Do hearing-impaired students learn mathematics in a different way than their hearing peers? – A review

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In the last few years, the concept of inclusion has become more and more prevalent in school education. Accordingly, teachers in mathematics classrooms have to face not just a wide range of heterogeneity related to social background, language skills and performance abilities, but also various impairments like physical, sensory and mental disabilities. To facilitate gainful inclusive mathematics education, it is important to understand the aspects of mathematical concept formation and of mathematics performance which differ between disabled and not-disabled children. The main focus in the current paper is on the differences between hearing and hearing-impaired students when doing mathematics. Empirical studies from the last two decades are summarized and some guidelines for inclusive mathematics settings with hearing and hearing-impaired children are derived.

Keywords: Inclusive education, hearing impairments, concept formation, mathematics education.

Background

There is an overwhelming amount of literature related to deaf children's learning, a significant part of which discusses aspects of mathematics performance. Even if it is not possible to seek completeness, a short overview on some well-selected articles can give a useful insight into the main research questions and already ascertained results. For this reason, 24 articles (mainly from two highly relevant journals, the Educational Studies in Mathematics and Journal of Deaf Studies and Deaf Education) regarding hearing-impaired students' learning in mathematics (mainly reports on empirical studies about differences between the performance of hearing and hearing-impaired students) were chosen predominantly from the last two decades, such that all education levels from kindergarten to college are represented. Additionally, some of the papers discuss adults' mathematical performance long after completing school education. The main goal in this paper is to identify relevant differences between hearing and hearing-impaired students when doing mathematics and to derive their possible influence on an inclusive classroom. According to Ziemen (2017), the term inclusion will be used in the current paper as overcoming of all kind of marginalization, discrimination and stigmatization, which includes especially the respect and appreciation of handicapped students in co-educated classrooms.

There is a common agreement in the relevant literature, that hearing-impaired students' performance in school mathematics is on average far below the average performance of their hearing peers and that this delay corresponds to a disadvantage of 2 to 4 school years. However, there is no consensus regarding when this delay first appears, or which parts of the language skills and cognitive abilities are affected. Three different areas of related research can be identified: Studies that mainly focus on detecting and describing cognitive differences between hearing and hearing-impaired pupils, studies that look for reasons for those disadvantages, and studies that suggest interventions for hearing-impaired students and measure their effectiveness. Table 1 shows an overview of all reviewed papers, categorized according to the main focus and the examined educational level. Please note that some of the papers include more than one study and therefore more than one of these aspects; it also happens, that a study was carried out on more than one education level. Thus, several papers are registered more than once. In the next sections, research results in the identified three areas (differences, reasons and interventions) will be summarized.

<table>
<tr><th>main focus</th><th>differences</th><th>reasons for differences</th><th>intervention</th></tr>
<tr><td>preschool</td><td>Ansell & Pagliaro, 2006;
Kritzer, 2009;
Pagliaro & Kritzer, 2013;
Zarfaty, Nunes, & Bryant, 2004</td><td>Kritzer, 2008</td><td></td></tr>
<tr><td rowspan="2">primary school</td><td>Ansell & Pagliaro, 2006;
Frostad & Ahlberg, 1999;
Nunes et al., 2009</td><td>Pagliaro & Ansell, 2002</td><td rowspan="2">Nunes & Moreno, 1998;
Nunes & Moreno, 2002;
Nunes et al., 2009</td></tr>
<tr><td colspan="2">Frostad, 1999; Zevenbergen, Hyde, & Power, 2001</td></tr>
<tr><td rowspan="2">secondary school</td><td>Blatto-Vallee et al., 2007;
Searle, Lorton, & Suppes, 1974</td><td>Kelly, Lang & Pagliaro, 2003; Lang et al., 2007</td><td rowspan="2"></td></tr>
<tr><td colspan="2">Zevenbergen et al., 2001</td></tr>
<tr><td rowspan="2">college, university</td><td>Blatto-Vallee, Kelly, Gaustad, Porter, & Fonzi, 2007</td><td>Kelly & Gaustad, 2007</td><td rowspan="2">Marshall, Carrano, & Dannels, 2016</td></tr>
<tr><td colspan="2">Bull, Blatto-Vallee, & Fabich, 2006;
Kelly, Lang, Mousley, & Davis, 2003;
Marschark et al., 2015</td></tr>
<tr><td>adults</td><td colspan="2">Korvorst, Nuerk, & Willmes, 2007; Kramer & Grote, 2009
Masataka, 2006</td><td></td></tr>
</table>

Table 1: Main focus of the reviewed articles categorized by education level

Before looking into the content of the papers it is important to remark that, although the term ´deaf´ is commonly taken to mean profound hearing loss, most of the papers use the term in a wider sense. For example, Nunes et al. (2009) examine the performance of children with moderate to profound hearing loss in their first study and the performance of children with mild to profound hearing loss in their second reported study. In Frostad’s (1999) case study the pupils have moderately severe to profound hearing loss, and Zarfaty, Nunes and Bryant (2004) also work with children with moderate to profound hearing loss. All these papers use the term ´deaf´. Others, for example Pagliaro and Kritzer (2013) emphasize the difference between partial and complete hearing loss and make use of the term ´deaf and hard-of-hearing´. Because of this inconsistent and confusing use of terms, in this paper the term ´hearing-impaired´ will be employed and used in the sense which incorporates all the meanings of related terms in the reviewed papers. So, by hearing-impaired will be meant all kinds of hearing loss, which exceed the threshold of normal hearing (hearing loss 15 dB at the most).

Differences between hearing and hearing-impaired students when doing mathematics

Because of the main consensus about the delayed performance of hearing-impaired students in school mathematics, researchers started to pay more attention to preschool mathematics in order to answer the question of whether those differences are already present before starting school. A surprising and also promising result from Zarfaty, Nunes and Bryant (2004) with children in the age range of 3 to 4 years, is that they could remember and reproduce numbers as well as their hearing peers when the task was offered in a sequence, but they outperformed the hearing children, when the task was organized spatially. Contradictory to these findings, hearing-impaired children did not benefit from the visual-spatial problem presentation in the study of Ansell and Pagliaro (2006). With respect to arithmetic story problems, preschool and primary school hearing-impaired children do not use linguistic markers in the story, instead reacting more to the assumed mathematics operation (Ansell & Pagliaro, 2006). Similarly, not only do 4- to 6-year-old hearing-impaired children already present a developmental delay in both informal and formal mathematics tasks relative to hearing peers, but also, not even participants with high mathematical ability could make relationships between the numbers and the story in a word problem (Kritzer, 2009). The results from Pagliaro and Kritzer (2013) can further differentiate the picture: In their study, 3- to 5-year-old hearing-impaired children showed strength especially in geometry.

In accordance with the results of Ansell and Pagliaro (2006), Frostad and Ahlberg (1999) found that hearing-impaired primary school children typically approach word problems as numbers and procedures without reflecting on the semantic relations in the text. Frostad (1999) found additionally that primary school hearing-impaired children do not use their knowledge base for deriving the answer, instead reverting to counting. Similarly, hearing-impaired primary and secondary school students rely on trigger words in the text as well as ignore key words in decontextualized problems (Zevenberger et al., 2001). In the study of Nunes et al. (2009), hearing-impaired primary school children under-performed their hearing peers in multiplicative reasoning tasks.

At the secondary school level, the findings of Zevenberger et al. (2001) are already reported above. In contrast to that, Searle, Lorton and Suppes (1974) found among grade 4-6 deaf and deprived students that terse and story-free problems are especially difficult for them to solve. Another relevant result in this study was that the length of the word problems did not affect the mathematical performance. Blatto-Vallee, Kelly, Gaustad, Porter and Fonzi (2007) examined the use of visual-spatial schematic and visual-spatial pictorial representations among secondary school and college students when solving word problems. Hearing-impaired students tended to utilize more visual-spatial pictorial representations, which encode only the visual appearance of objects described in the problem and are therefore on a lower cognitive level than visual-spatial schematic representations, which encode the spatial relationships described in the problem.

Kelly, Lang, Mousley and Davis (2003) examined the consistency hypothesis during solving arithmetic word problems among hearing-impaired college students. The hypothesis postulates that students perform better on word problems in which the order of the information is consistent with the order of the corresponding mathematical operation; supporting evidence has been found previously among hearing students. The results of the Kelly et al. (2003) study support the consistency hypothesis: Hearing-impaired students performed similarly. On the other hand, they made – regardless of reading ability – more goal monitoring mistakes than hearing peers. Bull, Blatto-Vallee and Fabich (2006) found that on subitizing tasks (instantaneous recognition of the cardinality of small sets), both hearing and hearing-impaired college students' performance have a similar pattern. Surprisingly, hearing-impaired students did not perform better on a special skew dot format, even if this was anticipated due to their assumed better visual-spatial skills. Accordingly, Marschark et al. (2015) report that hearing university students outperformed their hearing-impaired peers in visual-spatial tasks. Note, that this contradicts to the findings of Zarfaty et al. (2004) on the preschool level.

Masataka (2006) investigated the number sense of hearing-impaired adults and their hearing peers. In this study the hearing-impaired participants outperformed the hearing peers on tasks which used non-symbolic numerosity, but they did worse, when the same tasks were offered in a formal mathematical way. Kramer and Grote (2009) compared the performance of hearing-impaired adults on basic mathematics operations with that of hearing peers. Even if the performance of hearing-impaired adults is far below the performance of people with the lowest level certificate of secondary school in Germany, the two groups showed in a language-free test almost the same cognitive ability. The authors also found a performance benefit for deaf adults with deaf parents and concluded that the language (not exclusively sign language) used in mathematics classrooms has a negative effect on their learning. In accordance with this, Korvorst, Nuerk and Willmes (2007) found hearing-impaired adults' performance on complex numerical information-extracting tasks was quite similar to the performance of their hearing peers, when the tasks were offered in sign language.

Possible reasons for differences in mathematical performance

At the primary level, Pagliaro and Kritzer (2013) conclude that the detected delayed development of hearing-impaired children related to basic concepts in mathematics can be caused by absent, inappropriate, or misguided learning opportunities. Similarly, Kritzer (2008) found in a qualitatively analyzed case study with hearing-impaired 4- to 6-year-old children and their parents, that the four mathematically based concepts (numbers, quantity, time and/or sequence, categorization) were used more frequently by the parents of children with high mathematical ability than by parents of children with lower mathematical ability. The first group of children was also exposed to mathematically based concepts in a way that was more purposeful and meaningful.

Even if sign language number symbols have many of the characteristics of analogue representations, the efficiency of those numbers for counting can delay the development of conceptual knowledge in hearing-impaired primary school children (Forstad, 1999). So, the use of sign language is beneficial, but can also lead to disadvantages. Zevenbergen et al. (2001) described a similar dilemma: If the teacher reorganizes the word problems so as to make them more accessible for hearing-impaired pupils, students are not challenged cognitively and do not get access to the highly specific register of the discipline. Totally in accordance with these findings, Pagliaro and Ansell (2002) concluded based on a questionnaire with teachers of third-grade hearing-impaired students, that they do not encounter story problems early enough and often enough, so they are not provided sufficient opportunities to form problem-solving strategies.

Because the study of Zevenbergen et al. (2001) was made with first- to seventh-grade hearing-impaired children, the statement above is also valid for the secondary school level. Additionally, Kelly, Lang and Pagliaro (2003) found that not only do teachers of hearing-impaired students not challenge them cognitively in solving mathematics word problems, but also that they have low perceptions and expectations about the students' abilities and therefore do not offer them meaningful problem-solving situations. Also, the teachers associated limited English skills with a primary barrier to learn, and thus emphasized comprehension strategies rather than problem-solving strategies. However, difficulties can also be caused by other factors: In a case study with teachers, Lang et al. (2007) determined that visual representations of science concepts (among others technical science signs) may lead to misconceptions, but also, that for the majority of science terms there is no published or recorded sign.

There is some evidence that mathematics performance is affected by language abilities, especially at higher levels of education. Kelly et al. (2003) found that, even if the rate of goal-monitoring errors was much higher among hearing-impaired college students than among hearing peers, this rate nevertheless decreased with increasing reading ability. Marschark et al. (2015) examined the executive functioning behaviors (such as comprehension and conceptual learning, factual memory, attention and so on) of hearing and hearing-impaired first-year university students in everyday life with a self-report questionnaire. They found better scores for the hearing than for the hearing-impaired students, but also, that difficulties in executive functioning among participants with cochlear implant are the result of both, language delay and auditory deprivation. In accordance with this, Kelly and Gaustad (2007) could demonstrate, that specific morphological competencies in English in addition to reading ability level, are significantly related to mathematics performance. Bull et al. (2006) concluded that hearing and hearing-impaired students do not differ from each other in the format of numerical representation and the level of automatic activation of magnitude information. Thus, this aspect cannot be the reason for later difficulties with arithmetic.

Based on his study with hearing and hearing-impaired adults, Masataka (2005) concluded that, difficulties in mathematics are related to the formal, symbolic side of the discipline, and that this is modulated by the environment and the culture. Kramer and Grote (2009) came to a similar conclusion: They found the language used in mathematics classrooms to be responsible for the mathematics difficulties of deaf (native sign-language user) individuals, but they also mention missing and deficient opportunities for developing language skills and everyday-life-knowledge. In accordance with this and also with the findings of Bull et al. (2006) above, Korvorst et al. (2007) did not find evidence for core differences between hearing and hearing-impaired adults in solving bisection tasks, which require the extraction of complex numeric information, when hearing-impaired participants used (their native) sign language.

Interventional methods and their effectiveness

Nunes and Moreno (1998) applied a non-traditional method for calculating, namely the signed algorithm, in a mathematics classroom with solely hearing-impaired primary school children. While solving addition and subtraction problems, the pupils showed systematic errors similar to the ones in written computation related to place value understanding and the mechanics of written algorithms. Thus, the authors suggest to try out this method as an alternative and make use of the systematic errors to optimize teachers' instruction. Nunes and Moreno (2002) also developed an interventional program for hearing-impaired students in primary school which involved two main aspects: Giving opportunities to learn basic mathematical concepts, which can be learnt informally by hearing students, and promoting connections between informal and formal mathematical concepts. The interventional group performed significantly better in the posttest not just in comparison with the baseline group, but also with their own previously estimated performance. The intervention led also to motivational benefits. Furthermore, Nunes et al. (2009) adapted an intervention program on multiplicative reasoning – originally developed for hearing children at risk for difficulties in learning mathematics – for hearing-impaired students. However, the intervention was also applied to a hearing experimental group. Both hearing and hearing-impaired children benefited significantly from the intervention, but in a delayed posttest the performance of the hearing-impaired children decreased. A possible reason for this fact could be the long-term poorer problem-solving environment for hearing-impaired pupils.

Responding to missing everyday opportunities and problem-solving strategies of hearing-impaired students, Marshall, Carrano and Dannels (2016) developed an intervention program based on the concept of experimental learning, on best-practice experiments and on the concept of plan-do-check-act. During the lessons, hearing-impaired students become more and more familiar with the solving of real, work-related problems. The sessions featured sign-supported explanatory videos. Significant improvements were found between the performance in the pre- and posttest for the long term.

Conclusions for inclusive mathematics classrooms

According to Pagliaro and Kritzer (2013), Kritzer (2008) and also to Kramer and Grote (2009), the main focus in the preschool education should be on offering opportunities primarily to develop informal mathematics knowledge such as numbers, quantity, time, events in a sequence, categorization and to improve language skills.

In primary and secondary school, word problems seem to be the most challenging for hearing-impaired students. Intervention programs such as suggested by Nunes and Moreno (2002) and

Nunes et al. (2009) could and should be implemented and extended for other areas and for secondary level, for the following reasons: (1) Hearing-impaired students could compensate their deficient informal mathematics knowledge and language skills (2) both hearing-impaired and hearing students could develop high-level problem-solving strategies (3) these activities could also be beneficial for socioeconomically disadvantaged pupils and students at a risk for difficulties in learning mathematics. It is also important to make use of visual-spatial schematic representations when solving word problems (Blatto-Vallee et al., 2007) and to discuss story-free word problems (Searle et al., 1974). Children with profound hearing loss could use the sign algorithm as an alternative (Nunes & Moreno, 1998) and should use their native language (sign language) (Kramer & Grote, 2009; Korvorst et al., 2007), perhaps with the help of a native sign-user translator. At college and university education level, the support of language skills – including technical (sign) language – is recommended.

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