An efficient algorithm for simulation of forced deformable bodies interacting with 2D incompressible flows; Application to fish swimming

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Outline

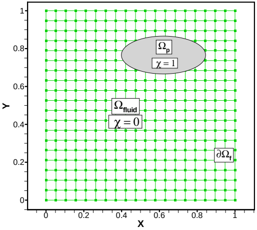
1. Fluid/solid interaction
2. Validation
3. Application
4. Conclusion

Motivation

Numerical simulation of Eel like-swimmers via the volume penalization method

- Biology/Ecology of the aquatic animals, migration ...
- Biologically inspired design
- Optimization
- Robo-fish control
- Improving the knowledge of fish-like swimmers control

Vorticity transport equation including penalization



$$\frac{\partial \omega}{\partial t} + (\mathbf{u} \cdot \nabla)\omega = \nu \nabla^2 \omega + \nabla \times \left(\frac{\chi(\mathbf{u}_P - \mathbf{u})}{\eta} \right) \quad , \quad \mathbf{x} \in \Omega \in \mathbb{R}^2 \tag{1}$$

$$\chi(\mathbf{x}, t) = \begin{cases} 1 & \mathbf{x} \in \Omega_b \\ 0 & \mathbf{x} \in \Omega_f \end{cases} \tag{2}$$

E. Arquis et J.P. Caltagirone. Comptes Rendus de l'Academie des Sciences, Paris, 2(299), 1-4, 1984.

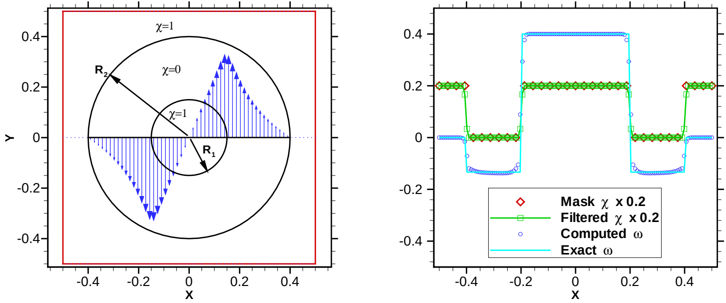
Numerical method

$$-\nabla^2\psi = \omega \tag{3}$$

- 4^{th}-order Compact FDM for spatial terms
- RK4 for parabolic vorticity Eq. (1)
- Direct sin FFT-FDM based method for Eq. (3)

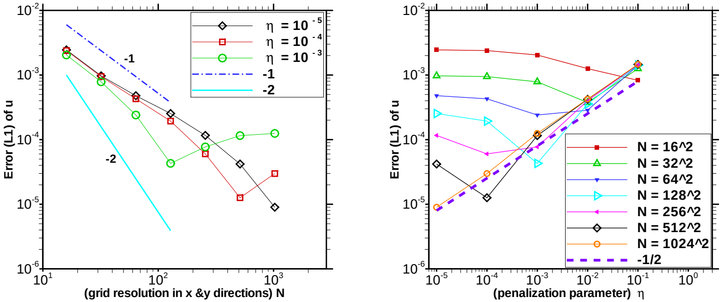
Numerical methods spatial error

Taylor-Couette flow ($Ta \approx 1$)

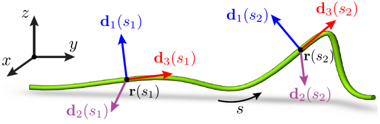


Numerical methods spatial error

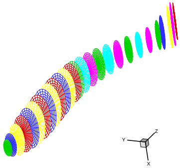
$$\|u^{\text{exact}} - u_\eta^{\text{num}}\| \leq \underbrace{\|u^{\text{exact}} - u_\eta\|}_{O(\sqrt{\eta})} + \underbrace{\|u_\eta - u_\eta^{\text{num}}\|}_{O(\Delta x^p)} \quad (4)$$



Fish's backbone as a 1D Cosserat medium



(a) picture from Lazarus et al. 2012



(b) A 3D example

$$\frac{\partial Q}{\partial s} = \frac{1}{2} M^{\vee}(K)\ Q \quad (5)$$

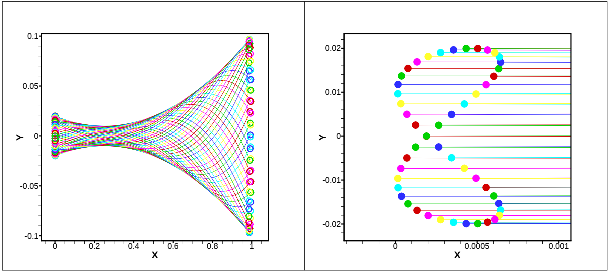
$$\frac{\partial \mathbf{X}}{\partial s} = Rot(Q)\ \Gamma \quad (6)$$

$$\frac{\partial}{\partial s}\begin{bmatrix} V \\ \Omega \end{bmatrix} = -\begin{bmatrix} K^{\vee} & \Gamma^{\vee} \\ 0 & K^{\vee} \end{bmatrix}\begin{bmatrix} V \\ \Omega \end{bmatrix} + \begin{bmatrix} \dot{\Gamma} \\ \dot{K} \end{bmatrix} \quad (7)$$

F. Boyer, M. Porez and W. Khalil. *IEEE Transactions on Robotics*, Vol. 22, No. 4, 763-775, August 2006.

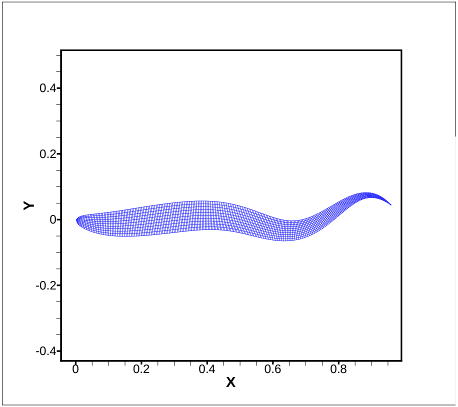
Anguilliform swimmers

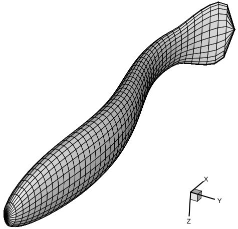
Motion of the head/tail over an butterfly-like curve



$$y(x(s),t) = f_{\text{decay}}(t)\ f_{\text{growth}}(t)\ a(x)\sin\left(2\pi(x/\lambda \pm ft)\right) \quad (8)$$

Lagrangian structured grid covering the body





Anguilliform swimmers undulate the majority of their body

Interpolation to Eulerian grid

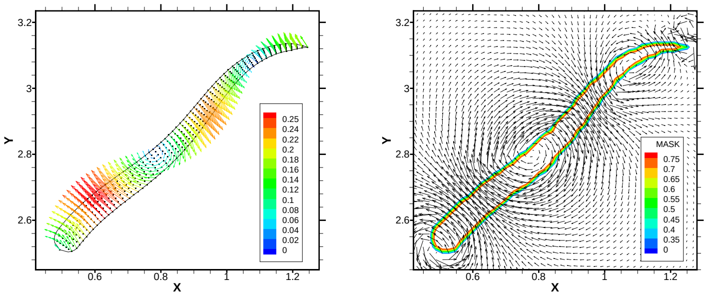
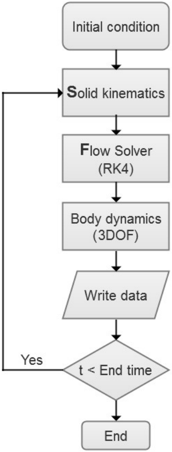


Figure: (left) Lagrangian structured grid. (right) Interpolated smoothed mask function $\bar{\chi}$ on the Eulerian grid.

Flowchart of the algorithm



Body dynamics 3DOF

Newton-Euler laws

$$\Sigma(\mathbf{F}_H + \mathbf{F}_G) = \frac{d}{dt}(m\dot{\mathbf{V}}) \tag{9}$$

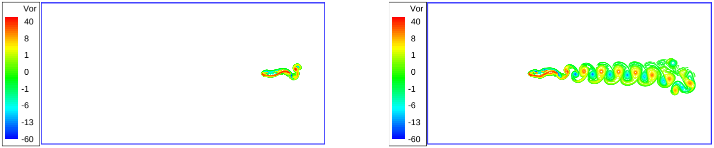
$$\Sigma M_{cg} = \frac{d}{dt}(I_{zz}\dot{\theta}) \tag{10}$$

Hydrodynamics coefficients

$$\mathbf{F}_H \approx \frac{\rho_f}{\eta}\int_{\Omega_b}\chi(\mathbf{u}-\mathbf{u}_P)\,\mathrm{d}s + \rho_f\; S\;\ddot{\mathbf{X}}_{cg} \tag{11}$$

$$M_{cg} \approx \frac{\rho_f}{\eta}\int_{\Omega_b}\chi(\mathbf{x}-\mathbf{x}_{cg})\times(\mathbf{u}-\mathbf{u}_P)\,\mathrm{d}s + \rho_f\frac{I_{zz}}{\rho_b}\ddot{\theta}_{cg} \tag{12}$$

Fish swimming in forward gait



(a) $t = 1$ (b) $t = 9$

Figure: Snapshots of vorticity isolines, $(x, y) \in [0\,, 10l_{\text{fish}}] \times [0\,, 5l_{\text{fish}}]$, resolution 2048×1024, $Re \approx 3800$.

Fish swimming in forward gait

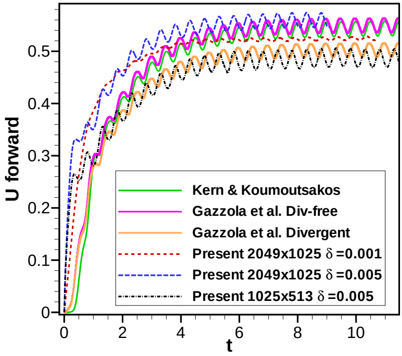
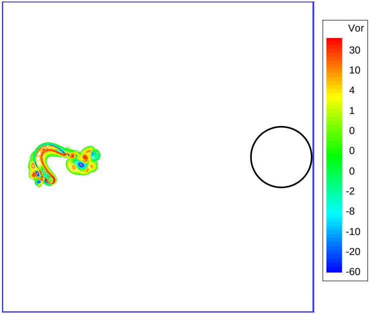


Figure: Forward velocity *U* comparisons of for 2D anguilliform swimmer.

Fish rotation - How the fish can turn quickly?

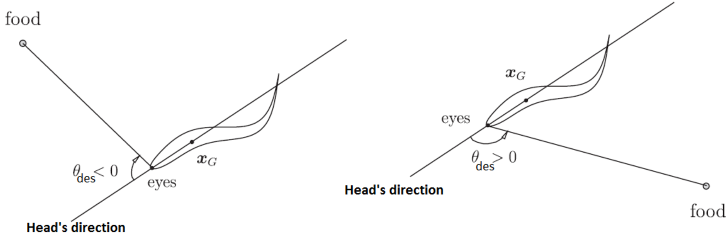


$$k_{total} = k_{propulsion} + k_{\text{des}}(\theta_{\text{des}}) \quad (13)$$

$$k_{propulsion} = a(x)\sin(2\pi(ft \pm x/\lambda)) \quad (14)$$

Fish rotation

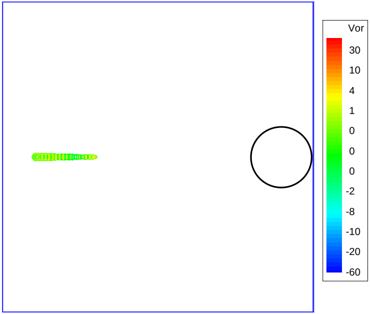
Curvature control law



$$k_{\text{des}}(\theta_{\text{des}}) = \begin{cases} -\text{sgn}(\theta_{\text{des}})\ k_{\text{max}} & |\theta_{\text{des}}| \geq \theta_{\text{limit}} \\ -\text{sgn}(\theta_{\text{des}})\ k_{\text{max}}\ \left(\frac{\theta_{\text{des}}}{\theta_{\text{limit}}}\right)^2 & \text{else} \end{cases} \quad (15)$$

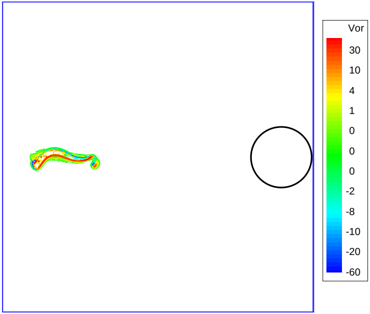
M. Bergmann and A. Iollo. *Journal of Computational Physics*, Vol. 230, 329-348, 2011.

Fish rotation



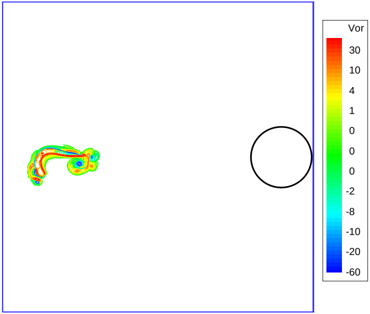
(a) $t = 0$

Fish rotation



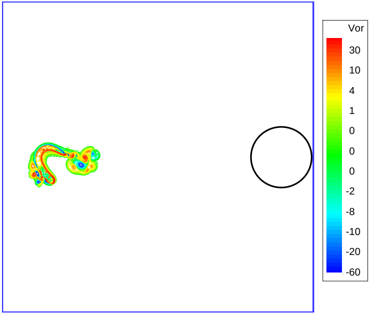
(b) $t = 1$

Fish rotation



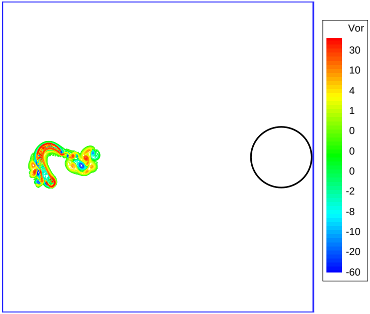
(c) $t = 2$

Fish rotation



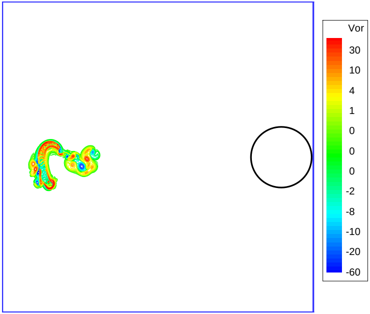
(d) $t = 2.3$

Fish rotation



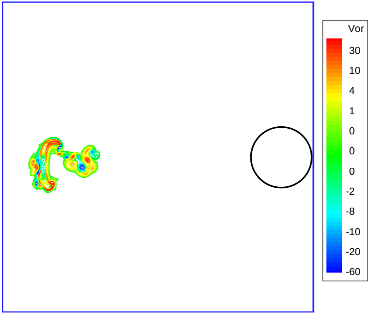
(e) $t = 2.4$

Fish rotation



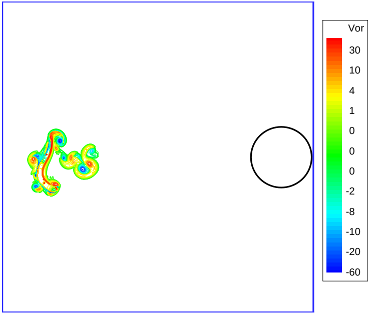
(f) $t = 2.5$

Fish rotation



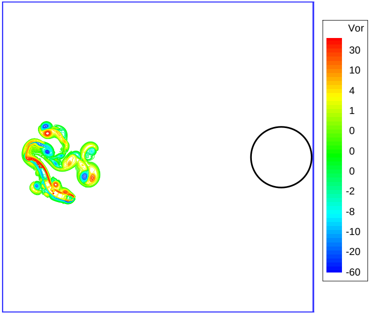
(g) $t = 2.6$

Fish rotation



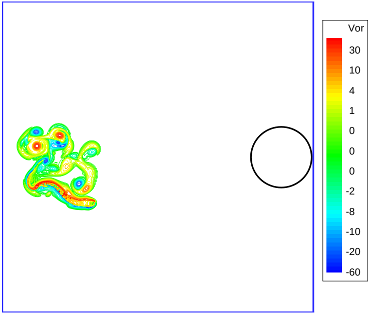
(h) $t = 3$

Fish rotation



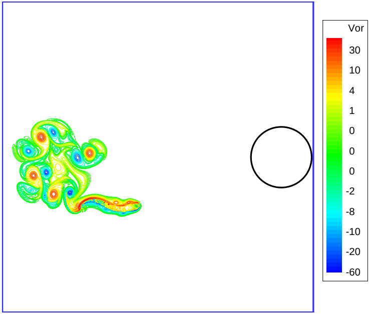
(i) $t = 4$

Fish rotation



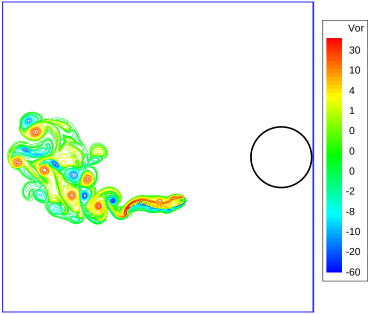
(j) $t = 5$

Fish rotation



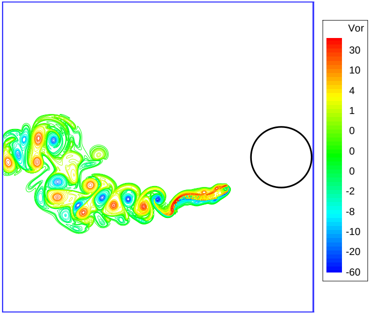
(k) $t = 7$

Fish rotation



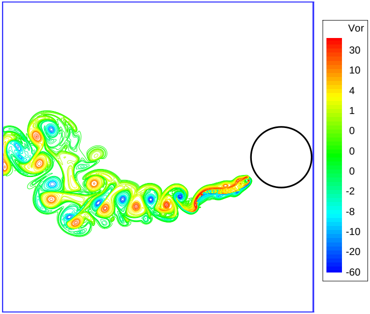
(l) $t = 9$

Fish rotation



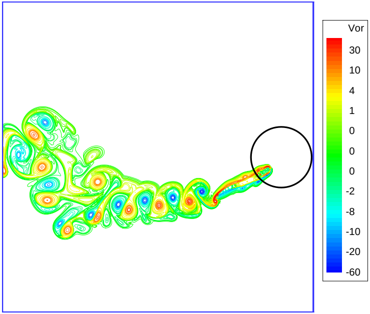
(m) $t = 11$

Fish rotation



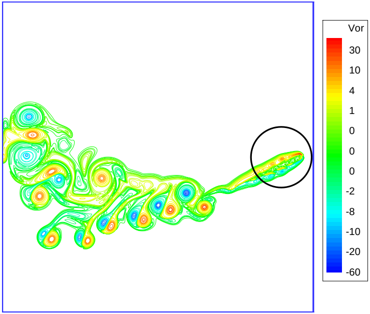
(n) $t = 12$

Fish rotation



(o) $t = 13$

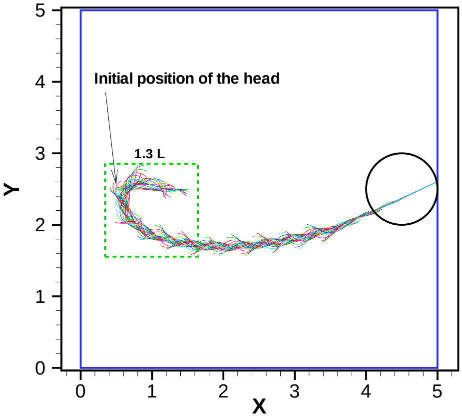
Fish rotation



(p) $t = 15$

Fish rotation

Trajectory of the backbone during the rotation toward the goal



Conclusion

- An efficient algorithm for simulation of deformable bodies interacting with 2D incompressible flows is proposed.
- The volume penalization method is applied to the N-S equations in vorticity stream-function formulation.
- An efficient law for curvature control of an anguilliform swimmer looking for food is proposed which is based on geometrically exact theory of nonlinear beams.
- Starting from rest the fish performs a 180^o rotation within an area of about 1.3 its length.
- The FORTRAN code is open source and is also accessible upon request.

Thanks for your attention