



# Multi-scale modeling of imperfect interfaces

Serge Dumont, Frédéric Lebon, Raffaella Rizzoni

## ► To cite this version:

Serge Dumont, Frédéric Lebon, Raffaella Rizzoni. Multi-scale modeling of imperfect interfaces. Multi-scale modeling and characterization of Innovative materials and structures, May 2013, Cetara, Italy. hal-00817555

**HAL Id: hal-00817555**

**<https://hal.science/hal-00817555>**

Submitted on 24 Apr 2013

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Multi-scale modeling of imperfect interfaces

Serge Dumont<sup>1,2</sup>, Frédéric Lebon<sup>2</sup>, Raffaella Rizzoni<sup>3</sup>

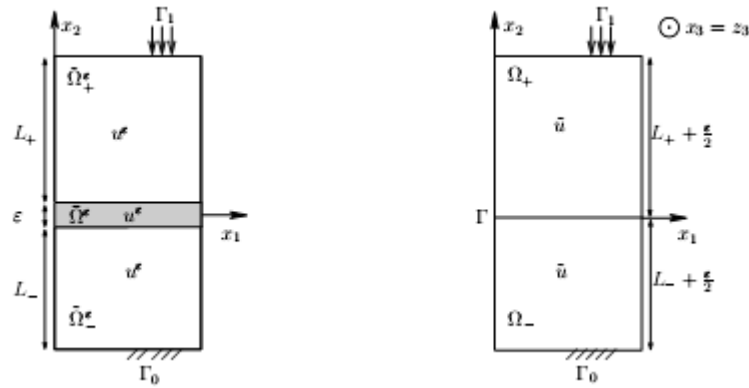
<sup>1</sup> Laboratoire Amiénois de Mathématique Fondamentale et Appliquée, CNRS UMR 7352, UFR des Sciences, 33, rue Saint-Leu, 80039 Amiens Cedex 1, France, [dumont@lma.cnrs-mrs.fr](mailto:dumont@lma.cnrs-mrs.fr)

<sup>2</sup> Laboratoire de Mécanique et d'Acoustique, CNRS UPR 7051, Université Aix-Marseille, 31 Chemin Joseph Aiguier, 13402 Marseille Cedex 20, France, [lebon@lma.cnrs-mrs.fr](mailto:lebon@lma.cnrs-mrs.fr)

<sup>3</sup> Dipartimento di Ingegneria, Università di Ferrara, Via Saragat 1, 44122 Ferrara, Italy, [rizzoni.raffaella@unife.it](mailto:rizzoni.raffaella@unife.it)

## Abstract

In [1–6] we study the mechanical behavior of thin films between elastic adherents. The analysis is based on the classic idea that a very thin adhesive film can be replaced by a contact law.

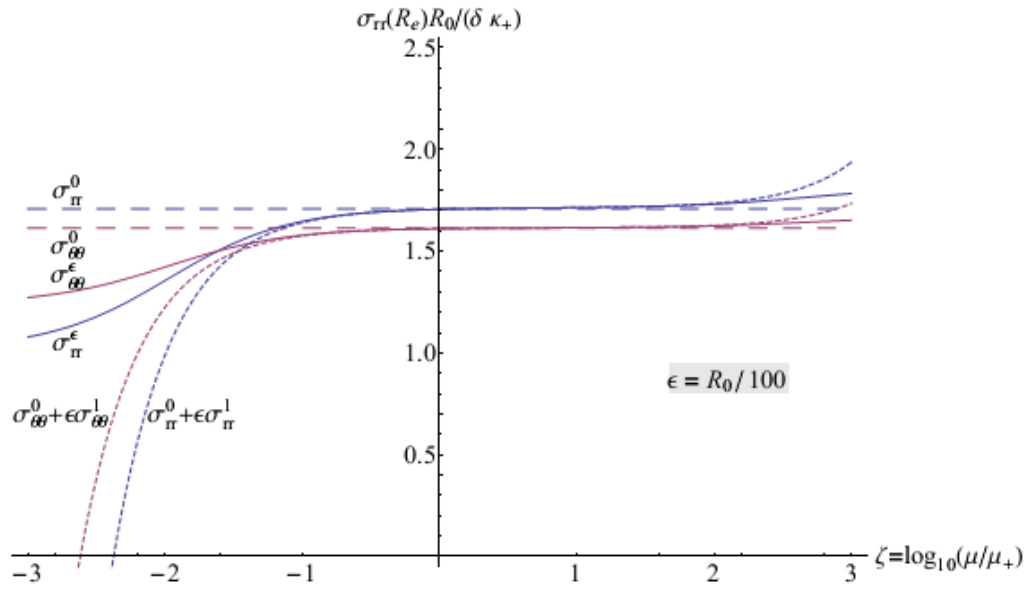


**Figure 1.** Initial and limit configuration of two solids glued together.

The contact law describes the asymptotic behavior of the film in the limit as its thickness goes to zero and it prescribes the jumps in the displacement (or rate of displacements) and traction vector fields at the limit interface. The formulation of the limit problem involves the mechanical and the geometrical properties of the adhesive and the adherents, and in [1–6] several cases were considered: soft films [1]; adhesive films governed by a non convex energy [2]; flat linear elastic films having stiffness comparable with that of the adherents and giving rise to imperfect adhesion between the films and the adherents [3,4]; joints with mismatch strain between the adhesive and the adherents [6]. Several mathematical techniques can be used to perform the asymptotic analysis:

- $\Gamma$ -convergence,
- Variational analysis,
- Matched asymptotic expansions,
- Energy approaches [5],
- Numerical studies [7].

In this lecture, we present some mathematical contributions in elasticity, with a focus on the energy approach, and numerical results in this field. In particular, it will be shown that the energy approach is very efficient to compute the singularities on the edge of the glue. Plane and curved interfaces will be analyzed, and it will be demonstrated the necessity of a “at least first order asymptotic theory” in the case of similar rigidity between the adherents and the glue.



**Figure 2.** Bonding of two spheres: comparative plots of the radial and circumferential stresses on the external surface calculated from the exact three-phase solution, the perfect (order 0) and the imperfect (order 1) interface approximations.

## References

- [1] F. Lebon, R. Rizzoni, S. Ronel-Idrissi, Analysis of non-linear soft thin interfaces, *Computers and Structures*, 82 (2004), 1929-1938.
- [2] F. Lebon, R. Rizzoni, Asymptotic study of a soft thin layer: the non convex case, *Mechanics of Advanced Materials and Structures*, 15 (2008), 12-20.
- [3] F. Lebon, S. Ronel-Idrissi, First-Order Numerical Analysis of Linear Thin Layer, *Journal of Applied Mechanics, Transaction of ASME*, 74 (2007), 824-828.
- [4] F. Lebon, R. Rizzoni, Asymptotic analysis of a thin interface: The case involving similar rigidity, *International Journal of Engineering Science*, 48 (2010), 473-486.
- [5] F. Lebon, R. Rizzoni, Asymptotic behavior of a hard thin linear interphase: An energy approach, *International Journal of Solids and Structures*, 48 (2011), 441-449.
- [6] R. Rizzoni, F. Lebon, Asymptotic analysis of an elastic thin interphase with mismatch strain, *European Journal of Mechanics*, 36, (2012), 1-8.
- [7] S. Dumont, F. Lebon, R. Rizzoni, An asymptotic approach to the adhesion of thin films, *Structural Engineering and Mechanics*, submitted
- [8] R. Rizzoni, F. Lebon, Imperfect interfaces as asymptotic models of thin curved elastic adhesive interphases, *Mechanics Research Communications*, in revision