Set-membership fault detection approach for a class of nonlinear networked control systems with communication delays

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Abstract In this paper, a Fault Detection (FD) problem for a class of Nonlinear Networked Control Systems (NNCS) in a set-membership framework is investigated. Under the assumption of bounded network-induced delays and process uncertainties (i.e. process disturbances and measurement noises), a residual generator is constructed based on a set-membership estimation-based predictor approach. Finally, a numerical example illustrating the performances of the proposed method is given.

Key words: Fault detection, Nonlinear Networked Control System (NNCS), unknown network delay, interval observer, predictor.

1 Introduction

The NCS are systems wherein some or all signals are transmitted among the system's components as information flows through a shared network [7]. Compared with conventional point-to-point architectures, the advantages of NCS are lighter wiring, lower installation costs and greater abilities in diagnostic, reconfigurability and maintenance [7]. Thanks to these distinctive benefits, application of NCS ranges over various industry's fields nowadays [7, 13]. However, using a shared network for data exchange make system control [10], monitoring [9] or diagnostic [11] more difficult where some communication constraints should be considered such as packets losses, sampling problems and network-induced delays [7]. The last mentioned is one of the most common problem in literature [6], especially in NCS FD. Intensive research addresses this challenging subject, one can see for example [6, 11] and the

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references therein. Filtering method, Markovian jump approach and observer-based approach are ones of the most used approaches dealing with this problem. In this paper, a FD technique is proposed in a set-membership framework for the NNCS with unknown communication delays and unknown external disturbances [5] by only knowing their bounds. The main idea consists in detect faults by generating interval residual signals. Then, FD is ensured through a belonging test of the zero signal to the interval delimited by upper and lower residual signals.
This paper is structured as follows. Section 2 introduces some preliminaries. Section 3 represents the studied system architecture. In Section 4, we develop the proposed fault detection technique. Then, Section 5 is devoted to simulation results proving the proposed FD strategy. Conclusions are given in Section 6.

2 Preliminaries

$\mathbb{R}$ and $\mathbb{N}$ represent the sets of real and natural numbers, respectively. The eigenvalues set of a matrix $A \in \mathbb{R}^{n\times n}$ is named $\lambda(A)$ and $\mathrm{Re}(z)$ is the real part of the complex number z. The set of Hurwitz matrices from the set $\mathbb{R}^{n\times n}$ is denoted by $\mathbb{H}$, i.e. $R \in \mathbb{H} \Leftrightarrow \mathrm{Re}(\lambda) < 0, \forall \lambda \in \lambda(R)$. We denote $\mathbb{M}$ as the set of Metzler matrices from the set $\mathbb{R}^{n\times n}$, i.e. $R = \{r_{ij}\}_{i,j=1}^{n} \in \mathbb{M} \Leftrightarrow r_{i,j} \geq 0$ for $i \neq j$. For a variable $x(t) \in \mathbb{R}^n$, the upper and lower bounds are denoted by $\overline{x}(t) \in \mathbb{R}^n$ and $\underline{x}(t) \in \mathbb{R}^n$, respectively, such that $\underline{x}(t) \leq x(t) \leq \overline{x}(t)$ and the relation $\leq$ should be interpreted elementwise for vectors and matrices, i.e. $A = (a_{i,j}) \in \mathbb{R}^{n\times m}$ and $B = (b_{i,j}) \in \mathbb{R}^{n\times m}$ such that $A \geq B$ if and only if $a_{i,j} \geq b_{i,j}$ $\forall$ $i \in \{1, ..., n\}$ and $j \in \{1, ..., m\}, i, j \in \mathbb{N}$. For a matrix $R \in \mathbb{R}^{n\times m}$, define $R^+ = \max\{0, R\}$ and $R^- = R^+ - R$. The matrix of absolute values of all elements of a matrix $M \in \mathbb{R}^{n\times m}$ is $|M| = M^+ + M^-$. The vector E_p is stated for $(p \times 1)$ vector with unit elements, and I_n denotes the identity matrix of $n \times n$ dimension. Superscript T denotes the transpose of a matrix or a vector. $\mathcal{K}$ is the set of continuous increasing functions $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ with $\gamma(0) = 0$. We refer by $\beta \in \mathcal{KL}$ if $\beta(\cdot, t) \in \mathcal{K}$ for all $t \geq 0$ and $\beta(r, \cdot)$ is continuous and strictly decreasing to zero for all $r > 0$. $\|.\|$ is the standard 2-norm.

Lemma 1 *[3] Let $\underline{x}, x, \overline{x} \in \mathbb{R}^n$ be vectors satisfying $\underline{x} \leq x \leq \overline{x}$ and $A \in \mathbb{R}^{n\times m}$ be a time-invariant matrix. Then, the inequalities below hold:*

$$A^+\underline{x} - A^-\overline{x} \leq Ax \leq A^+\overline{x} - A^-\underline{x}. \tag{1}$$

Lemma 2 *[2] Consider the following system:*

$$\begin{cases} \dot{x}(t) = Ax(t) + \psi(t), \\ y(t) = Cx(t), \end{cases} \tag{2}$$

where $\psi(t)$ is a continuous function and A, C are known matrices. Suppose that there exist two known continuous-time functions $\underline{\psi}(t)$ and $\overline{\psi}(t) : \mathbb{R} \to \mathbb{R}^n$ satisfied $\underline{\psi}(t) \leq \psi(t) \leq \overline{\psi}(t)$, $\forall$ $t \geq 0$.

If there exists a gain L such that $(A-LC) \in \mathbb{H} \cap \mathbb{M}$ *and* $\underline{x_0}, x_0, \overline{x_0} \in \mathbb{R}^n$, $\underline{x}_0 \leq x_0 \leq \overline{x}_0$, *then the system:*

$$\begin{cases} \dot{\overline{x}}(t) = A\overline{x}(t) + \overline{\psi}(t) + L(y(t) - C\overline{x}(t)) \\ \dot{\underline{x}}(t) = A\underline{x}(t) + \underline{\psi}(t) + L(y(t) - C\underline{x}(t)) \end{cases} \tag{3}$$

is an interval observer for (2) and $\underline{x}(t) \leq x(t) \leq \overline{x}(t)$, $\forall t \geq 0$.

Definition 1 Consider the nonlinear system

$$\dot{x} = f(x,u), \tag{4}$$

with $f(x,u) \in \mathbb{R}^n$, the system (4) is input-to-state Stable (ISS) if for any input $u \in \mathbb{R}^m$ and $x_0 \in \mathbb{R}^n$ there exist functions $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$ such that

$$|x(t,x_0,u)| \leq \beta(x_0,t) + \gamma(\|u\|), \quad \forall t \geq 0. \tag{5}$$

3 NCS architecture description and problem formulation

Consider the following NNCS:

$$\begin{cases} \dot{x}(t) = Ax(t) + F(u(t),y(t)) + w(t) + Hf(t), \\ y(t) = Cx(t) + v(t), \end{cases} \tag{6}$$

where $x \in \mathbb{R}^n$ denotes the state vector, $y \in \mathbb{R}^p$ the measurable output vector $u \in \mathbb{R}^m$ the known input vector, where $w \in \mathbb{R}^n$, $v \in \mathbb{R}^p$ and $f \in \mathbb{R}^n$ are the external disturbances and the additive faults to be detected. The functions u, v, w are continuous. The function $F(u(t),y(t)) \in \mathbb{R}^n$ is a globally Lipschitz nonlinear function. The matrices A, C and H are known matrices of compatible dimensions. Before proceeding further, we make some assumptions on the process matrices.

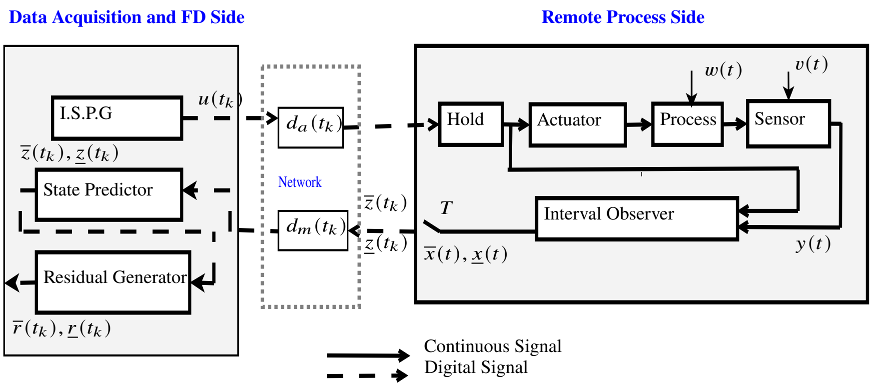

Fig. 1 NCS architecture.

Assumption 1 The pair (A, C) is detectable. □

Assumption 2 $w(t)$ and $v(t)$ are unknown but bounded functions with a priori known bounds, for $\overline{w}$, $\underline{w} \in \mathbb{R}^n$, $\overline{V} \in \mathbb{R}_+$:

$$\underline{w} \leq w(t) \leq \overline{w}, \quad |v(t)| \leq \overline{V} E_p, \quad \forall t \geq 0 \tag{7}$$

The network induces two uncertain delays: $d_a(t)$ and $d_m(t)$ which refer to the actuation and the measurement channels delay, respectively. These delay functions are unknown but bounded:

$$0 \leq \underline{d_a} \leq d_a(t) \leq \overline{d_a}, \quad 0 \leq \underline{d_m} \leq d_m(t) \leq \overline{d_m}, \quad \forall t, \tag{8}$$

where $\overline{d_a}$, $\underline{d_a}$ are respectively the upper and lower bounds of $d_a(t)$ and $\overline{d_m}$, $\underline{d_m}$ are respectively the upper and lower bounds of $d_m(t)$. Suppose that $\overline{d}$ and $\underline{d}$ are respectively the upper and lower bounds of the communication delays such that:

$$\overline{d} = \max\{\overline{d_a}, \overline{d_m}\}, \underline{d} = \min\{\underline{d_a}, \underline{d_m}\}. \tag{9}$$

Considering the communication delay in the actuation channel, the process (6) can be modeled as an input delayed system:

$$\begin{cases} \dot{x}(t) = Ax(t) + F(u(t - d_a(t)), y(t)) + w(t) + Hf(t) \\ y(t) = Cx(t) + v(t). \end{cases} \tag{10}$$

Assumption 3 We assume that $u(t - d_a(t))$ is bounded. □

Let $\{t_k, k \in \mathbb{N}\}$ be the sequence of sampling instants such that $t_{k+1} - t_k = T$ and $\lim\limits_{k\to\infty} t_k = \infty$, T is the sampling period and t_k is an increasing sequence such that $t_k = kT$. In a network environment, data sampling is needed. Therefore, the next assumptions are required.

Assumption 4 [1] The sampling communication delays $d_a(t_k)$ and $d_m(t_k)$ are unknown but bounded with a priori known bounds and the upper bound $\overline{d}$ is assumed to be a multiple of the sampling period T. □

Assumption 5 [1] The information on the control signal $u(t_k)$, the information on the output $y(t_k)$ and the information on the sampling upper and lower bounds $\underline{z}(t_k)$, $\overline{z}(t_k)$ could be stored and used $\forall\, t_k \in \left[t_k - \overline{d}, t_k\right)$. □

Each block of the NCS architecture shown in Fig. 1 performs the same function as described in [5]. However, the added Residual Generator block is implemented in the calculator for residual generation; after receiving predictor outputs; $\overline{z}(t_k)$ and $\underline{z}(t_k)$, Residual Generator calculates interval residual signals $\overline{r}(t_k)$ and $\underline{r}(t_k)$ which will be used for FD test detailed in Section 4.

3.1 Interval observer structure

In this section, the interval observer developed in [5] is tackled to estimate the unavailable states of the process under unknown input delay $d_a(t)$ and process uncertainties. In free faulty case, the system (10) is:

$$\begin{cases} \dot{x}(t) = Ax(t) + F(u(t - d_a(t)), y(t)) + w(t) \\ y(t) = Cx(t) + v(t). \end{cases} \tag{11}$$

Since it is not always possible to compute a gain L for the system (11) such that $A - LC \in \mathbb{H} \cap \mathbb{M}$, a change of coordinates $\xi = Sx$ with a nonsingular matrix S such that the matrix $S(A - LC)S^{-1} \in \mathbb{H} \cap \mathbb{M}$ is used to relax this restriction [8].

Theorem 1 *[5] Let Assumptions 1–3 be satisfied and $\underline{x_0} \le x_0 \le \overline{x_0}$. If there exists a change of coordinates $\xi = Sx$ satisfying $\mathcal{E} = S(A - LC)P \in \mathbb{H} \cap \mathbb{M}$, $P = S^{-1}$ so that the following system*

$$\begin{cases} \dot{\hat{x}}^+(t) = \mathcal{E}\hat{x}^+(t) + \overline{SF}(u(t - \underline{d}), y(t)) + SLy(t) + S^+\overline{w} - S^-\underline{w} + |SL|\, E_p\overline{V}, \\ \dot{\hat{x}}^-(t) = \mathcal{E}\hat{x}^-(t) + \underline{SF}(u(t - \underline{d}), y(t)) + SLy(t) + S^+\underline{w} - S^-\overline{w} - |SL|\, E_p\overline{V}, \end{cases} \tag{12}$$

where

$$\begin{cases} \overline{SF}(u(t - \underline{d}), y(t)) = \max\limits_{\alpha \in \left[0, \overline{d} - \underline{d}\right]} \{SF(u(t - \underline{d} - \alpha), y(t))\}, \\ \underline{SF}(u(t - \underline{d}), y(t)) = \min\limits_{\alpha \in \left[0, \overline{d} - \underline{d}\right]} \{SF(u(t - \underline{d} - \alpha), y(t))\}, \end{cases} \tag{13}$$

and the initial conditions are calculated as follows:

$$\hat{x}^+(0) = S^+\overline{x}_0 - S^-\underline{x}_0, \quad \hat{x}^-(0) = S^+\underline{x}_0 - S^-\overline{x}_0, \tag{14}$$

is input-to-state stable (ISS) interval observer for the system (11) satisfying [4]

$$\hat{x}^-(t) \le \xi(t) \le \hat{x}^+(t), \quad \forall t \ge 0 \tag{15}$$

where the bounds of the solution $x(t)$ are:

$$\overline{x}(t) = P^+\hat{x}^+(t) - P^-\hat{x}^-(t), \underline{x}(t) = P^+\hat{x}^-(t) - P^-\hat{x}^+(t)\,. \tag{16}$$

such that

$$\underline{x}(t) \le x(t) \le \overline{x}(t), \quad \forall t \ge 0. \tag{17}$$

Proof The proof of the above theorem is detailed in [5]. □

3.2 State predictor design

To compensate the large unknown communication delays $d_m(t_k)$, an interval predictor introduced in [5] is used. Based on the delayed data $\overline{z}(\frac{t_k}{T}-\frac{\overline{d}}{T})$, $\underline{z}(\frac{t_k}{T}-\frac{\overline{d}}{T})$, we will reconstruct $\overline{z}(\frac{t_k}{T})$, $\underline{z}(\frac{t_k}{T})$ after a finite time $t_k=\overline{d}$.

Assumption 6 T is selected such that the matrix $\Phi = I_n + T\mathcal{E}$ is positive. □

Theorem 2 *[5] If Assumptions (4)–(6) hold $\forall\ t_k \geq \overline{d}$, we get:*

$$\begin{aligned}\hat{z}^{+}(k) &= \Phi^{k_2}\hat{z}^{+}(k-k_2) + \sum_{j=1}^{k_2}\Phi^{k_2-j}T\overline{SF}(u(k-k_1-k_2+j-1),y(k-k_2+j-1))\\ &+ \sum_{j=1}^{k_2}\Phi^{k_2-j}L_1 y(k-k_2+j-1) + \sum_{j=1}^{k_2}\Phi^{k_2-j}\overline{\beta},\end{aligned} \tag{18}$$

$$\begin{aligned}\hat{z}^{-}(k) &= \Phi^{k_2}\hat{z}^{-}(k-k_2) + \sum_{j=1}^{k_2}\Phi^{k_2-j}T\underline{SF}(u(k-k_1-k_2+j-1),y(k-k_2+j-1))\\ &+ \sum_{j=1}^{k_2}\Phi^{k_2-j}L_1 y(k-k_2+j-1) + \sum_{j=1}^{k_2}\Phi^{k_2-j}\underline{\beta},\end{aligned} \tag{19}$$

where $j=1,2,3,\dots,k_2$, $k=\frac{t_k}{T}$, $k_1=\left\lfloor\frac{d}{T}\right\rfloor$ and $k_2=\frac{\overline{d}}{T}$, $\Phi=(I_n+T\mathcal{E})$, $L_1=TSL$, $\overline{\beta}=T(S^{+}\overline{w}-S^{-}\underline{w}+|SL|\,E_p\overline{V})$, $\underline{\beta}=T(S^{+}\underline{w}-S^{-}\overline{w}-|SL|\,E_p\overline{V})$ and

$$\overline{z}(k)=S^{+}\hat{z}^{+}(k)-S^{-}\hat{z}^{-}(k),\underline{z}(k)=S^{+}\hat{z}^{-}(k)-S^{-}\hat{z}^{+}(k) \tag{20}$$

are a predictor from the sampling instant of time $k-k_2$ to k for the process (11) *i.e. $\overline{z}\longrightarrow\overline{x}$ and $\underline{z}\longrightarrow\underline{x}$ $\forall\ t_k\geq\overline{d}$ and the following inclusion holds*

$$\underline{z}(\frac{t_k}{T}) \leq x(\frac{t_k}{T}) \leq \overline{z}(\frac{t_k}{T}), \quad t_k \geq \overline{d}. \tag{21}$$

Proof Please see the proof detailed in [5]. □

4 Fault detection

In this section, a procedure of fault detection is developed thanks to Residual Generator block as shown in Fig.1. The residual evaluation in a set-membership context is ensured via the following belonging test [12]: If the zero signal is enclosed by upper and lower bounds of the residual signal, it is a fault-free case. Otherwise, a fault is occurred. Two steps are required to indicate the presence of faults:

- **Step1: Residuals generation**: in this first step, the Residual Generator calculates upper and lower residuals defined as the gap between measured outputs and estimated outputs using stored information.
  From Lemma 1, the upper and lower bounds of the estimated output are then computed as follows:
  $$\begin{cases} \overline{y}(t_k) = C\overline{z}(t_k) + \overline{V}E_p = C^{+}\overline{z}(t_k) - C^{-}\underline{z}(t_k) + \overline{V}E_p \\ \underline{y}(t_k) = C\underline{z}(t_k) - \overline{V}E_p = C^{+}\underline{z}(t_k) - C^{-}\overline{z}(t_k) - \overline{V}E_p \end{cases} \tag{22}$$
  Then, upper and lower residuals are :
  $$\overline{r}(t_k) = \overline{y}(t_k) - y(t_k), \underline{r}(t_k) = \underline{y}(t_k) - y(t_k)\,. \tag{23}$$
- **Step 2: Residuals evaluation:** this second step is detailed as follows. When a fault is occurred, an inconsistency is detected shown that the estimated outputs are no more compatible with the measurements where:
  $$y(t_k) \notin [\underline{y}(t_k), \overline{y}(t_k)] \tag{24}$$
  The above belonging test is rewritten as follows:
  $$\begin{cases} 0 \notin [\underline{y}(t_k), \overline{y}(t_k)] - y(t_k) \\ 0 \notin [\underline{y}(t_k) - y(t_k), \overline{y}(t_k) - y(t_k)] \\ 0 \notin [\underline{r}(t_k), \overline{r}(t_k)] \end{cases} \tag{25}$$
  Then, the zero signal is enclosed by $\overline{r}$ and $\underline{r}$ in the fault free case. Otherwise, a fault is detected.

From (23) and using the fact that $C = C^{+} - C^{-}$, $\overline{r}$ and $\underline{r}$ are computed as follows:

$$\overline{r}(t_k) = \overline{y}(t_k) - y(t_k) = C^{+}(\overline{z}(t_k) - z(t_k)) + C^{-}(z(t_k) - \underline{z}(t_k)) - v(t_k) + \overline{V}E_p \tag{26}$$

$$\underline{r}(t_k) = \underline{y}(t_k) - y(t_k) = -C^{+}(z(t_k) - \underline{z}(t_k)) - C^{-}(\overline{z}(t_k) - z(t_k)) - v(t_k) - \overline{V}E_p \tag{27}$$

An augmented system is then defined as:

$$\begin{gathered} \begin{bmatrix} \overline{r} \\ \underline{r} \end{bmatrix} = \begin{bmatrix} C^{+} & C^{-} \\ -C^{-} & -C^{+} \end{bmatrix} \begin{bmatrix} \overline{e_z} \\ \underline{e_z} \end{bmatrix} + \begin{bmatrix} -v + \overline{V}E_p \\ -v - \overline{V}E_p \end{bmatrix}, \\ \overline{e_z}(t_k) = \overline{z}(t_k) - x(t_k), \underline{e_z}(t_k) = x(t_k) - \underline{z}(t_k) \end{gathered} \tag{28}$$

Or we have $\overline{z} \longrightarrow \overline{x}$ and $\underline{z} \longrightarrow \underline{x}$ $\forall\ t_k \geq \overline{d}$ (see Section 3.2), then

$$\begin{cases} \overline{e_z}(t_k) = \overline{z}(t_k) - x(t_k) \simeq \overline{x}(t_k) - x(t_k) = \overline{e}(t_k) \\ \underline{e_z}(t_k) = x(t_k) - \underline{z}(t_k) \simeq x(t_k) - \underline{x}(t_k) = \underline{e}(t_k) \end{cases} \tag{29}$$

Known that the measurement noise v is bounded, stability analysis of upper and lower residual signals is equivalent to assure the stability of the estimation errors.

From Theorem 1, we have upper and lower bound estimation errors $\overline{e}$ and $\underline{e}$ are ISS [5]. Therefore, from (29), one can prove that $\overline{e_z}$ and $\underline{e_z}$ are ISS and then the residual signals $\overline{r}(t_k)$ and $\underline{r}(t_k)$ are ISS $\forall\ t_k \geq \overline{d}$.

5 Numerical example

To prove the efficiency of the proposed FD strategy, the next system (6) is considered:

$$A = \begin{bmatrix} -3.5000 & 0 & 0.5000 \\ 0 & -2.7540 & 0 \\ 0 & 0 & -1.2000 \end{bmatrix}, F(u(t), y(t)) = \begin{bmatrix} \sin(u(t)) \\ 1.5\sin(u(t)y(t)) \\ 2\sin(u(t)) \end{bmatrix},$$

$$C = \begin{bmatrix} 0 \; 0 \; 1 \end{bmatrix}, H = [-3 \quad 2 \quad 1]^T$$

$$w(t) = [0.1\cos(2t) \quad 0.1sin(3t) \quad 0.1\cos(4t)]^T, \overline{w} = [0.1 \quad 0.1 \quad 0.1]^T, \underline{w} = -\overline{w}$$
$$\text{and } v(t) = 0.2\cos(t)\cos(5t)\sin(10t)\sin(20t) \text{ with } \overline{V} = 0.2.$$

On can see that the function $F(u(t), y(t))$ is globally Lipschitz.
Considering communication delays, system (6) will be modeled as (10); its output signal and the input signal delivered by the I.S.P.G are depicted in Fig.2.
Network proprieties: The network induces unknown but bounded delays as described by (8). These bounds are $\underline{d} = 0.7s$, $\overline{d} = 0.1s$ and the distribution of delays is shown in Fig.3 with the disturbances and the measurement noise. The sampling period is given by $T = 0.01s$.

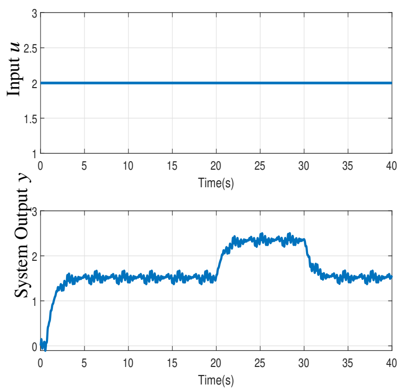

Fig. 2 System input and output.

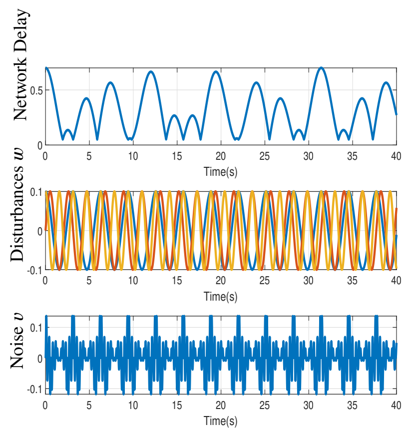

Fig. 3 Network delay, disturbances and noise.

Interval observer design: A gain $L = \begin{bmatrix} -10 \; 0 \; 0 \end{bmatrix}^T$ is computed satisfying $A - LC \in \mathbb{H}$. However, the matrix $A - LC$ is Hurwitz but is not Metzler. Then,

a transformation of coordinates as described in Section 3.1 is needed. We propose $S = \begin{bmatrix} 0.5005 & 0.0000 & -2.2800 \\ 0.000 & 1.0064 & 0.0000 \\ -0.0010 & -0.0012 & 5.5701 \end{bmatrix}$, and we can easily verify that $\mathcal{E} = S(A-LC)S^{-1} = \begin{bmatrix} -3.5000 & 0.0000 & 0.0020 \\ 0.0000 & -2.7540 & 0.0000 \\ 0.0049 & 0.0019 & -1.2000 \end{bmatrix} \in \mathbb{H} \cap \mathbb{M}$. Therefore, an interval observer as (12) can be designed. The initial conditions of system (10) are chosen such that $\underline{x}_0(t) \leq x_0(t) \leq \overline{x}_0(t)$; $\overline{x}(0) = \begin{bmatrix} 0.5 & 0.5 & 0.5 \end{bmatrix}^T$ and $\underline{x}(0) = \begin{bmatrix} -0.5 & -0.5 & -0.5 \end{bmatrix}^T$. Also the estimator (12) is initialized by $\hat{x}^{+}(0)$ and $\hat{x}^{-}(0)$ which are defined in (14).
State predictor design: For the sampling period T = 0.01s, we obtain
$\Phi = \begin{bmatrix} 0.9650 & 0.0000 & 0.0000 \\ 0.0000 & 0.9724 & 0.0000 \\ 0.00005 & 0.00002 & 0.9880 \end{bmatrix}$. The predictor described by (18) and (19) can be computed with Φ, $\tilde{L_1} = [-0.0500 \quad 0.0000 \quad 0.0001]^T$
and $\overline{\beta} = [0.0127 \quad 0.0010 \quad 0.0010]^T$, $\underline{\beta} = -\overline{\beta}$.
Fault evolution: The system is affected by an additive fault $f(t)$:

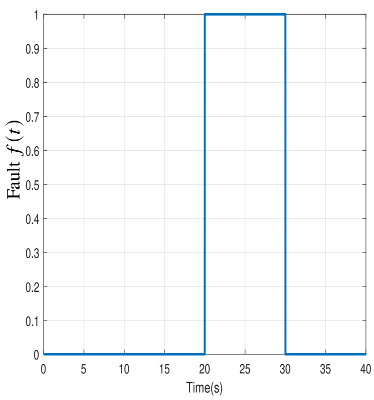

Fig. 4 Fault evolution.

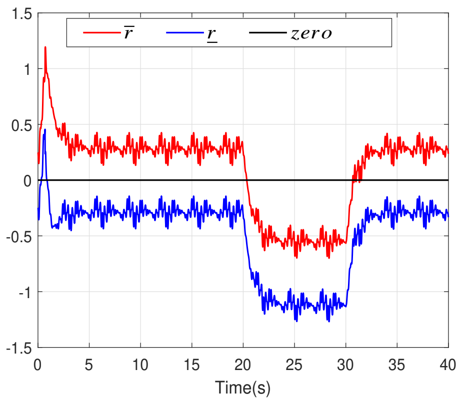

Fig. 5 Evolution of upper and lower residual signals.

$$\begin{cases} f(t) = 1 & 20s \leq t \leq 30s \\ f(t) = 0 & else \end{cases} \tag{30}$$

The fault evolution is presented in Fig.4. The evolution of the residual signals in faulty case is plotted in Fig.4 where black lines are the zero signal and red, blue lines represent the upper and lower residuals, respectively. As shown in Fig.5 upper and lower residual signals are sensitive to the fault f i.e. $0 \notin [\underline{r}, \overline{r}]$ when $20s \leq t \leq 30s$. Then we can conclude that the fault detection strategy is ensured and validated despite unknown communication delays and external process uncertainties.

6 Conclusion and future work

In this work, a set-membership FD technique is developed for NNCS subject to network delays and uncertainties. These uncertainties are assumed to be unknown but bounded with a priori known bounds. The main contribution consists in using upper and lower residuals for FD decision. A belonging test of the zero signal to the interval delimited by the upper and lower residuals is used to ensure the detection of faults. Theoretical results have been validated through the numerical example.
Nevertheless, the fault isolation problem for such NNCS is not investigated in this contribution. It will be the subject of a future work.

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