Integrated maintenance/production planning plan for a multiple-product manufacturing system with variable maintenance and changeover costs considerations

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Abstract—This paper deals with the problem of integrated maintenance/production strategy for multiple-product manufacturing system. We consider a manufacturing system consisting of one machine which ensures the production of several types of products in order to satisfy randomly demands corresponding to every type of product. At any given time, the machine is able to produce one type of product, and then switches to another. From the point of view of reliability, the manufacturing system is subject to random failures. The goal of this study is to establish an economical production planning followed by an optimal maintenance strategy, taking into account the influence of production rate on the system degradation. Compared to previous works dealing with the same subject, the first contribution of this study consists of the considering a variable setup costs according to the type of product to switch. More than that, we adopt an increased preventive maintenance cost according the system degradation. Analytical models are developed in order to minimize sequentially the production/holding cost and the total maintenance cost. Finally, a numerical example is presented to illustrate the usefulness of the proposed approach.

Keywords—Maintenance/production policy; multiple-product; optimization; setup cost.

I. Introduction and literature review

The Manufacturing companies have to cope successfully with several functional capacities, such as production, and maintenance. One of the keys to accomplishment consists of treating simultaneously all services. We note that the satisfaction of the customer is one of the important objectives of a business. Therefore, it becomes compulsory to develop a new integrated maintenance policies relating to production, which reduce the total costs integrating maintenance and production. This objective is needed in the case of multiple product manufacturing systems.

Establishing an optimal production planning and maintenance strategy has always been the great challenge for industrial companies. Moreover, during the last few decades, the integration of production and maintenance policies problem has attracted much research attention. In fact, integrated maintenance-production strategies which take into consideration subcontracting have been studied by [1]. They developed and optimized a maintenance policy incorporating subcontractor constraints. They demonstrated through a case study, the influence of the subcontractor constraints on the optimal integrated maintenance-production strategy.

An analytical model and a numerical procedure which allow to determine a joint optimal inventory control and an age based on preventive maintenance policy for a randomly failing production system was presented by [2].

The present study examined both of the problems of the optimal production planning formulation and the optimal maintenance strategy of a manufacturing system. The system considered is composed of a single machine which produces several products in order to meet corresponding several random demands. The problem is presented in (Fig. 1).

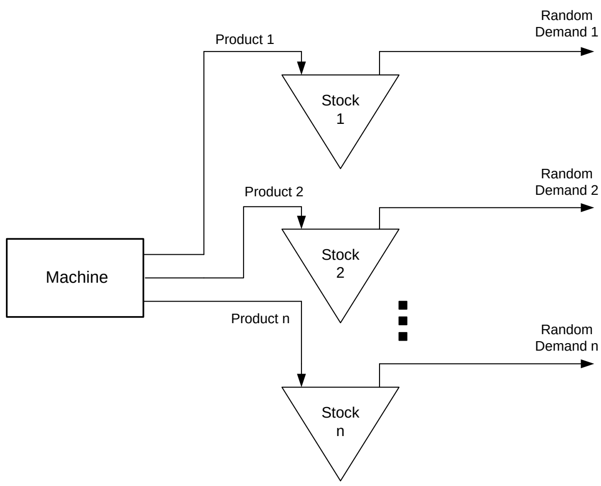

Fig. 1. Problem description

This type of problem, concerning multiple products or multiple machines, was treated by [3]. They presented an analysis of production control and corrective maintenance problem in a multiple-machine, multiple-product manufacturing system. They realized a near optimal control policy of the system through numerical techniques by controlling both production and repair rates. Sloan *el al.* [4] treated a Markov decision process model that simultaneously established maintenance and production schedules for a multiple-product, single-machine production system. They considered that the equipment condition can affect the yield of different product types differently.

In other hand, the literature on integrated maintenance policies, we noticed that the influence of the production rate on the system degradation over a finite planning horizon was rarely addressed in depth. Hajej et al. [5] took into account the influence of production plan on the equipment degradation, but in the case of a system composed by single machine producing one type of product under randomly failing and satisfying a random demand over a finite horizon. In the same context [6] presented a model, where the failure rate of a machine depends on its age; hence, maintenance policies are machine-age dependent.

Motivated by the work of [5], we treated the production and maintenance problem in another context that we consider a more complex and real industrial system, composed by one machine that produces several products during a finite horizon divided into periods. This study shows that it has a novelty and originality relative to this type of problem which reflects the influence of several products on the degradation state of the considered manufacturing system. More than keys study consists of, firstly, considering a variable setup cost between products; secondly we adopt an increased preventive maintenance cost according to the system degradation.

We note that, our problem can easily adopted to case of multiple machine with multiple product, and the same methodology will adopted considering every machine

This paper is organized as follows: In the next section we specify the targeted contributions of this work. Section 3 presents the notations and assumptions. The production and maintenance models are developed respectively in section 4 and 5. A numerical example is presented in section 6. Finally, the conclusion is included in Section 7.

II. Targeted contribution

The present study examined a problem of the optimal production planning formulation of a manufacturing system consisting of one machine producing several products in order to meet several random demands. Then we find the optimal maintenance strategy according to the optimal production plan established for every product. In fact we take into account the impact of the production rate on the system degradation. We note that we considered a variable setup cost between products; and we adopt an increased preventive maintenance cost according to the system degradation. In fact, the variable setup and maintenance costs, which present the industrial reality, are rarely treated in literature.

III. Notations and assumptions

In this paper, we use the following notations and assumptions:

A. Notations

$Cp(i)$	: The unit production cost of product i
$Cs(i)$	: The unit holding cost of one unit of product i during Δt
$St(i,l)$	: Changeover cost from product i to product l
Mc	: Corrective maintenance action cost
Mp	: Preventive maintenance action cost
H	: Total number of periods
n	: Total number of products
p	: Total number of sub-periods during each period
Δt	: Production period duration
U_{inom}	: Nominal production rate of product i during Δt
θ_i	: Probabilistic index (related to customer satisfaction) of product i
$\varphi^{-1}(.)$	: Inverse distribution function
$\hat{d}_i(k)$	: The average demand of product i during period k
$\sigma(.)$	: The standard deviation
$\hat{S}_{i,(k\times p)-(p-j)}$	: The average stock level of product i at the end of sub-period j of period k
$Z\ (U)$	: The total expected cost of production and holding over the finite horizon
$\Gamma(N)$	: The total cost of maintenance
$\lambda_{(k\times p)-(p-j)}(.)$	: Failure rate function at sub-period j of the period k
$\lambda_n(.)$	: Nominal failure rate
$\phi(.)$	: The average number of failures
N	: The number of actions of preventive maintenance
T	: The intervention period for preventive maintenance actions

Decision variables:

$U_{i,j,k}$	: The production rate of product i during sub-period j of period k
$\delta_{(k\times p)-(p-j)}$	: The duration of sub-period j at period k
$y_{i,j,k}$	: A binary variable, which is equal to 1 if product i is produced in sub-period j of the period k, and 0 otherwise
N	: The number of preventive maintenance actions during the finite horizon

B. Assumptions

To develop the production model, the following assumptions are specifically adopted:

- setup costs, holding and production costs of each product are assumed constant;
- Only a single product can be produced in each sub-period;
- As described in (Fig. 2), the period k is divided in exactly p sub-periods, in order to simplify the mathematical model developed;
- The standard deviation of demand $\sigma(di)$ and the average demand $\hat{d}_i$ for each product and each period k are known and constant.

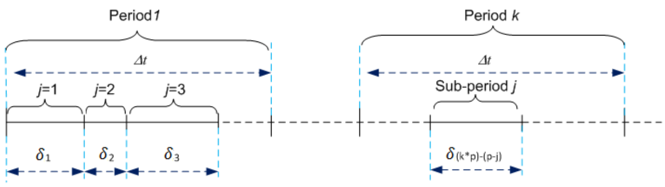

Fig. 2. Split of production horizon

To develop the maintenance model, it was assumed that

- Durations of maintenance actions are negligible;
- Mp and Mc costs incurred by the preventive and corrective maintenance actions are known and constant, with $Mc >> Mp$.
- Preventive maintenance actions are always performed at the end of the sub-periods of production.

IV. Production Policy

The aim of this section is to develop an analytical model that allows us to determine the economical production plan for a finite time horizon $H \times \Delta t$. The decision variables are the production rates $U_{i,j,k}$, the duration of sub-periods $\delta_{(k\times p)-(p-j)}$ and the binary variable $y_{i,j,k}$.

The mathematical formulation of the proposed problem is based on the extension of the model described by [7]. The difference is that we considered that the setup costs are variable.

$$Min\,(\mathrm{Z}(U)) \qquad U = U_{i,j,k}\ \forall\, \{i = 1 \dots n\}, \{j = 1 \dots p\}, \{k = 1 \dots H\} \tag{1}$$

With:

$$\mathrm{Z}(U) = \sum_{k=1}^{H}\sum_{j=1}^{P}\sum_{i=1}^{n}\left[\begin{array}{c} y_{i,j,k} \times \begin{pmatrix} St\left(i, q((k\times p)-(p-j)-1)\right) \\ +\left(Cp(i) \times (U_{i,j,k})^2\right) \end{pmatrix} \\ +Cs(i) \times \dfrac{\delta t_{(k\times p)-(p-j)}}{\Delta t} \times \left[\begin{array}{c} \sigma^2(S_{i,0}) \\ +\left(\sum_{Q=1}^{k-1}\sum_{l=1}^{p} Ent\left(\frac{l}{p}\right) \times \sigma^2(d_i(Q))\right) \\ +\left(\sum_{l=1}^{j} Ent\left(\frac{l}{p}\right) \times \sigma^2(d_i(k))\right) \\ +\left(\hat{S}_{i,(k\times p)-(p-j)}\right)^2 \end{array}\right] \end{array}\right] \tag{2}$$

Under the following constraints:

$$\hat{S}_{i,(k\times p)-(p-j)} = \hat{S}_{i,(k\times p)-(p-j)-1} + (y_{i,j,k} \times U_{i,j,k}) - Int\left[\frac{j}{p}\right] \times \hat{d}_i(k) \qquad \forall\, \{i = 1 \dots n\}, \{j = 1 \dots p\}, \{k = 1 \dots H\} \tag{3}$$

This first constraint represents the stock balance equation for each product i, during each sub-period j, of period k.

$$0 \le U_{i,j,k} \le \frac{\delta_{(k\times p)-(p-j)}}{\Delta t} \times U_{inom} \tag{4}$$

$$\forall\, \{i = 1 \dots n\}, \{j = 1 \dots p\}, \{k = 1 \dots H\}$$

The second constraint is defined by (4). This constraint expresses the upper production rate of the machine for each product i.

$$\sum_{j=1}^{p}(y_{i,j,k} \times U_{i,j,k}) \ge \sigma^2(d_i(k)) \times \varphi^{-1}(\theta_i) + \hat{d}_i(k) - S_{i,(k-1)\times p} \qquad \forall\, \{i = 1 \dots n\}, \{k = 1 \dots H\} \tag{5}$$

The above constraint refers to the satisfaction level of demand of product i in each period k. This equation is obtained on the basis of the stochastic formula below:

$$Prob\left(S_{i,(k\times p)} \ge 0\right) \ge \theta_i \qquad \forall\, \{i = 1 \dots n\}, \{k = 1 \dots H\} \tag{6}$$

Where $S_{i,(k\times p)}$ represents the stock level of product i at the end of sub-period j of period k.

$$\sum_{j=1}^{p} \delta_{(k\times p)-(p-j)} = \Delta t \qquad \forall\, \{k = 1 \dots H\} \tag{7}$$

$$0 < \delta t_{(k\times p)-(p-j)} \le \Delta t \quad \forall\, \{j = 1 \dots p\}, \{k = 1 \dots H\} \tag{8}$$

The aim of (7) and (8) is to divide each period k into p different sub-periods and the sub-periods duration must be between 0 and Δt.

$$q((k\times p)-(p-j)) = \sum_{i=1}^{n} y_{i,j,k} \times i \qquad \forall\, \{j = 1 \dots p\}\, \{k = 1 \dots H\} \tag{9}$$

The purpose of (9) is to memorize the treated product at each sub-period $j$of period k.

The constraints below should also be taken into account.

$$\sum_{i=1}^{n} y_{i,j,k} = 1 \qquad \forall\, \{j = 1 \dots p\}\ For\ \{k = 1 \dots H\} \tag{10}$$

$$\sum_{j=1}^{p} y_{i,j,k} = 1 \qquad \forall \{i = 1 \dots n\}\ For\ \{k = 1 \dots H\} \tag{11}$$

$$y_{i,j,k} \epsilon \{0,1\} \qquad \forall\, \{i = 1 \dots n\}, \{j = 1 \dots p\}, \{k = 1 \dots H\} \tag{12}$$

The equations (10) and (11) indicate that only one type of product will be produced in each sub-period j of period k.

The constraint (12) states that $y_{i,j,k}$ is a binary variable.

V. Maintenance Policy

A. Description

In this study we adopted a maintenance strategy with minimal repair. As illustrated in the figure below, the preventive maintenance actions are carried out at each instant $q \times T \ (q = 1, 2 \dots)$ in order to replace the system by a new one (as good as new). If the system fails between preventive maintenance actions, only minimal repair is implemented.

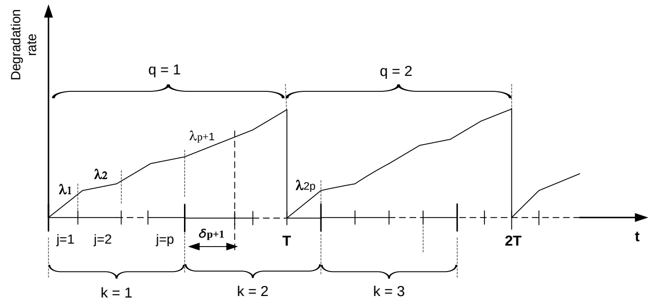

Fig. 3. The degradation rate

If we note Mc the cost of corrective maintenance actions, Mp the initial cost of preventive maintenance actions. It's clear that's difficult to make a perfect preventive maintenance action with restoring the system as good as new with adopting the usually the initial cost of preventive maintenance action Mp. That's why we will consider an increased preventive maintenance cost.

Formally we assume that the preventive maintenance action increased between successive actions according to the geometric sequence with constant rate α. Since that the total cost of maintenance is expressed as follows:

$$\Gamma(N) = Mc \times \phi_{(N,U)} + \sum_{i=1}^{N} Mp_i \tag{13}$$

The aim of this maintenance strategy is to determine the optimal number of preventive maintenance actions $N^* (N = 1, 2 \dots)$ minimizing the total cost of maintenance over a given horizon $H \times \Delta t$.

Nakagawa *et al.* [8] has proven that there is an optimal number of partitions N^* and therefore, the optimal preventive maintenance period T^*, in the case of increased failure rate.

B. The expression of the average number of failures

As illustrated in "Fig. 3", we assume that the actions of preventive maintenance are made at the end of sub-periods, in order to reduce the complexity of the generation of the optimal number of preventive maintenance.

Hence, the function of the intervention period for preventive maintenance actions is presented as follows:

$$T = Round\left[\frac{H \times p}{N}\right] \tag{14}$$

$Round[x]$: A round number of x

Lemma1:

$$\phi_{(U,N)} = \sum_{q=1}^{N+1} \left[\begin{array}{l} \displaystyle\sum_{j=((q-1)\times T+1)-\left(In\left[\frac{(q-1)\times T}{\Delta t}\right]\times p\right)}^{p} \int_0^{\delta_{\left(In\left[\frac{(q-1)\times T}{\Delta t}\right]+1\right)\times p-(p-j)}} \lambda_{\left(In\left[\frac{(q-1)\times T}{\Delta t}\right]+1\right)\times p-(p-j)}(t)dt \\ \displaystyle + \sum_{k=In_{sup}\left[\frac{(q-1)\times T+1}{\Delta t}\right]+1}^{In\left[\frac{q\times T}{\Delta t}\right]} \sum_{j=1}^{p} \int_0^{\delta_{(k\times p)-(p-j)}} \lambda_{(k\times p)-(p-j)}(t)dt \\ \displaystyle + \sum_{j=1}^{q\times T-In\left[\frac{q\times T}{\Delta t}\right]\times p} \int_0^{\delta_{\left(In\left[\frac{q\times T}{\Delta t}\right]+1\right)\times p-(p-j)}} \lambda_{\left(In\left[\frac{q\times T}{\Delta t}\right]+1\right)\times p-(p-j)}(t)dt \end{array} \right]$$

$In[x]$: Integer part of number x
$In_{sup}[x]$: Superior integer part of number x

Proof: See [7]

C. The expression of failure rate

Recall, that the failure rate is influenced by the variation of the production rates. As illustrated in "Fig. 3", the failure rate is reset after each $q \times T$.

Thus, the expression of the failure rate can be written as follows:

Lemma2:

$$\lambda_{(k\times p)-(p-j)}(t) = \left[\begin{array}{c} \left(\begin{array}{c} \lambda_0 \\ \displaystyle + \sum_{Q=1}^{k-1} \sum_{l=1}^{p} \sum_{i=1}^{n} \frac{U_{i,l,Q} \times \Delta t}{U_{imax} \times \delta_{(Q\times p)-(p-l)}} \times \lambda_n\left(\delta_{(Q\times p)-(p-l)}\right) \\ \displaystyle + \sum_{l=1}^{j-1} \sum_{i=1}^{n} \frac{U_{i,l,k} \times \Delta t}{U_{imax} \times \delta_{(k\times p)-(p-l)}} \times \lambda_n\left(\delta_{(k\times p)-(p-l)}\right) \end{array} \right) \\ \displaystyle \times \left(1 - In\left[\frac{(k\times p) - (p-j+1)}{q \times T}\right]\right) \\ \displaystyle + \sum_{i=1}^{n} \frac{U_{i,j,k} \times \Delta t}{U_{imax} \times \delta_{(k\times p)-(p-j)}} \times \lambda_n(t) \end{array} \right]$$

$$t \in \left[0, \delta_{(k\times p)-(p-j)}\right] \quad \forall \{k = 1 \dots H\}, \{j = 1 \dots p\}$$

Proof: See [7]

D. The expression of the total cost of maintenance

We recall that we considered we will consider an increased preventive maintenance cost to determine the total cost of maintenance.

Therefore:

$$Mp_{i+1} = Mp_i \times \alpha \quad (with\ \alpha > 1) \tag{15}$$

Supposed that the initial preventive maintenance cost is:

$$Mp_1 = Mp \tag{16}$$

The total cost of maintenance expressed in (13) can be represented as follows:

$$\Gamma(N) = Mc \times \phi_{(N,U)} + Mp_1 \times (1 + \alpha + \alpha^2 + \cdots + \alpha^N) \quad (17)$$

Thus:

$$\Gamma(N) = Mc \times \phi_{(N,U)} + Mp \times \frac{1-\alpha^N}{1-\alpha} \quad (18)$$

Using the average number of failure $\phi_{(U,N)}$ established in lemma 1, we can deduce that the analytical expression of the total maintenance cost is expressed as follows:

$$\Gamma(N) = \left[Mc \times \sum_{q=1}^{N+1} \left[\begin{array}{l} \sum_{j=((q-1)\times T+1)-\left(In\left[\frac{(q-1)\times T}{\Delta t}\right]\times p\right)}^{p} \int_0^{\delta_{\left(In\left[\frac{(q-1)\times T}{\Delta t}\right]+1\right)\times p-(p-j)}} \lambda_{\left(In\left[\frac{(q-1)\times T}{\Delta t}\right]+1\right)\times p-(p-j)}(t)dt \\ + \sum_{k=In_{sup}\left[\frac{(q-1)\times T+1}{\Delta t}\right]+1}^{In\left[\frac{q\times T}{\Delta t}\right]} \sum_{j=1}^{p} \int_0^{\delta_{(k\times p)-(p-j)}} \lambda_{(k\times p)-(p-j)}(t)dt \\ + \sum_{j=1}^{q\times T-In\left[\frac{q\times T}{\Delta t}\right]\times p} \int_0^{\delta_{\left(In\left[\frac{q\times T}{\Delta t}\right]+1\right)\times p-(p-j)}} \lambda_{\left(In\left[\frac{q\times T}{\Delta t}\right]+1\right)\times p-(p-j)}(t)dt \end{array} \right] + Mp \times \frac{1-\alpha^N}{1-\alpha} \right]$$

VI. Numerical Exemple

A simple example of a system that produces three different types of products is considered in order to satisfy three random demands according to every type of product.

Using the models described in previous sections, we will determine first the economical production plan followed by the optimal strategy of maintenance over a finite planning horizon: $H = 8\ trimesters$ (two years). We consider that the duration of periods $\Delta t = 3\ months$.

A. Numerical data

- The data relating to production:

We supposed that the standard deviation of demand of each product i, is the same for all periods $\left(\sigma(d_i(k)) = \sigma(d_i(k+1)) = \sigma(d_i)\right)$.

TABLE I. The average demands

	Average demand	Standard deviation
Product 1	200	1.5
Product 2	110	0.9
Product 3	320	1.2

The demands for each product by trimester are presented in the following table:

TABLE II. Demands

	Tr1	Tr2	Tr3	Tr4	Tr5	Tr6	Tr7	Tr8
Prod1	201	199	198	199	201	202	200	199
Prod2	111	119	108	201	111	112	110	119
Prod3	321	322	323	319	321	317	320	319

The setup costs are represented as follows:

TABLE III. Setup costs

		To		
		Product 1	Product 2	Product 3
From	Product 1	0	60	55
	Product 2	65	0	45
	Product 3	40	35	0

The other data are represented as following:

TABLE IV. Other data relating to production

	$S_{i,0}$ (pu)	U_{inom} $(pu/\Delta t)$	$Cp(i)$ (mu)	$Cs(i)$ (um/ut)	Θ_i (%)
Prod1	110	750	13	3	87
Prod2	85	530	17	5	95
Prod3	145	1150	9	2	90

- The data relating to maintenance:

System reliability law (Weibull) and preventive/corrective maintenance costs are defined by the following data:

TABLE V. Parameters of weibull function

Scale parameter (η)	Shape parameter (β)	Position parameter (γ)
12 months	2	0

The other data are given in the table below:

TABLE VI. Other data relating to maintenance

$Mp\ (mu)$	$Mc\ (mu)$	λ_0	α
800	1500	0	0.05

B. The economical production plan obtained

The production rates and the duration of sub-periods are given in the table below.

TABLE VII. The economical production plan

	Trimester 1			Trimester 2		
	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6
Durations	0.85	0.71	1.44	0.65	1.21	1.14
Prod1	0	129	0	0	298	0
Prod2	120	0	0	0	0	185
Prod3	0	0	507	230	0	0
	Trimester 3			**Trimester 4**		
	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6
Durations	0.18	1.18	1.01	0.43	0.74	1.83
Prod1	0	295	0	0	151	0
Prod2	134	0	0	0	0	312
Prod3	0	0	387	158	0	0

	Trimester 5			Trimester 6		
	δ_{13}	δ_{14}	δ_{15}	δ_{16}	δ_{17}	δ_{18}
Durations	1.82	0.87	0.31	0.56	0.55	1.89
Prod1	0	212	0	0	138	0
Prod2	0	0	52	58	0	0
Prod3	354	0	0	0	0	542
	Trimester 7			Trimester 8		
	δ_{19}	δ_{20}	δ_{21}	δ_{22}	δ_{23}	δ_{24}
Durations	0.76	1.11	1.13	1.05	0.77	1.18
Prod1	0	172	0	0	81	0
Prod2	0	0	92	130	0	0
Prod3	187	0	0	0	0	235

C. The optimal strategy of maintenance obtained

In the figure below, we represent the evolution of the total cost depending to the number of preventive maintenance actions.

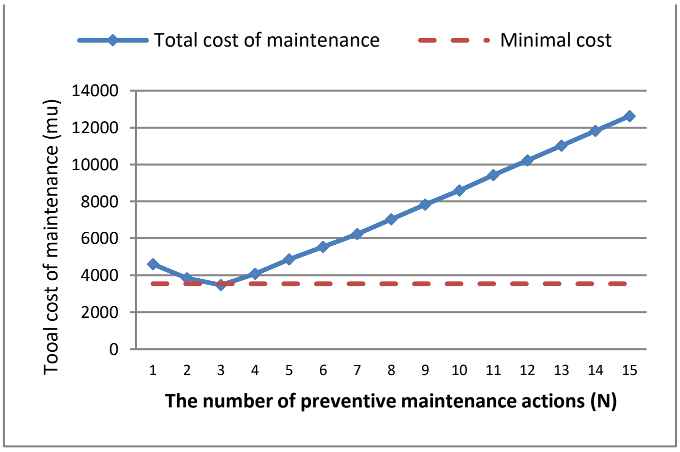

Fig. 4. The evolution of the total cost of maintenance

We deduce that the optimal number of preventive maintenance actions is $N^* = 3\, times$. Hence, the optimal period to apply the preventive maintenance is $T^* = 8\, months$, ensuring a minimal total cost of maintenance $\Gamma^*(N) = 3\,469\, mu$.

VII. Conclusion

In this paper we have discussed the problem of integrated maintenance/production strategy for a manufacturing system consisting of a single machine which ensures the production of several types of products in order to satisfy random demands according to every product. The machine is subject to random failures, with increased failure rate according both time and production rate. Consequently, preventive maintenance actions, bloc type with minimal repair, are adopted in order to improve the system reliability. At failure, a minimal repair is carried out to restore the system into the operating state without changing its failure rate. In contrast to several studies in the literature, the originality of our study is the consideration of a variable setup cost and an increased preventive maintenance cost between successive actions. A variable setup costs used in our study and according to different product play an import role in the decision making in order to choose the order of product to produce at every period. More than that, the majority of the studies in maintenance frame in literature use a fixed cost of preventive maintenance action and assume that the system is restored to the state “as good as new” after every preventive maintenance action. It’s clear that this assumption is unreal. That’s why in this study, we considered an increased preventive maintenance cost between successive actions. A mathematical model is presented in order to formulate this assumption.

According to the proposed problem we start by developing analytically a stochastic production problem. Solving the analytical model expressing production, storage and setup cost and respecting proposed constraints, we obtained the economical production plan, presenting the quantity to produce for every product in each sub-period over a finite horizon. In the second phase, taking into account the economical production plan obtained, we have studied and optimized the maintenance policy. An analytical model, expressing the total maintenance cost is developed and optimized in order to determine the optimal number of preventive maintenance to do over a finite horizon, ensuring a minimal maintenance cost. We note that the maintenance policy adopted is characterized by considering the influence of production rate variation on the system degradation in the case of multiple products and an increased preventive maintenance cost between successive actions.

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