oil prices either collects more information for a given unit price, say c_1, or accepts to purchase their optimal amount of information, say H_1^*, at a higher unit price.

---

[2] It is interesting to point out the analogy between the total information cost minimizer forecaster's behavior and the utility maximizer consumer's behavior.

[3] If the cost of the information obtained by imitating is less than the one supported when the information is purchased to its own price, then E_1 represents an isolated forecaster's equilibrium while E_2 corresponds to a mimetic agent's equilibrium.

To the optimal amounts of information $H_{z,t}^{j*}$, $H_{v,t}^{j*}$ and $H_{\varphi,t}^{j*}$ is associated the dynamics function the agent will choose to represent the change in prices. The higher (lower) the optimal amount of information of each type, the more complex (simple) the dynamics and the higher (lower) the number of the possible states of the nature considered by the agent. For instance, assume $H_{v,t}^{j*}=H_{\varphi,t}^{j*}=0$ and that $H_{z,t}^{j*}$ is weak: the agent is likely to refer only to a simple autoregressive dynamics to form their forecast, in which case their expectations will be extrapolative. To a larger $H_{z,t}^{j*}$ corresponds a higher ordered extrapolative process. Similarly, if $H_{\varphi,t}^{j*}$ is the only non-zero set of information, the agent will observe a series of random shocks which will lead them to form adaptive expectations. If now $H_{v,t}^{j*}$ is the optimal non-zero set of information, then the expected change in prices depends on the expected change of macroeconomic variables. Obviously, when various types of information are used, a source of complexity concerning the dynamics other than an intensive use of a given type of information arises.

Assume that the optimal sets of information $H_{z,t}^{j*}$, $H_{v,t}^{j*}$ and $H_{\varphi,t}^{j*}$ allow agent j to select at any time a given representation of prices, and let p_t^j be the value generated by this representation.[4] A general representation of prices which nests all individually optimal dynamics functions may be written as:

$$p_t^j - p_{t-1}^j = \lambda^{j}{}' z_{t-1}^j + \delta^{j}{}' v_t^j + \varphi_t^j - \alpha^j \varphi_{t-1}^j \qquad (8)$$

where λ^j and δ^j are (m,1) and (k,1) vectors of coefficients and α^j is a scalar ($\alpha^j \geq 0$)[5], z_{t-1}^j and v_t^j are vectors of variables (in levels, rates of change, spreads,…) of types z and v and φ_t^j an individual zero-mean stochastic variable representing the total effect at time t of economic shocks which have occured between t-1 and t. The fact that variables of type z are at least one-period lagged is a necessary condition for equation (8) to allow for forecasting. Conversely, the variables of type v and φ may be actual to account for a contemporaneous effect. The endogenous variable is the theoretical change in prices that is generated by the chosen dynamics function. Note that by retaining this function, the agent accepts p_t^j to differ from the observed prices p_t, the spread variable $p_t - p_t^j$ containing voluntarily uncollected costly information and corresponding in Figure 1 to the difference between $H_{\max}$ and H^*.

---

[4] According to the theory of information, the quantity of information related to a random variable X (here a variable of a given type) taking a finite number N of values (representing the different states of the nature) with probabilities $p_1, p_{2,\dots,} p_N$ is given by the coefficient of entropy $H_X = -k \sum_{i=1}^{N} p_i Log \, p_i \quad (k > 0)$. For a given N, the entropy is maximum when $p_i = 1/N \quad \forall i = 1,\dots, N$ (the state of the nature is unpredictable) and minimum, that is $H_X = 0$, when the probability of one of the N realizations is 1 (the state of the nature is fully predictable).

[5] For the sake of simplicity, the coefficient of the actual shock is assumed to equal one. Any positive value would keep the analysis unchanged.

We now determine the expectational process agent j will use on the basis of the dynamics they have chosen. Write equation (8) one period ahead. Expectations at time t of agent j are given by:

$$E_t^j p_{t+1}^j = p_t^j + \lambda^{j\,\prime} z_t^j + \delta^{j\,\prime} E_t^j v_{t+1}^j - \alpha^j \varphi_t^j \tag{9}$$

Equation (9) may also be written as:

$$\varphi_t^j = \frac{p_t^j - p_{t-1}^j - \lambda^{j\,\prime} z_t^j - \delta^{j\,\prime} v_t^j}{1 - \alpha^j L} \tag{10}$$

where L is the lag operator. Substituting (10) into (9) and rearranging we get:

$$E_t^j p_{t+1}^j - p_t^j = \alpha^j (E_{t-1}^j p_t^j - p_t^j) + \lambda^{j\,\prime} z_t^j + \delta^{j\,\prime} (E_t^j v_{t+1}^j - \alpha^j E_{t-1}^j v_t^j) \tag{11}$$

Equation (11) is the general representation of the expected change in oil price, which nests all the individual optimal processes. It states that the expected change in oil price depends on the last forecast error, on currently available information about history of prices and on actual and lagged macroeconomic forecasts. Note that for any lagged variable of type v, expected values correspond to actual or past values. Otherwise, macroeconomic forecasts entering equation (11) must be determined as well. We assume the macroeconomic type variables are themselves generated by a dynamics function similar to (8), thus involving their own past observed and expected values, other macroeconomic variables and macroeconomic shocks. Hence, their expected change can be modeled using a specification analogous to (11). This shows that processing macroeconomic type information is very costly and, to justify the cost, it must yield to a substantial marginal gain. Under the very strong condition that the quantity of information of type v is unlimited, solving the equation (11) for the expected change in p_t will yield to a full description of this forecast in terms of all relevant variables of the economy. This solution is consistent with the rational expectation hypothesis. Equation (11) also states that the agent's opinion about the future rate of change in p_t includes p_t^j and not the observed price. This is because, in a context where information is costly (i.e. $p_t - p_t^j \neq 0 \;\; \forall j$) and where this cost may vary across agents, the retained dynamics function (8) results from an individual choice and may differ from one agent to another. This explains why, in response to the question regarding the future rate of change in p_t, that is what will $E_t^j p_{t+1} - p_t$ be, the forecaster j will give their theoretical expectation $E_t^j p_{t+1}^j - p_t^j$ based on their own dynamics function (8). Hence, these two forecast concepts are equivalent in the mind of the agent, and this also holds for any expected macroeconomic type variable. Equation (11) can thus equivalently be written as:

$$E_t^j p_{t+1} - p_t = \alpha^j (E_{t-1}^j p_t - p_t) + \lambda^{j\,\prime} z_t^j + \delta^{j\,\prime} (E_t^j v_{t+1} - \alpha^j E_{t-1}^j v_t) \tag{12}$$

We show now how equation (12) encompasses all kinds of standard expectational behavior considered in the literature. In this perspective, assume that the vector of type z is limited to $m = 3$ variables, such that:

$$\lambda^{j\,\prime} z_t^j = \lambda_1^j (p_t - p_{t-1}) + \lambda_2^j (\overline{p}_t^j - \overline{p}_{t-1}^j) + \lambda_3^j (\overline{p}_{t-1}^j - E_{t-1}^j p_t) \tag{13}$$

where $\bar{p}_t^j$ is the target value of p_t according to an agent. The first two components describe the effects of changes in price and in the target. The third component states that the agent believes there exists a strength making the price converge towards a "normal" value represented by the target. Replacing (13) into (12), and for the following set of parameter identifications:

$$\alpha^j = \omega_1^j(1-\beta) + \omega_3^j$$
$$\lambda_1^j = \omega_2^j\gamma$$
$$\lambda_2^j = \omega_3^j\mu_1$$
$$\lambda_3^j = \omega_3^j\mu_2$$
$$\delta_1^j = \omega_4^j\rho_1$$
$$\vdots$$
$$\delta_k^j = \omega_{k+3}^j\rho_k$$

(14)

where the k+4 scalars $\beta, \gamma, \mu_1, \mu_2, \rho_1, ..., \rho_k$ denote some structural parameters assumed to be invariant across agents and ω_i^j $(i=1,...,k+3)$ are some weighting coefficients such that $\sum_{i=1}^{k+3}\omega_i^j = 1$ and $0 \leq \omega_i^j \leq 1$ $\forall i, \forall j$, equation (12) can be written in the form of a weighted average of alternative standard expectational processes:

$$E_t^j p_{t+1} - p_t = \omega_1^j \underbrace{(1-\beta)(E_{t-1}^j p_t - p_t)}_{adaptive\ component} + \omega_2^j \underbrace{\gamma(p_t - p_{t-1})}_{extrapolative\ component}$$
$$+ \omega_3^j \underbrace{[E_{t-1}^j p_t - p_t + \mu_1(\bar{p}_t^j - \bar{p}_{t-1}^j) + \mu_2(\bar{p}_{t-1}^j - E_{t-1}^j p_t)]}_{error-correction\ form\ regressive\ component}$$
$$+ \sum_{i=1}^{k} \omega_{i+3}^j \underbrace{\rho_i[E_t^j v_{i,t+1} - [\omega_1^j(1-\beta) + \omega_3^j]E_{t-1}^j v_{it}]}_{fundamentalist\ component}$$

(15)

with $0 \leq \mu_i \leq 1$ (i=1,2) and $0 \leq \beta \leq 1$. Although the theoretical sign of the parameter γ is more likely to be positive, a negative value is conceivable in so far as it reflects a naive regressive process (systematic turning tendency). The theoretical value of $|\gamma|$ should also be less than one for the expectations not to explode or collapse.

Equation (15) is a weighted average of a standard adaptive process, a standard extrapolative process, an error-correction form regressive process[6] and k fundamentalist processes. Concerning the error-correction form regressive process, notice that it leads to the same equilibrium value $\bar{p}_t^j$ that the one imbedded in the traditional regressive model. On the other hand, there is no reason here to consider the adaptive component as a particular case of the ECM.[7] For appropriate one or zero values given to the weights, model (15) can represent

---

[6] Rearranging the ECM $E_t^j p_{t+1} - E_{t-1}^j p_t = \mu_1(\bar{p}_{t-1}^j - p_{t-1}) + \mu_2(\bar{p}_t^j - \bar{p}_{t-1}^j)$ such that the endogenous variable becomes $E_t^j p_{t+1} - p_t$, we obtain the error-correction component of equation (15).

[7] In the expectational ECM, $E_t^j p_{t+1}$ converges towards its normal value $\bar{p}_t^j$, while in the expectational adaptive model, $E_t^j p_{t+1}$ converges towards the observed value p_t.

any traditional expectational behavior in the literature, including naive expectations (i.e. the change in prices is a martingale difference sequence) when all weights are zero. Moreover, for values of the weights lying between zero and one the model describes any process mixing behavior of the forecaster and in the case all relevant information is used it corresponds to the Muthian rationality. For all these reasons the model can be viewed as an encompassing model of expectation formation.

2.2.2. Aggregation of individual behavior

Let n be the number of forecasters concerned by oil price. Taking the average of (15) across all agents, the expected change in oil price at the aggregate level can be written in the following form:

$$E_t p_{t+1} - p_t = (\omega_1(1-\beta) + \omega_3)(E_{t-1} p_t - p_t) + \omega_2 \gamma (p_t - p_{t-1}) + \omega_3 \mu_1 (\overline{p}_t - \overline{p}_{t-1}) + \omega_3 \mu_2 (\overline{p}_{t-1} - E_{t-1} p_t)$$

$$+ \sum_{i=1}^{k} \omega_{i+3} \rho_i \left[E_t v_{i,t+1} - (\omega_1(1-\beta) + \omega_3) E_{t-1} v_{it} \right] + b_t \qquad (16)$$

where

$$E_t p_{t+1} = \frac{1}{n} \sum_{j=1}^{n} E_t^j p_{t+1},$$

$$E_t v_{i,t+1} = \frac{1}{n} \sum_{j=1}^{n} E_t^j v_{i,t+1} \qquad (i = 1,...k),$$

$$\overline{p}_t = \frac{1}{n} \sum_{j=1}^{n} \overline{p}_t^j,$$

$$\omega_i = \frac{1}{n} \sum_{j=1}^{n} \omega_i^j \qquad (i = 1,...k+3).$$

and where b_t represents the aggregation bias, expressed as:

$$b_t = (1-\beta) \operatorname*{cov}_{j=1,n}(\omega_1^j, E_{t-1}^j p_t) + (1-\mu_2) \operatorname*{cov}_{j=1,n}(\omega_3^j, E_{t-1}^j p_t) + (\mu_2 - \mu_1) \operatorname*{cov}_{j=1,n}(\omega_3^j, \overline{p}_{t-1}^j) + \mu_1 \operatorname*{cov}_{j=1,n}(\omega_3^j, \overline{p}_t^j)$$

$$+ \sum_{i=1}^{k} \rho_i [\operatorname*{cov}_{j=1,n}(\omega_{i+3}^j, E_t^j v_{i,t+1}) - \operatorname*{cov}_{j=1,n}(\omega_{i+3}^j \omega_3^j, E_{t-1}^j v_{it}) - (1-\beta) \operatorname*{cov}_{j=1,n}(\omega_{i+3}^j \omega_1^j, E_{t-1}^j v_{it})]$$

$$- \sum_{i=1}^{k} \rho_i E_{t-1}^j v_{it} [(1-\beta) \operatorname*{cov}_{j=1,n}(\omega_{i+3}^j, \omega_1^j) - \operatorname*{cov}_{j=1,n}(\omega_{i+3}^j, \omega_3^j)] \qquad (17)$$

Equation (17) states that the bias depends on covariances between individual weights on the one hand and opinion variables or other weights on the other hand. In a study devoted to aggregation bias, Prat (1995) shows, on the basis of stochastic simulations, that the aggregation of individual processes assumed to be alternatively adaptive, regressive and extrapolative does not alter the form of the process even by imposing strong heterogeneity. It follows that the aggregation bias does not hide any of the adaptive, extrapolative and regressive components. Moreover, the author finds that the average of individual parameters remain close to those of the aggregate model. Assuming that aggregation is still robust in the case of macroeconomic variables, we will admit that the part of the variance of $E_t p_{t+1} - p_t$ explained by the variance of b_t is negligible even if b_t has a non-zero mean.

Equation (16) can account for any partition of the population of forecasters, and the weights may be given different meanings depending on the partition. To illustrate this, we will consider two limit cases. In the first case, suppose that all agents refer to the same complex dynamics (8) with their own coefficients. There is thus a unique group of forecasters each of whom use a forecast function as in (15). We call this polar case the *pure individual weighting effect*, whose aggregation over all agents leads to equation (16). Assume now that the population of forecasters is split into $k+3$ groups of sizes n_i $(i=1,...k+3)$ respectively, with $n = \sum_{i=1}^{k+3} n_i$, each of them using one and only one of the $k+3$ components of the forecast function (15). In this other polar situation, which we call *the pure group-heterogeneity effect*, the weights ω_i^j are such that:

$\omega_1^j = 1; \quad \omega_2^j = 0; \quad \omega_3^j = 0; \quad \omega_4^j = 0; \quad ...\omega_{k+3}^j = 0$ for any adaptive agent (group 1);

$\omega_1^j = 0; \quad \omega_2^j = 1; \quad \omega_3^j = 0; \quad \omega_4^j = 0; \quad ...\omega_{k+3}^j = 0$ for any extrapolative agent (group 2);

$\omega_1^j = 0; \quad \omega_2^j = 0; \quad \omega_3^j = 1; \quad \omega_4^j = 0; \quad ...\omega_{k+3}^j = 0$ for any regressive agent (group 3);

$\omega_1^j = 0; \quad \omega_2^j = 0; \quad \omega_3^j = 0; \quad \omega_4^j = 1; \quad ...\omega_{k+3}^j = 0$ for any fundamentalist agent who forecasts on the basis of the expected value of the variable v_{1t}^j (group 4);

....

$\omega_1^j = 0; \quad \omega_2^j = 0; \quad \omega_3^j = 0; \quad \omega_4^j = 0; \quad ...\omega_{k+3}^j = 1$ for any fundamentalist agent who forecasts on the basis of the expected value of the variable v_{kt}^j (group k+3).

It then follows from the condition $\sum_{i=1}^{k+3} \omega_i^j = 1$ that $\omega_i = n_i / n \quad \forall i$. All the coefficients ω_i in equation (16) now represent the frequencies of agents belonging to each of the k+3 groups.[8] The expected change in p_t provided by the consensus may then be represented as the weighted average of the forecasts made by the k+3 groups. Here again, the average expectation is given by (16). It is interesting to note that since all the k+3 groups cannot make accurate forecasts simultaneously, model (16) necessarily generates a systematic aggregate forecast error. Hence, the occurrence of any set of heterogeneous groups of forecasters is sufficient for expectations not to be rational at the aggregate level. In fact, the group-heterogeneity and the individual mixing effects may act simultaneously so that there may exist a very large number of possible partitions of agents forming the consensus. Let $d \in N$ denote the number of different components of a mixed process. For any $d \in \{1, 2, ..., k+3\}$, there are at most C_{k+3}^d possible *d-mixed* processes candidate to describe the consensus at time t. In the case $d = 1$, there are at most $k+3$ basic possible processes, and this refers to our second polar example. In the case $d = k+3$, there is only one single group of agents who all use a combination of $k+3$ basic processes, and this refers to our first polar example. Since

---

[8] A well-known illustration introduced by Frankel and Froot (1987) involves two groups of forecasters, repectively called by the authors the "chartists" (assumed to adopt an extrapolative process) and the "fundamentalists" (assumed to follow a regressive process where the target may or may not include limited information).

various *d*-mixed processes with different values of d may coexist, there are at most $\sum_{d=1}^{k+3} C_{k+3}^{d}$ partitions of the consensus.

2.2.3. Generalizing for the forecast horizon

Hitherto, we set the whole framework on the implicit assumption that the forecast horizon is equal to the frequency of the observations (i.e. one unit of time). We now call τ the forecast horizon such that $\tau \geq 1$. The general form for any τ of the dynamics function generating the rate of change in p^j between t-τ and t is (the subscript τ will be omitted from parameters to simplify notation):

$$p_t^j - p_{t-\tau}^j = \sum_{q=1}^{m}\sum_{i=0}^{\tau-1}\lambda_{qi}^j z_{q,t-\tau-i}^j + \sum_{h=1}^{k}\sum_{i=0}^{\tau-1}\delta_{hi}^j v_{h,t-i}^j + \varphi_t^j - \sum_{i=1}^{\tau}\alpha_i^j \varphi_{t-i}^j \tag{18}$$

Following the same steps as (8) through (12) to find the individual forecast function and aggregating yield to the following general forecast function of the consensus:

$$\begin{aligned}
E_t p_{t+\tau} - p_t &= \sum_{i=1}^{\tau-1}\alpha_i (E_{t-i}p_{t+\tau-i} - p_{t-i}) + \alpha_\tau (E_{t-\tau}p_t - p_t) \\
&+ \sum_{q=1}^{m}\left\{ \sum_{i=0}^{\tau-1}(\lambda_{qi} - \sum_{l=1}^{i}\alpha_l \lambda_{q,i-l})z_{q,t-i} - \sum_{i=\tau}^{2(\tau-1)}(\sum_{l=i-\tau+1}^{\tau-1}\alpha_l \lambda_{q,i-l})z_{q,t-i} \right\} \\
&+ \sum_{h=1}^{k}\left\{ \sum_{i=0}^{\tau-1}\delta_{hi}\left[E_t v_{h,t+\tau-i} - \sum_{l=1}^{\tau-1}\alpha_l E_{t-l}v_{h,t+\tau-i-l} - \alpha_\tau [E_{t-\tau}v_{h,t-i} - v_{h,t-i}] \right] \right\} + b_t'
\end{aligned} \tag{19}$$

The terms z_{qt} $(q=1,..3)$ and v_{ht} $(h=1,...,k)$ are the averages of z_{qt}^j and of v_{ht}^j over all agents respectively,[9] and b_t' is an aggregation bias assumed to be negligible. Define now $m=3$ and $z_{1t}^j = p_t - p_{t-1}$, $z_{2t}^j = \overline{p}_t^j - \overline{p}_{t-1}^j$, $z_{3t}^j = \overline{p}_{t-1}^j - E_{t-1}^j p_{t-1+\tau}$ similarly to equation (13). If an identification system similar to (14) holds, the coefficients in (19) may be written as $\alpha_i = \omega_{1i}(1-\beta) + \omega_{3i}$ for $i=1,....,\tau$, $\lambda_{1i} = \omega_{2i}\gamma$, $\lambda_{2i} = \omega_{3i}\mu_1$, $\lambda_{3i} = \omega_{3i}\mu_2$, $\delta_{1i} = \omega_{4i}\rho_1,....$ $\delta_{ki} = \omega_{k+3,i}\rho_k$ for $i=0,....,\tau-1$, where ω_{si} $(s=1,...,k+3)$ is the weight corresponding to the i'th lagged value of the s'th expectational component. It is easy to check that for $\tau=1$ equations (18) and (19) simplify to (8) and (16) respectively.

3. Evidence on the expectational process used by experts

Assuming that the aggregate process chosen by experts is stable over time (i.e. the elements of equation (7) are time-invariant), we now attempt to identify empirically this process in the light of survey data.

---

[9] We limit the number of lags in the exogenous variables of equation (18) to the horizon for the sake of conformity with the special case $\tau=1$. However the number of lags for variables of types z and v can be easily extended beyond τ without changing the general form of the forecast function (19). If the φ's lags were extended, equation (19) would become extremely cumbersome.

3.1. The data

At the beginning of each month, « Consensus Forecasts » (CF) asks 180 or so economy and capital market specialists in approximately 30 countries to estimate future values of a large number of economic variables for 3-month and 12-month horizons. These variables are the production growth rate, inflation rate, unemployment rate, wage rates, new housing starts, company profits, interest rates, foreign exchange rates, West Texas Intermediate (WTI) oil price…. The WTI represents the international reference spot price of the US oil (US\$ per barrel), and its global benchmark role is reinforced by the rejection of the regionalisation hypothesis (Gülen, 1999). Towards the end of each month, CF sends each of the bodies (scattered throughout the world) who have agreed to participate in the survey, a questionnaire which asks for their opinion on the future numerical values of WTI oil price. The consensus is the arithmetic average of the individual expected values of oil price and is published in the monthly CF newsletter. These consensus time series are used in this paper over the period November 1989 to January 1998. The coefficient of variation at each point in time (i.e. the ratio of the standard deviation of individual answers over the consensus) lies between 4% and 8% for each horizon, except during the Gulf crisis where it increases almost up to 20%. This indicates that the heterogeneity of individual expectations is neither negligible, nor large enough for the consensus to be irrelevant.

Respondents are commercial or investment banks, industrial firms and forecast companies, whose forecasts influence many market participants' decisions. These experts are identified with a confidential code, which only mentions their country. They are only asked to reply when they are sufficiently concerned by an economic variable. Only two thirds of the 180 experts of CF answer the questions concerning future values of oil price. This confirms that experts who respond are those who are informed about the oil market and who are professionally concerned by the requested horizons. In this way, noise traders cannot bias the consensus because only informed agents are asked to respond. Since the individual answers are confidential (i.e. only the consensus is disclosed to the public with a time lag) and since each individual is negligible within the consensus, it is difficult to say that, for reasons which are inherent to speculative games, individuals might not reveal their « true » opinion. The CF requires a very specific day for the answers, i.e. at the beginning of the following month. As a rule, this day is the same for all respondents.[10] Finally, given that questions concern the expected levels of oil price, the expected change rate can only be determined with respect to the last spot price which is assumed to be known by the individuals the day they answer (reference price). It is thus clear that any mistake in the choice of the reference price date implies a mistake in the measurement of the expected change. However, the price values considered in this paper being dated from the day required by CF for the answers, the concentration of the answers on the same day implies that we can retain the same reference price for all respondents.

As shown in the theoretical analysis presented in section 2, the opinion variable for agent j is assumed to be the expected return of the oil price $\pi_{t,\tau}^{j} = E_t^j p_{t+\tau} - p_t$ (i.e. agents consider the distribution of $\pi_{t,\tau}^{j}$ prior to forming forecasts). Since agent is asked by Consensus Forecasts to give their opinion on the level (and not the log-level) of the future oil

---

[10] We notice that this day may fall anywhere between the 1st and the 8th of the month. The effective horizons however always remain equal to 3 and 12 months. If, for instance, the answers are due on the 3rd of May (which was the case in May 1993), the future values are asked for August 3, 1993 (3 month-ahead expectations) and for January 3, 1994 (12 month-ahead expectations).

price, they expresses their response as $P_{t,\tau}^{e\,j} = P_t \exp[\pi_{t,\tau}^j]$ for both $\tau = 3$ and $\tau = 12$ months. This gives, in logarithms, $p_{t,\tau}^{e\,j} = \ln P_{t,\tau}^{e\,j} = p_t + \pi_{t,\tau}^j = E_t^j p_{t+\tau}$ according to the expected return relationship. Thus, the logarithm of the forecast values of the price provided by the respondent j at time t for t+3 months and for t+12 months ($p_{t,3}^{e\,j}$ and $p_{t,12}^{e\,j}$ respectively) correspond to the expected value of the logarithm of the future price for these horizons ($E_t^j p_{t+3}$ and $E_t^j p_{t+12}$).

Thus, we can write at the aggregate level $\dfrac{1}{n}\sum_{j=1}^{n} p_{t,\tau}^{e\,j} = \dfrac{1}{n}\sum_{j=1}^{n} E_t^j p_{t+1} = E_t p_{t+1}$. However the consensus value published by Consensus Forecasts is an arithmetic (and not geometric) average of oil prices, i.e. $P_{t,\tau}^{e} = \dfrac{1}{n}\sum_{j=1}^{n} P_{t,\tau}^{e\,j}$. Hence, substituting $\ln P_{t,\tau}^{e}$ for $E_t p_{t+1}$ generates a systematic bias on the aggregate expected oil price. It can be shown that the wider the dispersion of individual expectations the larger is the bias. Because this dispersion is rather low and stable over time for the two horizons (see above), this bias will be supposed to be constant. A second and similar source of measurement bias may result from the aggregation of individual targets. A last type of bias may be due to the non-zero mean of the b_t term (equation (17)). We will therefore introduce in each process an intercept to capture the total bias, which allows us to replace $\ln P_{t,\tau}^{e}$ by $E_t p_{t+1}$ in the empirical analysis to be in conformity with equation (19). Furthermore, the error term of the expectational process may capture a stochastic measurement error in expectations.

3.2. Empirical results

All basic or mixed processes are derived from the general equation (19) for $\tau = 3$ and $\tau = 12$. The empirical exercise presented hereafter aims to identify the relevant process used by the respondents to Consensus Forecasts. We will first examine the relevance of the REH, which describes a situation in which the consensus is generated by an underlying process based on all available information (this process being unknown to the investigator).[11] We will begin by implementing ADF tests to the three variables of interest: p_t, $E_t p_{t+3}$ and $E_t p_{t+12}$. At a 1% level of significance (with an intercept and one lag), all variables were stationary. It follows that the unbiasedness tests may be run on the log-levels of the variables.[12] For the two horizons, we obtain the following results:

$\tau = 3$, sample period: February 1990 – October 1997 (N=93)

$$p_{t+3} = \underset{(3.25)}{0.38}\, E_t p_{t+3} + \underset{(5.42)}{1.87} + \hat{v}_t \qquad \overline{R}^2 = 0.094 \quad SE = 0.148 \quad DW = 0.41$$

---

[11] It is worth noting that if all individuals are rational we should find that the consensus is rational. However we may find rational consensus although none of agents is rational. For instance, if the consensus was made by two agents, and if one of them had underestimated the importance of a given information (with respect to what would have been necessary to forecast correctly the future price), while the other had overestimated this information, we would find a rational consensus.

[12] Note that the specification in level avoids statistical biases due to overlapping data that would occur with first differences. Moreover the choice of the observed price as the endogenous variable allows to estimate the slope without the bias which would result from the unknown forecast error variance of the rational expectation.

$\tau = 12$, sample period: February 1990 – January 1997 (N=84)

$$p_{t+12} = \underset{(0.45)}{0.07}\, E_t\, p_{t+12} + \underset{(5.88)}{2.77} + \hat{v}_t' \qquad \bar{R}^2 = -0.01 \quad SE = 0.119 \quad DW = 0.34$$

Generally, the hypothesis of unbiasedness involves testing the joint hypothesis that the slope is 1 and the intercept is zero.[13] Nevertheless, because of the possible measurement bias on the expected variable leading to a non-zero intercept, only the value of the slope accounts for the unbiasedness. Our results strongly reject the unbiasedness hypothesis for the consensus. Further, we implemented a Chow test on two equal sub-periods for the two horizons to verify the stability of these estimates. The test led us to accept the stability hypothesis of these parameters at the 1% level of significance, which suggests the absence of a learning mechanism allowing a convergence towards the REH: the barrier of costly information acts at any time. This confirms that aggregate expectations described by equation (19) are not rational.

However, these results do not preclude the existence of a significant group of rational forecasters in the consensus. If this group exists, we should be able to capture it empirically. In this case, the group uses all relevant information imbedded in (19) to produce unbiased expectations and the question is to determine whether the REH may be represented by a linear combination of variables included in (19). In this perspective, we regress the ex-post change in oil price $F_{t,\tau} = p_{t+\tau} - p_t$, which stands for the change expected by the rational group, on a large number of observed and expected variables and we find that for the two horizons, $F_{t,\tau}$ cannot be fit by any combination of these variables. However, rational agents may have used information other than that which we empirically exploited, as this information was unknown to us. To circumvent this methodological difficulty, we will consider the variable $F_{t,\tau}$ as a proxy of all information used by rational agents. Hence, the introduction of the variable $F_{t,\tau}$ in the aggregate expectational process (19) allows us to test whether or not a significant group of rational agents belongs to the consensus. In other words, within our group-heterogeneity framework it makes sense to combine the REH with any other expectational hypothesis to account for the behavior of the consensus. Obviously, there is no sense to consider such a combination at the individual level because if the latter hypothesis is relevant the two hypotheses are redundant.

In order to run calculations, we needed a hypothesis about the target value of the oil price as it appears in the regressive process for each horizon. For the three-month horizon, we supposed that the target value is given by the aggregate 12 month ahead expected oil prices provided by the survey data, that is:

$$\bar{p}_{t,3} = E_t\, p_{t+12} \tag{20}$$

whereas for the twelve month horizon we retained - after having tried several other alternatives – a constant (long-run) target value, which is given by its mean over the period:

$$\bar{p}_{t,12} = \frac{1}{T}\sum_{t=1}^{T} p_t = \bar{p} \tag{21}$$

---

[13] This condition is not sufficient to prove the REH but its failure is sufficient to reject the REH.

This assumption is in accordance with the fact that the time series p_t was found to be stationary over the sample period. A direct consequence of equation (21) is that when $\tau = 12$ months, the rate of change in the target in (19) is zero.

The estimation of equation (19) including the variable $F_{t,\tau}$ allows for the identification of relevant sets of information used by experts. For both horizons, the forecast error, the more-than-one-period lagged expected changes in p_t, the lagged regressive components and the actual or lagged macroeconomic components[14] did not appear to be significant. Furthermore, the more-than-two-periods lagged extrapolative components in the case $\tau = 3$ and the more-than-one-period lagged ones in the case $\tau = 12$ were also found to be insignificant. These variables are therefore ignored in the remainder. In fact, for the 3-month horizon, the estimate of $\lambda_{11} - \alpha_1 \lambda_{10}$ (composite coefficient of the one-period-lagged extrapolative component), the estimate of $\lambda_{12} - \alpha_1 \lambda_{11} - \alpha_2 \lambda_{10}$ (composite coefficient of the two-periods-lagged extrapolative component) and the sum of the estimates of α_1 and λ_{10} (coefficients of the first order autoregressive component and of the actual extrapolative component, respectively) were not found to be significantly different from each other. For the 12-month horizon, we found that the estimate of $\lambda_{11} - \alpha_1 \lambda_{10}$ is not significantly different from the sum of the estimates of α_1 and λ_{10}. Hence, writing the term $\lambda_{10} z_{1t}$ in the right-hand side as $(\alpha_1 + \lambda_{10}) z_{1t} - \alpha_1 z_{1t}$, we can write equation (19) equivalently as follows. First, the extrapolative component now defines the quarterly change in prices in the 3-month horizon model and the two-month change in the 12-month horizon model, instead of monthly changes. Second, the adaptive component is modified such that the last forecast is compared to the *actual* price and not to the lagged one. Although this is not the traditional adaptive behavior, we call it an *early reappraisal adaptive* behavior: agents do not wait until the end of the τ month horizon to revise their expectations, but they rather revise them in response to the first observed price. This particular adaptive behavior says that expectations are defined as a weighted average of actual and past monthly values of p_t and not an average of quarterly or annual values of it (depending on the forecast horizon) as the traditional adaptive process states.

Table 1 provides the significant components for the two horizons respectively. The estimate associated to $F_{t,\tau}$[15] is reported when it is significant at the 5% level, while a hyphen (-) is used to denote that this variable was insignificant and has therefore been excluded from

---

[14] These are observed and expected values of the rate of inflation, of the money stock, of the change in real GNP, of real investment, of the rate of interest and of the rate of unemployment revealed by the same CF survey. The weak influence of macroeconomic variables on asset price expectations also results from the survey by Gennotte and Leland (1990) who asked traders on NYSE market to provide the information their expectations are based on, and found that agents base their forecasts on observed stock prices rather than on fundamentals.

[15] Since the REH is represented as the sum of the variable $F_{t,\tau}$ and a white noise forecast error, a bias may be generated in the estimation of any model in which $F_{t,\tau}$ appears as a regressor. In order to appreciate the importance of this bias, we generate a new variable $F'_{t,\tau} = F_{t,\tau} + \upsilon_{t,\tau}$, where $\upsilon_{t,\tau}$ is drawn from $N(0, \sigma_\tau)$ allowing different values for σ_τ, and estimate the models by substituting $F'_{t,\tau}$ for $F_{t,\tau}$ for each value of σ_τ. Even for rather high values of σ_τ (including the standard error of $F_{t,\tau}$), we observe that the estimates are not significantly different from those obtained by using the regressor $F_{t,\tau}$ instead of $F'_{t,\tau}$. This result allows us to use $F_{t,\tau}$ in all regressions.

the presented regression. Stationarity (ADF) test and Ljung-Box serial correlation test have been performed to the residuals of each of the twelve regressions. When the intercept was not significantly different from zero, it was dropped in the course of the final estimation and again this has been indicated with a hyphen.

Our results call for the following comments. While the ADF tests do not allow us to reject any of the processes, the Ljung-Box test is very discriminating. Regressions 5 and 10 indicate the only process without serial correlation of the residuals for both horizons.[16] Since in this process all macroeconomic variables and $F_{t,\tau} = p_{t+\tau} - p_t$ are insignificant, we call it an extrapolative-regressive-adaptive model with limited information (ERAMLI).[17] In fact, the adaptive component being included in the regressive process, there is no way to identify econometrically between the extrapolative-regressive process and the ERAMLI. Regressions 5 and 10 may thus correspond to either of these two processes, but in the remainder we simplify by referring to the ERAMLI when talking about a 3-mixed process. The observation of Table 1 suggests that $F_{t,\tau}$ is significant only when the limited information model is misspecified. This leads to the conclusion that there is no significant group of rational agents in the consensus. The estimates of regressions 5 and 10 correspond to the parameters λ_{10}, α_1, λ_{20}, and λ_{30} of the theoretical equation (19) respectively and they all have the expected positive signs. It is important to note that the extrapolative component acts in the ERAMLI with a positive sign, contrary to what is generally obtained when this process is estimated alone. This result is satisfactory since it conforms to the intuitive idea that the extrapolation should maintain the past direction of the market. We also found that the intercept for each horizon is not significantly different from zero, and this suggests that the measurement biases if any compensate each other in average.

Another implication of the fact that the regressive process imbeds the adaptive component is that if the former was relevant by itself, the coefficient of the adaptive component should not be significantly different from one. According to regressions 4 and 9, this is not the case and thus these equations represent a regressive-adaptive 2-mixed process for both horizons.

To check for the colinearity between exogenous variables of the ERAMLI, we calculated the R^2 between each exogenous variable and the others and compared it to the one

---

[16] It should be noticed that the 3-month horizon ERAMLI (regression 5) involves on both sides not only the spot price at time t but also an expectational variable, that is the 3-month expected price on the left hand side and the 12-month expected price on the right hand side. This raises the question of whether or not there exists some possibility of spurious validation of the ERAMLI. To check this issue, we derived from the ERAMLI an equivalent equation in which all expectational variables dated at time t are gathered on the left hand side while actual and lagged spot variables and lagged expectational variables are gathered on the right hand side. There is thus no possibility for this last equation to be necessarily validated because of common information on both sides of the equation. By applying an iterative procedure, this transformed equation yielded to a set of estimates which is not significantly different from those of the regression 5. We then can conclude that these latter estimates cannot be attributed to any statistical necessity. For the 12-month horizon ERAMLI it should be noticed that regression 10 involves on both sides only the spot price. We then expressed the *level* of expected price in terms of actual price p_t and the four exogenous variables of the ERAMLI. The estimate of p_t appeared to be very significantly equal to 1, all the other parameters being insignificantly changed with respect to those of regression 10. This again confirms that the presence of p_t at both sides of the equation involves no spurious correlation.

[17] Abou and Prat (2000) and Prat and Uctum (2000) have found similar mixed models with stock market and foreign exchange market survey data respectively.

of the ERAMLI, as suggested by Johnston (1963, p. 295), and Kennedy (1985, pp. 150-153). For the three (twelve) month horizon, the R^2 values corresponding to each of the four (three) regressions were found to be less than 0.9 and less than the R^2 of the ERAMLI. These are the relevant conditions to conclude that colinearity does not introduce significant biases in the estimates.

For the 3-month and 12-month horizons respectively, the observed and the fitted values of the expected rate of change of oil price according to the ERAMLI are such that the main fluctuations are well-represented (Figures 2 and 3). We particularly observe that there are no systematic time lags between the fitted and observed values, which is the sign of a good specification of the ERAMLI. This remark holds especially for the twelve-month horizon.

We also performed White tests (not provided in Table 1) on the residuals of the ERAMLI to check for the presence of heteroskedasticity: the null hypothesis of homoskedasticity is rejected at the 5% level of significance since the p-values are found to be zero for both $\tau = 3$ and $\tau = 12$. Heteroskedasticity may be generated by aggregation biases not captured by the intercept or by the influence of (non-economic) extreme events such as the Gulf crisis. The abnormal spreads we observe during the Gulf crisis period (August 1990 - April 1991) between the observed and fitted values (Figures 2 and 3) are likely to be the source of the heteroskedasticity found over the whole period. Such extreme events may be accounted for by adding, in the dynamics function, a series of dummy variables $\psi_{i,t-i}$ (i=0,1,…τ) taking the value 1 when an extreme (and unique) event occurs between t and t-τ and zero otherwise. Adding thus $\sum_{i=0}^{\tau} \delta_i^j \psi_{i,t-i}$ in equation (18), where δ_i^j is the impact on the change in oil price of the i-month-lagged extreme event according to agent j, it can be verified that the aggregate forecast function (19) is modified so as to include also the effects of the war, that is $\sum_{i=0}^{\tau-1} (\alpha_\tau \delta_i - \alpha_i \delta_\tau)\psi_{i,t-i}$. In order to examine the importance of these effects at each time of the period of the crisis (August 1990 - April 1991 included), we introduce nine dummy variables $D9008, D9009,..., D9104$, which are equal to one on August 1990, September 1990,…, April 1991 respectively and zero elsewhere. By eliminating successively insignificant dummies, we found the following regressions for the two horizons:

$$\pi_{t,3} = 0.06(p_t - p_{t-3}) + 0.30(E_{t-1}p_{t+11} - E_{t-1}p_{t+2}) + 0.31(E_t p_{t+12} - E_{t-1}p_{t+11}) + 0.58(E_{t-1}p_{t+2} - p_t)$$
$$\text{(3.2)} \qquad\qquad \text{(8.6)} \qquad\qquad\qquad \text{(9.4)} \qquad\qquad\qquad \text{(24.1)}$$
$$+ 6.39 D9009 + 13.05 D9010 + 4.53 D9011 + 8.36 D9012 - 0.91 + e_{t,3} \quad \text{(Regression n°11)}$$
$$\text{(3.5)} \qquad\quad \text{(6.8)} \qquad\quad \text{(2.1)} \qquad\quad \text{(4.5)} \qquad\quad \text{(-4.8)}$$

$\overline{R}^2 = 0.915$, $SE = 1.53\%$, Ljung-Box probability = 62.60% (3 lags), ADF test: CI(1%) with 0 lag

$$\pi_{t,12} = 0.025(p_t - p_{t-2}) + 0.014(\overline{p} - E_{t-1}p_{t+11}) + 0.816(E_{t-1}p_{t+11} - p_t)$$
$$\text{(5.2)} \qquad\qquad \text{(2.1)} \qquad\qquad\qquad \text{(23.7)}$$
$$+ 1.69 D9009 - 1.42 D9010 - 2.43 D9011 + e_{t,12} \qquad\qquad \text{(Regression}$$
n°12)
$$\text{(3.5)} \qquad\quad \text{(-3.0)} \qquad\quad \text{(-4.7)}$$

$\overline{R}^2 = 0.968$, $SE = 0.42\%$, Ljung-Box probability = 97.71%(3 lags), ADF test: CI(1%) with 0 lag

Using these estimates, we calculated the part of the endogenous variable $\pi_{t,\tau}$ unexplained by the ERAMLI, that is:

$$\phi_{t,3} = \pi_{t,3} - [0.06(p_t - p_{t-3}) + 0.30(E_{t-1}p_{t+11} - E_{t-1}p_{t+2}) + 0.31(E_t p_{t+12} - E_{t-1}p_{t+11})$$
$$+ 0.58(E_{t-1}p_{t+2} - p_t) - 0.91]$$

$$\phi_{t,12} = \pi_{t,12} - [0.02(p_t - p_{t-2}) + 0.01(\overline{p} - E_{t-1}p_{t+11}) + 0.82(E_{t-1}p_{t+11} - p_t)]$$

It should be underlined that these two spread variables represent a « total shock », which may contain any exogenous influence due to intra-month historical events and any measurement or specification error in the ERAMLI. Although these effects are not distinguishable, it is interesting to present their overall evolution during the Gulf crisis. In this perspective, Figure 4 shows, for each horizon, the normalized values of $\phi_{t,3}$ and $\phi_{t,12}$ at the beginning of each month over the period June 1990 to December 1991, which contains the Gulf crisis. Between August 1990 and May 1991, the two spreads exhibit a higher volatility than they do outside of this period. This period starts with the crisis and ends around the end of the war. This suggests that, during the crisis, the ERAMLI and the extreme event effects explain complementarily expectations in the oil market.[18] We finally implemented the heteroskedasticity White test by transforming regressions 11 and 12 so as to eliminate from the endogenous variable the influence of the dummy variables. The test gives the NR^2 p-value = 0.010 for the 3-month horizon and p-value = 0.803 for the 12-month horizon, which suggests the absence of heteroskedasticity at the 1% significance level for the former horizon and 5% for the latter horizon. These findings show that the Gulf crisis is sufficient to explain the heteroskedasticity reported above. More generally, the well-behaved residuals in regressions 11 and 12 suggest that the total aggregation bias (see subsection 3.1) is not significant.[19]

---

[18] More specifically, increased tension among Gulf states pushed up oil price expectations sharply in the course of August 1990 (see the positive change of the two spreads between August and September), and the sign of this shock is conform with the high probability of a substantial fall in oil supply. It is worth noting that during September 1990, although the three month ahead expectations imbed a positive shock, the twelve month expectation exhibits an important negative shock. For the short term horizon, the fact that Iraq made it clear that the destruction of the Saudi oilfields would be a primary objective in the event of hostilities may explain the upward shock. In the long run, the panel of experts may have thought that, in all circumstances, the insufficiency in oil supply will vanish, and this may explain the negative slope of the 12-month oil price expectations. In October 1990, the perspective of a peaceful settlement to the Gulf crisis may have led also the 3-month expectations to sharply decrease. After the declaration of war (january 1991), the 3-months and 12-month spread variables exhibit negative and positive slopes, respectively : while in the short run the belief that the Allied forces would win in a short time may have led agents to foresee a reduction in their precaution oil storage, in the long run agents may have thought that the observed destruction of some oil wells will cause a rigiditiy of the oil production and hence increased prices. Stabilization of the spread variables is observed starting from May 1991. Overall, these historical events do not appear distinctly from the observation of expectations, but they do when we substract the fitted values provided by the ERAMLI from the observed expectations (i.e. when we consider the spread variables). This reinforces the relevance of this model.

[19] Hitherto, we ignored the feedback effect, that is the influence of price expectations on actual prices. The fact that the actual prices have been viewed as econometrically exogenous is unlikely to have altered the quality of the estimates presented above, in the extent that no specification bias has been detected with the tests implemented in this paper.

Expectations generated by the adaptive and regressive components of the ERAMLI are stabilizing in that by definition they converge to the spot price and the target respectively. The extrapolative process does not involve any convergence and therefore is not stabilizing. In order to appreciate the magnitudes of these two strengths, we decomposed the variance of the deterministic part of the endogenous variable corrected from the Gulf effect as the variance of the stabilizing factor (weighted sum of the regressive and of the adaptive components), the variance of the destabilizing factor (extrapolative component) and twice the covariance. These are respectively 37.29, 1.06, and –9.00 for $\tau = 3$ months and 6.15, 0.17 and –1.46 for $\tau = 12$ months. Hence, expectations can be viewed as exerting a stabilizing effect in the dynamics of the oil market for both horizons.

Recall that the ERAMLI applied to the three-month horizon (regression 5) depends on the actual value of the twelve-month expectations, which represents the target of the regressive process as supposed in equation (20). On the other hand, we have explored the determinants of this target, which is the ERAMLI applied to the twelve-month horizon (regression 10). It becomes then interesting to examine jointly these two hypotheses by testing the robustness of the estimates of the 3-month ahead expectation model where the target is replaced by its fitted values. Indeed, if the residuals of the target model have a constant but too large variance with respect to the ones of the regressive components on the 3-month model, we would expect the estimates of this model to be significantly worsened. Another advantage of such a « nested model » is to explain three-month expectations *only* with actual prices and the past values of expectations. Inferring the estimated target from regression 12 as $\hat{E}_t p_{t+12} = \hat{\pi}_{t,12} + p_t$ and testing the 3-month horizon ERAMLI with this estimated target yields:

$$\pi_{t,3} = \underset{(2.22)}{0.057}(p_t - p_{t-3}) + \underset{(7.29)}{.348}(\hat{E}_{t-1} p_{t+11} - E_{t-1} p_{t+2}) + \underset{(5.02)}{0.211}(\hat{E}_t p_{t+12} - \hat{E}_{t-1} p_{t+11}) + \underset{(17.67)}{0.507}(E_{t-1} p_{t+2} - p_t)$$

$$+ \underset{(2.90)}{7.70} D9009 + \underset{(4.75)}{12.65} D9010 + \underset{(1.53)}{4.83} D9011 + \underset{(4.14)}{10.47} D9012 - \underset{(-4.80)}{1.20} + e'_{t,3} \text{ (Regression n°13)}$$

$\overline{R}^2 = 0.842$, $SE = 2.07\%$, Ljung-Box probability = 83.24% (3 lags), ADF test: CI(1%) with 0 lag

It is clear that estimates in regressions 11 and 13 are not significantly different. Moreover, as with the 3-month horizon model, the nested model provides stationary and serially uncorrelated residuals. Heteroskedasticity White test gives NR^2 p-value = 0.181, which leads to accept the null. These results validate jointly the two models.

A final point we considered was the analysis of the time-stability of the ERAMLI by performing the (in-sample) Chow test on the three-month model, twelve-month (or target) model and the "nested" model. Because the Gulf crisis may be a source of instability of the structural parameters of the ERAMLI, the test has been implemented by eliminating from the endogenous variable the influence of the significant dummies over the crisis period. We considered 3 sub-periods of roughly the same sizes: March 1990 – September 1992 (N=31), October 1992 - April 1995 (N=32) and May 1995 – January 1998 (N=32). For each of the three models, the p-values obtained are 7.64%, 28.10% and 22.33%, respectively. We can thus accept the time stability hypothesis at the 5% level of significance whatever the model considered. This suggests that the retained hypotheses on the targets are econometrically consistent.
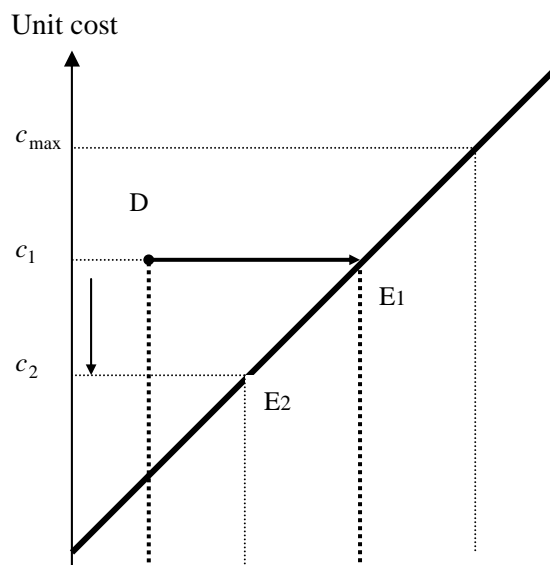
4. Concluding remarks

In this paper we attempt to analyze the formation of oil price expectations in an economically rational expectation framework, which states that a forecaster chooses the optimal quantity of information such that the marginal gain due to the decrease in the forecast error equals the unit cost. This quantity of information corresponds to a dynamics function of oil price, and allows the agent to specify an expectational process. Starting with a general form of dynamics, we propose an encompassing model which nests all kinds of expectational processes considered in the literature and which we generalize for any horizon. A central implication of the framework is that expectations may be biased although economically rational. We also show that the aggregation of individual expectational optimal processes leads to an aggregate mixed process, which may be interpreted as resulting from individual mixing effects and/or group heterogeneity effects. We then attempt to identify empirically the effective components of this aggregate process.

Using Consensus Forecasts survey data for three and twelve month horizons, we found that not only the rational expectation hypothesis is rejected - whether it is supposed to hold for all experts or only for a group of them - but also that no macroeconomic fundamental is significant. Nevertheless, expectation formation appears to be characterized by an empirical complementarity among adaptive, regressive and extrapolative processes for both horizons, i.e. a mixed expectational model. Moreover, we show that the twelve month expectations appear to be a valuable target in the three month model. An important property of this model is that expectations exert a stabilizing strength in the oil market. Generally speaking, the mixing behavior seems to explain why the basic processes are inappropriate to describing expectations and why rational expectation hypothesis is not validated by data.

REFERENCES

Abou A. and G. Prat, 2000, Modeling stock price expectations: lessons from microdata, in: F. Gardes and G. Prat, eds, Price expectations in goods and financial markets: new developments in theory and empirical research, Edward Elgar.

Agénor P.R. and P.J. Montiel, 1996, Development macroeconomics, Princeton University Press.

Baillie R. and P. MacMahon, 1992, The foreign exchange market: theory and econometric evidence, Cambridge University Press, Cambridge.

Feige E.C. and D.K. Pearce, 1976, Economically rational expectations: are innovations in the rate of inflation independent of innovations in measures of monetary and fiscal policy?, Journal of Political Economy, 84, 499-522

Garratt A. and S.G. Hall, 1997, E-equilibria and adaptive expectations: output and inflation in the LBS model, Journal of Economic Dynamics and Control, 21(7), 1149-71.

Gennotte G. and H. Leland, 1990, Market liquidity, hedging and crashes, American Economic Review, December.

Gilbert C.L. and T.B. Palaskas, 1990, Modelling expectations formation in primary commodity markets, in: L.A. Winters and D. Sapsford D., eds., Primary commodity prices: economis models and policy, Cambridge University Press, 44-69.

Gülen S.G., 1999, Regionalization in the world crude oil market: further evidence, Energy Journal, 20(1), 125-39.

Hammoudeh S., 1996, Oil price, mean-reversion and zone readjustments, Southern Economic Journal, 62(4), 916-29.

Hammoudeh S. and V. Madan, 1995, Expectations, target zones, and oil price dynamics, Journal of Policy Modeling, 17(6), 597-613.

Hawdon D., 1987, Short and long-run crude oil price expectations in 1986: results of a survey, in: P. Stevens, ed., Energy demand: prospects and trends, St. Martin's Press, New York, 166-76.

Hawdon D., 1989, Changing oil price expectations, in: D. Hawdon, ed., Oil prices in the 1990s, St Martin's Press, New York, 115-27.

Holden K., D.A. Peel and J.L. Thompson, 1985, Expectations: theory and evidence, St. Martin's Press, New York.

Johnston J., 1984, Econometric methods, MacGraw-Hill, Tokyo.

Kennedy P., 1985, A guide to econometrics, Southampton, Basil Blackwell.

Moosa I.A. and N.E. Al-Loughani, 1994, Unbiasedness and time varying risk premia in the crude oil futures market, Energy Economics, 16(2), 99-105.

Muth J.F., 1960, Optimal properties of exponentially weighted forecasts, Journal of the American Statistical Association, june, 299-306.

Prat G., 1995, La formation des anticipations et l'hypothèse d'un agent représentatif, Revue d'Economie Politique, 105(2), p.197-222.

Prat G. and R. Uctum, 2000, The evidence of a mixed expectation generating process in the foreign exchange market, in: F. Gardes and G. Prat, eds., Price expectations in goods and financial markets: new developments in theory and empirical research, Edward Elgar.

Rauscher M., 1988, OPEC behavior and the price of petroleum, Journal of Economics, 48(1), 59-78.

Walls M.A., 1987, Petroleum supply modeling in a dynamic optimization framework: forecasting the effects of the 1986 oil price decline, Energy and Materials Discussion Paper, 87-03, december.

Wirl F., 1991, Energy demand and consumer price expectations : an empirical investigation of the consequences from the recent oil price collapse, Resources and Energy, 13(3), 241-62.

Unit cost

M'   N   M

0   Marginal gain
                                                             $-\pi\,f_H^{'}$

H_1^* ....

        $\uparrow \pi$

H_2^* ....

H_D ....

H_{max} $\downarrow f_H^{"}$
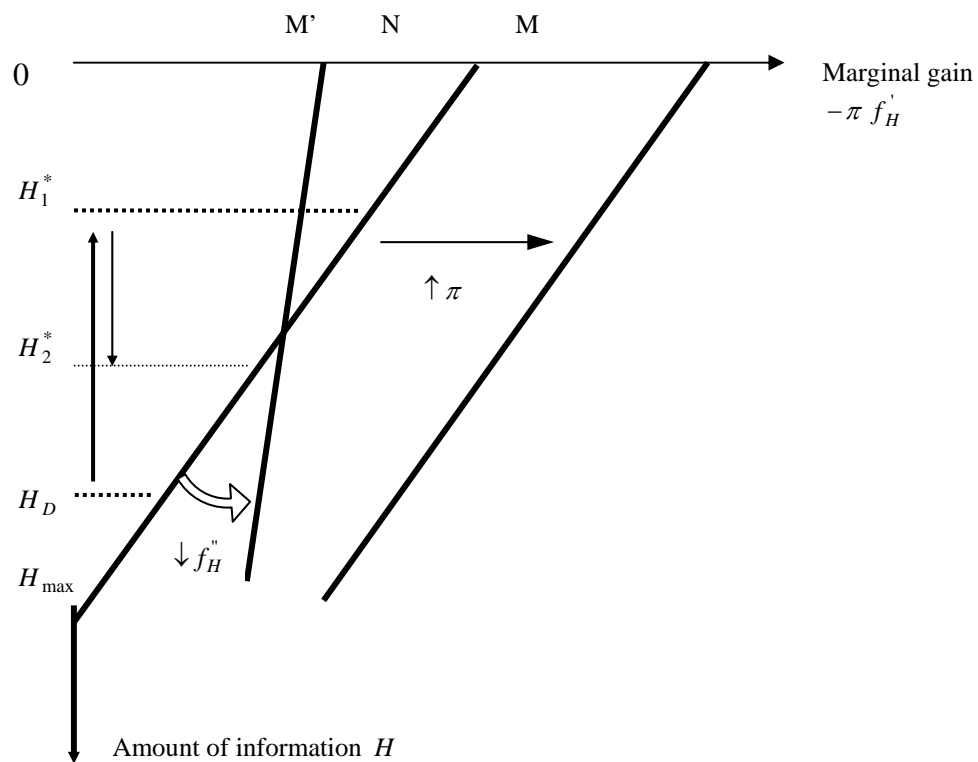
        Amount of information H

Figure 1: relations between the quantity of information, its unit cost and the marginal gain for a given agent in the Economically Rational Expectations framework

Figure 2. Three-month horizon expected rate of change in oil price: observed and fitted values according to the mixed model, February 1990 - January 1998

Observed

Fitted

Fitted values from regression V, Table1
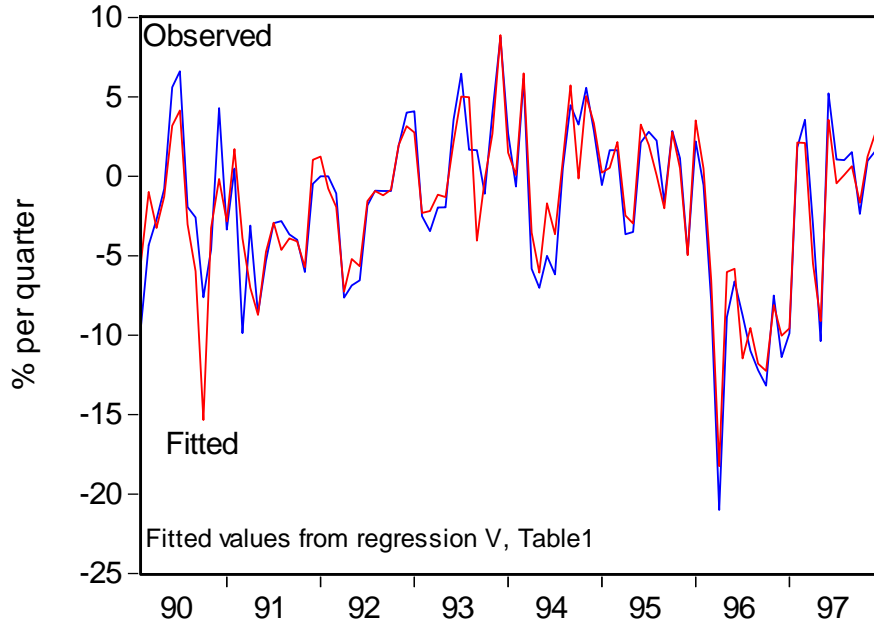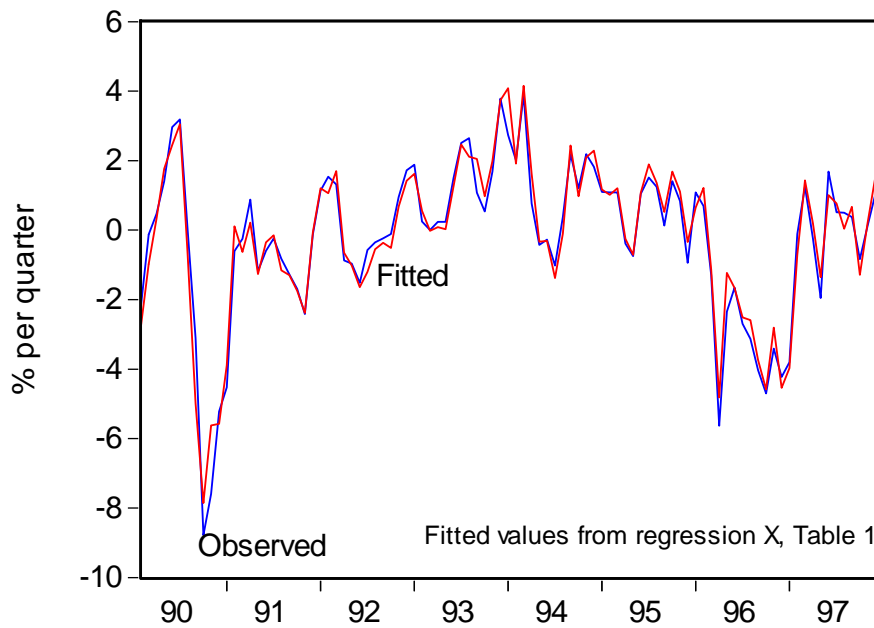
% per quarter

Figure 3. Twelve-month horizon expected rate of change in oil price: observed and fitted values according to the mixed model, February 1990 - January 1998

Fitted

Observed

Fitted values from regression X, Table 1

% per quarter

**Figure 4. Stardardized values of the spread variables,
June 1990 - December 1991**
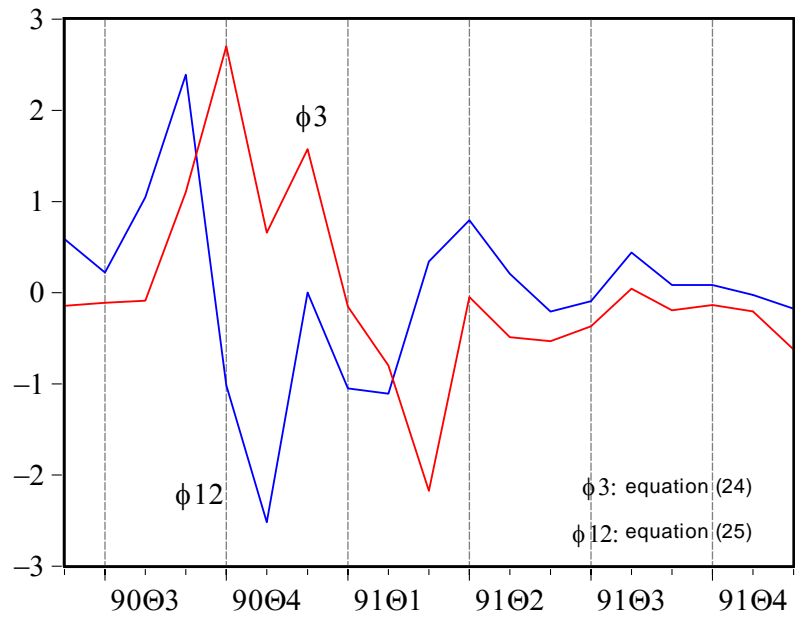
Table 1.   Determinants of oil price expectations

for the 3-month horizon										
$p_t - p_{t-3}$	$E_{t-1}p_{t+2} - p_t$	$E_{t-1}p_{t+11} - E_{t-1}p_{t+2}$	$E_t p_{t+12} - E_{t-1}p_{t+11}$	$p_{t+3} - p_t$	C	$\bar{R}^2$ (SE %)	ADF	Ljung-Box Probability	Sample period	Regression N°
-0.136 (-4.86)				—	-1.93 (-4.02)	0.192 (4.71)	CI 1% 0 lag	0.00%	90.02-98.01	1
	0.284 (12.13)			0.046 (2.31)	-1.23 (-3.49)	0.633 (3.33)	CI 1% 0 lag	0.00%	89.12-97.10	2
0.121 (4.32)	0.375 (11.77)			0.071 (3.76)	-0.79 (-2.44)	0.68 (2.98)	CI 1% 0 lag	0.2%	90.02-97.10	3
	0.527 (17.5)	0.11 (2.78)	0.376 (8.98)	—	-0.59 (-2.28)	0.817 (2.34)	CI 5% 1 lag	0.3%	89.12-98.01	4
0.117 (6.2)	0.615 (22.11)	0.181 (5.00)	0.353 (10.12)	—	—	0.854 (2.00)	CI 1% 0 lag	45.72%	90.02-98.01	5

for the 12-month horizon									
$E_{t-1}p_{t+11} - p_t$	$p_t - p_{t-2}$	$\bar{p} - E_{t-1}p_{t+11}$	$p_{t+12} - p_t$	C	$\bar{R}^2$ (SE %)	ADF	Ljung-Box Probability	Sample Period	Regression n°
-0.049 (-5.71)			0.274 (6.84)	—	0.542 (1.636)	CI 1% 0 lag	0.00%	90.02-97.01	6
	0.704 (24.81)		0.042 (2.15)	—	0.917 (0.694)	CI 1% 0 lag	0.25%	90.02-97.01	7
0.030 (7.73)	0.873 (32.94)		—	—	0.943 (0.550)	CI 1% 0 lag	26.07%	90.02-98.01	8
	0.687 (33.94)	0.052 (6.53)	—	-0.13 (-2.20)	0.936 (0.582)	CI 1% 0 lag	0.01%	90.02-98.01	9
0.022 (5.38)	0.812 (27.15)	0.030 (3.76)	—	—	0.949 (0.518)	CI 1% 0 lag	37.24%	90.02-98.01	10

Notes- The endogenous variable is the expected change of the oil price $\pi_{t,\tau} = E_t p_{t+\tau} - p_t$, $\tau = \{3,12\}$ representing the horizon. Whatever τ, are reported at the first column the extrapolative component and at the second column the adaptive component. The third and fourth columns for $\tau = 3$ and the third column for $\tau = 12$ provide the regressive component(s) in which targets are given by equations (20) and (21) respectively. Variable $p_{t+\tau} - p_t$ is a proxy of all information used by the rational agents if any. All variables are expressed in percent per quarter. C denotes an intercept. The sign « – » denotes a coefficient not significantly different from zero at the 5% level; the estimates provided by the table are those of the regression without the corresponding variable. The values in brackets under the estimates are the « t-values ». « SE » is the standard error of the regression. Augmented Dickey-Fuller test: « CI α % » or « NCI α % » means CoIntegration or No CoIntegration at the α % level of significance and with the indicated number of lags in the equation of change in residuals. Ljung-Box probability test: if this probability exceeds 5%, the null hypothesis stating that the residuals are independent is accepted (the test takes into account three lags).