Problem creating tasks to develop teacher's competencies

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In this presentation, an experience about posing new problems carried out by teachers in their classrooms is analysed. The teachers use a strategy specifically designed to change a given problem posed in a concrete class episode. Form such experience problem creating is found a way of contributing to the development of didactic and mathematics competencies of teachers is proposed.

Keywords: problem creating, professional development, teacher competencies, teacher training.

INTRODUCTION

Problem posing takes increasing attention in recent years. It was recommended that the teachers offering opportunities to know about formulating problems from a giving situation. The best way to do this is to adopt an inquiry-based learning approach. Obviously, teachers should have developed their problem creating skill to be able to work in this way with their students. Many authors underline the importance of the relationship between problem solving, problem posing and problem creating by including problem creating in teacher training programs (Singer, Ellerton & Cai, 2015). An open problem is how to articulate theoretical notions of Ontosemiotic perspective of constructing mathematical objects and processes with problem posing in teacher training courses in order to produce good mathematical and didactical problems promoting professional development.

The future teacher needs to be able to modify some proposed problems in order to get a richer mathematical activity, being aware of their mathematical benefits. It should be part of growing the capacity of analysing didactically the mathematics activity (Rubio, 2012). We expect that having in mind such tools for designing and didactical challenging problems motivate teachers' interest in creating problems and in developing their capacity for creating problems in ways that serve teaching and learning. Thus, the aim of the paper is to explore the use of a strategy for engaging pre-service and early career teachers to enrich mathematics problems after didactical and epistemic reflection by means of problem transformation. In particular, to find which mathematical and didactical benefits emerge when using instructional strategies in teacher training courses as a case study design. Thus, two questions are involved: (a) Which are the characteristics of a strategy that uses reflection as the core of promoting that better proposed problems appear, and (b) which mathematical and didactical benefits emerge when using a didactical analysis reflection.

THEORETICAL FRAMEWORK

Problem posing has been usually interpreted as the generation of a new problem or reformulation of a given problem; as the formulation of a sequence of mathematical problems from a given situation; or as a resultant activity when a problem is inviting the generation of other problems. Authors as Silver referred to problem posing as involving the creation of a new problem from a situation or experience, or the reformulation of given problems (Stoyanova & Ellerton, 1996). Instead of that we consider creating mathematics problems being a process which let us obtain a new problem from a given one (problem's *variation*); or from a situation (problem's *elaboration*). The situation can be as it is presented in the reality, or configured as a part of the problem's elaboration (Malaspina, 2013).

Some researchers try to integrate problem posing ideas and didactical analysis for teacher training purposes, analysing the benefits of qualified joint reflection and aspects associated with its development, using problems with fractions by using semantic analysis as a reflective analysis (Ticha and Hošpesová, 2013). We assume creating problems as related to complex processes considering knowledge base, task organisation, heuristics and schemes, group dynamics, and individual considerations (Koichu & Kontorovich, 2013). It is also important valuing aspects of the proposed problems in order to see a mathematical improvement (Sengül & Katranci, 2014) even because future teachers had difficulties for characterising conceptual aspects.

It is clear the power of transforming mathematical tasks according variations by promoting teachers being sensitive and recognising how to use in the classroom (Milinkovic, 2015). Mathematical content knowledge is necessary, but our hypothesis is that transformations in problem posing can improve content knowledge by means of didactical analysis, and also contributes to increase didactical competencies of teachers. We identify more deep approach when using suitability criteria proposed by OSA, considering the analysis of epistemic issues; cognitive; normative; interactional; emotional and ecological issues to influence task design (Gimenez, Font &Vanegas, 2013).

In this paper, problem's variation instruction (Malaspina, Font & Mallart, 2015) is a content analysis based strategy to integrate above proposals for improving competence of didactical analysis for future teachers or in service teachers. It consists of first exposing teachers to a class episode. When analysing the possible mathematical difficulties solving the problems included during the episode, we also notice the didactical requirements to improve the solving process. A *pre-problem* is a new proposal statement that try to satisfy such didactical needs. In order to develop this perspective of problem creating, we consider four key problem elements (Malaspina, 2013): Information, Requirement, Context and Mathematical Environment. As a second redesign, we introduced a more detailed epistemic analysis, using the tools of suitability criteria, in order to recognise the power of knowing the configuration of objects and processes following OSA. A *post-problem* is a new proposal to improve the problem by finding easier problems responsive to difficulties students had, and harder problems to challenge students to generalize key ideas beyond simply answering the initial problem. This global instructional strategy is called ERPRP because, it starts by facing a class episode (E), first reflection (R), producing a problem (P), introducing tools from didactical analysis (R), and producing a better problem (P). In such a framework, we would like to show that such strategy helps and stimulate the ability to create mathematics problems, through modifying a given problem, considering mathematical and didactic aspects. Therefore, problem creating by using transformations of previous problems is a contributing way to the professional development for future teachers.

METHODOLOGY

We have chosen a qualitative ethnography study with the proposal of a starting strategy, and exemplification as cases study from 2013 to 2015 with three groups of 25 prospective teachers participating in problem solving courses in Peru, Ecuador and Spain. The first step consisted of choosing a topic and designing some easy and motivating problems as starting points to create new problems by changing some math concepts or ideas assuming ERPRP strategy already described. All the proposed pre and post-problems are analysed by means of content analysis to see which are the mathematical and didactical new ideas learnt behind the proposals. We use next section to present some paradigmatic examples to reveal the power of the phases of the strategy used as a qualitative analysis, and some of the mathematical and didactical benefits drawn.

DISCUSSION

At the beginning, the problem creating experiences had been performed with pre-service teachers as a part of the mathematics courses with a strategy ERPP, in which initial reflection, pay attention to analyse mathematical difficulties, doing two steps of modification problem posing. The positive experiences of the individual work and of the group work were the basis to design the strategy above explained.

We spent two hours only on developing a starting problem creating workshop. We made a short exposition about problem creating, including some examples of problems created in previous workshops. We presented a previously elaborated problem to the students presenting a concrete class episode of a teacher T. In this episode, the trainer describe some of pupil reactions when solving the problem. Each future teacher created its Pre problems individually. Group discussion plays an important role of this first strategy. We redesigned such initial strategy in order to include another reflection moment using suitability tools to improve challenging pos-problems. In a second experience, a theoretical based reflection is a new phase. Pos-problems are the final step to be analysed. Let us see some research results of mathematical and didactical benefits by means of some examples.

The role of the initial Episode.

The main issue of proposing an episode instead a problem, is to see problem statement in a real professional class-context. In fact, the position of a teacher is not only being a problem solver, but a problem inquirer. We see it in the following example of proposal.

> The first week of July a shop called MARKET sold all the products without any discount; the second week, applied discount of 20% on all the items; and the third week, added discount of 15%. It was announced as the GREAT DISCOUNT OF 20%+15% ON ALL THE PRODUCTS. You have to study whether the third week of July the shop called ALFA sold products with 35 its % discount on prices of the first week is true or not. After a few minutes: Most of the students say yes, it is true. Juan and Carla say no because the discount of the third week was less than 35%. Maria says that the discount of the third week was 68%.

The role of individual reflecting. Future teachers usually explain that they have similar difficulties to the students in the episode. Just some of them can solve the problem discussing about the multiplicative structure of a discount.

The role of Pre-Problem creating and group discussion. The main value of pre-problem is to contribute to a better comprehension of the situation presented in the episode. It gives opportunities for starting a didactical analysis. Let us see some *pre problems* posed by teachers to help in the process of creating new problems after discussing the problem above cited. The research group analyse all the productions, to observe the hypothesis of mathematical and didactical purpose in each proposal. It gives opportunities for identifying the background of future teachers. The problems creation starts individually at the beginning and discussed afterwards in groups. All the groups of future teachers solve the problems created, and the explanation's resolution is part of a socialization process with all the participants. Following the examples, we notice that in some cases, the author's idea when the teacher posed the problem was considering a price very easy to calculate its percentage in order to help pupils focus their attention on the total discount

> Rosa pays a sum of 100 "new soles" for a shirt, with discount of 20% because of ending bargain sales and with an additional discount of 10% thanks to having the shop's card. What percentage did Mary take off on buying the shirt? (FT1)

In other cases, the author was interested in showing the students another point of view of the total discount. It is not only a simple sum of percentages. In order to achieve this objective, the author had chosen this problem because it posed a total discount (100%) very little intuitive.

> In a clearance sale, a shop applies discount of 50% on all its textiles during a week, and the following week applies an additional discount of 50%. What is the total percentage discount applied during the second week? (FT2)

In some cases, a group of future teachers develop a common problem trying to help pupils to distinguish between the money paid and the discount. This seemed to be the confusion of the student called Maria in the situation. Apparently, she had done well her calculations but she did not distinguish between the money paid (68% of the initial price) and the total discount (100 - 68 = 32%).

> Rosa pays a sum of 100 “new soles” for a shirt, with discount of 20% because of ending bargain sales and with an additional discount of 10% thanks to having the shop’s card.
> a) How much does the blouse cost to Maria if she buys it during the second week?
> b) What is the percentage representing the second week’s price with the blouse’s price without discount?
> c) What is the blouse's total percentage discount during the second week?

Observing the examples proposed, there is a need of focusing, on solving the problem giving a justified answer, but to understand the mathematical content or property distinguished or “specific” in such a proposed initial task. Only when the groups of future teachers think more than the mathematics topic involved, we can see didactical growing. In general, future teachers are worried about what they do not know, and they could learn from others. The pre-problem also help to recognise management aspects by doing a context analysis. Future teachers know about facing children’s difficulties.

The role of Post-Problem creating.

At the beginning of creating post-problems, many future teachers thought that it is important to conserve the structure by finding similar problems to the given problem, with other prices and in some cases, considering three successive discounts; basically, with quantitative modifications in the data. The future teachers imagined that the main aspect to modify is the computation problem and the particular process of solving the problem. One of the future teachers tell us “it is a problem of discounts”. However, they carried out more enriched problems via transformations when they formulated post- problems, even without a second reflection.

In some groups of future teachers, it appear the need that children should reinforce the comprehension of the fact that the total discount is not a simple sum.

Pedro and Juan bought a shirt each one. Pedro bought it with a discount of 20% plus another additional discount of 20%. Juan bought it with a discount of 30% plus another additional discount of 10%. Who did obtain the greater discount? (Gr 1)

Another group thought it was interesting to pose situations about cumulative percentage, considering charges and not only discounts. Its solution requires a better understanding of the percentage concept.

There is a shop where if you want to pay after enjoying the product 30 days with a card, the price increases 10% more. And if you want to pay after 31 days but before 35 days, there is a surcharge of 5%. If Juan bought in this shop on August 20th and paid on September 23rd, what did he pay for percentage of surcharge? (Gr 2)

Bearing in mind the importance of the socialization aspect, in all the cases we have kept an extra mathematical context but we need also to create problems in an intra mathematical context. Generalization allows us working in this context. Generalisation appear as being a new statement with higher mathematical value.

If the shop called BETA knocks off end of the season of p%, plus an additional of q%, what is the percentage of the total discount in relation to the price without any discount? (Gr 3)

In this case, or similar ones, the problem lets illustrate in an easy way the discount of r% applied to the sale price of a product (x) through a composition of linear functions: $f(x) = \left(1 - \frac{r}{100}\right)x$

It is clear the role of this phase is to see what is behind a problem in terms of promoting mathematics learning. If there is a discussion about "particularisation/ generalisation" or contextualisation/ de-contextualisation, we can see the teachers growing their didactical analysis competence. And consequently, promoting more rich problems.

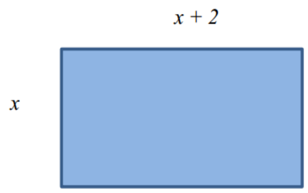
The power of discussion when analysing the content

Let us notice the influence of the mathematical content example and the classroom discussion showing the value of such reflection to improve mathematical content knowledge, by using successive problem transformations. Working with second degree it was presented the following short episode.

If we multiply the age of Charles 3 years ago, times the age that Charles will have after 5 years, we obtain 48. Which is actually the age of Charles? To solve the problem some students wrote (x + 9)(x– 7)= 0 to conclude that the actual age of Charles is 7 years old, because the other solution is negative.

One group wrote the factorisation as $f(x) = x^2 + 2x - 15 = (x + 5)(x - 3)$. They feel the main issue is to identify the second-degree equation to solve the problem, and then plan the following post-problem, introducing contextualised situation.

> Which can be the dimensions of a rectangular room if the area must be maximum of 15 square meters being the length two meters more than the other size? (post Gr 5). Then, they draw the following design



And then they wrote the following inequality $x(x+2) \leq 15$, and they conclude that the result are the points of the interval $]0\ ;\ 3]$ In such case, the teacher ask the participants to make explicit the relation between the solutions and the given function f. After some minutes without any proposal, we propose to use another register, not an algebraic equation, but a graphic register.

Therefore a variation of the problem was suggested.

> Which can be the dimensions of a rectangular room if the area must be maximum of 15 square meters being the length two meters more than the other size? Ilustrate the solution using the graphic of a quadratic function. We ask the teachers to create a new problem using the graphic of linear functions above cited.

The need of epistemic analysis. The adapted new scheme ERPRP.

During the first experiences, having the post-problem the trainer was almost satisfied. In fact, the mathematical object more difficult to analyse was the mathematical argument in front of expressions and terms. Only six future teachers distinguish the arguments used during the solving process. A half of the future teachers talk about propositions and procedures.

When we introduced an epistemic analysis during the redesign, the future teachers noticed more mathematical aspects than before. For instance, many of the future teachers talk about generalisation, and give explanations about the need for analysing maximum or minimum when the problem needs to use a second-degree equation. It is the case of the problem of second-degree, in which a sequence of new post-problems appear, to introduce the role of connecting representations when introducing mathematical objects. Let us see an example of starting problem (Malaspina, 2013).

> *Present the graphic of a function f given by the function $f(x) = x^2 + 2x - 15$ using the graphs of two affine functions.*

The future teachers used both graphs of g(x) = x+5 and h(x) =x– 3 as you can see, using geogebra. A first reaction was to obtain points of the product by multiplying the corresponding ordinates of the points of the graphics of g and h. Nevertheless, it was suggested to do a more wide and global analysis, and more qualitative, using key points.

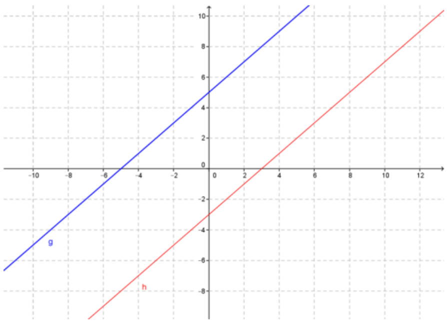


Figure 1. Observations about the function approach

The trainer asked to find some points of g and h in which we have points of the function f. They discover the graph must include the points (-5; 0), (3; 0) . Therefore g(-5) = 0 and they conclude that multiplying by every number the result must be zero. With similar argument, they found that f(-5) = 0 y f(3) = 0. It was also proposed to find the sign of points for f according the signs of g and h. They conclude that for $x \neq -5$ and something similar for $x \neq 3$. They tell us "when x < -5, the graphs of g and h are below the x axe. In consequence, the product is positive and the graph of f, when x<5, will be up the x-axe. Similar analysis give to the conclusion that when -5 <x<3 the graph will be below the y=0, and for x > 3, the graph of the function f will be above y=0

Using ideas about the continuity of f, they discover that the graph is a curve passing through (-5; 0) and (3 ; 0); decreasing till certain point of the interval]-5 ; 3[; increasing the points after. When they used geogebra to observe their intuitions, they found that the result is according the intuition.

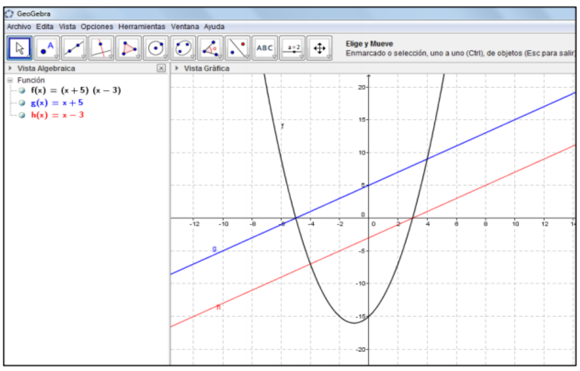


Figure 2. The parabola as a "product of two right lines".

After that, the future teachers identify that new statements can be proposed. Didactically speaking, the revised experiences offer also evidences in which the future teachers identify and notice tools for understanding students' practices and difficulties. According final master comments of pre-service teachers in Spain, we found many examples in which they learn from students, using the strategies related to problem creating activities. They told us about "how interesting was to see that now I understand why 14 years-old students have difficulties to understand that a parabola is the product of two right-lines". The percentage's theme and equations theme are very favourable to create problems in an extra mathematical context and it suggests a great diversity of imagined situations in the created problems. This reveals the authors' advances in the mathematical object management, in the reality observation and in elaborating tasks to go deeper into the subject to solve the problem created.

The cases presented are paradigmatic examples showing the emergence of empowerment of future teachers, using different kind of transformations: quantitative (changing numbers), qualitative (the problem deals with discounts and increases), relational (the information is shown to make easier the meditation over possible wrong answers) and in some cases, it is a piece of information added or the requirement is extended. Problem creating as a redesigned process related to one concrete theme contributes to deal it deeply. It provides opportunities to relate mathematical ideas and representations to get an insight to involve students into intra-mathematical connections. In our examples, creating problems within a reflective process gave opportunities for relating algebraic situations to geometrical graphical interpretations unknown for the teachers. Such interventions give opportunities of reflecting about intra and extra mathematical connections. But at the same time, future teachers talk about interactions, the role of contextualisation, to overcome the magisterial class, and the role of mathematical debates.

FINAL REMARKS

Creating problems give opportunities and benefits to challenge future teachers to claim for powerful understanding of connecting representations, assuming the role of problem posing as a positive way for critical math understanding.

As a part of the challenges posed by this research on creating math problems in mathematics education contexts, we see evidences in which creating math problems on a given topic activates new learning processes that favour intra mathematics connections with other fields of knowledge and reality. The intervention of the researcher contributed more to focus upon the theoretical perspectives of problem creating.

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