Erratum: Spectral analysis of semigroups and growth-fragmentation equations

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Abstract. We correct the article [S. Mischler, J. Scher, Spectral analysis of semigroups and growth-fragmentation equations. Ann. Inst. H. Poincar Anal. Non Linaire 33 (2016), no. 3, 849898].

In the article [2], there was an error in Step 1 in the proof of Theorem 2.1 that we were not able to fix, with fatal repercussions on most of the abstract spectral analysis results (Corollary 2.4, Theorem 3.1, Theorem 3.3 and Theorem 3.5 in [2]). However, we claim that Corollary 2.5 is correct and that we may slightly modify the statements of the other above mentioned abstract results in order to make them correct and then to use these variants in order to repair the proof of [2, Theorem 1.1].

We employ the notation and assumptions of [2], except that we replace assumption **(H2)** by the assumptions **(h2)** or **(h2′)** below. More precisely, we consider a Banach space X and the generator Λ of a strongly continuous semigroup S_Λ on X. We assume that Λ splits as

$$\Lambda = \mathcal{A} + \mathcal{B}$$

where $\mathcal{B}$ is the generator of a strongly continuous semigroup $S_\mathcal{B}$ and $\mathcal{A}$ is $\mathcal{B}$-bounded. We also assume that the operators $\mathcal{A}$ and $S_\mathcal{B}$ satisfy one of the two following regularizing properties of an iterated enough convolution product:

(h2) there exist $\zeta \in (0,1]$ and $\zeta' \in [0,\zeta)$ such that $\mathcal{A} \in \mathscr{B}(X_{\zeta'}, X)$ and there exists an integer $n \geq 1$ such that for any $a > a^*$, there holds

$$\forall\, t \geq 0, \qquad \|(\mathcal{A}S_\mathcal{B})^{(*n)}(t)\|_{\mathscr{B}(X,X_\zeta)} \leq C_{a,n,\zeta}\, e^{at}$$

for a constant $C_{a,n,\zeta} \in (0,\infty)$,

or

(h2′) there exist $\zeta \in [-1,0)$ and $\zeta' \in (\zeta, 0]$ such that $\mathcal{A} \in \mathscr{B}(X, X_{\zeta'})$ and there exists an integer $n \geq 1$ such that for any $a > a^*$, there holds

$$\forall\, t \geq 0, \qquad \|(S_\mathcal{B}\mathcal{A})^{(*n)}(t)\|_{\mathscr{B}(X_\zeta,X)} \leq C_{a,n,\zeta}\, e^{at}$$

for a constant $C_{a,n,\zeta} \in (0,\infty)$.

It is worth recalling that X_ζ stands here for the abstract Sobolev space associated to $\mathcal{B}$ (and thus also to Λ) and $\zeta \in \mathbb{R}$.

Replacing hypothesis **(H2)** in Theorem 2.1 by hypothesis **(h2)** or **(h2′)**, we denote as Theorem E.2.1 the resulting statement. We denote similarly Corollary E.2.4, Theorem E.3.1, Theorem E.3.3 and Theorem E.5.3.

We are going now to briefly explain how to modify the arguments presented in [2] in order to prove the above erratum results. More details and more general versions of these results are available in [1].

Sketch of the proof of Theorem E.2.1. We first consider the case when we assume **(h2)** instead of **(H2)**. We fix $a \in (a^*, a')$. On the one hand, from **(h2)**, we have

$$\forall\, z \in \Delta_a, \quad \|(\mathcal{A}\mathcal{R}_\mathcal{B}(z))^n\|_{X \to X_\zeta} \le C'_{a,n},$$

for a constant $C'_{a,n} \in (0,\infty)$ only depending on a, a^* and $C_{a,n,\zeta}$. On the other hand, using the same interpolation argument as in Step 1 in the proof of [2, Theorem 2.1], we have

$$\forall\, z \in \Delta_a, \quad \|\mathcal{R}_\mathcal{B}(z)\|_{X_\zeta \to X_{\zeta'}} \le C_{a,\varepsilon} \langle z \rangle^{-(\zeta-\zeta'-\varepsilon)},$$

for any $\varepsilon > 0$ and for a constant $C_{a,\varepsilon} \in (0,\infty)$ only depending on a, ε and the dissipativity estimate on $\mathcal{B}$. Writing

$$(\mathcal{A}\mathcal{R}_\mathcal{B}(z))^{n+1} = \mathcal{A}\,\mathcal{R}_\mathcal{B}(z)\,(\mathcal{A}\mathcal{R}_\mathcal{B}(z))^n$$

and using the available estimates on $(\mathcal{A}\mathcal{R}_\mathcal{B}(z))^n$, $\mathcal{R}_\mathcal{B}(z)$ and $\mathcal{A}$, we immediately obtain that

$$\forall\, z \in \Delta_a, \quad \|(\mathcal{A}\mathcal{R}_\mathcal{B}(z))^{n+1}\|_{\mathscr{B}(X)} \le C_{a,\alpha}\,\frac{1}{\langle z \rangle^\alpha},$$

for any $\alpha \in (0, \zeta - \zeta')$ and a constant $C_{a,\alpha} \in (0,\infty)$. That last estimate is nothing but [2, (2.12)] and thus concludes the proof of Step 1 in the proof of [2, Theorem 2.1]. The other steps are unchanged in the case we make the assumption **(h2)**.

We now explain how to adapt the proof in the case we assume **(h2′)**. More details are given in [3, 1]. The first step consists in writing

$$(R_\mathcal{B}(z)\mathcal{A})^{n+1} = (R_\mathcal{B}(z)\mathcal{A})^n\, R_\mathcal{B}(z)\,\mathcal{A}$$

and to prove in a similar way as above that

$$\forall\, z \in \Delta_a, \quad \|(R_\mathcal{B}(z)\mathcal{A})^{n+1}\|_{\mathscr{B}(X)} \le C_{a,\alpha}\,\frac{1}{\langle z \rangle^\alpha},$$

for any $\alpha \in (0, \zeta' - \zeta)$ and a constant $C_{a,\alpha} \in (0,\infty)$. The end of the proof is exactly the same as in the proof of [2, Theorem 2.1], except that we use the left factorization identity

$$\mathcal{R}_\Lambda(z) = \mathcal{R}_\mathcal{B}(z) - \mathcal{R}_\mathcal{B}(z)\mathcal{A}\mathcal{R}_\Lambda(z)$$

and the left factorization Duhamel formula

$$S_\Lambda = S_\mathcal{B} + S_\mathcal{B}\mathcal{A} * S_\Lambda$$

instead of the right factorization ones.

The other parts of the proof are unchanged. $\square$

Next, the proofs of Corollary E.2.4, [2, Corollary 2.5], Theorem E.3.1, Theorem E.3.3 and Theorem E.5.3 are exactly the same as the original ones except that we use Theorem E.2.1 instead of [2, Theorem 2.1].

For the fragmentation equation, assumption **(h2)** is fulfilled for the same splitting $\Lambda = \mathcal{A} + \mathcal{B}$ as introduced in [2, Section 4] because of [2, Lemma 4.8] and [2, Lemma 4.9]. For the fragmentation equation, the proof of [2, Theorem 1.1] is then unchanged (using the erratum versions of the abstract spectral analysis results).

We finally consider the age structured population equation [2, (1.42)-(1.44)] and we recall that it equivalently writes

$$\partial_t f = \Lambda f = \mathcal{A} f + \mathcal{B} f,$$

with $\mathcal{A}$ and $\mathcal{B}$ defined on $X := M^1(\mathbb{R})$ by

$$\begin{aligned}(\mathcal{A} f)(x) &:= \delta_{x=0} \int_0^\infty K(y)\, f(y)\, dy \\ (\mathcal{B} f)(x) &:= -\partial_x f(x) - f(x),\end{aligned}$$

with $K \in C^1_b(\mathbb{R}_+) \cap L^1(\mathbb{R}_+)$, $\|K\|_{L^1} > 1$. We observe that because of the regularity assumption made on K, we have $\mathcal{A} \in \mathscr{B}(X_{-1}, X)$. On the other hand, $\mathcal{B}$ is -1-dissipative in X from [2, Lemma 4.13]. Both together, we deduce that

$$\forall\, t \ge 0, \qquad \|S_\mathcal{B}(t)\mathcal{A}\|_{\mathscr{B}(X_{-1},X)} \le C\, e^{-t},$$

for a constant $C \in (0,\infty)$, and then that condition **(h2′)** holds true. For the age structured population equation, the proof of [2, Theorem 1.1] is then a consequence of the new abstract spectral analysis results Theorem E.3.3 and Theorem E.5.3 exactly as explained in [2, Section 4.3] and [2, Section 6.1]. More developments on the spectral analysis of the age structured population model are presented in [3, 1].

References

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