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# The Integrated Production-Inventory-Routing Problem of EOL products with simultaneous delivery and pickup

Zakaria CHEKOUBI<sup>1,2</sup>

<sup>1</sup>LGIPM - Université de Lorraine

<sup>2</sup>LIDRA - Universiapolis

Metz, France

[zakaria.chekoubi@univ-lorraine.fr](mailto:zakaria.chekoubi@univ-lorraine.fr)

Nathalie SAUER<sup>1</sup>

LGIPM

Université de Lorraine

Metz, France

[nathalie.sauer@univ-lorraine.fr](mailto:nathalie.sauer@univ-lorraine.fr)

Wajdi TRABELSI<sup>1,3</sup>

LGIPM

<sup>3</sup>ICN Business School

Metz, France

[wajdi.trabelsi@icn-artem.com](mailto:wajdi.trabelsi@icn-artem.com)

**Abstract**—Production, inventory and distribution with returns management are three key decisions in closed-loop supply chain planning. In order to achieve an effective operational performance level, it is important for these three decisions to be integrated, especially in closed-loop supply chains (CLSC) with remanufacturing operation. In this paper, we address the integrated optimization of Production, Inventory control and Routing decisions with remanufacturing of End-Of-Life (EOL) products (IPIRP-R) related to supplying several customers from a central plant. In particular, we consider a CLSC in which two production lines and a fleet of homogenous capacitated vehicle, who make multiple trips over the planning horizon, are available to manufacture new products, remanufacture EOL products, deliver final products and pickup EOL ones, respectively. The aim of solving the IPIRP-R model is to minimize jointly the total production, setup, inventory and routing cost over a finite planning horizon. The originality of our study lies in the fact of jointly treating the single-item capacitated lot-sizing with remanufacturing and the vehicle routing problem with simultaneous pick-up and delivery in a single framework. Computational experiment is conducted on a set of randomly generated instances. An illustrative example to illustrate the relevance of our model is given.

**Keywords**—Integrated Production-Inventory-Routing Problem, Dynamic lot-sizing, Remanufacturing, VRP with Simultaneous Pick-up and Delivery, Reverse Logistics, Mixed Integer Linear Programming (MILP).

## I. INTRODUCTION

Managing returns flow within a close-loop supply chain is considered as a business opportunity for many companies. Today, many firms have initiated efforts to incorporate the concept of reverse logistics in their regular production, inventory and distribution decision systems for several reasons such as the increasing concern for environment, restrictions of applicable government regulations and laws on recycled products and waste disposal, the growing energy consumptions, and the stiff competition between the firms. The integration of these three decisions on the context of reverse supply chains offers tremendous cost savings opportunities to firms. Moreover, integrating production-inventory-distribution decisions into a single problem is almost indispensable and relevant for some type of goods in particular perishable or time-sensitive goods [1]. In this direction, the Integrated Production-Inventory-Routing Problem (IPIRP), which aims to jointly minimize production, inventory, setup and routing costs, has drawn the attention of many researchers since the seminal work of [2] who studied the economic value of integrating and coordinating production and routing decisions. The authors found that integrating these decisions may lead to savings ranging from 3 % to 20 % in comparison with the traditional sequential approach, in which

routing decisions are made after the production plan has been determined [3]. Afterwards, several studies suggested both mathematical formulations and procedures to solve various variants of the problem. Most of the research on the IPIRP has focused on a scenario with a single production facility that produces one product and owns a limited fleet of homogeneous vehicles [3]. Solution methods for this problem include mathematical programming-based heuristics [4, 5], B&C algorithms [6, 7], Benders-based B&C [8], B&P-based heuristics [9, 10], Greedy Randomized Adaptive Search Procedure (GRASP) [11], Adaptive Large Neighborhood Search (ALNS) [12] and Genetic Algorithm with Population Management (GAPM) [13].

Besides integrating operations forward, closed-loop supply chain optimization showed a further reduction in environmental impact [14]. Return flow processes in a closed-loop supply chain usually consists of (1) product collection from consumers; (2) reverse logistics to take collected products back; (3) screening, assorting and disposal to specify the most economically attractive reuse alternatives; (4) remanufacturing; and (5) remarketing to produce and utilize new markets [15]. The remanufacturing operation consists on transforming end-of-life returned products into usable products through refurbishment, repair or upgrading [1]. Moreover, since the importance of considering remanufacturing in closed-loop supply chain was stressed by [14], the research on remanufacturing had mainly focused on inventory system with remanufacturing [16, 17], economic aspect of remanufacturing [18, 19], and marketing issues [20, 21]. However, little has been revealed when remanufacturing is not only involved with inventory decisions but also with routing decisions [22]. Furthermore, addressing the IPIRP in the context of closed-loop supply chain is important because in addition to economic benefits, environmental benefits due to extension of the product useful life, reduced energy and material consumption, pollution prevention, and other sustainability benefits can be expected [22].

Our goal through this paper is to bridge these gaps by extending our previous study [1] insofar we propose a novel MILP model for the IPIRP with remanufacturing (IPIRP-R) of EOL products considering a typical direct-inverse distribution with simultaneous pickups and deliveries performed by a fleet of homogeneous capacitated vehicle.

The recent work of [22] is considered as the first study that investigates the same problem as ours by introducing a mixed integer programming model for the integrated production routing problem with reverse logistics and remanufacturing. They proposed a novel branch-and-cut guided search algorithm as solution method. The difference of our study

compared to theirs, lies in the formulation of the proposed model and the fact that they consider a typical IPIRP integrating the Vendor Manager Inventory (VMI) policy, the thing we are not considering.

The contributions of this paper can be summarized as follows. First, we introduce a variant of the IPIRP with a direct-reverse distribution and remanufacturing. The direct-reverse distribution with simultaneous pickups and deliveries is now mixed with capacitated vehicle routing problems, which has never been investigated in the IPIRP literature. Second, we formulate a new MILP formulation for the IPIRP-R. Finally, we conduct extensive computational experiments on a set of randomly generated instances to evaluate the performance of the proposed MILP model and provide remarks and conclusions for future works.

The remainder of the paper is organized as follows: Section 2 briefly describes the considered problem as well as the assumptions considered during the development of our model. The MILP formulation integrating production with remanufacturing, inventory and routing decisions is presented in Section 3. Section 4 is devoted to computational experiments and the obtained results. Finally, concluding remarks and future research perspectives are highlighted in Section 5.

## II. PROBLEM DESCRIPTION

In this section, we provide a formal description of the Integrated Production, Inventory, Distribution problem with remanufacturing addressed in this paper, denoted IPIRP-R and present the assumptions we considered in the development of the MILP formulation model.

let  $G = (N, A)$  defines a complete directed graph network, where  $N = \{0, 1, 2, \dots, |N_C|\}$  is the set of nodes and  $A = \{(i, j) : i, j \in N, i \neq j\}$  is the set of arcs. The central plant is denoted by node  $\{0\}$  and  $N_C = N \setminus \{0\}$  denotes the set of customers. Each arc  $(i, j) \in A$  has a non-negative symmetric travelling cost denoted  $c_{ij}$  that represents the cost of reaching node  $j$  from node  $i$  and satisfying triangular inequality (i.e.,  $c_{ij} + c_{jk} \geq c_{ik}$ ). Each customer  $i \in N_C$  requires a dynamic demand for delivery and pickup in each period  $t \in T$ , where  $T = \{1, 2, \dots, |T|\}$  denoted respectively  $d_{it}$  and  $p_{it}$ . We assume that the central plant does not require any delivery or pickup ( $d_{0t} = p_{0t} = 0, \forall t \in T$ ). At the central plant are located two production systems, respectively for manufacturing and remanufacturing of serviceables products and returned EOL products. Each production line has a limited production capacity and has its own set-up cost. Both production line systems are based on the economic lot-sizing production strategy of a single product and have to fulfill all demand requirements of each period over the finite planning horizon either by manufacturing new products (serviceables) or by remanufacturing returned EOL products or both. It means that the remanufactured products will be of the same quality as new products and backlogging is not allowed. In addition, a unitary production cost occurs for manufacturing (resp., remanufacturing) a new product (resp., returned product) in each period. Also, manufacturing and remanufacturing operations lead times are supposed to be zero. Regarding customers, we consider that their requests for delivery and pickups are deterministic and known for all planning horizon periods. Also, we assume that the initial stocks of serviceables and returns are both null (zero initial stocks conditions) and there is a positive demand in the first

period. Moreover, new manufactured (resp., remanufactured) products are stored in the serviceables (resp., returns inventory) without exceeding its capacity and each inventory has its own holding cost. A fleet of homogeneous vehicles based at the central plant is available for distribution of the product to customers. We assume that each customer with no requests should not be visited ( $d_{it} > 0 \wedge p_{it} \leq Q$ ). Each vehicle departing from and returning back to the central plant, can visit several customers only once in a route without exceeding its capacity  $Q$ . Furthermore, a vehicle can only perform a single tour in each period and the split deliveries and pickups are not allowed. The studied system can be modelled as shown in FIG. 1.

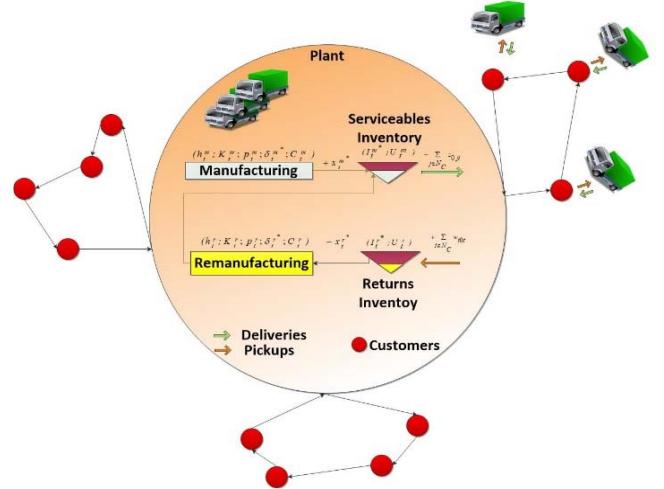


FIG. 1 ILLUSTRATION OF THE STUDIED CLOSED-LOOP SUPPLY CHAIN SYSTEM

The IPIRP-R problem consists of determining how much items to manufacture, remanufacture, store and distribute through optimal routes over a finite multi-period planning horizon. The objective is to jointly minimize the setup, manufacturing, remanufacturing, holding and routing costs.

## III. MATHEMATICAL FORMULATION

This section presents the main notation and a mathematical formulation that integrates production, inventory and routing decisions with remanufacturing in a single framework. Before describing the MILP model, let us introduce the following additional notation:

### A. Sets

$N$ : Set of all nodes, with  $N = \{0, 1, 2, \dots, |N_C|\}$ , where 0 represent the central plant;

$N_C$ : Set of customer nodes, indexed by  $i$  and  $j$ ,  $N_C = \{1, 2, \dots, |N_C|\}$ ,

$T$ : Set of time periods, indexed by  $t$ ,  $T = \{1, 2, \dots, |T|\}$ ;

$K$ : Set of vehicles, indexed by  $v$ ,  $K = \{1, 2, \dots, |K|\}$ .

### B. Parameters

$p^m$ : Unitary production cost for manufacturing at period  $t$

$K^m$ : Fixed manufacturing setup cost

$h^m$ : Unit inventory holding cost for manufacturing serviceable product

$C_t^m$ : Manufacturing production line capacity at period  $t$

$U_t^m$ : Maximum inventory level of serviceable products at period  $t$

$p^r$ : Unitary production cost for remanufacturing at period  $t$

$K^r$ : Fixed remanufacturing setup cost

$h^r$ : Unit inventory holding cost for remanufacturing return-product

$C_t^r$ : Remanufacturing production line capacity at period  $t$

$U_t^r$ : Maximum inventory level of return-products at period  $t$

$d_{it}$ : Delivery demand of customer  $i \in N_C$  at period  $t$

$p_{it}$ : Pickup demand of customer  $i \in N_C$  at period  $t$

$Q$ : Vehicle capacity

$fc$ : Vehicle fixed cost

$c_{ij}$ : Transportation cost from node  $i$  to node  $j$  (assume  $c_{ij} = c_{ji} \wedge c_{ii} = 0 \forall (i, j) \in A$ ).

### C. Decision variables

$x_t^m$ : Quantity of products manufactured at period  $t$ ;

$\delta_t^m$ : Binary variable which is equal to 1 if  $x_t^m > 0$ , and 0 otherwise

$I_t^m$ : Inventory level of serviceables at the end of period  $t$

$x_t^r$ : Quantity of products remanufactured at period  $t$

$\delta_t^r$ : Binary variable which is equal to 1 if  $x_t^r > 0$ , and 0 otherwise

$I_t^r$ : Inventory level of returns at the end of period  $t$

$z_{ijt}$ : Delivered demand up to node  $i$  and carried in arc  $(i, j)$  at period  $t$

$w_{ijt}$ : Picked-up demand up to node  $i$  and carried in arc  $(i, j)$  at period  $t$

$y_{ijvt}$ : Binary variable which is equal to 1 if vehicle  $v$  travels from location  $i$  to location  $j$  at period  $t$ , and 0 otherwise.

Using the above parameters and variables, the full integrated model can be formulated as follows (Model IPIRP-R):

$$\begin{aligned} \min z = & \sum_{t=1}^{|T|} (K^m \cdot \delta_t^m + p^m \cdot x_t^m + h^m \cdot I_t^m \\ & + K^r \cdot \delta_t^r + p^r \cdot x_t^r + h^r \cdot I_t^r) \\ & + \sum_{j \in N_C} \sum_{t=1}^T f_c \cdot y_{0jvt} \\ & + \sum_{i \in N} \sum_{j \in N} \sum_{v \in K} \sum_{t=1}^{|T|} c_{ij} \cdot y_{ijvt} \end{aligned} \quad (1)$$

s.t.

$$I_t^m = I_{t-1}^m + x_t^m + x_t^r - \sum_{j \in N_C} z_{0jt} \quad \forall t \in T \quad (2)$$

$$I_t^r = I_{t-1}^r - x_t^r + \sum_{i \in N_C} w_{i0t} \quad \forall t \in T \quad (3)$$

$$I_0^m = 0; I_0^r = 0 \quad (4)$$

$$0 \leq I_t^m \leq U_t^m \quad \forall t \in T \quad (5)$$

$$0 \leq I_t^r \leq U_t^r \quad \forall t \in T \quad (6)$$

$$x_t^m \leq \min \left\{ \sum_{i \in N_C} \sum_{l=t}^T d_{il}; C_t^m \right\} \cdot \delta_t^m \quad \forall t \in T \quad (7)$$

$$x_t^r \leq \min \left\{ \sum_{i \in N_C} \sum_{l=t}^T d_{il}; C_t^r \right\} \cdot \delta_t^r; I_{t-1}^r \quad \forall t \in T \quad (8)$$

$$\sum_{j \in N_C} y_{0jvt} \leq 1 \quad \forall t \in T; \forall v \in K \quad (9)$$

$$\sum_{i \in N} y_{i0vt} \leq 1 \quad \forall t \in T; \forall v \in K \quad (10)$$

$$\sum_{i \in N} \sum_{v \in K} y_{ijvt} = 1 \quad \forall j \in N_C; \forall t \in T \quad (11)$$

$$\sum_{j \in N} y_{ijvt} - \sum_{j \in N} y_{jivt} = 0 \quad \forall i \in N_C; \forall v \in K; \forall t \in T \quad (12)$$

$$\sum_{i \in N} z_{ijt} - \sum_{i \in N} z_{jvt} = d_{jt} \quad \forall j \in N_C; \forall t \in T \quad (13)$$

$$\sum_{i \in N} w_{ijt} - \sum_{i \in N} w_{jvt} = p_{jt} \quad \forall j \in N_C; \forall t \in T \quad (14)$$

$$z_{ijt} + w_{ijt} \leq Q \cdot \sum_{v \in K} y_{ijvt} \quad \forall (i, j : i \neq j) \in N; \forall t \in T \quad (15)$$

$$w_{0jt} = 0 \quad \forall j \in N_C; \forall t \in T \quad (16)$$

$$z_{i0t} = 0 \quad \forall i \in N_C; \forall t \in T \quad (17)$$

$$y_{ijvt} \in \{0, 1\} \quad \forall (i, j : i \neq j) \in N; \forall v \in K; \forall t \in T \quad (18)$$

$$\delta_t^m \in \{0, 1\}; \delta_t^r \in \{0, 1\} \quad \forall t \in T \quad (19)$$

$$x_t^m, x_t^r, I_t^m, I_t^r, z_{ijt}, w_{ijt} \in \mathbb{Z}^+ \quad \forall (i, j : i \neq j) \in N; \forall t \in T \quad (20)$$

Objective function (1) minimizes the total manufacturing, remanufacturing, inventory and routing cost. Equations (2)-(4) are the inventory balance equations and initial inventories conditions for serviceable and return products respectively. Inequalities (5)-(6) limit the maximum capacity of serviceable and returned products not to be exceeded at the end of each period. Inequalities (7)-(8) represents the manufacturing (resp., remanufacturing) capacity constraints and guarantee that there is manufacturing (resp., remanufacturing) of new products (resp. return-products) in

a period  $t$  only if the manufacturing production line (resp., remanufacturing production line) is set up at the beginning of the period  $t$ . Equations (9)-(12) are the classical vehicle routing constraints. Constraints (13)-(14) guarantee that the delivery and pickup demands at customer  $j \in N_C$  are satisfied. Constraint (15) guarantees that the total load on the vehicle does not exceed its capacity. Constraints (16)-(17) ensures that the vehicle leaves the central plant without any pickup demand load and reaches the depot after delivering all the demands. Finally, constraints (18)-(20) provides the domain of the variables.

#### D. Illustrative example

In order to illustrate some key features of our model, we provide a solution of an illustrative example with  $|N_C| = 5$ ,  $|T| = 3$  and  $|K| = 5$ . FIG. 2 shows, for each period, serviceable inventory level, returns inventory level, manufactured lot-sizes, remanufactured lot-sizes, delivery quantities to be distributed and pickup quantities to be collected. FIG. 3 shows the total cost and its distribution according to the integrated operations considered. Finally, FIG. 4 shows the routing decisions for each period.

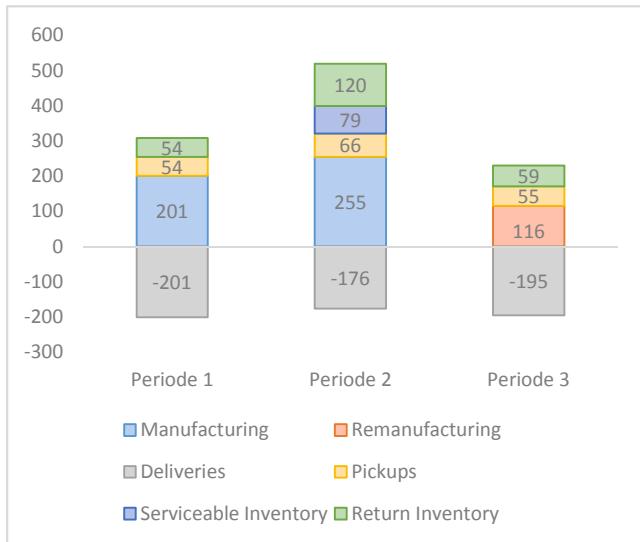


FIG. 3 INTEGRATED PRODUCTION-INVVENTORY-ROUTING PLAN OF THE ILLUSTRATE EXAMPLE

For example, at the beginning of period one the manufacturing lot-sizes and the quantities to be delivered are of the same values, namely 201 units, which means that we manufacture the same amount of requested deliveries to meet all delivery demands occurring in this period. Therefore, the inventory level of serviceable products at the end of the first period is  $0 + 201 - 201 = 0$  (assuming that initial inventory level of serviceables product to be 0 ( $I_0^m = 0$ )), which corresponds to the initial inventory of serviceable products in the second period. Regarding the level of return-products inventory, the same analysis applies. Thereby, the inventory level of inventory products at the end of the first period is  $0 + 54 - 54 = 0$  (assuming that initial inventory level of return inventory to be 0 ( $I_0^r = 0$ )), which corresponds to the initial inventory of return inventory in the second period. We can observe that no operation of remanufacturing was carried out, which is quite normal because the EOL product are collected and returned after the vehicles goes back to the central plant at the end of period one.

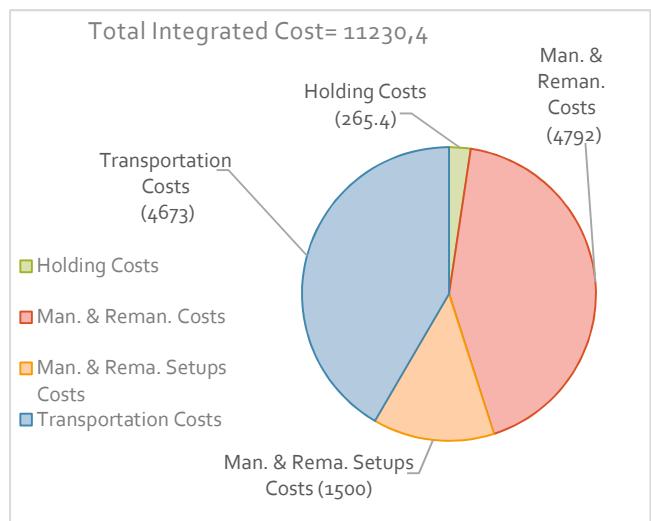


FIG. 2 TOTAL INTEGRATED COST OF THE ILLUSTRATE EXAMPLE

By performing the same analysis and calculations, one can obtain the manufacturing lot-sizes, remanufacturing lot-sizes, inventory levels for both serviceables and return inventories also the amounts of deliveries and pickups to be distributed and to be collected for the remaining of periods.

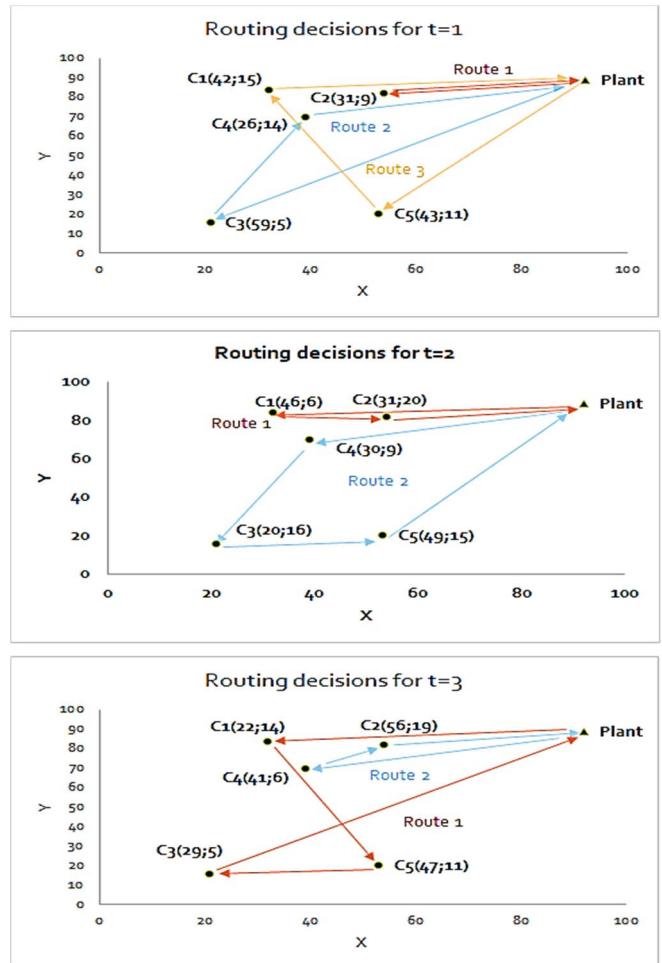


FIG. 4 ROUTING DECISIONS OF THE ILLUSTRATE EXAMPLE

Note that no manufacturing operation is performed, instead a remanufacturing one occurs in period three, and therefore the final inventory level of serviceables products is null. From figure 3 it is easy to notice that the costs related to production and transport operations are significant and represent respectively 42.67% and 41.61% of the total integrated cost against the costs related to holding and setups operations who come in third and fourth place respectively and represent 13.35% and 2.32% of the total integrated cost, respectively. From figure 4 we can notice that only 3 vehicles were used in period one and two vehicles in period 2 and 3 to perform routes among the 5 vehicles allowed on the entire planning horizon.

#### IV. COMPUTATIONAL EXPERIMENTS

##### A. Instances Generation

In this section we provide computational experiments conducted on a set of a randomly generated instances. The MILP model was coded in Eclipse Java Oxygen and solved using IBM ILOG CPLEX 12.7 solver on a PC with a CPU Core i5 2.40 GHz processor and 4 GB RAM. An acceptable computational time of 1 hour is set for each instance. Furthermore, all CPLEX parameters were set up to their default values for all numerical tests.

The conducted tests were performed using several randomly generated instances based on some adapted data sets from the literature of the single lot-sizing with remanufacturing problem and those from the integrated production-routing problem in particular research works of [23, 24]. The generated data instances, consist of problems with  $|T| = \{3, 6, 9\}$  and  $|N_C| = \{5, 10, 15, 20\}$ . We generate 10 instances for each combination of customers number  $|N_C|$  and number of time periods  $|T|$  according to the patterns shown in TABLE 1, resulting in a total of 12 problems with 120 instances.

TABLE 1 PARAMETERS USED TO GENERATE RANDOM INSTANCES

<i>Unitary production cost for manufacturing new product</i>	$p^m = 10$
<i>Fixed manufacturing setup cost</i>	$K^m = 500$
<i>Unit inventory holding cost for manufacturing serviceable product</i>	$h^m = 1$
<i>Manufacturing production line capacity at period t</i>	$C_t^m = 2 \cdot \frac{\sum_{t \in T} \sum_{i \in N_C} d_{it}}{T}$
<i>Maximum inventory level of serviceable products at period t</i>	$U_t^m = 2 \cdot \frac{\sum_{t \in T} \sum_{i \in N_C} d_{it}}{T} \cdot  N_C $
<i>Unitary production cost for remanufacturing a returned product</i>	$p^r = 2$
<i>Fixed remanufacturing setup cost</i>	$K^r = 500$
<i>Unit inventory holding cost for remanufacturing return-product</i>	$h^r = 0.8$

<i>Remanufacturing production line capacity at period t</i>	$C_t^r = 2 \cdot \frac{\sum_{t \in T} \sum_{i \in N_C} p_{it}}{T}$
<i>Maximum inventory level of return-products at period t</i>	$U_t^r = 2 \cdot \frac{\sum_{t \in T} \sum_{i \in N_C} p_{it}}{T} \cdot  N_C $
<i>Delivery demand of customer <math>i \in N_C</math> at period t</i>	$d_{it} \in U[20, 60]$
<i>Pickup demand of customer <math>i \in N_C</math> at period t</i>	$p_{it} \in U[5, 22]$
<i>Capacity of each vehicle</i>	$Q = 100$
<i>Vehicle fixed cost for using a vehicle</i>	$fc = 500$
<i>Coordinates of node i</i>	$(X_i, Y_i) \in [0, 100]$
<i>Travel cost from node i to node j</i>	$c_{ij} = \sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2 + 0.5}$
<i>Maximum number of vehicles over the planning horizon</i>	$ K  = \left\lceil \frac{\max(\sum_{t \in T} \sum_{i \in N_C} d_{it}; \sum_{t \in T} \sum_{i \in N_C} p_{it})}{Q} \right\rceil$

##### B. Numerical Results

In order to evaluate the efficiency and the limits of our integrated model (IPIRP-R), this section provides the computational results on the random instances described in the previous section.

The obtained results using CPLEX solver are shown in TABLE 2. Columns 2-4 show the number of customers, number of periods and the maximum number of vehicles to be used, respectively. Columns 5 shows the total cost average calculated based on instances solved to optimality. Finally, columns 6 shows the average of CPU time, in seconds. The symbol “†” means that CPLEX Solver failed to return a feasible solution for the corresponding instances within the 1-hour time limit.

TABLE 2 SUMMARY OF COMPUTATIONAL RESULTS FROM CPLEX SOLVER ((\*) SIGNIFICATES THAT THE AVERAGE VALUE WAS CALCULATED BASED ONLY ON THE FEASIBLE SOLVED INSTANCES)

Prob#	$ N_C $	$ T $	$ K $	Av. Total cost	Av. CPU (s)
1		3	6	11502,44	5,3163
2	5	6	12	22522,56	201,35
3		9	18	33654,7	959,1457
4		3	13	21042,5 <sup>(*)</sup>	>3600
5	10	6	24	56744 <sup>(*)</sup>	>3600
6		9	37	60957,11 <sup>(*)</sup>	>3600
7		3	18	29986,4 <sup>(*)</sup>	>3600
8	15	6	36	56793,5 <sup>(*)</sup>	>3600
9		9	54	†	>3600
10		3	25	40007,22 <sup>(*)</sup>	>3600
11	20	6	50	†	>3600
12		9	72	†	>3600

In general, we can easily observe that CPLEX solver can only solve to optimality instances with  $|N_C| = 5$  customers for a planning horizon with  $|T| = \{3, 6, 9\}$  time periods after 1-hour of computing time. Moreover, in some cases, the solver may have found an optimal solution but it is not able to prove it because of the poor quality of the lower bounds and needs more time to prove it. This is the case, for example, for the instances with  $|N_C| = \{10, 15\}$  costumers for most period times, whose feasible solutions was found in one hour. Another disadvantage of solving the integrated model is that CPLEX solver is unable to provide feasible solutions for most of the instances with  $|N_C| \geq 20$  for all time periods of the planning horizon. This gives us a strong proof about the complexity of the integrated model that can be explained by the fact that each of the considered problems is NP-hard [1]. Therefore, the use of approximate resolution approaches seems logical and essential to use, especially the heuristic and metaheuristics methods to solve the integrated model in order to achieve good quality solutions within reasonable times for all medium and large instances.

## V. CONCLUDING REMARKS AND FUTURE WORKS

In this paper, we proposed a new MILP formulation to represent and solve the IPIRP with remanufacturing of EOL products. The new model considered a new feature namely a fleet of homogenous capacited vehicles. We used a commercial solver to solve a set of 120 randomly generated instances. Numerical results that the solver was able to solve optimally the instances with  $|N_C| = 5$  and  $|T| = \{3, 6, 9\}$ , and near-optimal solutions were found for most of the instances with  $|N_C| = 10$  customers and  $|T| = \{3, 6, 9\}$  periods.

Based on the results obtained, an interesting perspective for future research is to study the complexity of the IPIRP-R problem and develop an effective heuristic procedure to solve it by combining mathematical programming and heuristic methods in order to find better solutions for medium and large-instances. Another interesting line of research is to extend the MILP formulation in order to include the case of stochastic requests for deliveries and pickups as well as multiple products, heterogeneous fleet of vehicles and time windows. Considering these three new features would result in a more complex scenario, which is often found in industrial real-life applications.

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