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Ductile fracture modeling considering plasticity and damage evolution combined with the strong discontinuity method

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Abstract. *In this paper we present a modeling tool used for representing ductile fracture. A coupled damage-plasticity model has been implemented. The model induces a softening behavior and the strong discontinuity method was implemented as a regularization method. In addition, a linear displacement jump has been introduced in a four node quadrilateral finite element.*

Keywords: ductile fracture; damage, strong discontinuity.

1 INTRODUCTION

Ductile materials are characterized by a severe non-linear behavior that induces two distinct phases of the global response to failure analysis : the pre-peak phase in which plasticity and damage are evolving, and the softening post-peak phase leading to a macro-crack growth responsible for the failure of the structure. The Lemaitre model[1], in which plasticity and damage are evolving, is used to represent the different dissipative mechanisms during the pre-peak phase. With regard to the post-peak phase, regularization is carried out using the strong discontinuity method whose efficiency has been demonstrated in the description of brittle failure [2]. Most works on strong discontinuity analysis consider a constant jump along the discontinuity surface within an element and this leads, in some cases, to the well known "locking phenomenon". Furthermore, this hypothesis does not allow the representation of a potential heterogeneous stress state within an element. In order to improve the description of a displacement jump that enables the objective characterization of the post-peak phase, a four node quadrangular element enhanced with a linear displacement jump is considered [3]. The determination of the local unknown variables is carried out by the resolution of the discretized weak form of the equation of continuity of tractions, at the element level. The global resolution is performed by performing a few modifications of the elementary tangent stiffness matrix in order to consider the evolution of the discontinuity surface. Another contribution of this work is the consideration of a coupled damage-elastoplastic model often used in the characterization of ductile fracture but never combined with the strong discontinuity method that enables the regularization of the post-peak phase. This model enables the discontinuity introduction criterion to be based on a physical consideration such as a damage threshold. The present work is limited to the study of quasi-static loadings assuming both plane strain and axisymmetric hypothesis. The regularization aspect of the method and the benefit of the linear jump on the discontinuity are proved with a numerical three-point bending test.

2 THEORETICAL FORMULATION

In this section, we briefly present the models considered to represent both pre-peak and post-peak parts of the material behavior, this means both bulk model and surface model used on the discontinuity.

2.1 Bulk material model : Lemaitre damage model

With the aim of representing the pre-peak phase of the material behavior, we consider the coupled damage-plasticity Lemaitre model [1] widely used in the literature to model ductile metal failure. We limit the study to the consideration of isotropic damage (the scalar variable D is the isotropic damage) and isotropic hardening ($\bar{\xi}$ represents the accumulated plastic strain and \bar{q} its dual variable). In this case, the yield function is very close to the Von Mises yield function constitutive model taking into account the effective stress $\tilde{\sigma} = \sigma/(1 - D)$, and is written as follow :

$$\bar{\phi}(\sigma, D, \bar{q}) = \sqrt{\frac{3}{2} dev(\tilde{\sigma}) : dev(\tilde{\sigma}) - (\sigma_{y_0} + \bar{q}(\bar{\xi}))} \quad (1)$$

where σ_{y_0} is the elastic limit, \bar{q} is the hardening variable associated to the accumulated plastic strain $\bar{\xi}$. In the Lemaitre model, the damage evolution is written as follow :

$$\dot{D} = \frac{\dot{\gamma}}{1 - D} \left(\frac{-Y}{S} \right)^r \quad (2)$$

where Y is the thermodynamic force associated to the damage variable D , $\dot{\gamma}$ is the plastic multiplier, and S and r are the two Lemaitre parameters that characterize damage evolution.

2.2 Surface material model : strong discontinuity

Because of the damage evolution, the above described model presents a softening phase that requires a specific treatment in order to ensure the objectivity of the solution. We choose to use the strong discontinuity method [6, 2] in which strain band localization is taken into account by introducing a discontinuity surface of the displacement jump.

The constitutive law chosen to represent the discontinuity surface behavior is a multi-surface damageable law [2]. The associated yield function takes the form :

$$\begin{cases} \bar{\phi}_n(\mathbf{t}_{\Gamma_s}, \bar{q}) = \mathbf{t}_{\Gamma_s} \cdot \mathbf{n} - (\bar{\sigma}_f - \bar{q}_n) \\ \bar{\phi}_m(\mathbf{t}_{\Gamma_s}, \bar{q}) = |\mathbf{t}_{\Gamma_s} \cdot \mathbf{m}| - (\bar{\sigma}_s - \bar{q}_m) \end{cases} \quad (3)$$

where \mathbf{n} and \mathbf{m} represent, respectively, the unit normal and tangential vectors to the discontinuity surface.

3 NUMERICAL IMPLEMENTATION

Introducing a displacement field discontinuity requires an enhancement of the standard shape functions with incompatible shape functions enabling to interpolate the displacement jump on the discontinuity. The enhancement being based on the incompatible modes method, only the strains interpolations are modified. As specified above, we consider a quadrangular element with four nodes and a linear displacement jump. On the basis of the works of [3], the displacement jump is projected on a basis (n_0, n_1, m_0, m_1) that corresponds to the different modes of separation of a crack, in the linear case : a constant normal separation mode n_0 , a linear normal mode n_1 equivalent to the rotation of the discontinuity around the discontinuity midpoint \mathbf{x}_{Γ_s} , a constant tangential separation mode m_0 and a linear tangential separation mode m_1 .

The kinematics proposed (described in figure 1) allow to represent every possible separation modes in two dimensions, in the case of a linear displacement jump.

Derivation of the interpolated displacement field gives the following strain interpolation :

$$\begin{aligned} \epsilon^h(\mathbf{x}) = \sum_{a=1}^{a=4} \mathbf{B}_a(\mathbf{x}) \mathbf{u}_a & - \left[\sum_{a \in \Omega^+} \mathbf{B}_a(\mathbf{x}) - \mathbf{N} \right] [\alpha_{n0}, \alpha_{m0}]^T \\ & + \alpha_{n1} \left[- \sum_{a \in \Omega^+} \mathbf{B}_a(\mathbf{x}) \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} (\mathbf{x}_a - \mathbf{x}_{\Gamma_s}) + \xi_{\Gamma_s} \mathbf{N} \cdot \mathbf{n} \delta_{\Gamma_s} \right] \\ & + \alpha_{m1} \left[- \sum_{a \in \Omega^+} \mathbf{B}_a(\mathbf{x}) ((\mathbf{x}_a - \mathbf{x}_{\Gamma_s}) \cdot \mathbf{m}) \mathbf{m} + \mathbf{M} \mathbf{m} \mathcal{H}_{\Gamma_s} + ((\mathbf{x} - \mathbf{x}_{\Gamma_s}) \cdot \mathbf{m}) \mathbf{N} \mathbf{m} \delta_{\Gamma_s} \right] \end{aligned} \quad (4)$$

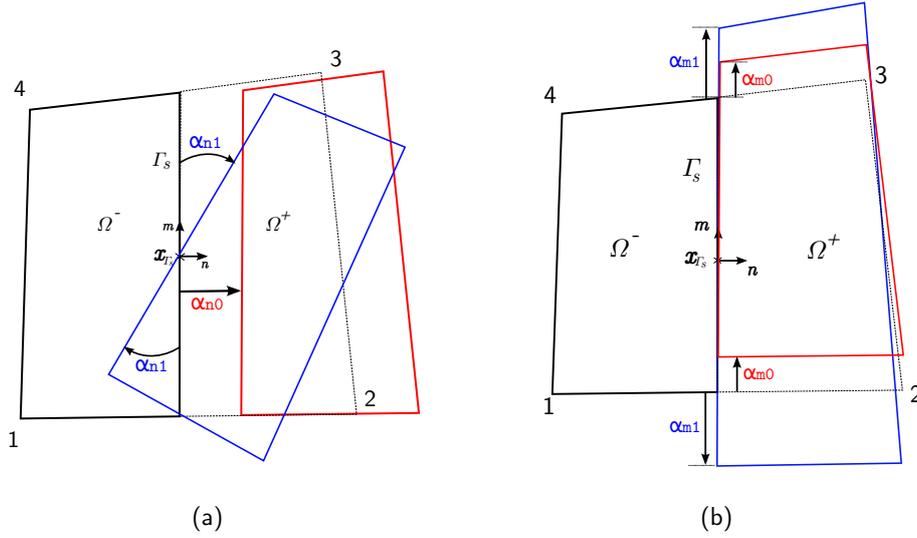


FIGURE 1: (a) constant (red) and linear (blue) normal modes, (b) constant (red) and linear (blue) tangential modes

where $\xi_{\Gamma_s} = (\mathbf{x} - \mathbf{x}_{\Gamma_s}) \cdot \mathbf{m}$ ($\mathbf{x} \in \Gamma_s$) represents the curvilinear abscissa along the discontinuity, $\mathbf{N} = \begin{bmatrix} \mathbf{n}_x & 0 \\ 0 & \mathbf{n}_y \\ \mathbf{n}_y & \mathbf{n}_x \end{bmatrix}$

(same as \mathbf{M}) and δ_{Γ_s} represents the Dirac distribution on the discontinuity surface.

Both real and virtual strain interpolations can be put in the general form :

$$\epsilon(\mathbf{x}, t) = \mathbf{B}\mathbf{u} + \bar{\mathbf{G}}_{\mathbf{r}}\boldsymbol{\alpha} + \bar{\bar{\mathbf{G}}}_{\mathbf{r}}\boldsymbol{\alpha}\delta_{\Gamma_s} \quad ; \quad \delta\epsilon(\mathbf{x}, t) = \mathbf{B}\delta\mathbf{u} + \bar{\mathbf{G}}_{\mathbf{v}}\delta\boldsymbol{\alpha} + \bar{\bar{\mathbf{G}}}_{\mathbf{v}}\delta\boldsymbol{\alpha}\delta_{\Gamma_s} \quad (5)$$

where $\boldsymbol{\alpha} = \langle \alpha_{n_0}, \alpha_{n_1}, \alpha_{m_0}, \alpha_{m_1} \rangle^T$, $\bar{\mathbf{G}}_{\mathbf{v}}$ and $\bar{\bar{\mathbf{G}}}_{\mathbf{v}}$ are based on $\bar{\mathbf{G}}_{\mathbf{r}}$ and $\bar{\bar{\mathbf{G}}}_{\mathbf{r}}$ (matrix forms of the interpolations written in (4)) in order to fulfil the patch-test.

Considering these chosen interpolations for both real and virtual strain fields, the discretized problem may be expressed in the form of the equations obtained using the incompatible modes method ([4]) :

$$\left\{ \begin{array}{l} \sum_{e=1}^{Nel} [\mathbf{f}^{int,e}(t) - \mathbf{f}^{ext,e}(t)] = \mathbf{0} \\ \mathbf{h}^e(t) = \int_{\Omega^e \setminus \Gamma_s} \bar{\mathbf{G}}_{\mathbf{v}}^T \boldsymbol{\sigma} d\Omega + \int_{\Gamma_s} \bar{\bar{\mathbf{G}}}_{\mathbf{v}}^T \boldsymbol{\sigma}|_{\Gamma_s} d\Gamma = \mathbf{0}, \quad \forall e \in [1, Nel] \end{array} \right. \quad (6)$$

The global equilibrium equation is completed by a local equilibrium equation written for each localized element, and corresponds to the projection on the different separation modes of the weak form of the continuity of tractions equation along Γ_s .

After linearisation and static condensation at elementary level, the problem resolution is reduced to the resolution of the standard equation :

$$\widehat{\mathbf{K}}^{(i)} \Delta \mathbf{d}^{(i)} = \mathbf{f}^{ext} - \mathbf{f}^{int(i)} \quad (7)$$

where $\widehat{\mathbf{K}}^{(i)}$ is the modified tangent stiffness matrix that takes into account the material behavior at the localization bands scale, in the "macro" level.

3.1 Numerical result

A three-point centered bending numerical test was undertaken on a notched specimen. Parameters for the bulk were calibrated for an AISI 1010 low carbon steel in a rolled state by Benallal et al. [7]. The model and the corresponding meshes used and the results in terms of force/displacement are presented in figure 2. The influence of the activation of the linear mode on the global response has been studied for two topologically identical meshes, but with different element sizes.

We note that results differ slightly when we take into consideration the linear mode, and that the peak is higher when only a constant displacement jump is considered. This may be due to the "locking phenomenon" that occurs in the

case of the constant jump alone. The interest in the linear mode is obvious for this test, since we observe that a stress concentration occurs near the notch that induces a heterogeneous stress state for the neighboring elements. Thus, a linear opening of the crack allows for a more correct representation of the traction state along the interface for these elements than with only a simple constant crack opening.

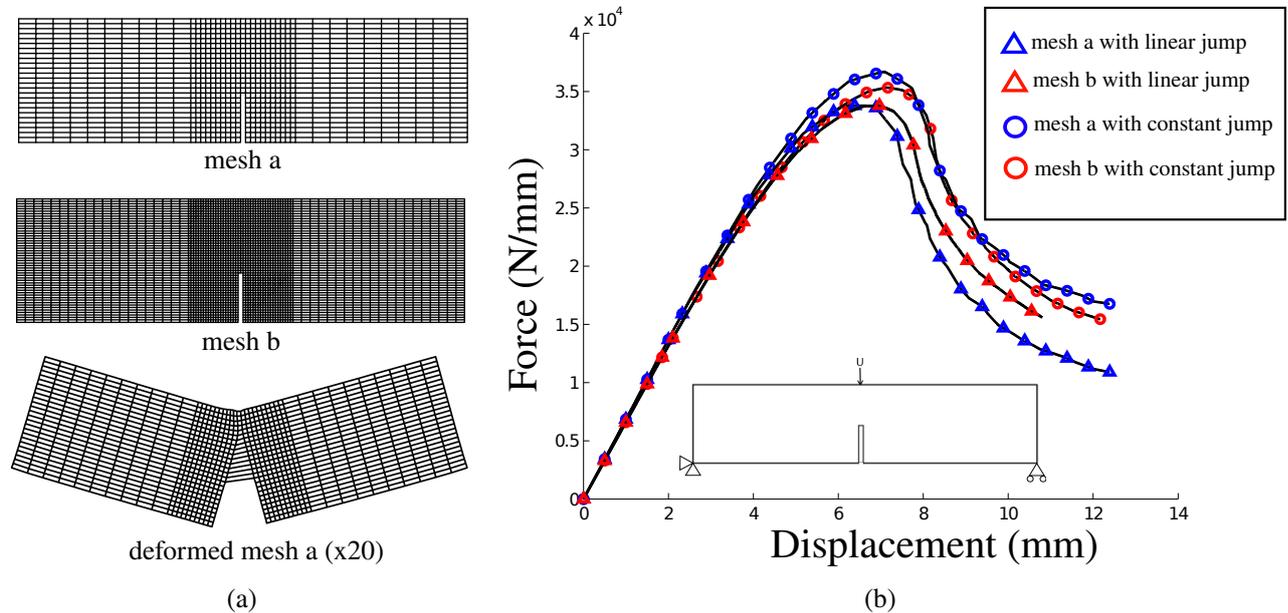


FIGURE 2: (a) Considered meshes and deformed mesh, (b) Force/displacement curve

4 CONCLUSIONS

A ductile fracture modeling method using the strong discontinuity method for the regularization of the post-peak phase and Lemaitre's damage model for the pre-peak phase has been presented. Results demonstrating the interest in the use of a linear displacement jump in the case of a three-point bending test were shown. Ductile fracture often induces large strain states and so an adaptation of the developments in this hypothesis will be undertaken shortly, in addition to considering the third invariant of the stress tensor in order to take into account the effect of this parameter on the crack propagation direction. Those results should be presented at the conference.

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