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Eric Goncalvès da Silva

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Turbulence modelling in cavitating flows

Eric Goncalves

LEGI, University of Grenoble-Alpes, France

7-11 July 2014
Outline of the speech

Outline

1. General considerations
2. One-fluid RANS equations
3. Limitation of eddy viscosity
4. Compressibility terms
5. Wall models
6. Improved models - hybrid RANS/LES
7. One-fluid filtered equations and LES
8. Conclusions
General considerations
A large quantity of opened questions

Questions

- Kolmogorov spectrum, slope $\neq -5/3$
- Kolmogorov scale / size of two-phase structures.
- Induced turbulence or pseudo-turbulence.
- Compressibility effects on turbulence.
- Anisotropy of the Reynolds tensor.
- Increase or decrease of the turbulence intensity.
- Cavitation-turbulence interaction at small scales.

Remarks

- no DNS data or turbulent quantities measurements.
- non universality of two-phase flows.
Example of flows

Cavitation pockets

Cavitation in vortex
Turbulence scales

- DNS: all scales are resolved
- LES: large scale are resolved and small scales are modelled
- RANS: mean flow is resolved and turbulence is modelled
- Hybrid RANS/LES: model adapts to the mesh resolution

Phase scales

- DNS: resolution of each fluid + interface
- pseudo-DNS: resolution of each fluid + tracking of the interface (VoF, level set...)
- filtered (LES) and averaged (RANS) models:
  - two-fluid model,
  - reduced models,
  - one-fluid model
Averaged approach - homogeneous mixture models

Averaged one-fluid equations (RANS)
- Temporal averaged or ensemble averaged equations
- The same operator for two-phase structure and turbulent structure
- Boussinesq analogy used similarly to single-phase flows
- Transport-equation models: $k - \varepsilon$, $k - \omega$, ...

Filtered one-fluid equations (LES)
- First works in incompressible flow without phase transition
- Lots of sub-grid terms - problem of modelling
- Link with the DNS
Averaged equations
Average operator

**Temporal phase average**

Averaging over a time $T$:

$$\overline{\phi_k} = \frac{1}{T} \int_T \phi_k(x, \tau) \, d\tau$$

Averaging over the time $T_k$ of presence of the phase $k$:

$$\overline{\phi_k} = \frac{1}{T_k} \int_{T_k} \phi_k(x, \tau) \, d\tau$$

Void fraction: $\alpha(x, t) = \frac{T_k}{T} = \overline{\phi_k} / \phi_k$
Decomposition of variable

Mass-weighted average (Favre average)

For a variable $\rho \phi$, a mass-weighted average:

$$\tilde{\phi}_k = \frac{\rho_k \phi_k}{\rho_k} = \frac{\rho_k \phi_k}{\rho_k} \quad \text{and} \quad \tilde{\phi} = \frac{\sum \alpha_k \rho_k \phi_k}{\sum \alpha_k \rho_k} = \frac{\sum \rho_k \phi_k}{\sum \rho_k}$$

Mass-weighted decomposition

An average part and a fluctuation (two-phase and turbulent contribution):

$$\rho_k = \overline{\rho_k} + \rho'_k, \quad u_k = \tilde{u}_k + u''_k, \quad \phi_k = \tilde{\phi}_k + \phi''_k$$

$$\overline{\rho'_k} = 0, \quad \overline{\rho_k u''_k} = 0, \quad \overline{\rho_k \phi''_k} = 0$$
Conservative equations for the phase $k$

**Mass conservation:**

\[
\frac{\partial \alpha_k \bar{\rho}_k}{\partial t} + \text{div} \left( \alpha_k \bar{\rho}_k \bar{u}_k \right) = \Gamma_k
\]

**Momentum conservation:**

\[
\frac{\partial \alpha_k \bar{\rho}_k \bar{u}_k}{\partial t} + \text{div} \left( \alpha_k \bar{\rho}_k \bar{u}_k \otimes \bar{u}_k + P_k \right) = \text{div} \left( \alpha_k (\bar{\tau}_k + \tau_{tk}) \right) + \bar{M}_k
\]

**Total energy conservation:**

\[
\frac{\partial \alpha_k \bar{\rho}_k (\bar{E}_k + k_k)}{\partial t} + \text{div} \left( \alpha_k \rho_k (\bar{E}_k + k_k) \bar{u}_k \right) = \text{div} (\alpha_k P_k \bar{u}_k) + \text{div} (\alpha_k (\bar{\tau}_k + \tau_{tk}) \bar{u}_k)
\]

\[
- \text{div} (\alpha_k (\bar{q}_k + q_{tk})) + \bar{Q}_k
\]

New unknowns: Reynolds stress tensor $\tau_{tk}$ and turbulent heat flux $q_{tk}$
Fluctuating fields

The velocity fluctuating field is not divergence free (even with incompressible phases):

$$\frac{\partial u'_{k,l}}{\partial x_l} = -\frac{\partial \bar{u}_{k,l}}{\partial x_l} = \frac{1}{\alpha_k} \dot{u}'_k \cdot n_k \delta_l$$

It leads to supplementary unknowns, called "compressible terms".

Equation for the momentum fluctuation:

$$\rho_k \frac{\partial u''_{k,i}}{\partial t} + \rho_k \bar{u}_{k,l} \frac{\partial u''_{k,i}}{\partial x_l} + \rho_k u''_{k,i} \frac{\partial u''_{k,i}}{\partial t} + \rho_k \bar{u}'_{k,l} \frac{\partial \tilde{u}_{k,l}}{\partial x_l}$$

$$= \frac{\partial \bar{T}_{k,il}}{\partial x_l} + \rho_k \bar{u}'_{k,l} \frac{\partial \bar{u}_{k,l}}{\partial x_l}$$

$$+ \frac{\rho_k}{\bar{\rho}_k \alpha_k} \left[ -\frac{\partial \alpha_k \bar{T}_{k,il}}{\partial x_l} - M_k + \frac{\partial}{\partial x_l} \left( \alpha_k \bar{T}_{k,il} u''_{k,i} u''_{k,l} \right) \right]$$

$$+ \frac{\rho_k}{\bar{\rho}_k \alpha_k} \left[ \bar{u}_{k,i} \left( \frac{\partial \alpha_k \bar{T}_{k,il}}{\partial t} + \bar{u}'_{k,l} \frac{\partial \alpha_k \bar{T}_{k,il}}{\partial x_l} \right) \right]$$
Turbulent kinetic energy equation for the phase $k$

It is possible to write the equation for the phasic Reynolds stress and the phasic TKE $k_k$

$$\frac{\partial \alpha_k \bar{\rho}_k k_k}{\partial t} + \text{div} (\alpha_k \bar{\rho}_k k_k \tilde{u}_k) = \alpha_k \bar{\rho}_k (P_k - \varepsilon_k + \Pi_k + M_k + D_k) + \Gamma_k K^\Gamma$$

with:

- Production term
  $$\bar{\rho}_k \alpha_k P_k = -\alpha_k \bar{\rho}_k u''_{k,i} \frac{\partial \tilde{u}_k,i}{\partial x_l}$$

- Dissipation rate
  $$\bar{\rho}_k \alpha_k \varepsilon_k = \alpha_k \tau'_{k,il} \frac{\partial u''_{k,i}}{\partial x_l}$$

- Pressure-dilatation term
  $$\bar{\rho}_k \alpha_k \Pi_k = \alpha_k p'_k \frac{\partial u''_{k,i}}{\partial x_i}$$

- Mass flux term
  $$\bar{\rho}_k \alpha_k M_k = \alpha_k u''_{k,i} \left[ -\frac{\partial \bar{p}_k}{\partial x_i} + \frac{\partial \bar{\sigma}_{k,il}}{\partial x_l} \right]$$

- Diffusion term
  $$\bar{\rho}_k \alpha_k D_k = -\frac{\partial}{\partial x_l} \left[ \alpha_k P'_k u''_{k,i} \delta_{il} - \alpha_k \tau'_{k,il} u''_{k,i} \right] - \frac{\partial \alpha_k \rho_k \frac{u''_{k,i} u''_{k,i}}{2}}{\partial x_l}$$

- Mass transfer term
  $$\Gamma_k K^\Gamma$$
Homogeneous mixture equations or one-fluid model (1)

Mixture mean quantities

- mixture density and pressure: \( \rho_m = \sum \alpha_k \bar{\rho}_k \), \( P_m = \sum \alpha_k \bar{P}_k \)
- mixture internal energy: \( \rho_m e_m = \sum \alpha_k \bar{\rho}_k \tilde{e}_k \)
- mass center velocity: \( \rho_m u_{m,i} = \sum \alpha_k \bar{\rho}_k \tilde{u}_{k,i} \)
- mixture viscosity: \( \mu_m = \sum \alpha_k \bar{\mu}_k \)
- mixture viscous stress tensor: \( \tau_{m,ij} = \sum \alpha_k \bar{\tau}_{k,ij} \)
- mixture heat flux: \( q_m = \sum \alpha_k \bar{q}_k \)

Mixture turbulent quantities

- mixture turbulent kinetic energy: \( k_m = \sum \alpha_k k_k = \sum \alpha_k \bar{u}_{k,i}''^2 / 2 \)
- mixture Reynolds stress tensor: \( \tau_{m,ij}^t = -\sum \alpha_k \rho_k \bar{u}_{k,i}'' \bar{u}_{k,j}'' \)
- mixture eddy viscosity: \( \mu_{tm} = \sum \alpha_k \mu_{tk} \)
- mixture turbulent heat flux: \( q_{m}^t = \sum \alpha_k \bar{q}_{k}^t \)
- mixture dissipation rate: \( \varepsilon_m \), mixture specific dissipation \( \omega_m \)...
Conservative equations

Mass equation:
\[
\frac{\partial \rho_m}{\partial t} + \text{div}(\rho_m u_m) = 0
\]

Momentum equation:
\[
\frac{\partial \rho_m u_m}{\partial t} + \text{div}(\rho_m u_m \otimes u_m + P_m) = \text{div}(\tau_m + \tau^t_m)
\]

Energy equation:
\[
\frac{\partial \rho_m (E_m + k_m)}{\partial t} + \text{div}(\rho_m (E_m + k_m) u_m) = \text{div}(-P_m u_m) - \text{div}(q_m - q^t_m) \\
+ \text{div}[(\tau_m + \tau^t_m) u_m]
\]
Mixture turbulent kinetic energy equation

\[
\frac{\partial \rho m k_m}{\partial t} + \text{div} (\rho m k_m u_m) = \rho m P_m + \rho m \Pi_m - \rho m \epsilon_m + \rho m M_m + \rho m D_m + \Gamma_m K_m^\Gamma
\]

It is assumed that each phase shares the same fluctuating velocity \(u_i''\), the same fluctuating pressure \(P'\), and the same fluctuating viscous stress \(\tau_{ij}'\): 

\[
\rho m P_m = \tau_{m,il}' \frac{\partial u_m,i}{\partial x_l}; \quad \rho m \Pi_m = P' \frac{\partial u_i''}{\partial x_i}; \quad \Gamma m K_m^\Gamma = 0
\]

\[
\rho m M_m = \overline{u_i''} \left[ - \frac{\partial \rho m}{\partial x_i} \right] + \overline{u_i''} \left[ \frac{\partial \sigma_m,il}{\partial x_l} - \sum_k \overline{\sigma_{k,il}} \frac{\partial \alpha_k}{\partial x_l} \right]
\]

\[
\rho m D_m = - \frac{\partial}{\partial x_l} \left[ P' \overline{u_i''} \delta_{il} - \sigma_{il}' \overline{u_i''} \right] - \frac{\rho m \overline{u_i'' u_i''}}{2} \frac{\partial \rho m}{\partial x_l}
\]
Main features

- A fluctuation is associated to a mixture quantity!
- The fluctuating velocity field is not divergence free $\rightarrow$ supplementary terms, difficult to model.
- The pressure-dilatation term $\rho_m \Pi_m$ is null in mean but not instantaneously.
- The mixture dissipation $\varepsilon$ is not only solenoidal. Inhomogeneous and dilatational (or compressible) contributions.

\[
\rho \varepsilon \approx 2\mu \frac{\omega'_{ik} \omega'_{ik}}{\rho \varepsilon_s} + 2\mu \frac{\partial}{\partial x_k} \left[ \frac{\partial u'_k u'_l}{\partial x_l} - 2 u'_k \s'_{ll} \right] + \frac{4}{3} \mu \frac{s'_{kk} s'_{ll}}{\rho \varepsilon_d}
\]

- The diffusion term is modelled with a gradient formulation.
- The mixture dissipation equation $\varepsilon$ is completely modelled following the single-phase formulation.
Boussinesq assumption

Boussinesq analogy and mixture eddy viscosity assumption $\mu_{tm}$:

$$
\tau_{tm,ij} = \mu_{tm} \left[ \frac{\partial u_{m,i}}{\partial x_j} + \frac{\partial u_{m,j}}{\partial x_i} - \frac{2}{3} \text{div} u_m \delta_{ij} \right] - \frac{2}{3} \rho_m k_m \delta_{ij}
$$

Evaluation of the mixture eddy viscosity with transport-equation models.

Turbulent Fourier law

Fourier law analogy and mixture thermal conductivity $\lambda_{tm}^t$:

$$
q_{tm}^t = -\lambda_{tm}^t \text{grad} T_m
$$

Assumption of constant turbulent Prandtl number $Pr_t$:

$$
\lambda_{tm}^t = \sum \alpha_k \lambda_{tk}^t = \sum \alpha_k \frac{\mu_{tk}^t C_{pk}}{Pr_t} \quad \text{approximated by} \quad \lambda_{tm}^t \approx \frac{\mu_{tm}^t C_{pm}}{Pr_t}
$$
Turbulence modelling (3)

Usual model, $k - \varepsilon$ model for a mixture

Transport-equation models equivalent to single-phase turbulent models. All supplementary terms are neglected. Only the solenoidal dissipation is taken into account.

\[
\frac{\partial \rho k}{\partial t} + \text{div} \left[ \rho u k - \left( \mu + \frac{\mu_t}{\sigma_k} \right) \text{grad} k \right] = \rho P_k - \rho \varepsilon
\]

\[
\frac{\partial \rho \varepsilon}{\partial t} + \text{div} \left[ \rho u \varepsilon - \left( \mu + \frac{\mu_t}{\sigma_{\varepsilon}} \right) \text{grad} \varepsilon \right] = c_{\varepsilon 1} \frac{\varepsilon}{k} \rho P_k - \rho c_{\varepsilon 2} f_2 \frac{\varepsilon^2}{k}
\]

\[
\mu_t \approx \rho \frac{k^2}{\varepsilon s}
\]

Remarks

- A large quantity of assumptions.
- Introduction of wall treatment (damping functions or wall functions).
- A large quantity of problems!
Limitation of the eddy viscosity
Turbulent eddy corrections

The Reboud correction

\[ \mu_t = f(\rho) C_\mu \frac{k^2}{\epsilon} \]

\[ f(\rho) = \rho_v + \left( \frac{\rho_v - \rho}{\rho_v - \rho_l} \right)^n (\rho_l - \rho_v) \]

\( n \) is usually set to 10

SST correction - Bradshaw’s assumption for 2D boundary layer

\[ \mu_t = \frac{\rho k / \omega}{\max\left(1, \frac{\Omega F_2}{a_1 \omega} \right)} ; \; a_1 = 0.3 ; \; \Omega = \sqrt{2\Omega_{ij}\Omega_{ij}} \text{ with } \Omega_{ij} = \frac{1}{2} \left( \frac{\partial \tilde{u}_i}{\partial x_l} - \frac{\partial \tilde{u}_l}{\partial x_i} \right) \]

Realisability constraints of Durbin

\[ \mu_t = \min\left( C_\mu^0 ; \frac{c}{s\sqrt{3}} \right) \frac{\rho k^2}{\epsilon} ; \; 0 \leq c \leq 1 ; \; C_\mu^0 = 0.09 ; \; s = \frac{k}{\epsilon} S \text{ with } S = 2S_{ij}S_{ij} - \frac{2}{3}S_{kk}^2 \]
Experimental conditions

Operational point:
- $U_{inlet} = 10.8 \text{ m/s}$
- $\sigma_{inlet} = \frac{P_{inlet} - P_{vap}}{0.5 \rho U_{inlet}^2} \approx 0.55$

Observations:
- A quasi stable cavitation of 0.70 to 0.85 m length
- An unsteady closure region with vapour cloud shedding and a liquid re-entrant jet

Measurements:
- Time-averaged longitudinal velocity and void ratio
- Time-averaged wall pressure evolution and RMS fluctuations
Reboud limiter

Results using the Spalart-Allmaras model, density gradient

Limitation of the eddy viscosity $\mu_t$

SA

SA + Reboud limiter

station 3

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Turbulence modelling in cavitating flows
Eddy viscosity limiter, profiles at stations 3 and 4

Time-averaged void fraction (left) and velocity (right) profiles
The $k-\ell$ model + SST correction

Time-averaged void fraction (left) and velocity (right) profiles

Contour of the density gradient: $a_1 = 0.2$ (left) and $a_1 = 0.1$ (right)
Compressibility corrections
Closure relations for turbulence compressible terms

**Pressure-dilatation, Sarkar formulation**

\[
\rho \Pi = P' \frac{\partial u''_i}{\partial x_i} = -\alpha_2 \rho P M_t + \alpha_3 \rho \varepsilon_s M_t^2
\]

\(\alpha_2, \alpha_3\) are constants to calibrate. \(M_t = \frac{\sqrt{k}}{2c}\) is the turbulent Mach number.

**Dilatational dissipation, Sarkar formulation**

\[
\varepsilon_d = \frac{4}{3} \mu \frac{\partial s'_{kk} s'_{ll}}{\partial x_i} = \alpha_1 \varepsilon_s M_t^2 \quad \text{with } \alpha_1 \text{ to calibrate}
\]

**Mass flux, Jones formulation**

\[
\rho M = \frac{\rho' u''_i}{\rho} \left( \frac{\partial \bar{P}}{\partial x_i} - \frac{\partial \bar{\sigma}_{il}}{\partial x_l} \right) = -\frac{\mu_t}{\rho^2 \sigma_p} \frac{\partial \rho}{\partial x_i} \frac{\partial \bar{P}}{\partial x_i}
\]

\(\sigma_p\) is a turbulent Schmidt number, which has to be calibrated.
$k - \varepsilon$ compressible model

$k - \varepsilon \ + \ \Pi, \ \alpha_2 = 0.15, \ \alpha_3 = 0.001$ (left) and $\alpha_3 = 0.025$ (right)

$k - \varepsilon \ + \ mass \ flux \ term, \ \sigma_p = 1$ (left) and $\sigma_p = 0.0001$ (right)
$k - \varepsilon$ compressible model (2)

$k - \varepsilon + \Pi + M + \varepsilon_d$, Sarkar values, at 2 instants

Time-averaged wall pressure (left) and RMS fluctuations (right)
Wall functions
Opened questions

- Existence of an universal velocity profile.
- Instantaneous logarithmic area.
- Cavitating law of the wall ($\kappa$ function of $\alpha$).
- Turbulence damping functions.
- Modifications of turbulent properties downstream a pocket.
- Compressibility effects.

Numerical study

- Computations using various meshes: $y^+$ from 1 to 50.
- Comparison of wall functions: two-layer model versus TBLE model.
Formulation - similar to single-phase flows

\[ u^+ = y^+ \quad \text{if} \quad y^+ < 11.13 \]

\[ u^+ = \frac{1}{\kappa} \ln y^+ + 5.25 \quad \text{if} \quad y^+ > 11.13 \]

where \( \kappa = 0.41 \) is the von Karman constant

For unsteady flows, validity of the velocity profile at each instant.

Turbulent quantities: the production of \( k \) or directly \( k \) is fixed following the formulation by Viegas and Rubesin:

\[ P_k = \frac{1}{y} \int_0^y \tau_{xy} \frac{\partial u}{\partial y} \, dy \]

The second variable is computed through a length scale.
Thin boundary layer equations TBLE

Formulation

Simplified momentum equation:

\[
\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} + \frac{1}{\rho} \frac{dP}{dx_i} = \frac{\partial}{\partial y} \left[ (\mu + \mu_t) \frac{\partial u_i}{\partial y} \right]
\]

Use of an embedded grid between the first grid point and the wall.

Discretization and integration of TBL equations in the embedded mesh. Iterative solving (Newton algorithm) for the variable \( \tau_w \). The number of nodes in the embedded grid is \( N = 30 \).
Venturi simulations

Mesh influence near wall, station 3

Two-layer model versus TBLE model
The experimental frequency is around 45 Hz. The inlet cavitation parameter $\sigma_{inlet} \sim 2.15$.

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<th>mesh</th>
<th>$\sigma_{inlet}$</th>
<th>frequency (Hz)</th>
<th>cost for 100 ite. (s)</th>
<th>ratio</th>
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<td>387</td>
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Improved model and hybrid RANS/LES
### Framework

Starting point: the $k - k\ell$ model of Rotta (1972) where $k\ell = \frac{3}{16} \int_{-\infty}^{\infty} R_{ii}(\vec{x}, r_y) \, dr_y$

Correlation tensor $R_{ij}(\vec{x}, r_y) = U_i(\vec{x}) \, U_j(\vec{x} + r_y)$

velocity correlation between a fixed point and a moving point in direction $y$

Transport equation for $\Psi = k\ell$ involving the integral quantity $I$:

$$I = -\frac{3}{16} \int_{-\infty}^{\infty} \frac{\partial U(\vec{x} + r_y)}{\partial y} R_{12} dr_y$$

Expansion in Taylor series gives:

$$I \approx \frac{\partial U(\vec{x})}{\partial y} \int_{-\infty}^{\infty} R_{12} dr_y + \frac{\partial^2 U(\vec{x})}{\partial y^2} \int_{-\infty}^{\infty} R_{12} r_y dr_y + \frac{1}{2} \frac{\partial^3 U(\vec{x})}{\partial y^3} \int_{-\infty}^{\infty} R_{12} r_y^2 dr_y + \ldots$$

The second order derivative is neglected (assumption of homogeneous flow). The third order derivative is difficult to model and Rotta neglected also this term.
Menter modelling

Menter proposed a model using the second derivative of the velocity (indicator of the heterogeneity of the flow) and the von Karman scale $L_{vk}$

$$\frac{\partial^2 U}{\partial y^2} \int_{-\infty}^{\infty} R_{12} r_y dr_y \approx P_k \frac{k\ell}{k} \left( \frac{\ell}{L_{vk}} \right)^2$$

and

$$L_{vk} = \kappa \left| \frac{U'}{U''} \right|$$

New term in the transport equation for $\Psi$ (S-A term), driven by a constant $\xi$. $\xi = 1.47$ following the calibration of Menter.

Scale-adaptive model

- The characteristic length scale is self-adaptive, function of the von Karman scale for a standard model, the length scale is proportional to $\delta$
- It allows to adjust the solve of turbulent structures → behaviour close to a LES
- BUT : problem in the near-wall area, the log zone is not respected → the S-A term is not activated in the near-wall area
- calibration of $\xi$ in cavitating flows.
Comparison between Reboud and S-A $k - \ell$ models

- time-averaged profiles quasi similar
- better pressure fluctuations with S-A model

Density gradient, Schlieren visualization

$k - \ell$ SAS

$k - \ell$ Reboud
Detached Eddy Simulation of Spalart

\[
\frac{\partial \rho \tilde{\nu}}{\partial t} + \text{div} \left[ \vec{V} \rho \tilde{\nu} - \frac{1}{\sigma} (\mu + \rho \tilde{\nu}) \text{grad} \tilde{\nu} \right] = c_b \left( 1 - f_{t2} \right) \tilde{S} \rho \tilde{\nu} + \frac{c_b^2}{\sigma} \text{grad} \rho \tilde{\nu}. \text{grad} \tilde{\nu}
\]

- \left( c_\omega f_\omega - \frac{c_b f_{t2}}{\kappa^2} \right) \rho \frac{\tilde{\nu}^2}{\tilde{d}^2}

with \( \tilde{d} = \min (d, C_{\text{DES}} \Delta) \) and \( \Delta = \max (\Delta x, \Delta y, \Delta z) \).

\( C_{\text{DES}} \) is a constant evaluated for the decay of isotropic turbulence = 0.65.

In equilibrium area: \( \tilde{\nu} = C_{\text{DES}}^2 \Delta^2 S \)

Drawbacks

- Grid induced separation.
- Transition between RANS-mode and LES-mode: "grey" zone.
- Calibration of the constant \( C_{\text{des}} \).
2D simulations of the Venturi 4°

Influence of $C_{DES}$. RANS regions (black) and LES (white)

$C_{des} = 0.65$

$C_{des} = 0.9$

Wall pressure and RMS fluctuations

EXP

SA-DES-c065-sigma065
SA-DES-c08-sigma06
SA-DES-c09-sigma0588
SA-DES-c09-sigma0579
Comparison between $k - \ell$ S-A and DES simulations

gradient density visualization at two different instants: $k - \ell$ S-A (left) and DES (right)
Comparison between $k - \ell$ Scale-Adaptive and DES simulations

Iso-surface of the void fraction for the value of 60% at two different instants:

- $k - \ell$ S-A (left)
- DES (right)
Comparison between 2D and 3D simulations

- Good agreement between models and experiment (mid-span profiles)
- 3D computations provides void fraction lower than 2D computations
Comparison between 2D and 3D simulations

The level of RMS pressure fluctuations is largely overestimated by the 3D computations.

Time-averaged wall pressure and RMS fluctuations
Dynamic behaviour - oblique mode using S-A simulations

Density gradient visualization

→ a transversal instability at a low frequency : 6 Hz.
One-fluid filtered equations and LES
Scales

- Micro-scales, scales which are small enough to describe individual bubble shapes.
- Meso-scales, which are comparable to bubble sizes.
- Macro-scales, which entail enough bubbles for statistical representation.

When LES is applied at a micro-scale, combination with interface tracking methods (see Lakehal).
When LES is applied at a macro-scale, the interface resolution is not considered.

The scale separation is mathematically obtained by applying a convolution product using a large-scale-pass filter (function $G$).
For a quantity $\phi$, the filtered variable is defined as: $\overline{\phi} = G \circ \phi$

The Favre filtered variable: $\tilde{\phi} = \frac{\rho \phi}{\overline{\rho}}$

The filtered phase indicator function $\overline{X_k} = G \circ X_k = \alpha_k$ can be interpreted as a filtered volume fraction of phase $k$.

The phase indicator function is defined as:

$$X_k(M(x, t)) = \begin{cases} 1 & \text{if phase } k \text{ is present in point } M(x, t) \text{ at } t \\ 0 & \text{otherwise} \end{cases}$$
Assumptions: the filtering operator commutes with time and spatial derivatives. The mass transfer is assuming to be proportional to the velocity divergence through a constant $C$.

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho \tilde{u}_i)}{\partial x_j} &= 0 \\
\frac{\partial \rho \tilde{u}_i}{\partial t} + \frac{\partial (\rho \tilde{u}_i \tilde{u}_j + \bar{P} \delta_{ij})}{\partial x_j} &= \text{div} \left(2\mu \tilde{S} + 2\tau_{\mu S} - 2\tau_{\rho S} - \tau_{\rho uu}\right) \\
\frac{\partial \rho \tilde{E}}{\partial t} + \frac{\partial (\rho \tilde{H} \tilde{u}_i + \bar{Q}^\prime)}{\partial x_j} &= \text{div} \left(2\tau_{\mu S} - 2\mu \tilde{V} \tau_{\rho S} - 2\mu \tilde{S} \tau_{\rho uu}\right) \\
&\quad + \text{div} \left(4\mu \tau_{\rho uu} \tau_{\rho S} + \bar{\rho} \tilde{H} \tau_{\rho uu} - \tau_{\rho Hu}\right) \\
\frac{\partial \alpha}{\partial t} + \tilde{V} \cdot \nabla \alpha - C \frac{\partial \tilde{u}_i}{\partial x_j} &= -\tau_{u\alpha} - C \text{div} \left(\tau_{\rho uu}\right)
\end{align*}
\]
Subgrid terms

\[
\begin{align*}
\tau_{u\alpha} & = \bar{V} \cdot \nabla X_v - \bar{V} \cdot \nabla \alpha \\
\tau_{\mu S} & = \mu \bar{S} - \bar{\mu} \bar{S} \\
\tau_{\rho H u} & = \rho \bar{H} V - \bar{\rho} \bar{H} \bar{V} \\
\tau_{\rho uu} & = \bar{\rho} \left( \bar{V} \otimes \bar{V} - \bar{\bar{V}} \otimes \bar{\bar{V}} \right)
\end{align*}
\]

- The subgrid term \( \tau_{u\alpha} \) is specific to two-phase flows.
- The influence and the hierarchy of all these terms have never been investigated in cavitating flows.
- The magnitude of the different subgrid terms was \textit{a priori} evaluated in the case of phase separation flows and turbulence bubble interaction (Labourasse,Vincent) \( \rightarrow \) the influence of \( \tau_{u\alpha} \) is highly dependent on the flow configurations and/or on the chosen two-phase description.
Modelling difficulties

- Lots of assumption in models.
- Lack of experimental data or DNS.
- Compressibility turbulent closure due to the no divergence free fluctuating velocity field
- Advanced turbulence models and hybrid turbulence model to improve the level of resolved scales
- A challenge: LES in cavitating flows.