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On the Parameter-dependent H_∞ control for MEMS gyroscopes: synthesis and analysis

Fabrício Saggin, Jorge Ayala-Cuevas, Anton Korniienko, Gérard Scorletti

Ampère laboratory UMR CNRS 5005
Ecole Centrale de Lyon, Ecully, France

Abstract

This document provides further details on the paper “Parameter-dependent H_∞ control for MEMS gyroscopes: synthesis and analysis”. These details concern the choice of the weighting function parameters and the system model in pseudo-continuous time (PCT).

1 Background and objectives

The to-be-controlled system is the drive mode of a MEMS, whose model, in continuous-time (CT), is given by:

$$G_{\omega_0}(s) = \frac{y(s)}{u(s)} = \frac{k}{(s/\omega_0)^2 + (s/\omega_0)/Q + 1}, \tag{1}$$

where y is the displacement of the drive mode, u is the input force, k is the static gain, Q is the quality factor, and ω_0 is the resonance frequency (in rad/s), which slowly ranges $[\omega_{0min},\ \omega_{0max}]$.

The control objectives are:

- tracking of a sinusoidal reference signal y_r of frequency ω_0;
- minimization of the control effort u;
- robust stability.

Moreover, to ensure high performance, the controller depends on ω_0. In Saggin et al. (2020), this problem is solved either for an analog or for a digital implementation of the ω_0-dependent controller. The solutions are based on time/frequency normalization and on the H_∞ synthesis.

In this report, we detail the choice of the H_∞ criterion, more specifically, the choice of the weighting function parameters, which is presented in Section 2. For the specific problem of a digital implementation of the controller, its design is based on the gyroscope model in the pseudo-continuous time (PCT). This model is developed in Section 3.

2 Choice of the weighting function parameters

In the H_∞ synthesis, the control specifications are expressed through the choice of the weighting functions and of the weighted closed-loop transfer functions. We consider the criterion presented in Fig. 1, where we include an input disturbance d, a measurement noise n and weighting functions $W^x_{\omega_0}$, and we define $\varepsilon = y_r - y_m$ and $y_m = y + n$.

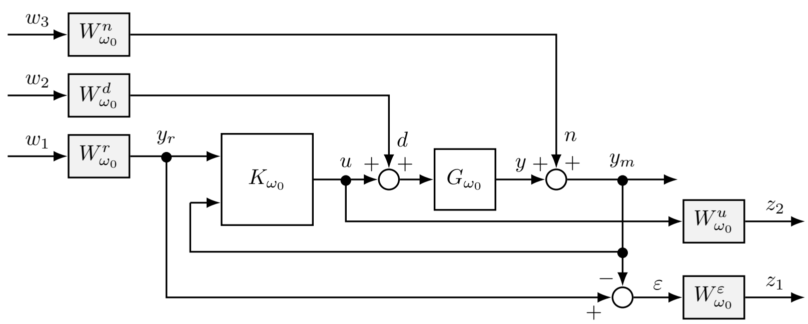

Figure 1: H_∞ criterion.

The H_∞ problem is then: given a performance level $\gamma > 0$, compute a controller K_{ω_0}, if there is any, such that $\|P_{\omega_0} \star K_{\omega_0}\|_\infty < \gamma$. If this problem has a solution for $\gamma = 1$, then the following H_∞ criterion is also ensured:

$$\left\| \begin{matrix} W^\varepsilon_{\omega_0} T_{y_r\to\varepsilon} W^r_{\omega_0} & W^\varepsilon_{\omega_0} T_{d\to\varepsilon} W^d_{\omega_0} & W^\varepsilon_{\omega_0} T_{n\to\varepsilon} W^n_{\omega_0} \\ W^u_{\omega_0} T_{y_r\to u} W^r_{\omega_0} & W^u_{\omega_0} T_{d\to u} W^d_{\omega_0} & W^u_{\omega_0} T_{n\to u} W^n_{\omega_0} \end{matrix} \right\|_\infty < 1. \tag{2}$$

Then, with the following weighting functions

$$W^\varepsilon_{\omega_0}(s) = \frac{1}{M_\varepsilon} \frac{(s/\omega_0)^2 + (s/\omega_0)\,\alpha_\varepsilon + 1}{(s/\omega_0)^2 + (s/\omega_0)\,\alpha_\varepsilon A_\varepsilon/M_\varepsilon + 1},$$

$$W^u_{\omega_0}(s) = M_u \frac{(s/\omega_0)^2 + (s/\omega_0)\,\alpha_u A_u/M_u + 1}{(s/\omega_0)^2 + (s/\omega_0)\,\alpha_u + 1},$$

$$W^r_{\omega_0}(s) = k_r, \quad W^d_{\omega_0}(s) = k_d \quad \text{and} \quad W^n_{\omega_0}(s) = k_n,$$

the choice of the parameters $A_\varepsilon \leq 1$, $M_\varepsilon \geq 1$, α_ε, $A_u \leq 1$, $M_u \geq 1$, α_u, k_r, k_d and k_n ensures the desired specifications, as follows Skogestad, Postlethwaite (2001):

- *Reference tracking:* (2) implies that

$$\forall\omega, \quad |T_{y_r\to\varepsilon}(j\omega)| \leq \frac{1}{|W^\varepsilon_{\omega_0}(j\omega)W^r_{\omega_0}(j\omega)|}, \tag{3}$$

  which ensures the tracking of the sinusoidal reference signal y_r by y_m with a frequency equal to ω_0, error bounded by A_ε/k_r and convergence speed constrained by α_ε.

- *Control limitation:* (2) implies that

$$\forall\omega, \quad |T_{y_r\to u}(j\omega)| \leq \frac{1}{|W^u_{\omega_0}(j\omega)W^r_{\omega_0}(j\omega)|}, \tag{4}$$

$$\forall\omega, \quad |T_{d\to u}(j\omega)| \leq \frac{1}{|W^u_{\omega_0}(j\omega)W^d_{\omega_0}(j\omega)|}, \tag{5}$$

$$\forall\omega, \quad |T_{n\to u}(j\omega)| \leq \frac{1}{|W^u_{\omega_0}(j\omega)W^n_{\omega_0}(j\omega)|}, \tag{6}$$

  which constrains by α_u the bandwidth of the controller and by A_u (for frequencies close to ω_0) and M_u (for low and high frequencies) the control signal amplitude and, therefore, its power.

- *Robust stability:* finally, (2) also implies

$$\forall\omega, \quad |T_{d\to\varepsilon}(j\omega)| \leq \frac{1}{|W^\varepsilon_{\omega_0}(j\omega)W^d_{\omega_0}(j\omega)|} \tag{7}$$

$$\text{and} \ \ \forall\omega, \quad |T_{n\to\varepsilon}(j\omega)| \leq \frac{1}{|W^\varepsilon_{\omega_0}(j\omega)W^n_{\omega_0}(j\omega)|},$$

  which are respectively used to avoid pole-zero compensations and to enforce a lower bound on the modulus margin $\mathcal{M}$, which is defined as $\mathcal{M} \triangleq 1/\|T_{n\to\varepsilon}\|_\infty$. Moreover,

$$\|T_{n\to\varepsilon}\|_\infty < M_\varepsilon/k_n. \tag{8}$$

  Hence, (2) also implies $\mathcal{M} > k_n/M_\varepsilon$.

Please note that the weighting functions are parameterized by ω_0, expressing the control specifications in continuous-time. Nevertheless, the above discussion also holds for the weighting functions in a normalized or pseudo-continuous space.

In (Saggin et al., 2020, Section 6), the control specification are:

1. track a reference signal $y_r(t) = Y_r \sin(\omega_0 t)$ with an error $\varepsilon(t) = y_r(t) - y_m(t)$ such that $|\varepsilon(t)| < 10^{-4} \cdot Y_r$ in steady-state;
2. the control signal amplitude is less than $0.02 \cdot Y_r$ in steady-state;
3. the closed-loop system is stable and has a modulus margin $\mathcal{M} > 1/2$.

The first control specification demands $|T_{y_r \to \varepsilon}(j\omega_0)| < 10^{-4}$, which is bounded by A_ε / k_r, see (3). Then, we choose $k_r = 1$ and $A_\varepsilon = 5 \cdot 10^{-5}$.

The third control specification demands $\|T_{n \to \varepsilon}\|_\infty < 2$, which is bounded by M_ε / k_n, see (8). Then, we choose $M_\varepsilon = 2$ and $k_n = 1$.

To avoid pole-zero compensation, $|T_{d \to u}|$ has to be bounded where the plant presents a resonance peak. Then, we choose $k_d = 0.05$, see (7).

We choose $M_u = 400$ to minimize the control signal amplitude at low and high frequencies, see (4), (5) and (6). Moreover, we choose $A_u = 0.004$ to allow high controller gains around ω_0. Note that the control signal amplitude at ω_0 (second specification) is structurally given by $Y_r / |G(j\omega_0)|$ and cannot be modified by the choice of the weighting functions.

Finally, α_ε and α_u are tuned ($\alpha_\varepsilon = 0.2$ and $\alpha_u = 1632$) to select adequate response times.

3 Details on the PCT model

Here, we intend to provide further details on G_{ω_0}, see (1), when analyzing its equivalent discrete-time (DT) model, $G^d_{\omega_0}$. We highlight that the latter one takes the presence of the zero-order holder into account.

For $Q > 1$, the equivalent DT system $G^d_{\omega_0}$, with sampling period T_s, is given by

$$G^d_{\omega_0}(z) = k \frac{b_1 z + b_2}{z^2 + a_1 z + a_2}$$

with

$$
\begin{aligned}
a_1 &= -2e^{-\omega_0 T_s/(2Q)} \cos(\omega_0 T_s) \\
a_2 &= e^{-\omega_0 T_s/Q} \\
b_1 &= 1 - e^{-\omega_0 T_s/(2Q)} \left(\frac{\sin(\omega_0 T_s)}{\sqrt{4Q^2-1}} + \cos(\omega_0 T_s) \right) \\
b_2 &= e^{-\omega_0 T_s/Q} + e^{-\omega_0 T_s/(2Q)} \left(\frac{\sin(\omega_0 T_s)}{\sqrt{4Q^2-1}} - \cos(\omega_0 T_s) \right)
\end{aligned}
$$

Then, by applying the bilinear transform, we obtain

$$
G^p_{\varpi_0}(s_p) = k \frac{(1 - s_p T_s/2)(1 + s_p z_1)}{s_p^2 + \varpi_0 s_p/\mathcal{Q} + \varpi_0^2}
$$

with

$$
\begin{aligned}
\varpi_0^2 &= \frac{4}{T_s^2} \frac{1 - 2e^{-\omega_0 T_s/(2Q)} \cos(\omega_0 T_s) + e^{-\omega_0 T_s/Q}}{1 + 2e^{-\omega_0 T_s/(2Q)} \cos(\omega_0 T_s) + e^{-\omega_0 T_s/Q}} \\
\frac{\varpi_0}{\mathcal{Q}} &= \frac{2}{T_s} \frac{1 - e^{-\omega_0 T_s/Q}}{1 + 2e^{-\omega_0 T_s/(2Q)} \cos(\omega_0 T_s) + e^{-\omega_0 T_s/Q}} \\
z_1 &= \frac{T_s}{2} \frac{1 - 2e^{-\omega_0 T_s/(2Q)} \frac{1}{\sqrt{4Q^2-1}} \sin(\omega_0 T_s) - e^{-\omega_0 T_s/Q}}{1 - 2e^{-\omega_0 T_s/(2Q)} \cos(\omega_0 T_s) + e^{-\omega_0 T_s/Q}}
\end{aligned}
$$

For $Q \gg 1$, we may approximate $e^{-\omega_0 T_s/(2Q)} \approx 1$ and $\xi^2 \approx 0$. Thus,

$$
\begin{aligned}
\varpi_0^2 &\approx \frac{4}{T_s^2} \frac{1 - \cos(\omega_0 T_s)}{1 + \cos(\omega_0 T_s)} \implies \varpi_0 \approx \frac{2}{T_s} \tan\left(\frac{\omega_0 T_s}{2}\right), \\
\frac{\varpi_0}{\mathcal{Q}} &\approx \frac{\omega_0}{(2Q)(1 + \cos(\omega_0 T_s))} \implies \mathcal{Q} \approx Q \,\mathrm{sinc}(\omega_0 T_s)
\end{aligned}
$$

$$
\text{and} \quad z_1 \approx \frac{T_s}{2} \frac{-\sin(\omega_0 T_s)}{(2Q)(1 - \cos(\omega_0 T_s))} = \frac{1}{2Q\varpi_0}.
$$

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