



**HAL**  
open science

## A preliminary design formula for the strength of stiffened curved panels by design of experiment method

Khan Le Tran, Cyril Douthe, Karam Sab, Julien Dallot, Laurence Davaine

### ► To cite this version:

Khan Le Tran, Cyril Douthe, Karam Sab, Julien Dallot, Laurence Davaine. A preliminary design formula for the strength of stiffened curved panels by design of experiment method. *Thin-Walled Structures*, 2014, 79, pp. 129-137. 10.1016/j.tws.2014.02.012 . hal-00960346

**HAL Id: hal-00960346**

**<https://hal.science/hal-00960346>**

Submitted on 18 Mar 2014

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# A preliminary design formula for the strength of stiffened curved panels by design of experiment method

K. L. Tran<sup>a</sup>, C. Douthe<sup>b,\*</sup>, K. Sab<sup>c</sup>, J. Dallot<sup>a</sup>, L. Davaine<sup>d</sup>

<sup>a</sup>*SNCF, Direction de l'Ingénierie, 6, av. F. Mitterrand, F-93574 La Plaine St Denis*

<sup>b</sup>*Université Paris-Est, IFSTTAR, F-77447 Marne-la-Vallée*

<sup>c</sup>*Université Paris-Est, Laboratoire Navier (UMR 8205), ENPC, IFSTTAR, CNRS, F-77455 Marne-la-vallée Cedex 2*

<sup>d</sup>*Ingérop, Expertises et Structure, 168/172, bd de Verdun, F-92408 Courbevoie*

---

## Abstract

In bridge construction, the use of stiffened plates for box-girder or steel beams is common day to day practice. The advantages of the stiffening from the economical and mechanical points of view are unanimously recognized. For curved steel panels, however, applications are more recent and the literature on their mechanical behaviour including the influence of stiffeners is therefore limited. Their design with actual finite element software is possible but significantly time-consuming and this reduces the number of parameters which can be investigated to optimise each panel. The present paper is thus dedicated to the development of a preliminary design formula for the determination of the ultimate strength of stiffened cylindrical steel panels. This approximate formula is developed with help of a design of experiment method which has been adapted from the current statistical knowledge. This method is first presented and its feasibility as well as its efficiency are illustrated through an

---

\*Corresponding author

*Tel:* +33 1 81 66 81 36; *E-mail:* [cyril.douthe@ifsttar.fr](mailto:cyril.douthe@ifsttar.fr)

application to the reference case of unstiffened curved panels. Then, the case of stiffened curved panels is investigated and a preliminary design formula is developed. The ease of use of this formula for preliminary design is finally illustrated in a cost optimisation problem.

*Keywords:* Design of computer experiments, Response surface, Cylindrical curved panels, Stiffeners, Stability, GMNIA.

---

## 1. Introduction

The interest of stiffening steel plates or panels to increase their strength under compression has been known for almost a century [1]. In the field of structural engineering, the use of such panels is a common practice, for example in bottom flanges of box-girder bridges. Recent developments of the curving process allowed for the use of curved panels in civil engineering structures where they offer attractive aesthetic and aerodynamic possibilities. The verification of these panels is yet difficult due to a lack of specifications, especially in European Standards: EN 1993-1-5 [2] gives specifications for flat or slightly curved panels with the condition  $R \geq R_{lim} = b^2/t_p$  (where  $R$  is the curvature radius of the panel,  $b$  its width and  $t_p$  its thickness) and EN 1993-1-6 [3] deals only with revolution cylindrical shells. Nevertheless the curved panels in bridges have characteristics exactly between these two conditions, as illustrated in the case of the Confluences bridge in Angers, France 2011 (Fig. 1), whose radius  $R = 80 \text{ m}$  is much smaller than the limit of EN 1993-1-5:  $R_{lim} = 1440 \text{ m}$  (with  $b = 4.8 \text{ m}$  and  $t = 16 \text{ mm}$ ) and for which EN 1993-1-6 is not applicable neither because these curved flanges are not full revolution cylinders.

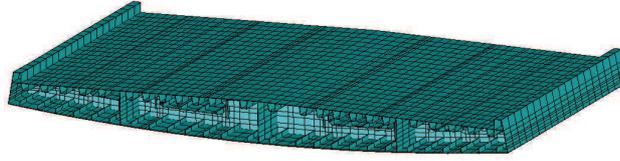


Figure 1: Stiffened curved panel of the Confluences Bridge in Angers (France, 2011)

19 From an academic point of view, the articles related to the buckling theory  
20 of curved panels are not so numerous due to the complexity of the studied  
21 problem and also due to its late application in the bridge construction. First  
22 investigations were conducted in the forties by Batdorf & Schildcrout [4] and  
23 Schildcrout & Stein [5] who showed that the stiffeners and the curvature  
24 increase the critical buckling strength. A state of the art on curved stiffened  
25 panels was then proposed by Becker [6] in 1958 in its handbook on structural  
26 stability. Based on experimental results (provided by Gall [7], Lundquist  
27 [1] and Ramberge *et al.* [8]), he confirmed that, when a stiffened flat panel  
28 is bent to a circular curve, its buckling stress is slightly increased (around  
29 6% for the tested specimen which is relatively few compared to the effect of  
30 stiffening alone or curvature alone). More recent parametric studies based  
31 on numerical examples and the finite element modelling (e.g. Cho *et al.* [9],  
32 Khedmati & Edalat [10] or Park *et al.* [11]) investigated and quantified the  
33 influence of the main parameters on the ultimate strength of curved stiffened  
34 plates. They however did not lead to a practical criterion for the evaluation  
35 of the resistance of such panels which is therefore still an open question.

36 In a former study, the authors [12] had investigated the case of unstiff-  
37 ened cylindrical curved panels under axial compression and established a set

38 of formulas for the evaluation of the ultimate strength (which were confirmed  
39 by [13]). These semi-analytic formulas had been fitted on a total of 524 com-  
40 binations of the main parameters. Each calculus involved Geometrical and  
41 Material Non-linearity with Imperfection Analysis (GMNIA) and required  
42 between 5 and 10 minutes depending on the refinement of the mesh. Consid-  
43 ering the fact that in the case of stiffened panels the number of parameters  
44 is considerably larger, re-employing the same methodology seemed unreal-  
45 istic. It appeared hence that there is a need for a robust strategy for the  
46 choice of the set of tested models and for the measure of the approximated  
47 model accuracy. Such a strategy exists for the design of physical experiments  
48 as well as for that of computer experiments, they are known as "design of  
49 experiments methods".

50 In the following, the authors present thus first the characteristics of com-  
51 puter experiment strategies. Afterwards the feasibility and ease of use of the  
52 methodology as well as its efficiency are illustrated through an application  
53 to the reference case of unstiffened curved panels. Then, the case of stiffened  
54 curved panels is investigated and a preliminary design formula is developed.  
55 The interest of this formula for early stages of design is finally illustrated by  
56 a short example of cost optimisation.

## 57 **2. Design of computer experiments**

### 58 *2.1. Background of the design of experiments method*

59 Design of experiment (DOE) methods exist since the beginning of sci-  
60 entific experiment. The first formal theory for the design of experiments  
61 in a "modern sense" was published by Fisher [14] in the 1920s and 1930s,

62 while working on improving agricultural yield. Since the 1940s, various re-  
63 searchers have promoted and developed the use of experiments strategies in  
64 many other areas [15]. In the late 1970s, the theory of Taguchi [16] on qual-  
65 ity improvement made the design of experiment widely used in the industrial  
66 environment. In the past 20 years, advances in computational power have led  
67 to the study of physical process through computer simulated experiments,  
68 which tends to replace physical experiments in cases where the number of  
69 variables is too large to consider performing a physical experiment or where it  
70 is simply economically prohibitive to run an experiment on the scale required  
71 to gather sufficient information.

72 Computer experiments differ from traditional physical experiment in their  
73 deterministic character, meaning that the computer produces identical an-  
74 swers for the same set of experimental parameters. The error in computer  
75 experiments is no longer due to random effects which derive from the vari-  
76 ability in experimental units, the order of experiments or the locations of  
77 the tests. However, it was shown that in many cases, the systematic error  
78 between a deterministic model and its approximation has a normal distribu-  
79 tion, so that standard statistical techniques can still be applied [17]. Several  
80 authors [17, 18, 19] also insisted on the fact that the selection of parameter's  
81 values for computer runs is still an experimental design problem of primary  
82 importance, especially considering the quantification of uncertainty of the  
83 model on a statistical point of view. Indeed, as not every combination of  
84 parameters can be tested, uncertainty and hazard enter the deterministic  
85 process through the choice of tested combinations. The design of a computer  
86 experiment is hence at the border of a physical and a statistical problem

87 which specificities are emphasized in the following section.

## 88 *2.2. General progress of the design of computer experiment method*

89 Schematically, a numerical model can be considered as a process: the user  
90 specifies the combinations of (input) variables to the computer simulator from  
91 which the responses (output) are generated. Fig. 2 illustrates this process in  
92 the simple case where there are only two input values ( $X_1$  and  $X_2$ ) and one  
93 response  $Y$ . Each variable can take a value from “low” to “high”. The set  
94 of all domains of variation forms the “region of interest”. In correspondence  
95 with each input variable ( $X_1^i, X_2^i$ ), the computer program will provide one  
96 result  $Y^i$ . A set of  $n$  responses will then generate by extrapolation a response  
97 surface. In practice, the explicit formula for this surface is not known. The  
98 aim of DOE method is to provide approximated models (response surfaces)  
99 that are sufficiently accurate to replace the true response and can be used to  
100 facilitate design space exploration, optimisation or reliability analyses.

101 The general steps of computer experiments are generally similar to those  
102 encountered in classical experiments [20] and can be summarized as follow:

- 103 • **Step 1:** Statement of the problem.
- 104 • **Step 2:** Choice of the model for the response surface.
- 105 • **Step 3:** Selection of the input data points.
- 106 • **Step 4:** Evaluation of the approximated model.
- 107 • **Step 5:** Validation of the accuracy of the response model.
- 108 • **Step 6:** Selection of most significant terms and conclusion.

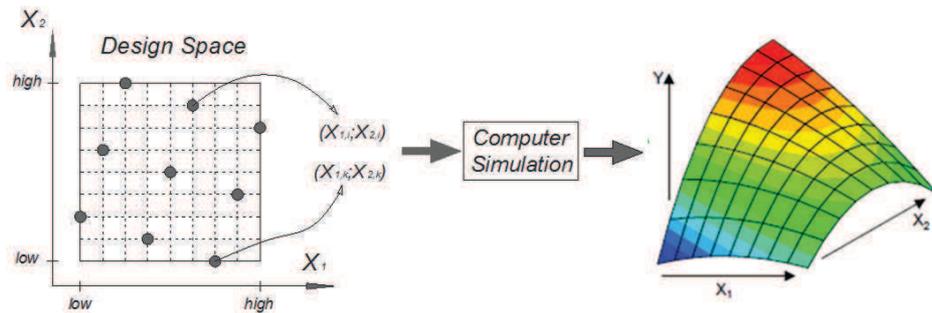


Figure 2: Principle of the computer experiment process and response surface.

109 This process is not necessarily linear and could be applied iteratively if the  
 110 predicted model does not meet the desired accuracy. In the above process, the  
 111 selection of the input variables (**step 3**) and the technique for approximating  
 112 the response (**step 4**) are the two main issues that differ between physical  
 113 and numerical experiments due to the deterministic property of computer  
 114 experiments. These two issues will be developed in the following paragraphs.

### 115 2.3. Selecting sampling points

116 A good experimental design should minimize the number of runs needed  
 117 to acquire information with a given level of accuracy. The experimental  
 118 design techniques were initially developed for physical experiments. Due to  
 119 the discrepancy associated with physical experimentation, classical DOEs  
 120 will focus on parameter settings near the perimeter of the region of interest  
 121 and take multiple data points (replicates) as shown in Fig. 3(Left). Computer  
 122 experiments are determinists and are not subjected to this necessity. The  
 123 objective of computer experiments is hence mainly to uniformly distribute  
 124 the sampling points in the region of interest (such a design is called “space  
 125 filling”) as seen in Fig. 3(Right).

126 Within the available methods of sampling [21], the following three are  
 127 the most common and efficient: the Monte Carlo method (MC), the Latin  
 128 Hypercube Sampling method (LHS) and the Quasi-Monte Carlo methods  
 129 (QMC) which can be viewed as deterministic versions of MC methods be-  
 130 cause they use deterministic points rather than random samples. Blatman  
 131 *et al.* [22] showed that QMC overperforms MC and LHS, when used with  
 132 polynomial response surfaces with a mean computational gain factor of 10 in  
 133 order to reach a given accuracy. The QMC methods are also termed as low  
 134 discrepancy procedures: sampling points are selected in such a way that the  
 135 error bound is as small as possible.

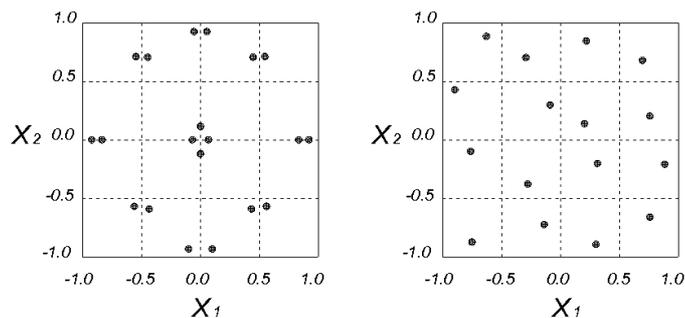


Figure 3: (Left) “Classical” and (Right) “Space Filling” designs.

136 There are many ways to construct a QMC sequence but the Sobol’ se-  
 137 quences are the most widely used because they are quick to construct and fast  
 138 to converge [23]. They also have the advantage of preserving the uniformity  
 139 of the distribution when the dimension increases: a Sobol’ sequence can be  
 140 constructed from a shorter one by adding points to the shorter sequence, on  
 141 the contrary to LHS, where the entire sampling process must be run again.

142 *2.4. Response model regression*

143     After selecting the appropriate experimental points and performing the  
144 necessary computer runs, the next step is to choose an approximated model  
145 and a fitting method. The approximated model must be simple and represent  
146 adequately the response of the studied problem. In recent years, a lot of work  
147 has been done on approximated models: polynomial response surfaces, neural  
148 networks, kriging or multivariate adaptive regression splines [24]. Despite  
149 the variety of approximations that is available, comparative studies of these  
150 approaches are limited [25]. Depending on the complexity of the problem, one  
151 of the aforementioned method might be suitable. However, the polynomial  
152 response surface model is by far the simplest; it has been used efficiently in  
153 a wide variety of applications and has provided good approximated solutions  
154 to even very complex problems [26]. Beside, the use of polynomial response  
155 surface for furthers studies such as reliability [27] and optimisation [28] is  
156 relatively easy.

157 *2.5. Statistic tools for adequacy checking*

158     As mentioned in the section 2, model adequacy checking is an important  
159 part of the data analysis procedure. Indeed it is necessary to ensure that  
160 the fitted model provides an adequate approximation of the true system and  
161 to verify that none of the model assumptions is violated. In most cases, the  
162 regression model is a linear function of some unknown coefficients which are  
163 identified thanks to the least square method which will be used here for its  
164 simplicity and reliability.

165 **3. Application to cylindrical curved panels under uniform axial**  
166 **compression**

167 In a former study, the authors [12] investigated the case of unstiffened  
168 cylindrical curved panels under axial compression (see Fig. 4) and proposed  
169 a set of formulas for the evaluation of the ultimate strength. These formulas  
170 were established following the general European Standards procedure for all  
171 kind of stability verification and will be used as a reference case to validate  
172 the accuracy and relevance of the methodology proposed in previous section.  
173 The strength of the panel  $\chi$  was hence given as a function of the relative  
174 slenderness  $\bar{\lambda}$  and three parameters  $\bar{\lambda}_0$ ,  $\beta$ ,  $\alpha_Z$  depending on the relative  
175 curvature:

$$\chi = \frac{2\beta}{\beta + \bar{\lambda} + \sqrt{(\beta + \bar{\lambda})^2 - 4\beta(\bar{\lambda} - \alpha_Z(\bar{\lambda} - \bar{\lambda}_0))}} \quad (1)$$

176  
177 These simulations, as well as those which will be conducted here, involved  
178 non-linear material and second-order analyses with imperfection (GMNIA).  
179 They were conducted with the software Ansys version 13 and the standard  
180 quadrilateral 4-nodes element [29]. Panels were made of elasto-plastic steel  
181 with linear hardening as indicated in EN 1993-1.5 C.6.c) (S355,  $E = 210GPa$ ,  
182  $\nu = 0.3$  and a slope of  $E/100$  ). The cylindrical panels were assumed simply  
183 supported on all edges and loaded by a uniform longitudinal compression  
184 along the curved edges. An initial imperfection with the shape of the first  
185 buckling mode and with a maximum amplitude of  $1/200^{th}$  of the smallest  
186 edge was also added. The study was limited to square panels, so that only  
187 the thickness, the width and curvature of the panels were varied.

188 *3.1. Step 1: Statement of the problem*

189 The aim of this step is to identify in an exhaustive manner the parameters  
190 of the problem and to select among them the ones which will have an influence  
191 on the response and which are liable of variations in practical applications.  
192 Here the quantity of interest in the panel response (the output) is the ultimate  
193 strength of the panel. Basic structural engineering tells us that it is influenced  
194 by the geometry of the panels (including their imperfections), their material  
195 properties, their boundary conditions and the nature of the loading. All  
196 these parameters could be included in the experimental program, but in this  
197 first example, the objective is to validate the method and to illustrate its  
198 pertinence, so that the same restrictions as in [12] will be observed:

- 199 • the imperfections are chosen following EN 1993-1-5 [2] (i.e. their shape  
200 is that of the first buckling mode and their amplitude is  $1/200th$  of the  
201 width of the panel)
- 202 • the steel grade is S355 as generally used in modern bridges;
- 203 • the panels are simply supported on all edges;
- 204 • the longitudinal compression is uniform along the curved edges.

205 The only varying input factors are thus the dimensions of the panels:  
206 their length  $a$ , width  $b$ , thickness  $t_p$  and radius of curvature  $R$  (see Fig. 4).  
207 Applying the Buckingham-Vaschy's theorem, it can be demonstrated that  
208 the ratio of the ultimate strength of the panel and the yield stress ( $\sigma_{ult}/f_y$ )  
209 depends on three independent dimensionless parameters:

$$\frac{\sigma_{ult}}{f_y} = f \left( \frac{a}{b}; \frac{t_p}{b}; \frac{b}{R} \right) \quad (2)$$

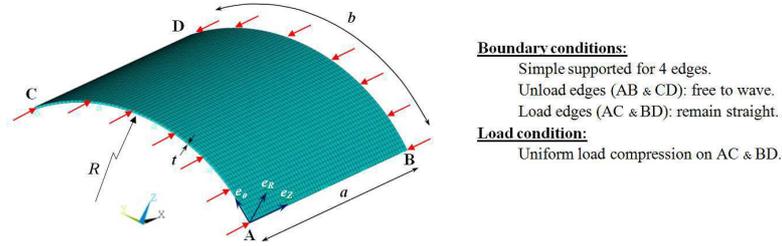


Figure 4: Cylindrical curved panel under uniform axial compression (after [12]).

211 Now that the parameters have been defined, it is essential to define the  
 212 range in which these parameters will vary. Every feasible configuration has  
 213 to be included but the range of variation has to be kept as small as possible:  
 214 it is directly linked with the precision of the approximated expression found  
 215 at the end of the process. Here the ranges given in table 1 seem reasonable  
 216 to cover most applications of such panels in bridge engineering.

Table 1: Design variables for unstiffened curved panels

Variable	Description	Variation range	Design variable
$a/b$	Aspect ratio	$0.6 \leq a/b \leq 1.6$	$X_1 = 2 a/b - 2.2$
$t_p/b$	Slenderness	$0.01 \leq t_p/b \leq 0.04$	$X_2 = 66.7 t_p/b - 1.67$
$b/R$	Curvature (Angle)	$0 \leq b/R \leq 1$	$X_3 = 2 b/R - 1$

217 As the order of magnitude of the variations of these three parameters is  
 218 different, it is preferable to transform the physical parameters into centred  
 219 variables  $X_i$ , ranging from -1 (low value) to 1 (high value). Their comparative  
 220 influence on the response will hence be easier to catch. The three adimen-  
 221 sional parameters  $X_1$ ,  $X_2$  and  $X_3$  used in the coming paragraphs are thus  
 222 given in the last column of table 1.

223 *3.2. Step 2: Choice of the response surface*

224 The choice of the response surface is based on two issues: the knowledge  
 225 of the physics of the problem and the desired accuracy of the approximation.  
 226 Here the target response is the ultimate strength of the plate, namely the  
 227 maximum load that the plate can bear when accounting for the elasto-plastic  
 228 behaviour of the material. From existing standards (EC3), it is known that  
 229 the strength of a flat plate is related to the slenderness by a second order  
 230 polynomial which was first proposed by Winter [30]:

$$\frac{\sigma_{ult}}{f_y} = \frac{1}{\bar{\lambda}} - \frac{0.22}{\bar{\lambda}^2} \quad (3)$$

231 In (3), the slenderness  $\bar{\lambda}$  is directly related to  $a/b$  and  $t/b$  (which means  
 232 to  $X_1$  and  $X_2$ ) by:

$$\bar{\lambda} = \sqrt{f_y \frac{12(1-\nu^2)}{\pi^2 E}} \cdot \sqrt{\frac{1}{k_{a/b}} \cdot \frac{b}{t_p}} \quad (4)$$

233 where  $k_{a/b}$  is a function of  $a/b$

$$k_{a/b} = \begin{cases} \left(\frac{a}{b} + \frac{b}{a}\right)^2 & \text{if } \frac{a}{b} \leq 1 \\ 4 & \text{if } \frac{a}{b} \geq 1 \end{cases} \quad (5)$$

234 It can thus be concluded that a second order polynomial should provide  
 235 a good approximation of the strength of a curved plate and that it will be  
 236 meaningful from a physical point of view. The response surface will thus be  
 237 investigated in the following form:

$$\widehat{Y} \left( = \frac{\sigma_{ult}}{f_y} \right) = \beta_0 + \sum_{i=1}^3 \beta_i X_i + \sum_{j=1}^3 \sum_{i=1}^j \beta_{ij} X_i X_j \quad (6)$$

238 where  $\widehat{Y}$  is the approximated response,  $X_i$  are the three input variables and  
 239  $\beta_{(\cdot)}$  are the ten unknown parameters.

240 *3.3. Step 3: Selection of the input data or sampling points*

241 The selection of the input data points covers the choice of the number  
242 of points and of their distribution in the investigated domain which is here  
243 a hypercube in the three dimensional space. The generation of a set of  
244 sampling points using a QMC method in this cube can be made easily using  
245 a common statistical tool providing a function generating a Sobol' sequence  
246 (Matlab here). Noting that with the Sobol' sequence, the extreme values  
247 of the parameters can only be reached for an infinite number of variables,  
248 some additional points located at the corners of the cube can be added to  
249 the sequence to give more weight to the boundary of the domain.

250 The key issue is thus the definition of the minimal number of experiments  
251 to be conducted to get a response with the desired accuracy. The number of  
252 simulations  $n$  depends on the complexity of the studied phenomenon as well  
253 as of the complexity of the approximated model. Yet there is not a unani-  
254 mously agreed method relating the number of observations versus the number  
255 of independent variables in the model. Some authors suggest  $3m + 1$  points  
256 [31] for a second-order polynomial approximation where  $m = \frac{(p+1)(p+2)}{2}$  is the  
257 number of unknown coefficients and  $p$  is the number of input variables. Fol-  
258 lowing this suggestion for the present example which has 3 input parameters,  
259 a second order polynomial approximation will have 10 unknown parameters  
260 which could be evaluated with a good accuracy with 31 experiments. Adding  
261 the corner points (in total 7 additional points as the point  $(-1; -1; -1)$  is  
262 by construction always part of the sequence) of the investigated domain, the  
263 total number of sampling points is set to 38. An illustration of such a set is  
264 shown in Fig. 5.

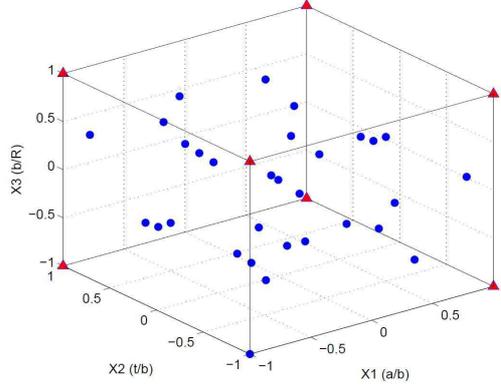


Figure 5: Sampling points generated by the Sobol' sequence (circles) and additional corner points (triangles).

265 **3.4. Step 4: Parameters evaluation**

266 The approximated response surface is here looked for in the form of a  
 267 second-order polynomial in the three variables  $X_i$  given by Eq. 6. The un-  
 268 known parameters  $\beta_{(\cdot)}$  have to be identified from the numerical experiments  
 269 (here  $n = 38$ ) which is here done by the least square method. So, from the  
 270 38 sampling points shown in Fig. 5, the ultimate strength of a curved plate  
 271 can be approximated by the following expression:

$$\begin{aligned} \hat{Y} = & 0.879 + 0.002X_1 + 0.212X_2 + 0.052X_3 - 0.001X_1X_2 - 0.063X_2X_3 \\ & - 0.004X_3X_1 - 0.037X_1^2 - 0.100X_2^2 - 0.003X_3^2 \end{aligned} \quad (7)$$

272 **3.5. Step 5: Evaluation of the accuracy of approximated model**

273 To evaluate the accuracy of the approximated model, conducting an  
 274 analysis of variance (ANOVA) is very useful. The coefficient of determina-  
 275 tion is first determined  $R^2 = 0.977$  and then the cross-validation coefficient  
 276  $Q^2 = 0.969$ . The fact that  $R^2$  is very close to 1 indicates that the regression

277 model fits well the data. The small difference between  $R^2$  and  $Q^2$  indicates  
278 that most observations have an influence on the regression equation and that  
279 the approximation model predicts well the observations. Moreover, the same  
280 model has been identified on a sample without additional points in the cor-  
281 ners, leading to  $R^2 = 0.971$  and  $Q^2 = 0.942$ , the small diminution of these  
282 coefficients is a direct consequence of the diminution of the number of sam-  
283 pling points (31 instead of 38), not to the fact that the discarded points were  
284 located in the corners of the domain. It is thus concluded that, the addi-  
285 tion of corner points is not necessary to get an accurate estimation of the  
286 regression coefficients.

287 *3.6. Step 6: Selection of most significant terms and conclusion.*

288 The second-order formula presented in Eq. 7 for the evaluation of the  
289 ultimate strength of curved steel panels under axial compression provides  
290 a good and best possible approximation of the real capacity of the panel.  
291 However, it is remarked that not every coefficient in Eq. 7 have the same  
292 order of magnitude. So, rather than trying to explain the model with all its  
293 terms, it can naturally be asked if some terms could be excluded from the  
294 initial model without altering significantly the accuracy of the whole model.

295 A criterion of exclusion should hence be fixed. If normality assumptions  
296 are verified (as in the present case), the t-test provides a fully reliable criterion  
297 as it relates the value of each coefficient to its estimated standard error.  
298 More simple criteria, such as arbitrary thresholds of significance are also  
299 very effective. Indeed, as the parameters all vary between -1 and 1, the  
300 contributions of the various terms can directly be analysed by comparing the  
301 coefficients which might then be neglected if their value is bellow a certain

302 absolute value (for example 2% of the sum of the coefficients absolute values  
 303 or 5% of the maximum value of the coefficients). Fixing here this threshold to  
 304 2 % of the sum of the coefficients, the terms ( $\beta_1, \beta_{12}, \beta_{13}$  and  $\beta_{33}$ ) are found  
 305 not significant (a criterion based on t-test and a 90% two sided interval would  
 306 give the same results). The new model is thus given by Eq. 8; it preserves  
 307 good precision with high value of  $R^2 = 0.976$  and  $Q^2 = 0.9674$ .

$$\widehat{Y} = 0.879 + 0.212X_2 + 0.052X_3 - 0.063X_2X_3 - 0.037X_1^2 - 0.100X_2^2 \quad (8)$$

308 Introducing the physical parameters of table 1 into Eq. 8, the ultimate  
 309 strength of unstiffened cylindrical curved panels under axial compression is  
 310 given by:

$$\frac{\sigma_{ult}}{f_y} = (-0.09 + 0.326(a/b) - 0.148(a/b)^2) + (40.6 + 0.314Z)(t_p/b) - (444 + 8.40Z)(t_p/b)^2 \quad (9)$$

311 where  $Z$  is the curvature parameter defined by  $Z = b^2/Rt_p$ . Eq. 9 is very  
 312 similar to the classical expression of the stability problem, where the ultimate  
 313 strength is represented as a polynomial function of the slenderness  $t/b$ .

314 Fig. 6 shows how well the expressions given by the DOE method (red  
 315 squares) and by the semi-analytical method [12] (green triangles) are able to  
 316 predict the numerical results (given by F.E. model). For most input values,  
 317 the two models have less than 5 % of discrepancy (in absolute value) to  
 318 the true numerical value. However a few observations (No. 19, No. 22 and  
 319 No. 28) predicted by DOE method have higher discrepancy (from 8 % to  
 320 10 %) on the contrary to the semi-analytical method whose error remain  
 321 below 5.5 %. This might be explained by the fact that, although the semi-  
 322 analytical model is not richer (the calibration of  $\overline{\lambda}_0, \beta, \alpha_Z$  involves only 7

323 parameters), its physical bases are finer which proves the crucial importance  
 324 of the choice of the parameters and response surface.

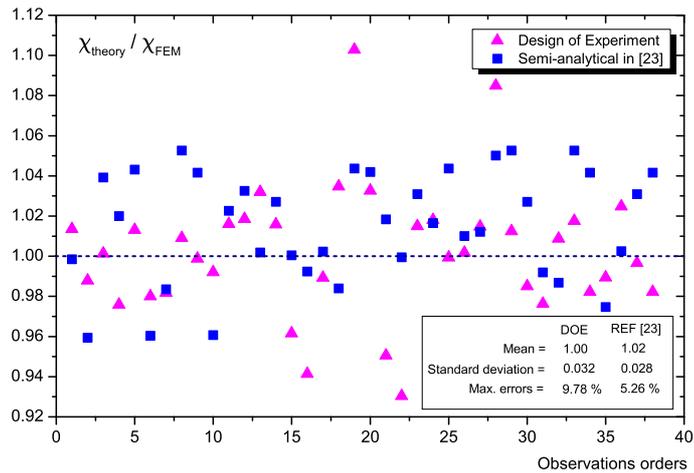


Figure 6: Comparison of FEM results with DoE and semi-analytical method [12].

325 These values of error should be relativised by the fact that in many com-  
 326 plex structures such as stiffened plates, the difference between Eurocode pre-  
 327 dictions and the results of numerical simulations might reach until 20 %,   
 328 sometimes in favor of safety, sometimes not [32, 33]. Moreover, as the so-  
 329 called characteristic value of a member is obtained by dividing its design  
 330 value (e.g. Eq. 9) by some safety factor (often taken as  $\gamma_{M1} = 1.1$ ) the  
 331 present discrepancy is indeed acceptable.

332 **4. Application to stiffened curved panels under uniform axial com-**  
 333 **pression**

334 The behaviour of stiffened curved panels is a more complex problem,  
 335 especially due to the interaction of different parameters (curvature, relative  
 336 rigidity of stiffeners and plate, imperfection, etc.) for which no semi-analytic  
 337 expression exists. So, as it has just been shown that the design of computer  
 338 experiment method is well adapted for studying the stability of curved plates,  
 339 it will be used for the development of a preliminary design formula (i.e suited  
 340 for hand-calculation) for the ultimate resistance of stiffened curved panels.

341 *4.1. Finite element modelling*

342 The stiffened panels are modelled and analysed using the commercial  
 343 finite element software Ansys [29]. The panels are supposed to be simply  
 344 supported on all edges of the panel ( $u_r = 0$  in the cylinder coordinate system  
 345 of Fig. 7) but not on the stiffeners (unfavourable condition).

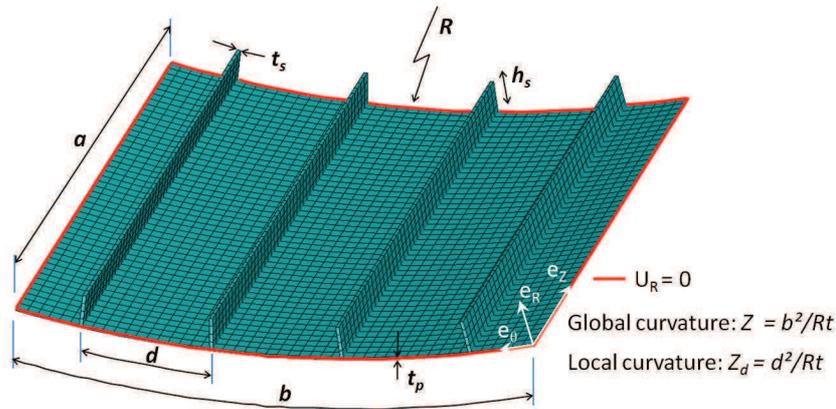


Figure 7: Boundary conditions of simply supported on all edges

346 For loading conditions, the study is here limited to a uniform compression  
 347 in the longitudinal direction as it is the dominant loading in bottom flange  
 348 panels. It is applied not only to the main panel, but also to the stiffeners due  
 349 to their participation in the overall behaviour of the structure (Fig. 8). In  
 350 fact, in a bridge, the compressive forces acting on the flange come through  
 351 the diaphragms and webs that connect the upper and lower panels of the  
 352 box girder. By construction, the stiffeners, in most cases, are continuous and  
 353 attached by welding to diaphragms: therefore they are also subjected to the  
 354 compressive load.

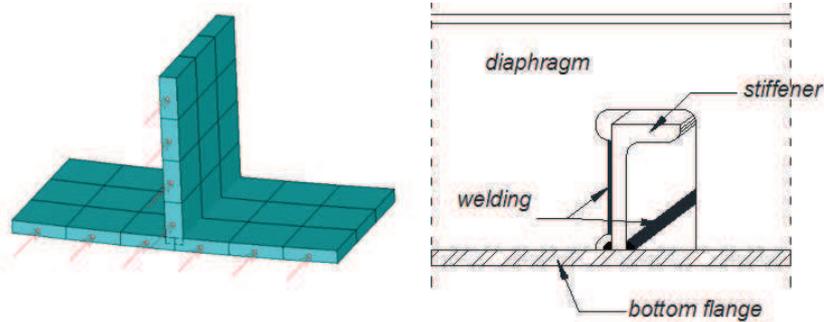


Figure 8: Loading condition and scheme of the connection stiffener/diaphragm by welding

355 The curved panels are meshed with eight-nodes shell elements which use  
 356 an advanced shell formulation that accurately incorporates initial curvature  
 357 effects (this element is called SHELL-281 in [29]). They are well-suited for  
 358 linear, large rotation and large strain non-linear applications and offers im-  
 359 proved accuracy in curved shell structure simulations and faster convergence  
 360 than plate elements as one can see in figure 9 which represents the conver-  
 361 gence study from [34]. A fine mesh with more than 30 elements per panel  
 362 edges is used to reduce the discretisation error.

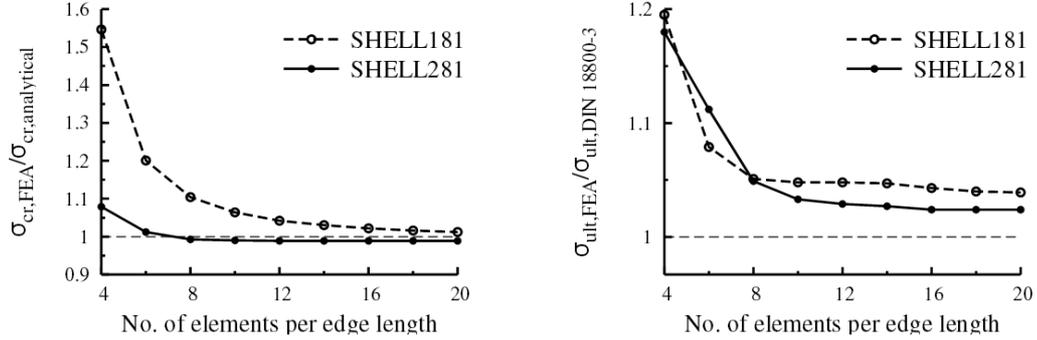


Figure 9: Convergence study of SHELL-281 element: linear bifurcation analysis (left), non linear buckling analysis (right)

363 The panels are all made of steel which is assumed to be elasto-plastic  
 364 with linear strain hardening as indicated in EN 1993-1-5 C.6 for the material  
 365 non-linear second-order analyses with initial imperfections (GMNIA). The  
 366 Young modulus  $E$  and Poisson's ratio are taken equal to  $210\text{ GPa}$  and  $0.3$   
 367 respectively. The steel grade is S355 with a yield steels equal to  $355\text{ MPa}$ .

#### 368 4.2. Evaluation of the ultimate strength

369 This study is limited to the case of stiffened curved panels under ax-  
 370 ial compression with open section stiffeners (simple flat plates) because the  
 371 curvature makes it difficult to realise a close form section of stiffener (boxed  
 372 rib). Therefore, the number of input parameters is here restricted to seven as  
 373 presented in table 2. The ranges of variation of these parameters are chosen  
 374 in order to cover most panels used in bridge construction. As the orders of  
 375 magnitude of the parameters variations are different, they are transformed  
 376 the physical parameters into centred variables  $X_i$ , ranging from  $-1$  to  $1$ .

377

Table 2: Design variable (dimension in meter)

Variable	Description	Variation range	Design variable
$a$	Length of the panel	$4 \leq a \leq 6$	$X_1 = a - 5$
$b$	Width of the panel	$4 \leq b \leq 6$	$X_2 = b - 5$
$t_p$	Thickness of panel	$0.01 \leq t_p \leq 0.02$	$X_3 = 200 \cdot t_p - 3$
$1/R$	Curvature of panel	$0 \leq 1/R \leq 0.1$	$X_4 = 20/R - 1$
$d$	Distance between stiffeners	$0.3 \leq d \leq 0.8$	$X_5 = 4 \cdot d - 2.2$
$h_s$	Height of stiffener	$0.1 \leq h_s \leq 0.2$	$X_6 = 20 \cdot h_s - 3$
$t_s$	Thickness of stiffener	$0.01 \leq t_s \leq 0.02$	$X_7 = 200 \cdot t_s - 3$

378 The approximated model is searched in the form of a second order poly-  
379 nomial. The total number of experiments, as suggested in section 3.3, is  
380  $n = 3 \cdot m + 1 = 109$  where  $m = 36$  is the number of unknown coefficients  
381 (1 constant, 7 linear and 28 quadratic terms). Their distribution in the re-  
382 gion of interest is generated by a Sobol' sequences and the coefficients are  
383 obtained by the least square method, supposing that errors are independent  
384 and normally distributed. Like previously, the selection of significant coeffi-  
385 cients is made based on the t-test (the limit value being given for 109 tests  
386 and the bilateral 5%-95% fractile). Then all remaining coefficients (here 18  
387 coefficients) are re-evaluated using the least square method a second time.  
388 The resulting approximated model (in MN) is thus the following:

$$\begin{aligned}
\hat{Y} = & +17.09 & (10) \\
& -0.47X_1 + 3.58X_2 + 4.24X_3 + 7.32X_4 - 3.87X_5 + 4.83X_6 + 2.33X_7 \\
& +1.65X_2X_4 - 1.72X_2X_5 + 1.71X_2X_6 + 0.89X_2X_7 + 1.33X_3X_4 \\
& -0.76X_3X_5 - 1.73X_4X_5 + 0.81X_4X_6 - 1.18X_5X_6 + 0.94X_6X_7
\end{aligned}$$

389 The coefficient of determination  $R^2 = 0.986$  (close to 1) indicates that  
 390 the value predicted by the model fits very well the data (see Fig. 10). Also,  
 391 the small difference between the values of  $Q^2 = 0.978$  and  $R^2$  shows that  
 392 there are few undue influence on the regression equation. The accuracy of  
 393 the response is also checked by the relative mean absolute error (RMAE):  
 394 4.2 % (which indicated that a mean error of 4 % is expected), the relative  
 395 minimum and maximum error:  $-14.0$  % and  $+13.5$  % respectively. The  
 396 above equation is thus fairly acceptable in bridge constructions and residual  
 397 errors can be easily covered by using a safety factor.

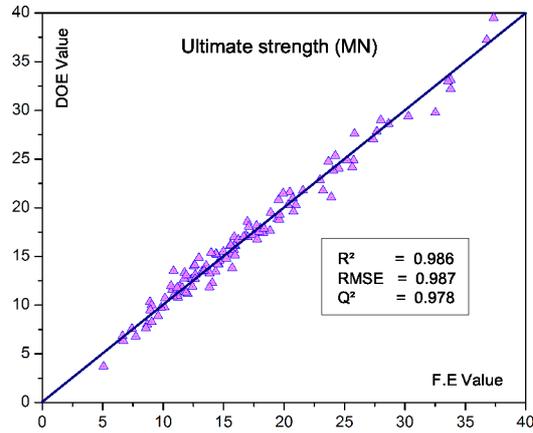


Figure 10: Comparison of the ultimate strength of the FEM and DOE

398 Considering now every coefficients independently, it is found that the 5 %  
 399 two-sided confidence interval of the regression coefficients is  $\pm 0.20$  MN for  
 400 the constant term,  $\pm 0.36$  MN for the linear terms and  $\pm 0.62$  MN for the  
 401 quadratic terms. As generally observed, the uncertainty on linear terms is  
 402 almost twice smaller than that on quadratic terms: the direct influences of  
 403 the parameters are better known than those of their interactions.

404 These statistical remarks being made, it is remarkable that all parameters  
 405 are found significant in Eq. 10. In decreasing order, the most significant  
 406 parameters are the curvature ( $X_4$ ), the height of the stiffeners ( $X_6$ ), the  
 407 thickness of the panel ( $X_3$ ), the distance between stiffeners ( $X_5$ ), the width  
 408 of the panel ( $X_2$ ), the thickness of the stiffeners ( $X_7$ ) and finally the length  
 409 of the panel ( $X_1$ ) whose influence is very limited (no more than  $\pm 3$  % of the  
 410 total strength). Quite obviously, increasing the curvature, the thickness of  
 411 the plate or the height and thickness of the stiffeners increases the strength  
 412 of the panel, while increasing the length of the panel or the distance between  
 413 stiffeners decreases it. Then the fact that the strength grows with the width  
 414 is not so immediate but can be easily understood considering that when the  
 415 width of the panel increases, the distance between the centre of gravity of  
 416 the panel and the curved plate increases due to curvature and by there the  
 417 global inertia of the curved panel increases.

418 There are then multiple interactions which combine effects are more dif-  
 419 ficult to analyse. Indeed increasing the curvature, the height and thickness  
 420 of the stiffeners or the thickness of the panel has always a positive effect  
 421 on the strength because, for these parameters, the linear term dominates  
 422 clearly the quadratic terms. Then concerning the distance between the stiff-  
 423 eners, in most cases diminishing it leads to an increase of the strength but  
 424 not mandatory as for slender panels with small curvature and small stiff-  
 425 eners it might lead to a smaller strength (indeed the coefficient of  $X_5$  is  
 426  $-3.87 - 1.72X_2 - 0.76X_3 - 1.73X_4 - 1.18X_6$  and varies between  $-9.26 MN$   
 427 and  $1.52 MN$ ). In the same way, in most cases increasing the width of the  
 428 panel leads to an increased strength but not for slender panels when the spac-

429 ing between stiffeners is too large (the coefficient of  $X_2$  being  $3.58 + 1.65X_4 -$   
 430  $1.72X_5 + 1.71X_6 + 0.89X_7$ , it varies between  $-2.39 MN$  and  $9.55 MN$ ). Very  
 431 likely in the last two cases, these changes of the coefficient sign correspond to  
 432 a change in the buckling mode from column to plate or from global to local.

433 Anyway, it must be recalled that even in these extreme cases, the error in  
 434 the prediction of the strength is not larger than in other cases (cf. Fig. 10) and  
 435 that according to all statistical criteria mentioned above, the approximated  
 436 model given by expression (10)) is able to predict correctly the ultimate  
 437 strength of stiffened curved panels in the interested domain. It can thus be  
 438 easily inserted in an optimisation pattern as illustrated in the coming section.

#### 439 4.3. Cost optimisation of curved stiffened panels

440 The cost optimisation scheme proposed here is based on a cost objective  
 441 function similar to the ones used by [35]. It assumes that the manufacturing  
 442 cost of a stiffened curved panel  $K$  defined by the parameters  $X_i$  is the sum of  
 443 the material costs  $K_m$  (the steel cost) and of the fabrication costs  $K_f$  which  
 444 can be defined as follow:

$$K(X_i) = K_m + K_f = k_m \rho V + k_f \sum T_i \quad (11)$$

445 where  $\rho$  is the steel density,  $V$  is the total volume of the curved panel,  $k_m$  and  
 446  $k_f$  are characteristic coefficients of material and fabrication costs.  $T_i$  denotes  
 447 manufacturing times:

- 448 •  $T_1$ : time for preparing, cutting and assembling the pieces:

$$T_1 = \Theta_d \sqrt{\kappa \rho V} \quad (12)$$

449 with  $\Theta_d$  a factor characterising the impediment for welding and  $\kappa$  the  
 450 number of elementary pieces to be welded;

451 •  $T_2$ : time for welding and  $T_3$ : additional time for maintenance of the  
 452 machine which might be considered as  $0.3T_2$ , so that:

$$T_2 + T_3 = 1.3 \sum C_i a_{wi}^2 L_{wi} \quad (13)$$

453 where  $L_{wi}$  is the length of the  $i^{\text{th}}$  weld,  $a_{wi} = \max(0.4t_s, 4mm)$  its width  
 454 and  $C_i$  a coefficient depending on the welding technique which is here  
 455 taken equal to 0.2349 for Shielded Metal Arc Welding.

456 The constraint equation is then given by the stability requirement of the  
 457 panel:

$$g(X_i) = \frac{N_{app}}{N_{ult}/\gamma_{M1}} - 1 \leq 0 \quad (14)$$

458 where  $N_{app}$  is the applied load,  $N_{ult}$  the capacity of the panel estimated by  
 459 Eq. (10) and  $\gamma_{M1}$  is a safety factor.

460 The panel which is proposed here for optimisation has fixed overall di-  
 461 mensions: its length  $a$  is 6  $m$ , its width  $b$  is 4  $m$  and its curvature radius  
 462  $R$  is 20  $m$ ). It is subjected to a uniform axial compression  $N_{app} = 12 MN$ .  
 463 The objective is thus to determine the parameters (thickness of the panel  $t_p$ ,  
 464 thickness  $t_s$  and height  $h_s$  of the stiffeners and distance between stiffeners  $d$ )  
 465 which will minimize the cost of the panel (11) and verify the constraint equa-  
 466 tion (14). To make the problem more realistic, it is also considered that the  
 467 variables are not continuous but discrete (which poses no problem to Matlab  
 468 optimisation algorithm), so that the solution is looked for in the following  
 469 domain:

- 470 •  $t_p \in [0.01; 0.02]$  by steps of 1  $mm$ ;
- 471 •  $d \in [0.3; 0.8]$  by steps of 5  $cm$ ;

472 •  $h_s \in [0.1; 0.2]$  by steps of 1 *cm*;

473 •  $t_s \in [0.01; 0.02]$  by steps of 1 *mm*;

474 Concerning then the definition of cost coefficients, as no precise data were  
475 available for  $k_f$  and  $k_m$ , it was decided to present the results in an adimen-  
476 sional form considering different values of the ratio  $k_f/k_m$ . For  $k_f/k_m = 0$ ,  
477 only material cost is taken into account, while for large values of  $k_f/k_m$ , man-  
478 ufacturing cost prevail (reasonable values in northern countries lie between  
479 1 and 2). The results of the optimisation procedure are shown in table 4.3  
480 (where  $n$  is the total number of stiffeners).

Table 3: Results of the optimisation procedure for  $N_{app} = 12$  MN

$k_f/k_m$	$t_p$	$d$	$n$	$h_s$	$t_s$	$K$
0.0	0.014	0.55	7	0.16	0.015	3450
0.5	0.015	0.75	5	0.18	0.016	4250
1.0	0.016	0.90	4	0.19	0.017	5000
2.0	0.018	0.90	4	0.17	0.015	6200

481 About the method first, it must be noticed here that the set of optimised  
482 parameters corresponding to each ratio  $k_f/k_m$  was obtained almost imme-  
483 diately thanks to the preliminary design formula developed in section 4.2  
484 whereas it would have taken hours or even days using directly finite element  
485 simulations. Concerning the results then, following remarks can be drawn:

486 • The number of stiffeners is higher when only material costs are con-  
487 sidered. It is however not maximal ( $n_{max} = 12$ ) which shows that

488 increasing reasonably the thickness of the plate is very efficient from a  
489 weight point of view.

490 • For higher values of  $k_f/k_m$ , stiffeners becomes logically stiffer to reduce  
491 their number and the number of welds.

492 • It is often more economical to increase the panel thickness than to  
493 increase the number of stiffeners which confirms the conclusion of [33].

## 494 5. Conclusion

495 Stiffened curved panels in civil engineering structures have high sensitivity  
496 to instability phenomenon. Analytical or semi-analytical studies are often  
497 not feasible as the problem depends on many parameters such as the panel's  
498 curvature or the panel configuration with its stiffener and semi-rigid supports.  
499 There is hence a need for a robust strategy when attempting to develop  
500 approximated models for such problems. The proposition of such a strategy  
501 was the aim of the first part of the present paper and this, through a turnkey  
502 methodology based on the theory of the design of experiment method. The  
503 efficiency of the method was first reviewed. Some particular points which  
504 differentiate the ordinary physical experiments from computer experiments  
505 were discussed. Afterwards this methodology was applied to the case of  
506 unstiffened curved panels for which solutions were already available in order  
507 to evaluate the accuracy of the method and its relevance. A huge gain of time  
508 was noticed when using the DOE method: only 38 simulations were needed  
509 in the first application against 524 observations in [12] for determining the  
510 capacity of a curved panel. Also the general accuracy of the model in the

511 form of a second-order polynomial was comparable to that obtained with  
512 more standard heuristic methods. Moreover, as the experiment designer had  
513 existing knowledge of the problem, the input values were adequately chosen  
514 and the physical interpretation of the results was easy and satisfying, despite  
515 the simplicity of the model. The strategy proposed here provides thus a  
516 reliable alternative method for the prediction of the ultimate strength of  
517 curved panels.

518 Confident in the methodology, the authors then developed a fully reliable  
519 formula for preliminary design of stiffened curved panels. The accuracy of  
520 the formula was demonstrated and the influence of various design parameters  
521 was discussed. A simple cost optimisation problem was finally presented to  
522 illustrate the potential of the formula.

### 523 **Acknowledgement**

524 The authors would like to acknowledge here Prof. B. Sudret from ETH  
525 Zürich for the kind advices that he provided during the maturation of this  
526 paper.

527 **Nomenclature**

528	$\alpha_Z$	Parameter characterising the imperfections sensitivity
529	$\beta$	Parameter characterising the asymptotic behaviour of the panel
530	$\beta_0$	Constant term and average value of the approximated response
531	$\beta_i$	Coefficient characterising the effect of the variable $X_i$
532	$\beta_{ij}$	Coefficient characterising the interaction of the variables $X_i$ and $X_j$
533	$\chi$	Reduction factor for the panel buckling according to EC3
534	$\bar{\lambda}$	Relative slenderness of the panel according to EC3
535	$\bar{\lambda}_0$	Slenderness separating plastic buckling from elasto-plastic buckling
536	$\rho$	Steel density
537	$\sigma_{ult}$	Ultimate strength of the panel
538	$a$	Length of the panel
539	$b$	Width of the panel
540	$d$	Distance between stiffeners
541	$f_y$	Yield stress of the panel
542	$h_s$	Height of stiffeners
543	$K_f$	Fabrication costs
544	$k_f$	Fabrication cost per volume unit

545	$K_m$	Material costs (steel cost)
546	$k_m$	Material cost per volume unit
547	$k_{a/b}$	Parameter characterising the influence of the aspect ratio
548	$m$	Number of unknown coefficients in the approximated model
549	$n$	Number of simulations or numerical experiments
550	$N_{app}$	Normal force applied to the panel
551	$N_{ult}$	Capacity of the panel
552	$p$	Number of input variables
553	$R$	Curvature radius of the panel
554	$T_i$	Manufacturing time of the $i^{th}$ operation
555	$t_p$	Thickness of the panel
556	$t_s$	thickness of stiffeners
557	$V$	Total volume of the curved panel
558	$X_i$	Generic name of the $i^{th}$ input variable
559	$X_i^j$	$j^{th}$ value of the $i^{th}$ input variable
560	$Y, \hat{Y}$	Response and approximated response
561	$Y^j$	$j^{th}$ value of the response
562	$Z$	Curvature parameter defined by $Z = b^2/Rt_p$

563 **References**

- 564 [1] Lundquist E. Comparison of three methods for calculating the compres-  
565 sive strength of flat and slightly curved sheet and stiffener combinations.  
566 Tech. Rep.; Nat. Advisory Committee for Aeronautics, TC 455; 1933.
- 567 [2] EN1993-1-5 . Eurocode 3 design of steel structures part 1-5: Plated  
568 structural elements. 2007.
- 569 [3] EN1993-1-6 . Eurocode 3 design of steel structures part 1-6: Strength  
570 and stability of shell structures. 2007.
- 571 [4] Batdorf S, Schildcrout M. Critical axial-compressive stress of a curved  
572 rectangular panel with a central chordwise stiffener. Tech. Rep.; 1948.
- 573 [5] Schildcrout M, Stein M. Critical axial-compressive stress of a curved  
574 rectangular panel with a central longitudinal stiffener. Tech. Rep.;  
575 NACA Technical Note 1879; 1949.
- 576 [6] Becker H. Handbook of structural stability. Part VI: Strength of stiffened  
577 curved plates and shells. Tech. Rep.; New York University, Washington;  
578 1958.
- 579 [7] Gall H. Compressive strength of stiffened sheets of aluminum alloy.  
580 Ph.D. thesis; Massachusetts Institute of Technology; 1930.
- 581 [8] Ramberg W, Levy S, Fienup K. Effect of curvature on strength of  
582 axially loaded sheet-stringer panels. Tech. Rep.; NACA-Technical note  
583 944; 1944.

- 584 [9] Cho S, Park H, Kim H, Seo J. Experimental and numerical investigations  
585 on the ultimate strength of curved stiffened plates. In: Proc. 10<sup>th</sup> Int.  
586 Symp. Practical design of ships and oth. floating str. 2007,.
- 587 [10] Khedmati M, Edalat P. A numerical investigation into the effects of  
588 parabolic curvature on the buckling strength and behaviour of stiffened  
589 plates under in-plane compression. Latin American J of Solids and Str  
590 2010;7(3).
- 591 [11] Park J, Iijima K, Yao T. Estimation of buckling and collapse behaviours  
592 of stiffened curved plates under compressive load. International Society  
593 of Offshore and Polar Engineers, USA; 2008,.
- 594 [12] Tran K, Davaine L, Douthe C, Sab K. Stability of curved panels under  
595 uniform axial compression. Journal of Constructional Steel Research  
596 2012;69(1):30–8.
- 597 [13] Martins J, da Silva LS, Reis A. Eigenvalue analysis of cylindrically  
598 curved panels under compressive stresses - extension of rules from {EN}  
599 1993-1-5. Thin-Walled Structures 2013;68:183 –94.
- 600 [14] Fisher R. The design of experiments. Edinburgh, Scotland: Oliver and  
601 Boyd; 1935.
- 602 [15] Kleijnen J. Design and analysis of computational experiments:  
603 Overview. Experimental Methods for the Analysis of Optimization Al-  
604 gorithms 2010;:51–77.
- 605 [16] Taguchi G, Chowdhury S, Wu Y. Taguchi's quality engineering hand-  
606 book. John Wiley and Sons Hoboken, NJ; 2005.

- 607 [17] Sacks J, Welch W, Mitchell T, Wynn H. Design and analysis of computer  
608 experiments. *Statistical science* 1989;4(4):409–23.
- 609 [18] Jourdan A. Design of numerical experiments. *Département de*  
610 *Mathématiques - Revue MODULAD* 2005;(in French).
- 611 [19] Santner T, Williams B, Notz W. The design and analysis of computer  
612 experiments. Springer Verlag; 2003.
- 613 [20] Montgomery D. Design and analysis of experiments. John Wiley and  
614 Sons Inc; 2008.
- 615 [21] Franco J. Design of numerical experiments in exploratory stage for com-  
616 plex phenomena. Ph.D. thesis; Ecole Nationale Supérieure des Mines;  
617 2008. (in French).
- 618 [22] Blatman G, Sudret B, Berveiller M. Quasi random numbers in stochastic  
619 finite element analysis. *Mechanics and Industries* 2007;8(3):289–97.
- 620 [23] Krykova I. Evaluating of path-dependent securities with low discrepancy  
621 methods. Ph.D. thesis; Worcester Polytechnic Institute; 2003.
- 622 [24] Simpson T, Poplinski J, Koch P, Allen J. Metamodels for computer-  
623 based engineering design: survey and recommendations. *Engineering*  
624 *with computers* 2001;17(2):129–50.
- 625 [25] Simpson T, Lin D, Chen W. Sampling strategies for computer exper-  
626 iments: design and analysis. *International Journal of Reliability and*  
627 *Applications* 2001;2(3):209–40.

- 628 [26] Allen T, Bernshteyn M, Kabiri-Bamoradin K. Constructing metamodels  
629 for computer experiments. *J Quality Technology* 2003;35:264–74.
- 630 [27] Zheng Y, Das PK. Improved response surface method and its appli-  
631 cation to stiffened plate reliability analysis. *Engineering Structures*  
632 2000;22(5):544–51.
- 633 [28] Roux W, Stander N, Haftka R. Response surface approximations for  
634 structural optimization. *International Journal for Numerical Methods*  
635 *in Engineering* 1998;42(3):517–34.
- 636 [29] ANSYS . User’s theory manual v13. 2009.
- 637 [30] Winter G. Strength of thin steel compression flanges. *Transaction ASCE*  
638 1947;112:527–54.
- 639 [31] Bowman K, Sacks J, Chang Y. Design and analysis of numerical exper-  
640 iments. *Journal of the atmospheric sciences* 1993;50(9):1267–78.
- 641 [32] André I, Degée H, De Ville De Goyet V, Maquoi R. Effect of initial  
642 imperfections in numerical simulations of collapse behaviour of stiffened  
643 plates under compression. In: *Proceedings of the Third European Con-*  
644 *ference on Steel Structures*. 2002, p. 503–12.
- 645 [33] De Ville De Goyet V, Maquoi R, Bachy F, André I. Ultimate load of  
646 stiffened compressed plates: Effects of some parameters and discussion  
647 concerning the ec3 rules. In: *Proceedings of the Third European Con-*  
648 *ference on Steel Structures*; vol. 1. 2002, p. 591–600.

- 649 [34] Braun B. Stability of steel plates under combined loading. Ph.D. thesis;  
650 University of Stuttgart; 2010.
- 651 [35] Jarmai K, Snyman J, Farkas J. Minimum cost design of a welded  
652 orthogonally stiffened cylindrical shell. Computers & Structures  
653 2006;84(12):787 –97.