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Random linear multihop relaying in a field of interferers using spatial Aloha

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Abstract

In our basic model, we study a stationary Poisson pattern of nodes on a line embedded in an independent planar Poisson field of interfering nodes. Assuming slotted Aloha and the signal-to-interference-and-noise ratio capture condition, with the usual power-law path loss model and Rayleigh fading, we explicitly evaluate several local and end-to-end performance characteristics related to nearest-neighbour packet relaying on this line. We study how these metrics depend on the density of relaying nodes and interferers, tuning of Aloha and on the external noise level. We consider natural applications of these results in a *vehicular ad-hoc network*, where vehicles are randomly located on a straight road. We also propose to use this model to study a "typical" route traced in a (general) planar ad-hoc network by some routing mechanism. Such a decoupling of a given route from the rest of the network in particular allows us to quantitatively evaluate the non-efficiency of long-distance routing in "pure ad-hoc" networks, previously observed in [1], and the need for a well-tuned structure of "fixed" relaying nodes. We consider several extensions of our basic Poison-line-in-Poisson field-model, notably as *Poisson-line ad-hoc network* in which all nodes (including interfering ones) are randomly located on a Poisson process of lines (routes). In this case our analysis rigorously (in the sense of Palm theory) corresponds to the typical route of this network.

Index Terms

MANET, VANET, Aloha, SINR, MAC, routing, end-to-end delay, packet speed, Poisson, Poisson-line process, doubly-stochastic Poisson process, lattice.

I. Introduction

The idea of mobile ad-hoc networks (MANETs) — spontaneous wireless networks, operating without a backbone infrastructure, whose users/nodes relay packets for each other in order to enable multihop communications — continues to inspire practitioners and poses interesting theoretical questions regarding its performance capabilities. Vehicular ad-hoc networks (VANETs) may well be currently one of the most promising incarnations of MANETs. Promoters of VANETs believe that these networks will both increase safety on the road and provide value-added services. Numerous challenging problems must however be solved in oder to be able to propose these services e.g. efficient and robust physical layers, reliable and flexible medium access protocols, routing schemes and optimized applications.

An almost ubiquitous stochastic assumption in the theoretical studies of these problems in MANETs is that the nodes of the network are distributed (at any given time) as points of a planar (2D) Poisson point process. In conjunction with the Aloha medium access (MAC) scheme — simpler but less efficient than CSMA usually considered in this context by practitioners — 2D Poisson MANET models allow for quite explicit evaluation of several performance metrics; cf. Section I-C. However, they mostly regard local (one-hop) transmissions. In fact, introducing routing even to the simplest 2D Poisson-Aloha model is very difficult and it is hard to obtain rigorous theoretical results regarding truly multihop performances of MANETs; cf. again Section I-C. One reason for this is that, while the source-node can be considered as a typical node in the MANET and the powerful Palm theory of point process can be used in the analysis of the first hop, further relay nodes on a given path (traced by the Dijkstra algorithm or any reasonable local routing on a 2D Poisson MANET) cannot be seen as typical nodes of the MANET. In fact, the route followed by a packet is a random subset of the MANET's point pattern (depending on the routing algorithm) and the typical point "seen" by the packet on a long route is not the typical point of the whole MANET in the sense of Palm theory[1], cf *routing paradox* in [3, § 21.7.2.2].

[1]Unless a high node mobility can be assumed, which results in completely independent re-sampling of the node placement after each hop, which might be reasonable in the context of delay-tolerant networks [2], but which we do not want to assume in this paper

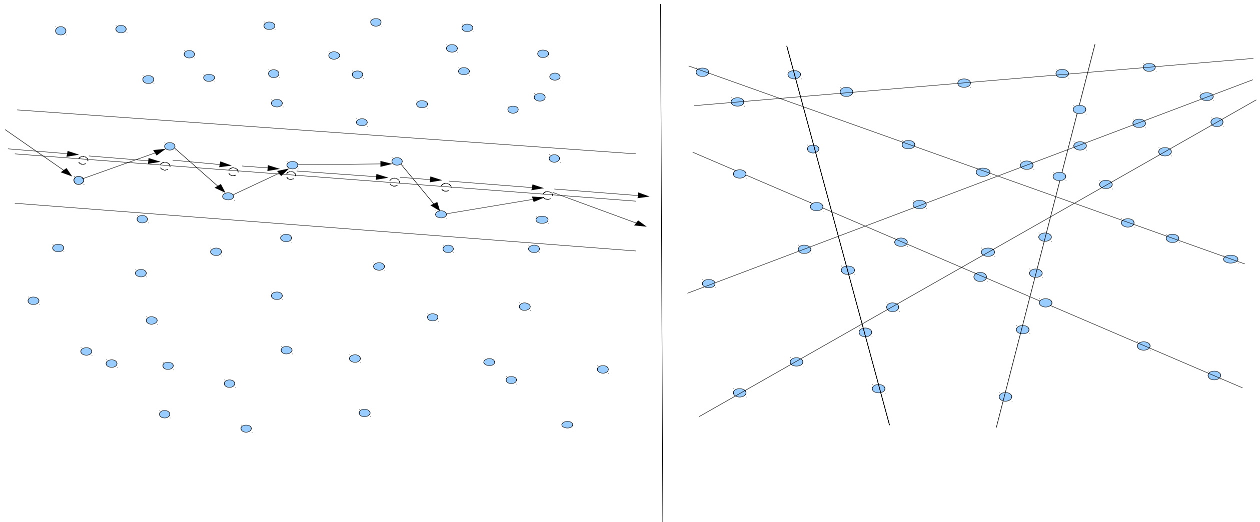

Fig. 1. Left : nodes relaying in given direction and in a given strip approximated by a line of nodes. Right: nodes on roads forming Poisson-line process

A. *Route-in-MANET model*

In order to overcome this impasse and to propose an analytically tractable multihop model, in this paper we propose to "decouple" a tagged route from the remaining (external to the route) part of the MANET. More precisely, we consider two stochastically independent point patterns to model respectively: some given route on which packets of a tagged flow are relayed and nodes which are only sources of interference for packet transmissions on this route. [2] Moreover, we assume that the tagged route is modeled by a linear (1D) stationary point process. These assumptions allow us to distinguish the typical node of the route from the typical point of the MANET, and appropriately use Palm theory to manipulate the two objects. The advantage of the 1D-modeling of the tagged route is that packets relayed on it through the nearest neighbor transmissions "see", at any relaying node, the typical (1D Palm) distribution of the whole route. This is a well-known *point-shift invariance property of the Palm distribution in 1D*, which does not have a natural extension in higher dimensions. Another advantage of our decoupling of the two parts of the MANET is that we can go beyond Poisson assumptions in both of them. Specifically, we will also consider a *Poisson route appended with a lattice structure of relaying nodes*, which turns out to be crucial to improve the routing performance on long routes in the presence of external interference and/or noise. Regarding external interference, we are able to study the impact of the clustering of nodes, considering e.g. *Poisson-line MANET* (to be explained in what follows).

Before describing our main results, let us further justify our route-in-MANET model. Firstly, it is a quite natural scenario in VANETs, where vehicles are randomly located on a road and subject to external sources of interference. In the context of a general 2D MANET, the linearity (1D-pattern) of routing is clearly a simplification. In this case we think of packets as being relayed in a given direction, e.g. in a strip as shown on the left sketch in Figure 1. We can approximate the "real" route by a "virtual" one by taking the orthogonal projections of the real nodes on the line joining the source and the destination. Such a situation represents an approximation of a geographic routing. Last but not least, in this paper we consider a 2D *Poisson-line MANET* model, in which all the nodes are randomly located on a (background) Poisson process of lines, and form the so-called *doubly-stochastic Poisson process* (or Cox process) see on the right in Figure 1. In this case our decoupled, tagged route is rigorously (in the sense of Palm theory) the typical route in this Poisson-line MANET.

B. *Summary of the results*

We consider slotted Aloha MAC to be used both by the nodes of the tagged route and of the interfering part of the MANET and the Signal-to-Interference-and-Noise Ratio (SINR) capture/outage condition, with the power-law path loss model and Rayleigh fading. We assume that all the nodes of the MANET are backlogged, i.e., they always have packets in their buffers to transmit. Moreover, we are interested in the routing performance of a tagged

[2]In a sense, this is an extension of the well-accepted *bipolar MANET model* in which a tagged pair emiter-receiver is considered in the field of interferers; cf. [3, Sec. 16.2.1].

packet relayed by successive nodes on the route with priority in all queues on the route (cf. a discussion of these assumptions in Section I-C). The main results of this paper are the following:

- In the absence of external (to the route) noise and interference we evaluate the *mean local delay* on the Poisson route, i.e. the expected number of time slots required for the packet to be transmitted by the typical node of the route to its nearest neighbor in a given direction. The inverse of this delay is intrinsically related to the *speed of the packet progression on asymptotically infinite routes*, which can also be related to the *route transport capacity* (number of bit-meters pumped per unit length of the route).
- The local mean delay is minimized (equivalently, the progression speed is maximized) at a unique value of the Aloha medium access probability p. Moreover, the routing is unstable — the delay is infinite (the speed is null) — for p larger than some critical value. This observation is fully compliant with the phase-transition phenomenon of the mean local delay discovered in the 2D Poisson MANET model in [4].
- We evaluate the *mean end-to-end delay* of the packet routing between two given nodes of the route. It shows a cut-off phenomenon, namely that *routing on distances shorter than some critical value is very inefficient in terms of exploiting the route transport capacity*.
- Next, we study the impact of noise and external interference on our previous observations. Confirming the theoretical findings of [1] in 2D Poisson MANETs, we observe that the routing on a Poisson route over long distances is unfeasible: *the speed of packet progression on such routes is null* due to the existence of large hops in random (Poisson) paths. Evaluating the end-to-end delay on finite distances with noise, we identify another, double cut-off phenomenon: the existence of *critical values of end-to-end distance and noise power, beyond which the speed of packet progression is close to 0*, again making the routing inefficient in terms of exploiting its transport capacity.
- In order to allow efficient routing over long routes one can complete the Poisson route with a fixed lattice of (equidistant) relaying nodes. For this model, we evaluate the mean local delay and show how the route transport-capacity can be maximized by an optimal choice of the inter-relay distance of the lattice structure.
- We explicitly evaluate the Poisson-line MANET model and compare it to that in the (basic) Poisson-line-in-Poisson-field model. We show that the Poisson-line exhibits a larger coverage probability but also a larger end-to-end delay. This confirms previous general observations regarding the impact of clustering in interference; [5].
- Regarding applications to VANETs, we evaluated delays of some emergency and broadcast communications.

C. Related work and model assumptions

Local (one-hop) characteristics in 2D Poisson MANETS, such as SINR outage probability, the related density of packet progress, mean local delay and many others, have been extensively studied in the literature; cf. e.g. among others [4, 6–10]. Poisson models also allow one to discover some intrinsic theoretical limitations of MANETs, regarding e.g. the scaling of the capacity of dense and/or extended networks [11, 12], or the speed of packet progression on long routes [1, 13].

In [14, 15] the authors study delay and throughput in a model with multihop transmissions with relays which are placed equidistantly on the source-destination line. The model is very similar to ours but with the following differences. In the model presented in [14, 15] the topology of relaying nodes is regular (they are equidistant) and the pattern on interferers is re-sampled at each slot. The delays include both the service times and waiting times in the buffers on the given route. A combined TDMA/ALOHA MAC protocol with intra-route TDMA and inter-route ALOHA is employed. In contrast, we use simple Aloha, ignore queueing, and focus on the performance issues caused by the *randomness of the topology of relaying nodes*. We are interested in intrinsic limitations of the performance of *long-distance routing* in wireless networks with an irregular topology. We believe that the observed limitations remain valid for networks employing CSMA. This is because they primarily depend on the existence of (arbitrarily) long hops. CSMA, which copes better with interference, cannot improve upon this situation. In contrast, we show that using a regular structure of relay nodes superposed with irregular MANET routes leads to better performance. This solution is also evidently necessary for the stability of queuing processes (not covered in our paper).

The remaining part of this paper is organized as follows. In Section II, we present our nearest neighbor routing model with slotted Aloha. In Section III we compute routing delays in deterministic networks. In Section IV we study the end-to-end delays on a Poisson route when the interference is limited to the interfering nodes on the Poisson route. In Section V we study the impact of external noise and interference. Section VI concludes the paper.

II. LINEAR NEAREST NEIGHBOR ROUTING MODEL IN A SPATIAL MANET WITH SLOTTED ALOHA

A. A tagged route

Let us denote by $\mathcal{R} = \{X_i, i = \dots, -1, 0, 1, \dots\}$, with $X_i \in \mathbb{R}^2$, the locations of nodes participating in the routing of some *tagged flow of packets* from X_i to X_{i+1}. This route is assumed not to change on the time scale considered in this paper.

The following assumptions regarding $\mathcal{R}$ will be considered.

1) Deterministic route: Although it is not the main scenario of this paper, we begin by considering a deterministic, fixed, finite pattern of nodes $\mathcal{R} = \{X_0, \dots, X_M\}$.

2) Poisson-line route: In this scenario we suppose that $\mathcal{R} = \Phi$ forms a Poisson point process of intensity λ, on the line $\mathbb{R}$. In this model the notational convention is such that $X_i < X_{i+1}$ and packets are sent by any given node X_i to its nearest-to-the-right neighbor X_{i+1}. Note that the Poisson assumption means that the 1-hop distances $X_{i+1} - X_i$ are independent (across i) exponential random variables with some given mean $1/\lambda$. We will call this scenario *nearest neighbor (NN) routing on Poisson line*. We will also consider a version of this model, where the packet is transmitted to the *nearest (available) receiver (NR)* to the right on $\mathcal{R}$. This is an opportunistic routing allowing for longer hops, as will be explained in Section II-F.

3) Poisson-line route with fixed relay nodes: In this model, the tagged route consists of a superposition $\mathcal{R} = \Phi \cup \mathcal{G}$, where the Poisson route Φ is completed with equidistantly located "fixed" relay nodes $\mathcal{G} = \{n\Delta + U_\Delta, n \in \mathbb{Z}\}$; $\Delta > 0$ is some fixed parameter and U_Δ is a uniform random variable on $[0, \Delta)$, independent of Φ, making $\mathcal{R}$ stationary. In this model we also consider NN routing, i.e., packets are always transmitted to the nearest neighbor to the right in $\mathcal{R}$.

B. A Route in a MANET

We consider $\mathcal{R}$ as some "tagged" route obtained in a MANET by some routing mechanism. A simple way of extending $\mathcal{R}$ to a 2D MANET model consists in embedding $\mathcal{R}$ in an external field of nodes $\Psi = \{Y_i\}$, on the plane $\mathbb{R}^2$. When doing so we will always consider that $\mathcal{R}$ and Ψ are independent.

The following assumptions regarding Ψ will be considered.

1) Fixed pattern of interferers: One may consider a *fixed, deterministic* pattern $\Psi = \{Y_i\}$, although, once again, it is not the main scenario considered in this paper.

2) Poisson field of interferers: In this model, we assume that Ψ is a Poisson point process of some given intensity μ on $\mathbb{R}^2$. The Poisson linear route $\mathcal{R} = \Phi$ embedded in a Poisson process of interferers will be our default *Poisson-line-in-Poisson field* model.

3) Poisson-line interferers: We will also consider a case where the interferers are located on a *Poisson process of lines* (roads) on $\mathbb{R}^2$ of rate ν representing the total line-length per unit of surface. Assuming that on each line of this process there is an independent Poisson process of points of intensity λ' nodes per unit of line length, we obtain a doubly stochastic Poisson point process on $\mathbb{R}^2$ with intensity $\mu = \lambda'\nu$ nodes per unit of surface; see [16]. In particular, assuming $\lambda = \lambda'$, one can rigorously consider the Poisson liner route $\mathcal{R} = \Phi$ embedded in such a Poisson line process of interferers, as the typical route of the *Poisson-line MANET*, see Figure 1 on the right-hand side.

C. Aloha MAC

We assume that the *all* nodes of $\mathcal{R}$ and Ψ try to access the channel (transmit packets) according to the Aloha scheme, in perfectly synchronized time slots $n = 1, 2, \dots$. Each node in each time slot independently tosses a coin with some bias for heads, which will be referred to as the *Aloha medium access probability* (Aloha MAP). Nodes whose outcome is heads transmit their packets, the others do not transmit. We denote by p the MAP of nodes in $\mathcal{R}$, and by p' the MAP of nodes in Ψ. The above situation will be modeled by marking the points $X_i \in \mathcal{R}$ with random, Bernoulli, medium access indicators $e^n_{X_i}$ equal to 1 for the nodes which are allowed to emit in the slot n and 0 for the nodes which are not allowed to emit. We have $\mathbf{P}(e^n_i = 1) = p$ for all i, n. When there is no ambiguity, we will skip the time index n in the notation and also use the notation $e_{X_i} = e_i$. Similarly, we mark the interfering nodes by independent Bernoulli, medium access indicators with parameter p'.

At each time slot n, Aloha splits $\mathcal{R}$ into two point processes $\mathcal{R}^1 = \mathcal{R}^{1,n}$ of emitters (having a MAC indicator e equal to 1 at time n) and (potential) receivers $\mathcal{R}^0 = \mathcal{R}^{0,n}$. It is known that when $\mathcal{R} = \Phi$ is a Poisson process of intensity λ then $\mathcal{R}^1$ and $\mathcal{R}^0$ are independent Poisson processes with intensity λp and $\lambda(1-p)$, respectively.

D. Signal propagation

Each transmitting node uses the same transmission power, which without loss of generality is assumed to be equal to 1. The signal-power path-loss is modeled by the power-law function $l(r) = (Ar)^\beta$ where $A, \beta > 0$ are some constants and r is the distance between the transmitter and the receiver.

Signal-power is also perturbed by random fading F which is independently sampled for each transmitter-receiver pair at each time slot n. Thus, the actual signal-power received at y from x at time n is equal to $F^n_{(x,y)}/l(|x-y|)$. In this paper we will restrict ourselves to an important special case where F is *exponentially distributed*, which corresponds to the situation of independent *Rayleigh fading*. By renormalization of A, if required, we can assume without loss of generality that F has mean 1.

E. SINR capture

When a node located at x transmits a signal to a node located at y, then successful reception depends on the signal-to-interference-and-noise ratio (SINR)

$$\text{SINR}_{(x,y)} = \text{SINR}^n_{(x,y)} = \frac{F^n_{(x,y)}/l(|x-y|)}{W^n(y) + I_{\mathcal{R}^{1,n}}(y)}, \tag{2.1}$$

where $I_{\mathcal{R}^{1,n}}$ is the shot-noise process of $\mathcal{R}^{1,n}$: $I_{\mathcal{R}^{1,n}}(y) = \sum_{X_i \in \mathcal{R}^{1,n}} F^n_{(y,X_i)}/l(|y-X_i|)$ representing the interference created by the nodes of route $\mathcal{R}$ and $W^n(y)$ represents an external (to $\mathcal{R}$) noise. This external noise can be a constant ambient noise $W^n(y) = W$ or a random field. In particular, $W^n(y)$ may comprise the interference created by the external field of interferers Ψ, which do not belong to the route $\mathcal{R}$. In this case:

$$W^n(y) = W + I_{\Psi^{1,n}}(y) = W + \sum_{Y_i \in \Psi^{1,n}} F^n_{(y,Y_i)}/l(|y-Y_i|)\,, \tag{2.2}$$

where $\Psi^{1,n}$ is the subset of nodes of Ψ transmitting at time n.

Throughout this paper we adopt the common assumption throughout this paper is that the *noise process $W^n(y)$ is independent of the route-interference process $I_{\mathcal{R}^{1,n}}(y)$* and that *the process $W^n(y)$ is stationary in n*.

In this paper we assume a fixed bit-rate coding, i.e., y successfully receives the signal from x if

$$\text{SINR}_{(x,y)} \geq T\,, \tag{2.3}$$

where $\text{SINR}_{(x,y)}$ is given by (2.1) and T is the SINR-threshold related to the bit-rate given some particular coding scheme.

F. NN versus NR routing

Recall that in our NN routing model each transmitter in $\mathcal{R}^1$ transmits to the nearest node on its right in $\mathcal{R}$, without knowing whether it is authorized to transmit at this time. Successful reception requires that this selected node is *not* authorized by Aloha to transmit (i.e., it is in $\mathcal{R}^0$), and that the SINR condition (2.3) for this transmitter-receiver pair is satisfied. This corresponds to the usual separation of the routing and MAC layers. In contrast, NR routing consists in transmitting to the nearest node (in the given direction) on $\mathcal{R}$ *which, at the given time slot, is not authorized by Aloha to transmit*. The NR model hence might be seen as an opportunistic routing, which requires some interplay between MAC and the routing layer. As we shall see, both routing schemes allow for quite explicit performance analysis in our basic Poisson-line-in-Poisson-field model.

G. Numerical assumptions

The default assumption in our numerical examples throughout the paper is $\lambda = 0.01$ nodes per meter in the Poisson line model, i.e. the mean distance between two consecutive nodes on the line is 100 m. We will also use $A = 1$, $\beta = 4$ and $T = 10$. The fading F is Rayleigh, F is exponentially distributed of mean 1, $E(F) = 1$.

III. Preliminaries: calculus of routing delays in deterministic networks

Unless otherwise specified, in this section we assume that the locations of the nodes in the network are known and fixed (deterministic). The only source of randomness is Aloha MAC and independent Rayleigh fading between any two given nodes. In what follows we present a simple computation which allows us to express coverage and routing delays in such networks. Our observations will be useful in the remaining part of this paper, when we will study random routes in random MANETs.

1) Coverage probability: Let us consider a transmitter located at $x \in \mathbb{R}^2$ communicating to a receiver located at $y \in \mathbb{R}^2$ in the presence of a field $\Psi = \{y_i \in \mathbb{R}^2\}$ of interferers, with all nodes subject to Aloha MAC with map p. We assume that $x, y, \notin \Psi$. We denote by $\Pi(x, y, \Psi)$ the *probability of successful transmission from x to y in one time slot*. This probability accounts for the likelihood of x being authorized by Aloha to transmit, y not to being authorized to transmit and, given such circumstances, the probability of achieving the SINR larger than T at the receiver.

We define two functions:

$$h(s, r) = 1 - \frac{p}{\frac{1}{T}(s/r)^\beta + 1} \quad s, r \geq 0, \tag{3.4}$$

$$w(s) = \exp(-TW(As)^\beta) \quad s \geq 0. \tag{3.5}$$

For reasons which will become clear in what follows, we call $h(s, r)$ the *interference factor* and w the *noise factor*.

Lemma 3.1: *We have*

$$\Pi(x, y, \Psi) = p(1-p)w(|x-y|) \prod_{z \in \Psi} h(|z-y|, |x-y|)\,.$$

Remark 3.2: Let us note that Π satisfies the following recursion when adding an interferer z to the field Ψ:

$$\Pi(x, y, \emptyset) = p(1-p)w(|x-y|)\,, \tag{3.6}$$

$$\Pi(x, y, \Psi \cup \{z\}) = \Pi(x, y, \Psi)h(|z-y|, |x-y|)\,. \tag{3.7}$$

In other words, the external noise reduces the probability of successful transmission over the distance r by the factor $w(r)$, which in the absence of noise and interference is equal to $p(1-p)$ (because of the Aloha scheme). Moreover, adding an interfering node z to Ψ causes a decrease in the successful transmission probability by the factor $h(s, r)$, where s is the distance of the new interferer to the receiver.

Proof of Lemma 3.1: We have

$$\begin{aligned}\Pi(x, y, \{z\}) &= p(1-p)\mathbf{P}\{\, F_{(x,y)}/l(|x-y|) \geq T(W + e_z F_{(z,y)}/l(|y-z|)\,\} \\ &= p(1-p)e^{-TWl(|x-y|)}\mathbf{E}\big[e^{-Te_z F_{(z,y)} l(|x-y|)/l(|z-y|)}\big]\end{aligned}$$

and

$$\mathbf{E}\big[e^{-Te_x F_{(z,y)} l(|x-y|)/l(|z-y|)}\big] = (1-p) + p\mathbf{E}\big[e^{-TF_{(z,y)} l(|x-y|)/l(|z-y|)}\big] = 1 - \frac{p}{\frac{1}{T}\frac{|z-y|^\beta}{|x-y|^\beta} + 1}\,,$$

where we use the assumption that F is an exponential variable. The proof follows by induction. ■

A. Local routing delay

$\Pi(x, y, \Psi)$ is the probability that node x can successfully send a tagged packet to node y in a single transmission. However, a single transmission is not sufficient in Aloha. Hence, after an unsuccessful transmission, the transmitter will try to retransmit the packet with Aloha, possibly several times, until the packet's reception. We denote by $L(x, y, \Psi)$ the expected number of time slots required to successfully transmit a packet from x to y (considering the previous scenario of $x \in \mathbb{R}^2$ transmitting to $y \in \mathbb{R}^2$ in the presence of a field $\Psi = \{y_i \in \mathbb{R}^2\}$ of interferers). Under our assumptions this number is a geometric random variable of parameter $\Pi(x, y, \Psi)$ and hence its expected value is equal to:

$$L(x, y, \Psi) := \frac{1}{\Pi(x, y, \Psi)}\,.$$

We call $L(x, y, \Psi)$ the *local (routing) delay*. By Lemma 3.1

$$L(x, y, \Psi) = \frac{1}{p(1-p)} w^{-1}(|x-y|) \prod_{z \in \Psi} h^{-1}(|z-y|, |x-y|)\,. \tag{3.8}$$

and an analogous recurrence to this of Remark holds for L with the reciprocals of the noise and interference factors.

B. Route delays

Let us now consider a route consisting of nodes $\mathcal{R} = \{x_0, x_1, \ldots, x_n\}$ in the field of interfering nodes Ψ (we assume that $\mathcal{R} \cap \Psi = \emptyset$). The average delay to send a packet from x_0 to x_n using the route $\mathcal{R}$ is simply the sum of the local delays on successive hops

$$L(\mathcal{R}, \Psi) := \sum_{k=0}^{n-1} L(x_k, x_{k+1}, \Psi \cup \mathcal{R} \setminus \{x_k, x_{k+1}\})\,.$$

Note that for the hop from x_k to x_{k+1} other nodes of the route $\mathcal{R} \setminus \{x_k, x_{k+1}\}$ act as interferers. We call $L(\mathcal{R}, \Psi)$ the *(routing) delay* on $\mathcal{R}$.[3] Using (3.8) we obtain

$$L(\mathcal{R}, \Psi) = \frac{1}{p(1-p)} \sum_{k=0}^{n-1} \prod_{z \in \Psi \cup \mathcal{R} \setminus \{x_k, x_{k+1}\}} h^{-1}(|z - x_{k+1}|, |x_k - x_{k+1}|) w^{-1}(|x_k - x_{k+1}|)\,.$$

We also denote by

$$V(\mathcal{R}, \Psi) := \frac{|x_n - x_0|}{L(\mathcal{R}, \Psi)}$$

the mean speed of packet progression on the route $\mathcal{R}$.

C. Introducing stochastic geometry

The above expressions allow for a relatively simple, explicit analysis of routing delays in fixed networks, i.e. in networks where the locations of the nodes are fixed and known. In the case when such information is not available, one adopts a stochastic-geometric approach averaging over possible geometric scenarios regarding $\mathcal{R}$ and/or Ψ. For example, assume that the route $\mathcal{R}$ is given and fixed, but the number and precise locations of interferers are not given. Assuming some statistical hypothesis regarding the distribution of Ψ (which becomes random point process) one can calculate the average route delay $\mathbf{E}_\Psi[L(\mathcal{R}, \Psi)]$ where the expectation regards the distribution of Ψ. Specifically, for a given couple $(x, y) \in \mathbb{R}^2$ we define a function

$$H_{x,y}(z) := \log(h(|z-y|, |x-y|), \qquad z \in \mathbb{R}^2\,. \tag{3.9}$$

We denote by $\mathcal{L}_\Psi$ the Laplace transform of Ψ, i.e., for a given (say non-negative) function $f(\cdot)$ defined on $\mathbb{R}^2$, $\mathcal{L}_\Psi(f) := \mathbf{E}_\Psi\Big[\exp\big(-\sum_{Y_i \in \Psi} f(X_i)\big)\Big]$. We then have:

$$\mathbf{E}_\Psi[\Pi(x, y, \Psi)] = p(1-p) w(|x-y|) \mathcal{L}_\Psi(H_{x,y})\,, \tag{3.10}$$

$$\mathbf{E}_\Psi[L(x, y, \Psi)] = \frac{1}{p(1-p)} w^{-1}(|x-y|) \mathcal{L}_\Psi(-H_{x,y}) \tag{3.11}$$

and

$$\mathbf{E}_\Psi[L(\mathcal{R}, \Psi)] = \frac{1}{p(1-p)} \sum_{k=0}^{n-1} \mathcal{L}_\Psi(-H_{x_k, x_{k+1}}) \prod_{z \in \mathcal{R} \setminus \{x_k, x_{k+1}\}} h^{-1}(|z - x_{k+1}|, |x_k - x_{k+1}|) w^{-1}(|x_k - x_{k+1}|)\,. \tag{3.12}$$

In the next section we will consider a scenario when the route $\mathcal{R}$ is also modeled as a point process.

[3] Observe that the routing delay does not take into account packet queuing at the relay nodes.

IV. End-to-end delays on a Poisson route in the route-interference limited case

In this section we assume Poisson route $\mathcal{R} = \Phi$ and no external interferers ($\Psi \equiv \emptyset$). We also assume that the external noise process $W^n(y) \equiv 0$ is negligible with respect to the interference created by the nodes participating in the routing. In this scenario we will consider NN and NR routing schemes; cf II-A2. We begin with a simple calculation of the capture probability.

A. Capture probability

We consider a typical node on the route $\mathcal{R}$, that is of the Poisson point process Φ. By the Slivnyak theorem, it can be seen as an "extra" node located at the origin $X_0 = 0$, with the other nodes of the route distributed according to the original stationary Poisson process Φ. All marks (Bernoulli MAC indicators, fading, etc) of these extra nodes are independent of the marks of the points of Φ. We denote by $\Pi_{NN}(p)$ and $\Pi_{NR}(p)$ the probability of successful transmission of the typical node $X_0 = 0$ in a given time slot to the receiver prescribed by the NN and NR routing schemes, respectively. In the capture probability $P_{NN}(p)$ and $P_{NR}(p)$ in the two routing schemes considered, we assume that the typical node is authorized to transmit. Obviously $\Pi_{NN}(p) = pP_{NN}(p)$ and $\Pi_{NR}(p) = pP_{NR}(p)$.

For arbitrary $a, b > 0$ we denote $C(a,b) = \int_a^\infty 1/(u^b + 1)\,\mathrm{d}u$ and

$$C(b) = C(0,b) = \frac{\pi}{b\sin(\pi/b)}\,. \tag{4.13}$$

Moreover, for given T and β let us denote:

$$\mathcal{C}_1 = \mathcal{C}_1(T,\beta) = T^{1/\beta}(C(T^{-1/\beta},\beta) + C(\beta)) \tag{4.14}$$

$$\mathcal{C}_2 = \mathcal{C}_2(T,\beta) = 2T^{1/\beta}C(\beta) = \frac{2T^{1/\beta}\pi}{\beta\sin(\pi/\beta)}\,. \tag{4.15}$$

Proposition 4.1: *The probability of successful transmission by a typical node of Poisson route $\mathcal{R} = \Phi$ authorised by Aloha to transmit to its relay node in the NN routing model without noise is equal to:*

$$P_{NN}(p) = \frac{1-p}{1+p\,\mathcal{C}_1}\,. \tag{4.16}$$

and similarly in the NR receiver model

$$P_{NR}(p) = \frac{1-p}{1+p(\mathcal{C}_2 - 1)}\,. \tag{4.17}$$

Remark 4.2: It is easy to see that $\mathcal{C}_2 - 1 \le \mathcal{C}_1$ and hence $P_{NR} \ge P_{NN}$, i.e., the opportunistic choice of the receiver pays off regarding the probability of successful transmission.

Proof of Prop. 4.1: Note directly from the form of the SINR in (2.1) that $P_{NN}(p)$ and $P_{NR}(p)$ do not depend on A. Hence in the remaining part of the proof we take $A = 1$.

We consider first the NN model. The scenario with the typical user $X_0 = 0$ located at the origin corresponds to the Palm distribution $\mathbf{P}^0$ of the Poisson route $\mathcal{R} = \Phi \cup \{0\}$. By the known property of the Poisson point process, the distance R from X_0 to its nearest neighbor to the right, X_1, has an exponential distribution with parameter λ. Moreover given $R = r$, all other nodes of the Poisson route $\mathcal{R}$ form a Poisson point process of intensity λ on $(-\infty, 0) \cup (r, \infty)$. Conditioning on $R = r$ we can thus use (3.10) with $\Psi = \Phi^1 \cap ((-\infty,0)\cup(r,\infty)) =: \Phi^1_r$ (recall that Φ^1 is a Poisson process of intensity λp of nodes transmitting in a given time slot) to obtain:

$$\begin{aligned} P_{NN}(p) &= \frac{\lambda}{p}\int_0^\infty e^{-\lambda r}\Pi(0,r,\Phi^1_r)\,\mathrm{d}r \\ &= \lambda(1-p)\int_0^\infty e^{-\lambda r}\mathcal{L}_{\Phi^1_r}(H_{0,r})\,\mathrm{d}r\,. \end{aligned} \tag{4.18}$$

Using the well known formula for the Laplace transform of the Poisson p.p. (see e.g., [3, (16.4)]) we obtain:

$$
\begin{aligned}
\mathcal{L}_{\Phi_r^1}(H_{0,r}) &= \exp\Big\{-\lambda p \int_{(-\infty,0)\cup(r,\infty)} \Big(1-e^{H_{0,r}(s)}\Big)\,\mathrm{d}s\Big\} \\
&= \exp\Big\{-\lambda p \int_{(-\infty,0)\cup(r,\infty)} \Big(1-h(s,r)\Big)\,\mathrm{d}s\Big\} \\
&= \exp\Big\{-\lambda p \int_{(-\infty,0)\cup(r,\infty)} \Big(\frac{1}{1+(s/r)^\beta/T}\Big)\,\mathrm{d}s\Big\} \\
&= \lambda p T^{1/\beta}(C(T^{-1/\beta},\beta)+C(\beta))\,.
\end{aligned}
$$

Inserting this expression in (4.18) we obtain

$$
P_{NN}(p) = \lambda(1-p)\int_0^\infty e^{-\lambda r\Big(1+pT^{1/\beta}(C(T^{-1/\beta},\beta)+C(\beta))\Big)}\,\mathrm{d}r\,, \tag{4.19}
$$

which boils down to the right-hand side of (4.16).

In order to obtain expression (4.17) for the NR model, we follow the same process, with the following modifications. The distance to the nearest *receiver* to the right has the exponential distribution of parameter $\lambda(1-p)$. Moreover the distribution of the point process of emitters is Poisson with parameter λp and independent of the location of this receiver. Note also that by the very choice of the receiver, it is not authorized to emit in the given time slot, hence there is no $(1-p)$ factor in the numerator. ∎

B. Local delays

Let us denote by L_0^{NN} the *local delay* at the tagged node in the NN routing scheme, i.e. the number of consecutive time slots needed for the tagged node to successfully transmit a given packet to the receiver designated by the given routing scheme.

In what follows we will give the expressions for the expected local delay $\overline{L}_{NN} = \mathbf{E}^0[L_0^{NN}]$ at the typical node of the Poisson route, which we call the *average local delay*, where the averaging regards all possible Poisson configurations of other nodes Φ. As we will see, this expectation is finite only if p is sufficiently small (for any given T and β).

Let us denote

$$
\begin{aligned}
\mathcal{D}_1(p) &= \mathcal{D}_1(p;T,\beta) \\
&= T^{1/\beta}\Big(\int_{T^{-1/\beta}}^\infty \frac{1}{u^\beta+1-p}\,\mathrm{d}u + \int_0^\infty \frac{1}{u^\beta+1-p}\,\mathrm{d}u\Big)\,.
\end{aligned}
$$

Proposition 4.3: *Under the assumptions of Proposition 4.1, the mean local delay in the NN routing model is equal to*

$$
\overline{L}_{NN} = \frac{1}{p(1-p)(1-p\,\mathcal{D}_1(p))}
$$

provided

$$
p\,\mathcal{D}_1(p) < 1 \tag{4.20}
$$

and $\overline{L}_{NN}=\infty$ otherwise.

Proof: Conditioning on the distance to the nearest neighbor as in the proof of Proposition 4.1 and using (3.11), we have:

$$
\overline{L}_{NN} = \frac{\lambda}{p(1-p)}\int_0^\infty e^{-\lambda r}\mathcal{L}_{\Phi_r^1}(-H_{0,r})\,\mathrm{d}r\,.
$$

with

$$
\begin{aligned}
\mathcal{L}_{\Phi_r^1}(-H_{0,r}) &= \exp\Big\{-\lambda p \int_{(-\infty,0)\cup(r,\infty)} \Big(1-e^{-H_{0,r}(s)}\Big)\,\mathrm{d}s\Big\} \\
&= \exp\Big\{-\lambda p \int_{(-\infty,0)\cup(r,\infty)} \Big(1-h^{-1}(s,r)\Big)\,\mathrm{d}s\Big\}
\end{aligned}
$$

$$= \frac{\exp\{\lambda pr\, \mathcal{D}_1(p)\}}{p(1-p)}\,.$$

Consequently, we have :

$$\overline{L}_{NN} \;=\; \frac{\lambda}{p(1-p)} \int_0^\infty e^{-\lambda r(1-p\,\mathcal{D}_1(p))}\,\mathrm{d}r\,. \tag{4.21}$$

It is easy to see that this integral is infinite if $p\,\mathcal{D}_1(p) \ge 1$ and $1/((1-p)(1-p\,\mathcal{D}_1(p)))$ otherwise. This completes the proof. ∎

Remark 4.4: Similar phase transition (existence of the critical p) for the mean local delay was discovered in 2D MANETs in [4]. Note that $\mathcal{D}_1(p) \ge \mathcal{C}_1$ with the equality attained when $p \to 0$. Hence, for small p we have $\mathcal{D}_1(p) \approx \mathcal{C}_1$, and the critical p for which the average expected delay explodes is approximately $1/\mathcal{C}_1$.

Expression (4.21) can be used to tune p to the value which minimizes the mean local delay. In the next section we will see that this value of p also *optimizes the performance of NN routing on long routes, for almost any realization of the Poisson route*.

C. Long-distance speed of packet progression

We call long-distance speed the speed at which a packet progresses over a long distance. Let us denote by D_i, $i = 1,\ldots$ the local delay of the packet on its hop from X_i to X_{i+1}. More specifically, then $\sum_{i=0}^{k-1} D_i$ is the end-to-end delay on k hops on the route that starts at $X_0 = 0$. We denote by $v = \lim_{k\to\infty} |X_k|/\sum_{i=0}^{k} D_i$ the *long-distance speed of the packet progression* (expressed in the units of distance per slot). Note also that v multiplied by the number of bits carried by one packet corresponds to the *route transport capacity* defined as the number of bit-meters "pumped" by the network per slot.

Proposition 4.5: *Under the assumptions of Proposition 4.1, the mean long-distance speed of the transmission of packets in NN routing model is equal to:*

$$v = \frac{p(1-p)(1-p\,\mathcal{D}_1(p))}{\lambda}$$

provided $p\,\mathcal{D}_1(p) < 1$ and 0 otherwise.

Proof: The mean empirical speed during the first k hops is:

$$\frac{X_k}{\sum_{i=0}^{k-1} D_i} = \frac{(\sum_{i=0}^{k-1}(X_{i+1}-X_i))/k}{(\sum_{i=0}^{k-1} D_i)/k}\,.$$

By the ergodic property when k tends to ∞ the numerator tends almost surely to $\frac{1}{\lambda}$ and the denominator tends almost surely to $\mathbf{E}[L_0]$. This completes the proof. ∎

Example 4.6: Figure 2 shows the mean long-distance speed for $\lambda = 0.01$, $\beta = 4$ and $T = 10$. We observe that there is an optimal value of p which maximizes the long-distance speed of packet progression and that this speed drops to 0 for p larger than some critical value.

Remark 4.7: Existence of the critical value of p can be attributed to hops, traversed by a tagged packet on the infinite route, being statistically too long. Indeed, analysing the expression in (4.19) one sees that the "rate" at which hops of length r occur it is equal to $e^{-\lambda r}$ while the rate at which the packet passes them is $e^{-\lambda r p D_1(p)}$. Thus, when $pD_1(p) > 1$ the packet delay becomes infinite.

D. Density of progress

Some MANET models (in particular the bipolar one) use the *mean density of progress* d to optimize the performance of the model with respect to p (cf e.g. [7]). Recall that in a 2D MANET d is defined as the expected total progress of all the successful transmissions per unit of surface. In our 1D model, by Campbell's formula d can be expressed as:

$$d = p(1-p)\mathbf{E}^0\Big[|X_1|\mathbb{1}(\mathrm{SINR}_{(0,X_1)} \ge T)\Big]\,.$$

The density of progress, can be also seen as quantifying the number of bit-(or packet)-meters "pumped" per unit length of a route.

Proposition 4.8: *Under the assumptions of Proposition 4.1 the density of progress in the NN receiver model is equal to:*

$$d = d_{NN}(p) = \frac{p(1-p)}{(1+p\,\mathcal{C}_1)^2} \tag{4.22}$$

and is maximized for p equal to:

$$p^*_{NN} = \frac{\mathcal{C}_1 + 1 - \sqrt{\mathcal{C}_1^2 - 1}}{2\,\mathcal{C}_1}\,.$$

Moreover, $0 < p^*_{NN} < 1$.

Proof: Using Campbell's formula, the density of progress in the NN model can be expressed as

$$d_{NN}(p) = \lambda p(1-p)\mathbf{E}\Big[R\mathbf{P}\{\,\mathbb{1}(F \geq TI_{\Phi^1} l(r)\,|\,R = r\,\}\Big]$$

with the notation as in the proof of Proposition 4.1. Following the same arguments as in this latter proof we obtain:

$$d_{NN}(p) = \lambda^2 p(1-p)\int_0^\infty r e^{-\lambda r\left(1+p\,\mathcal{C}_1\right)}\,\mathrm{d}r\,,$$

which is equal to the right-hand side of (4.22). Taking the derivative of the latter expression in p we find that its sign is equal to that of the polynomial $P(p) = 1 - p(2+\mathcal{C}_1) - 2p^2\,\mathcal{C}_1^2 + 2p^3\,\mathcal{C}_1^2$. Note that $P(-\infty) = -\infty$, $P(0) = 1$, $P(1) = -1 - \mathcal{C}_1 < 0$ and $P(\infty) = \infty$. Hence in the interval $(0,1)$, $P(\cdot)$ has a unique root: p^*_{NN} which maximizes $d_{NN}(p)$. The explicit expression for this root follows from the general formulas for the roots of cubic equations. ■

Remark 4.9: Note that density of progress is a "static" quantity, calculated with respect to one slot. It can also be easily evaluated for the NR model in contrast to the mean local delay. However, it fails to discover the existence of the critical value of p for the performance of the MANET revealed by the analysis of the local delay and packet progression speed. Also, one may be tempted to approximate this speed by d/λ. Let us note, however, in Figure 2, that the optimization of this quantity in p cannot be directly related to the maximization of the packet progression speed. For this reason we will not consider the density of progress any further in this paper.

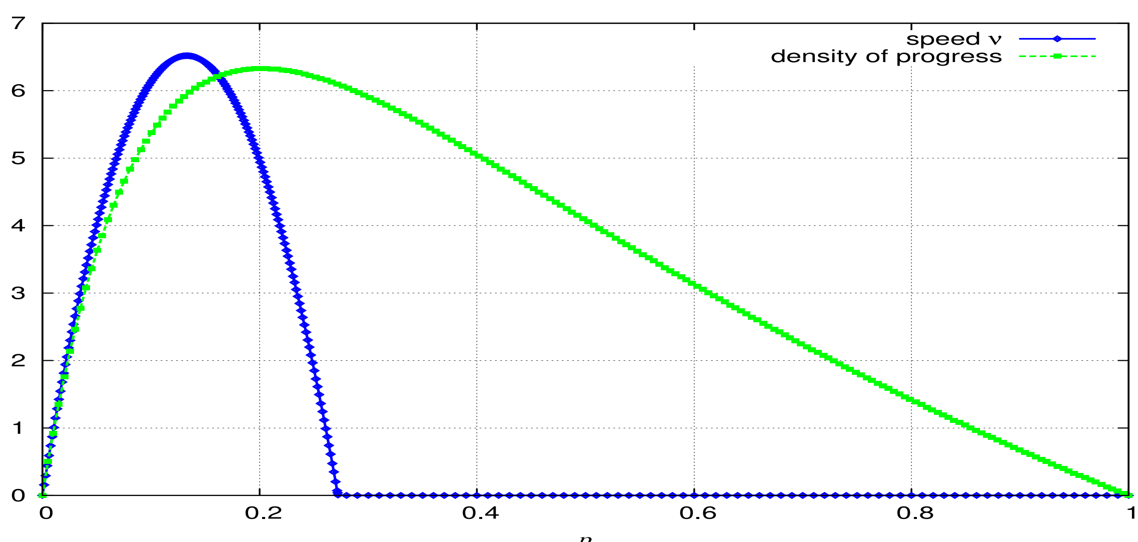

Fig. 2. Comparison of mean speed in the NN model and density of progress

E. Mean end-to-end delay and speed of packet progression on finite segments of routes

In this section we will studythe issues of delays and the speed of packet progression on finite segments of routes. In this regard, we assume that in addition to the node located at the origin $X = 0$ (also called O), a second node, being the final destination of a given packet is located at a fixed position $M > 0$ on the line. Mathematically, this corresponds to the Palm distribution $\mathbf{P}^{0M}$ of the Poisson pp, given these two fixed points. We want to compute the mean end-to-end delay for a given packet to leave the node 0 and reach node M following NN routing. The situation proposed is shown in Figure 3. We denote this end-to-end delay by $L_{0M} = \sum_{X_i \in [0,M)} D_i$.

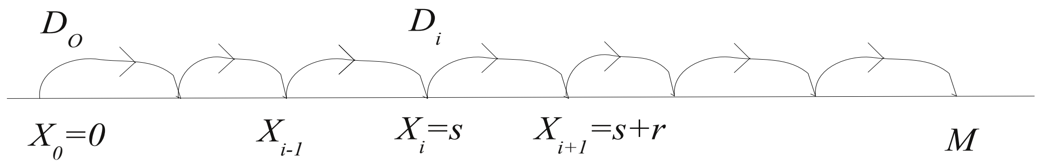

Fig. 3. Transmission delay from node 0 to node M

Proposition 4.10: *Under the assumptions of Proposition 4.1 the mean end-to-end delay in the NN routing on the distance M is equal to*

$$\mathbf{E}^{0M}\Big[\sum_{X_i\in[0,M)} D_i\Big] \tag{4.23}$$

$$= \frac{1}{p(1-p)}\bigg(e^{-\lambda M}E(M) \tag{a}$$

$$+ \int_0^M \lambda e^{-\lambda r}E(r)G_M(0,r)\mathrm{d}r \tag{b}$$

$$+ \lambda\int_0^M\int_0^{M-s} E(r)G_0(s,r)G_M(s,r)\lambda e^{-\lambda r}\mathrm{d}r\mathrm{d}s \tag{c}$$

$$+ \lambda\int_0^M E(M-s)G_0(s,M-s)e^{-\lambda(M-s)}\mathrm{d}s\bigg) \tag{d}$$

with $E(r) = e^{\lambda pr\,\mathcal{D}_1(p)}$, $G_0(s,r) = h(s+r,r)^{-1}$ and $G_M(s,r) = h(M-s-r,r)^{-1}$.

Proof: The mean sum of the delays from the source node 0 to the destination node $X = M$ under $\mathbf{E}^{0M}$ can be expressed using Campbell's formula as

$$\mathbf{E}^{0M}[L_{0M}] = \mathbf{E}^{0M}[D(0)] + \lambda\int_0^M \mathbf{E}^{0Ms}[D(s)]\mathrm{d}s\,,$$

where the first term corresponds to the average exit time from node 0 and $\mathbf{E}^{0Ms}[D(s)]$ is the average exit time from a "current" node located at $s \in (0,M)$. These two expectations differ from $\mathbf{E}^0[L_0]$, expressed in (4.21), because there is a fixed node at M which acts as an additional interferer but which also limits the hop length. Moreover, for the transmission from the current node $s \in (0,M)$, the node at 0 also acts as an additional interferer. We will show how the terms in (a)–(d) of (4.23) reflect these circumstances. Let us first remark first from (4.21) that the function $E(r)$ is the expected exit time from a typical node, given its receiver is located at the distance r and given no additional (non-Poisson) interfering nodes. Now, it is easy to conclude that the term (a) gives the expected end-to-end delay when it is necessary to make the direct hop from 0 to M (when there is no Poisson relay node between them). The term (b) gives the expected exit time from 0 to its nearest receiver when it is located at some $r \in (0,M)$:

$$\mathbf{E}^{0M}[D(0); r < M]$$
$$= \frac{1}{p(1-p)}\int_0^M \lambda e^{-\lambda r}e^{\lambda pt\,\mathcal{D}_1(p)}h(M-r,r)^{-1}\mathrm{d}r\,,$$

where the factor $h(M-r,r)^{-1} = G_M(0,r)$ (not present in (4.21)) is due to the fact that M acts as an additional interferer for the transmission from 0 to $r < M$. Similarly, the term (d) corresponds to the direct hop from the running node at s to M (when there is no Poisson relay node between them) and term (c) corresponds to the delay of going from s to its nearest receiver $r \in (s,M)$. The node at 0 interferes with both these transmissions, which is reflected by the factors $G_0(s,M-s)$ and $G_0(s,r)$ in the terms (d) and (c), respectively. The node at M also interferes with the transmission from s to $r \in (s,M)$, whence $G_M(s,r)$ in (c). ∎

Corollary 4.11: *The average speed of the packet progression on distance M is equal to*

$$v_{[0,M)} = M\Big(\lambda \mathbf{E}^{0M}\Big[\sum_{X_i\in[O,M)} D_i\Big]\Big)^{-1},$$

where $\mathbf{E}^{OM}(\sum_{X_i\in[O,M)} D_i)$ is given by proposition 4.10.

Example 4.12: In Figure 4 we present the speed of the packet progression on distance M assuming $T = 10$ and $\beta = 4$, for M varying from 100 to 2000. The calculation is performed for the value of p that maximizes the asymptotic long-distance speed calculated in Proposition 4.5, presented as the horizontal line in Figure 4. The existence of the destination node in a finite horizon has a double impact on the packet progression. On the one hand it "attracts" the packet reducing the negative impact of long hops (cf Remark 4.7). Indeed, no hop can be longer than the direct hop to M. On the other hand, it "repels" the packet, because it creates an additional interference, which is more significant when the packet is close to the destination. Figure 4 shows two phases related to these two "actions". One can interpret these observations by saying that *routing on distances shorter than some critical values ($M \gtrsim 500$) is very inefficient in terms of exploiting of the route transport capacity.*

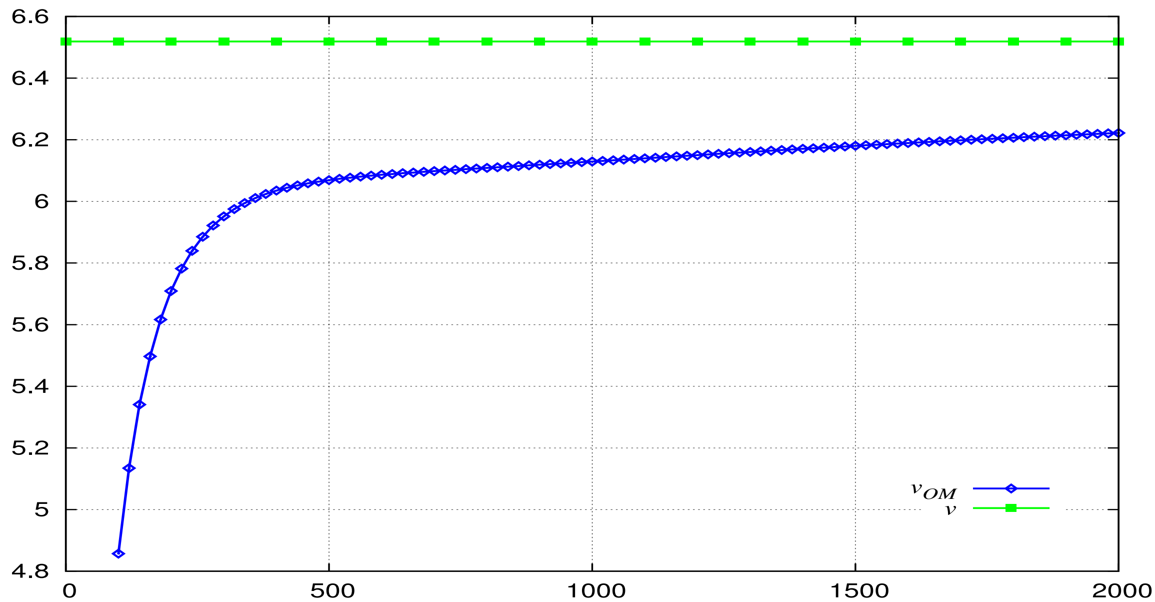

Fig. 4. Mean short-distance speed v_{OM} with respect to M in the NN model and long-distance speed v.

V. Impact of external noise and interference

In the previous section we assumed that the external noise is negligible ($W = 0$). In this section we study the impact a non-null external noise field $W^n(y)$. Recall that we assume that the noise field $\{W^n(y) : y \in \mathbb{R}\}$ is independent of the route process $\mathcal{R}$ and that it is stationary in n; i.e., $W^n(\cdot)$ is equal in distribution to $W^0(\cdot) = W(\cdot)$. We denote by $\mathcal{L}_{W(r)}$ the Laplace transform of $W(r)$ with $r \in \mathbb{R}$.

A. Capture probability

In is easy to extend the results of Propositions 4.1 and 4.8 to the case of an arbitrary noise field $W^n(y)$.

Proposition 5.1: *The probability of successful transmission by a typical node of Poisson route $\mathcal{R} = \Phi$ authorised by Aloha to transmit to its relay node in the NN routing model with noise field $W(\cdot)$ is equal to*

$$p_{NN} = \lambda(1-p)\int_0^\infty e^{-r\lambda(1+p\mathcal{C}_1)}\mathcal{L}_{W(r)}(TW(r)(Ar)^\beta)\,\mathrm{d}r\,. \tag{5.24}$$

Proof: The proof is straightforward and follows the same lines as in the proof of Proposition 4.1. ■

B. Long routes

The following results show a very negative impact of arbitrarily small noise on the performance of NN routing on long routes.

Proposition 5.2: *The mean local delay in the Poisson $\mathcal{R} = \Phi$ NN routing model with a noise field satisfying* $\Pr(W(r) > w) \geq \epsilon$ *for some* $w, \epsilon > 0$*, is infinite* $\mathbf{E}[L_0] = \infty$*. Consequently the speed of packet progression on long routes is almost surely null.*

Proof: As in the proof of proposition 4.3 we can show that

$$\begin{aligned}\mathbf{E}[L_0] &= \frac{\lambda}{p(1-p)} \int_0^\infty e^{-\lambda r} e^{\lambda pr\,\mathcal{D}_1(p)} \mathcal{L}_{W(r)}(-TW(Ar)^\beta)\,\mathrm{d}r \\ &\geq \frac{\epsilon\lambda}{p(1-p)} \int_0^\infty e^{-\lambda r(1-p\,\mathcal{D}_1(p))+Tw(Ar)^\beta}\,\mathrm{d}r = \infty.\end{aligned}$$

■

Remark 5.3: The reason why the long distance speed is 0 lies in the fact that the hops in a Poisson route can be arbitrarily long and that such hops slow down (to zero) the packet progression in the presence of noise. One method to cope with overly long hops consists in adding "fixed" points to the Poisson route $\mathcal{R}$. In fact, we will show in Section V-C that adding only the fixed destination node M is sufficient to make the end-to-end delay finite in the presence of noise. For infinite routes one needs, however, a "fixed" regular grid of relaying nodes; cf Section V-D.

C. Short routes

Our setting is now as in Section IV-E, i.e., we consider the Poisson route Φ under Palm with fixed origin and destination nodes 0 and M, and we compute the end-to-end delay assuming some external noise field of the form (2.2) created by a stationary pattern of interfering nodes Ψ. We assume that this field is stationary and stochastically independent from the Poisson route Φ. Moreover, we assume that the nodes of Ψ use Aloha MAC with probability p'.

We denote by $B(r) = e^{TW(Ar)^\beta}$ and $E'(r) = \mathcal{L}_\Psi\Big(\log h_p(\cdot, r)\Big)$ the Laplace transform of the point process Ψ taken for the function $h_{p'}(\cdot, r)$ given by (3.4) with p replaced by p'.

Proposition 5.4: *Under the assumptions of Proposition 4.1 and in the presence of the external noise of the form (2.2), as explained above, the mean end-to-end delay from* 0 *to* M *in Poisson NN routing is given by (4.23) with* $E(\cdot)$ *replaced by* $E(\cdot) \times E'(\cdot) \times B(\cdot)$*.*

Proof: Observe first that the conditional capture probability given the receiver in Φ at distance r, and given the interfering pattern Ψ, which we denote by $\pi(r, \Phi, \Psi)$, is equal to

$$\begin{aligned}&\pi(r,\Phi,\Psi) \\ &= (1-p)\exp\Big\{\sum_{X_i\in\Phi}\log\Big(1-\frac{p}{1/T(|X_i|/r)^\beta+1}\Big)\Big\} \\ &\quad\times\exp\Big\{\sum_{Y_i\in\Psi}\log\Big(1-\frac{p'}{1/T(|Y_i|/r)^\beta+1}\Big)\Big\} \\ &\quad\times\exp(TW(Ar)^\beta)\,;\end{aligned}$$

cf the proof of Proposition 4.3. Hence, by independence, the mean local delay, given the distance to the receiver $R = r$, is equal to

$$\frac{1}{p}\mathbf{E}^0\Big[\frac{1}{\pi(r,\Phi,\Psi)}\Big|R=r\Big] = \frac{E(r)E'(r)B(r)}{p(1-p)}\,.$$

The remaining part of the proof follows the same lines as the proof of Proposition 4.10. ■

The function E' (corresponding to the Laplace transform of Ψ) can be explicitly given in the case of Poisson and Poisson-cluster point processes. This allows us to study the impact of the clustering of interference in a MANET. In particular, we have the following expressions.

Proposition 5.5: *For Poisson field of interfering nodes of* Ψ *with intensity* μ *on the plane we have*

$$E'(r) = E'_P(r) = \exp\Big(\frac{2\pi^2\mu p' T^{2/\beta}}{\beta(1-p')^{1-2/\beta}\sin(\frac{2\pi}{\beta})}r^2\Big)\,.$$

For a Poisson-line interferers (cf Section II-B3) we have

$$E'(r) = E'_{PL}(r) = \exp\Big(-2\nu r\int_0^\infty \Big(1-\exp(2\lambda' r \times \int_0^\infty \frac{p'}{\frac{1}{T}(s^2+t^2)^{\beta/2}+1-p'}\mathrm{d}t)\Big)\mathrm{d}s\Big)$$

Proof: Both expressions follow from the evaluation of the Laplace transforms of the respective point processes. For a Poisson process this gives

$$E'_P(r) = \exp\Big(2\mu\pi r^2\int_0^\infty \frac{p'}{\frac{1}{T}s^\beta+1-p'}\mathrm{d}s\Big)$$

and can be evaluated to the required expression (which also appeared in [3, Equation 17.43]. [16] gives the following formula for the Laplace transform of the Poisson-line field of interferers

$$\mathcal{L}_\Psi(f) = \exp\Big(-2\nu\int_0^\infty 1-e^{-2\lambda'\int_0^\infty 1-\exp(-f(\sqrt{s^2+t^2}))\mathrm{d}t}\mathrm{d}s\Big)\,,$$

from which our expression for $E'_{PL}(r)$ follows. ■

Remark 5.6: Note that

$$E'_{PL}(r) \geq \exp\Big(8\pi\lambda'\nu r^2\int_0^\infty \frac{p'}{\frac{1}{T}s^\beta+1-p'}\mathrm{d}s\Big).$$

Hence, for the same intensities of points $\mu=\lambda'\nu$, $E'_{PL}(r)\geq E'_P(r)$. This implies that the mean end-to-end delay is larger in the Poisson-line field of interferers than in the homogeneous Poisson field. As the former process shows more clustering, we can conclude that *clustering in the interfering field increases end-to-end delays*.

Example 5.7: The average speed of the packet progression on the distance from O to M can be expressed as in Corollary 4.10. We now present some numerical illustrations how this speed depends on the model's parameters. We first assume only constant noise W. Figure 6 shows the speed of packet progression with respect to W. We have assumed $T=10$, $\beta=4$ and $p=0.15$, the value which maximizes the average long-distance speed without noise. The curves correspond to different routing distances $M=100,1000,10000$. Figure 5 shows the same speed of packet progression with respect to M for $W=10^{-11},10^{-13},10^{-15}$. For $M=100,1000,10000$ we keep $p=0.15$ the value which maximizes the average long-distance speed without noise. Both Figures 5 and 6 show a sharp cut of phenomenon: the speed of packet progression drops rapidly to zero beyond some critical noise level and/or route length. The impact of the noise is stable below its critical value (for a given distance). Comparing Figure 4 to Figure 5 we see that after the "fast acceleration" the packet reaches a constant speed which can be maintained on routes shorter than some critical length depending on the level of noise. Figure 7 shows the mean end-to-end speed with respect to the spatial density of the field of Poisson interferers λ' for two values of the Aloha parameter $p'=0.15$ and $p'=0.015$ used by the interferers. As in the case of constant noise, the speed rapidly falls from an almost constant value (corresponding to the case without interferers) to zero. Figure 8 shows the mean end-to-end speed with respect to the spatial density of the field of thePoisson-line process of rate ν and λ' for $p'=0.15$ and $p'=0.015$. We observe that this speed is smaller than the mean speed in a field of Poisson interferers with a comparable spatial density $\nu\times\lambda'$ (cf. Figure 7); this confirms remark 5.6.

D. Equipping long Poisson routes with fixed relay stations

As previously mentioned, in order to solve the problem of long hops slowing down the packet progression in the presence of external noise, one can add fixed relay stations, thus allowing efficient routing on long Poisson routes. In this section we will study this problem.

We assume regularly spaced "fixed" relay nodes $\mathcal{G}=\{n\Delta+U_\Delta, n\in\mathbb{Z}\}$, where $\Delta>0$ is some fixed distance. These nodes are added to the Poisson route Φ, they use the same Aloha parameter p and the NN routing is considered on $\Phi\cup\mathcal{G}$.

We wish to compute the mean local delay on $\Phi\cup\mathcal{G}$ in the presence of noise, and thus characterise the speed of packet progression on long routes equipped with fixed relay stations. This will allow us to optimise the performance of the network in the parameter Δ. It should be noted that a small value of Δ makes the fixed relay nodes too

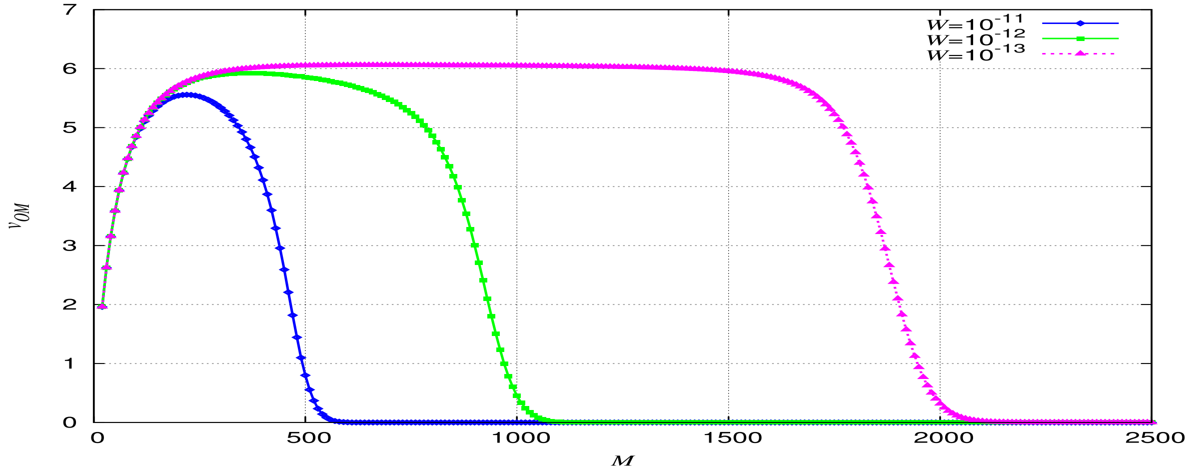

Fig. 5. Mean short-distance speed with respect to the distance M in the NN model, $W = 10^{-11}, 10^{-12}, 10^{-13}$.

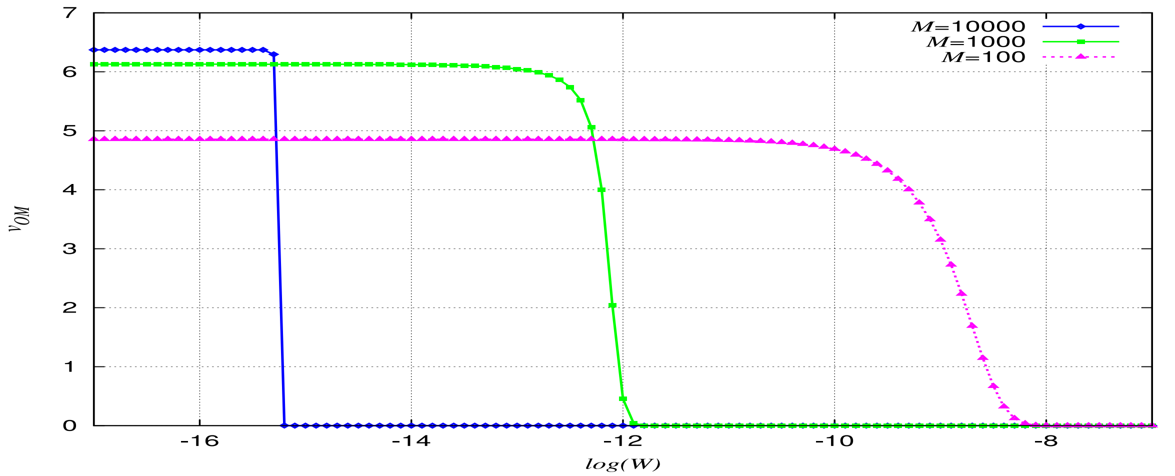

Fig. 6. Mean short-distance speed with respect to noise W in the NN model.

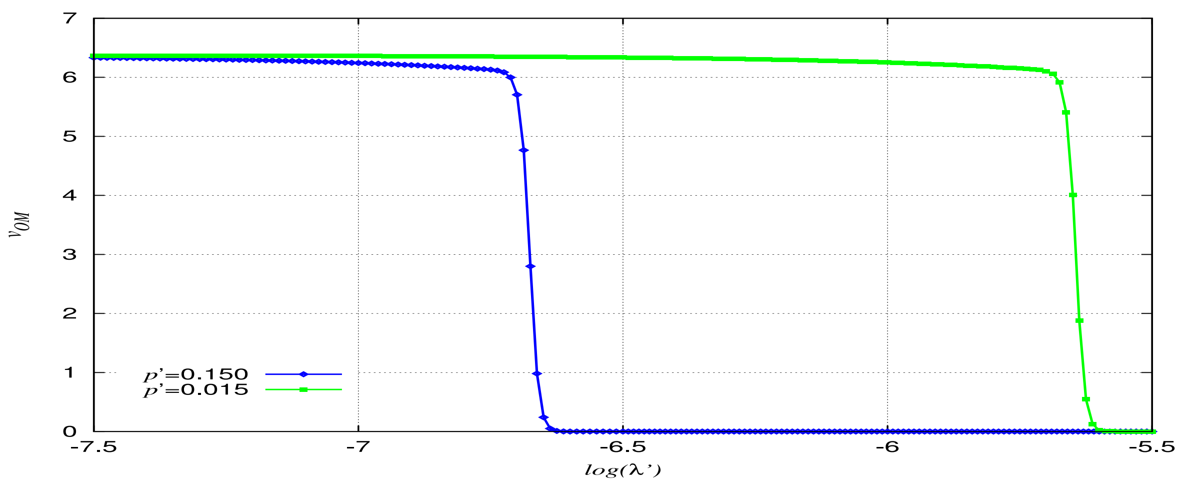

Fig. 7. Mean short-distance speed with respect to the spatial intensity of the field of interferers λ', $M = 10000$

dense, creating a lot of interference and making the routing hops very small. On the other hand, a large value of Δ leads to long hops which slow down the packet progression.

Proposition 5.8: *Assume $\mathcal{R} = \Phi \cup \mathcal{G}$ to be a Poisson route equipped with fixed relay stations with the inter-relay distance Δ, as described above. The mean local delay in the NN routing on this model in the presence of external noise of the form (2.2) as in Proposition 5.4 is equal to:*

$$\mathbf{E}^0[L] \tag{5.25}$$

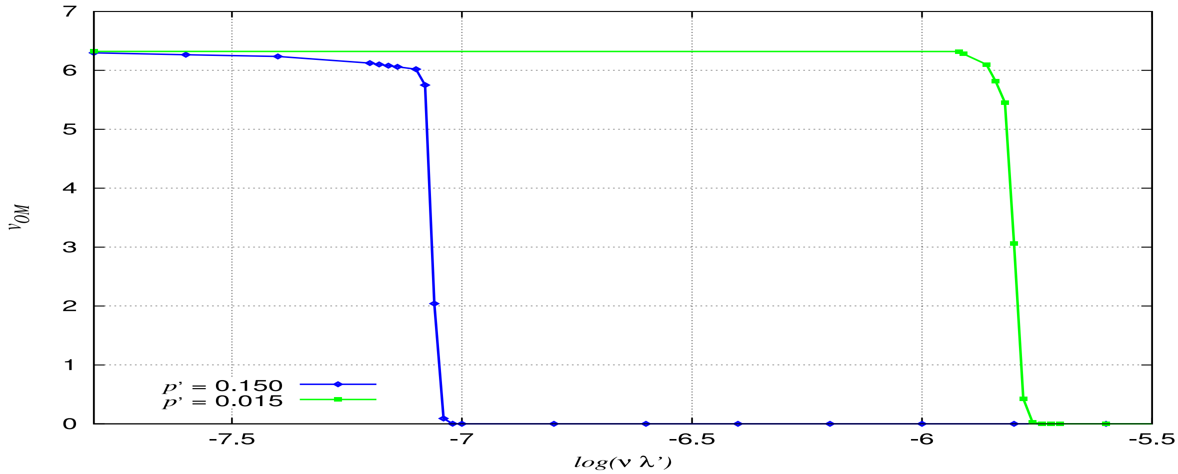

Fig. 8. Mean short-distance speed with respect to the spatial intensity $\nu\lambda'$ of Poisson-line process, $M = 10000$

$$
\begin{aligned}
= \; & \frac{\lambda}{p(1-p)(\epsilon+\lambda)}\Bigg(\frac{1}{\Delta}\int_0^{\Delta} E(z)E'(z)e^{-\lambda z}e^{H(0,z)}\mathrm{d}z \\
& +\frac{\lambda}{\Delta}\int_0^{\Delta}\int_0^{z} E(r)E'(z)e^{-\lambda r}e^{H(z-r,r)+\log(h(z-r,r))}\mathrm{d}r\mathrm{d}z\Bigg) \\
& +\frac{\epsilon}{p(1-p)(\epsilon+\lambda)}\Bigg(\frac{\lambda}{\Delta}\int_0^{\Delta} E(z)E'(z)e^{-\lambda z}e^{H(-z,z)}\mathrm{d}z \\
& +e^{-\lambda\Delta}B(\Delta)E(\Delta)e^{H(0,\Delta)-\log(h(\Delta,\Delta))}\Bigg)
\end{aligned}
$$

with $\epsilon = \frac{1}{\Delta}$ *and* $H(z,r) = \sum_{n\in\mathbb{Z},n\neq 0}\log(h(n\Delta+z,r))$.

Proof: Let us suppose that a typical node 0 of $\Psi \cup \mathcal{G}$ is at $X = 0$. We know that this typical node belongs to Ψ with probability $\frac{\lambda}{\epsilon+\lambda}$ and to $\mathcal{G}$ with probability $\frac{\epsilon}{\epsilon+\lambda}$. Thus we have

$$\mathbf{E}^0[L] = \frac{\lambda}{\epsilon+\lambda}\mathbf{E}^0[L|0\in\Psi] + \frac{\epsilon}{\epsilon+\lambda}\mathbf{E}^0[L|0\in\mathcal{G}].$$

Let us now assume that the typical node 0 is in Ψ and we use the fact that the nodes in Ψ are independent of the nodes in $\mathcal{G}$. If its closest node (towards the right) is in $\mathcal{G}$, the average delay to leave node 0 is : $B(z)E(z)e^{H(0,z)}$. Conditioning by the fact there is no node of Ψ in $[0,z]$ (that occurs with probability $e^{-\lambda z}$), we find the first integral in the first line of (5.25). We still assume that the typical node is in Ψ but now we also assume that the closest node (towards the right) is in Ψ. Computing the average delay in this case, we find the second double integral of the first line of (5.25); $\lambda e^{-\lambda r}$ is actually the density of the right-hand neighbor's position. The contribution $e^{\log(h(z-r,r))}$ corresponds to the fact that the node in $\mathcal{G}$ located at z is not taken into account by $e^{H(z-r,r)}$ since the summation in H is for $n \neq 0$.

The second part of the contribution for $\mathbf{E}^0[L]$ corresponds to the case where node 0 is in $\mathcal{G}$. If we assume that the right-hand neighbor of 0 is in Ψ, the contribution of the delay corresponds to the integral of the proposition in the second line of $\mathbf{E}^0[L]$. There is no correction needed for $H(-z,z)$ since the typical node 0 is in $\mathcal{G}$ and does not contribute to the delay as it is in $H(-z,z)$. The last term of the second line in (5.25) corresponds to the case where node 0 and its right-hand neighbor are in $\mathcal{G}$. This occurs with probability $e^{-\lambda\Delta}$. The node in $\mathcal{G}$ located at Δ does not contribute to the delay since we analyze the delay from 0 and its right-hand neighbor in $\mathcal{G}$ (thus located at Δ). Thus $H(0,\Delta)$ must be corrected by $e^{-\log(h(\Delta,\Delta))}$. ■

Example 5.9: In Figure 9, we plot the mean long-distance speed $v = 1/(\lambda\mathbf{E}^0[L])$ with respect to Δ with fixed noise of levels $W = 10^{-11}$ and $W = 10^{-12}$. Comparing Figure 5 to Figure 9 one can observe that the optimal inter-relay distance Δ of the fixed structure corresponds to the smallest length M of the route on which the mean speed of packet progression attains its maximum without the fixed infrastructure. In other words, the relay stations should "break down" long routes into segments of the length optimal for the end-to-end routing given the noise level.

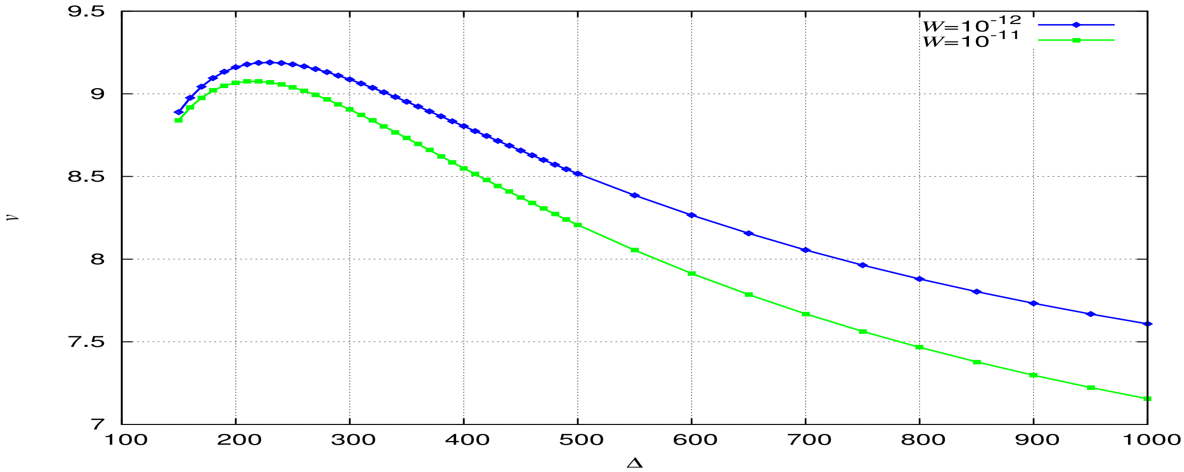

Fig. 9. Mean long-distance speed with respect to Δ for $W = 10^{-11}$ and $W = 10^{-12}$.

VI. Concluding Remarks

We have studied performances of end-to-end routing in MANETs, using a linear nearest-neighbor routing model embedded in an independent planar field of interfering nodes using Aloha MAC. We have developed numerically tractable expressions for several performance metrics such as the end-to-end delay and speed of packet progression. They show how the network performance can be optimized by tuning Aloha and routing parameters. In particular, we show a need for a well-tuned lattice structure of wireless relaying nodes (not back-boned) which help to relay packets on long random routes in the presence of a non-negligible noise. Our analysis does not take into account packet queuing at the relay nodes, which we believe can be introduced into our model in future work.

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