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## Modélisation du contact et de l'usure

27 et 28 mai 2010 – Albi – France

### **A NUMERICAL MODEL OF WEAR TAKING ACCOUNT OF MULTIDIRECTIONAL EFFECTS**

### **UN MODÈLE NUMÉRIQUE D'USURE TENANT COMPTE DES EFFETS MULTIDIRECTIONNELS**

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#### **Résumé**

De nombreux travaux ont montré que le changement de direction de glissement était un facteur aggravant l'usure entre deux corps en contact. L'objectif de cette étude est de proposer un modèle d'usure capable de rendre compte de ces observations. Ce modèle essaie de montrer qu'une part des effets multidirectionnels est le résultat de la variation de l'usure en fonction du nombre de cycles. Les résultats présentés dans ce papier sont issus de simulations numériques. Ils montrent que l'utilisation d'un coefficient d'usure dépendant de la distance de glissement permet, dans le cas d'un trajet rectangulaire de frottement, d'obtenir un volume d'usure plus de deux fois supérieur à celui obtenu pour un trajet rectiligne.

#### **Abstract**

*Many work showed that the change of sliding direction is a factor worsening wear between two bodies in contact. The objective of this study is to propose a model of wear able to reply these observations. This model tries to show that part of the multidirectional effects is due to the variation of wear rate according to the number of cycles. The results presented in this paper come from numerical calculations. They show that using a wear coefficient depending on the sliding distance, one can obtain, for a rectangular motion pattern, a volumetric wear up to 2 times more important than for a rectilinear motion pattern..*

#### **1. INTRODUCTION**

The works of Wang [1], Laurent et al.[2] and Turell et al.[3] clearly show the multidirectional effects of wear by comparing rectilinear motion pattern with rectangular motion pattern. They underline a significant increase in volumetric wear when the friction path changes of direction. For the same total distance from friction and the same load, volumetric wear can be up to 10 times greater depending on the form of the motion pattern. These observations are essential to a better understanding of the behaviour of wear. They suggest the need for particular care in the experimental and numerical modeling of wear mechanisms. To explain these multidirectional phenomena, Laurent et al.[2] propose an approach based on the dependence of wear on the sliding angle, whereas Turell et al.[3] propose a model of wear depending on the dimensions of the motion pattern. The physics of these multidirectional effects is likely to be varied in nature. It depends on the couple of materials in contact, their surface quality, roughness, thermomechanical properties, etc. This paper, proposes to allot part of this volumetric wear increase to the dependence of wear rate on the number of cycles and more precisely to the sliding distance. Indeed, many studies on

wear [4], [5], [6], [7], show that wear rate drops when the number of cycles increases. The results show that the abrasive potential of the bodies in contact gradually decreases with repeated passages. Part of the multidirectional effects of wear is accounted for by using the preceding observation. To do so, a numerical model of wear is presented, based on previous works [8], [9] and [10], which was adapted to take into account a wear rate depending on the sliding distance. The dependence of wear rate is calculated by a variation of the wear coefficient. The numerical results, obtained with this new model of wear, are then presented. The numerical tests reproduce the experimental protocol suggested by Turell et al.[3].

## 2. EXPERIMENTAL OBSERVATIONS

The first point raised relates to the experiments performed by Turell et al.[3] on multidirectional effects. The experimental protocol and results are presented, followed by a geometrical analysis of the zones of friction. The second point shows that wear rate drops as the number of cycles increases.

### 2.1. Multidirectional effects

The works of Turell et al.[3] highlight the multidirectional effects on the behavior of wear. Their works come from biomechanics investigations and validates a theory suggested 3 years earlier by Wang[1]. The material couple used was composed of Ultra-High Molecular Weight PolyEthylene (UHMWPE) and of Chromium- Cobalt (Co-Cr). The tests were carried out on a multidirectional machine of OrthoPOD type. The bodies in polyethylene were 9 mm diameter cylindrical pin. Six different rectangular motion patterns were tested. Curves of figure 1, after Turell et al.[3], represent, for each motion pattern, volumetric wear according to the number of cycle. One notes a significant difference between the various tests.

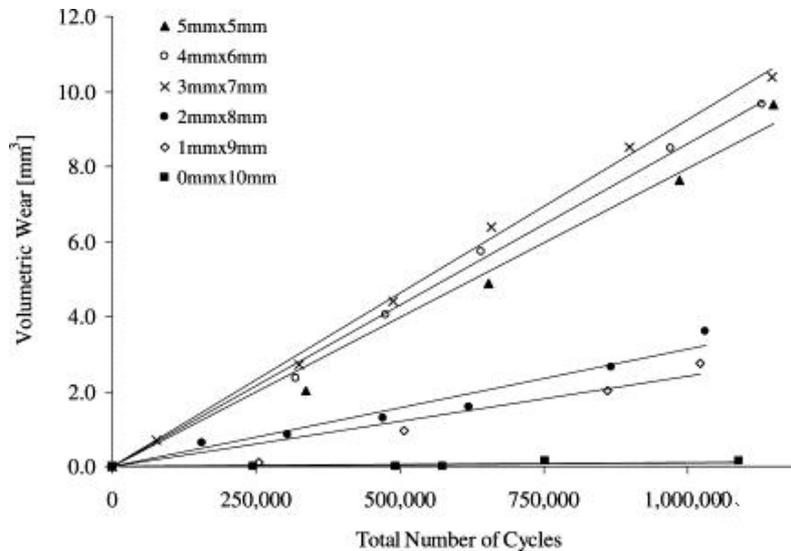


Fig. 1 Comparison of volumetric wear rate for five different rectangular motion path patterns and linear tracking. Turell et al.[3]

Now let us look more closely at the surface of wear on the plate of Co-Cr. In the diagram of figure 2, the discs represent the surface in contact under the UHMWPE pin at various steps of the cycle for a rectangular motion pattern. Numbers located in the various zones of figure 2 represent the number of times that these zones are rubbed during a cycle.

It is noted that for one cycle, there are zones less rubbed than others. The dimensions of these zones vary with the geometrical characteristics of the rectangular motion pattern.

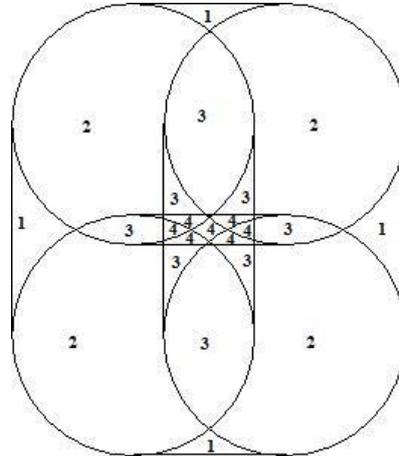


Fig. 2 Contact surfaces at various stages of the cycle for a rectangular motion pattern

## 2.2. Nonlinear wear rate

For all materials in contact the wear rate decreased when the number of cycles increased. This is supported by the works of [4], [5], [6], and [7]. A representative result is that obtained by Shi et al [4]. The figure 3 represents UHMWPE weight loss according to the number of cycles. It is noted clearly that, for this material couple, the wear rate drops when the number of cycles increases. This means that the abrasive properties of the rubbed zone decrease when the number of cycles increases. This decrease may come from a smoothing of counterface roughness. This is what is observed in figure 10 of paper of Samyn et al [11], after large-scale sliding.

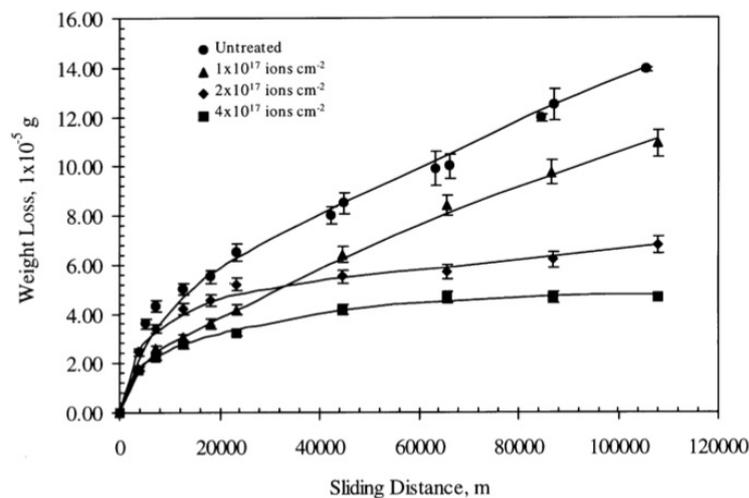


Fig. 3 Variation of the UHMWPE weight loss as a function of the number of cycles and plasma immersion ion implantation. [4]

According to the purely geometrical observations of figure 2 and knowing that the abrasive properties of the rubbed zone decrease when the number of cycles increases, it will be

shown that part of the multidirectional effect is due to the variation of wear rate according to the cumulative sliding distance. Indeed, for rectangular motion patterns the total rubbed surface is much greater than for a rectilinear motion pattern and this increase in rubbed surface corresponds to less rubbed zones that are therefore more abrasive. Wear must thus be greater. This assumption now needs to be checked and the increase must be quantified. This is the subject of the following paragraphs.

### 3. LAW OF WEAR

Archard's law [12] proposes a linearization of the experimental results presented in the preceding paragraph (figure 3 ), in the following form:

$$W = KsF$$

where  $W$  is the volume of wear,  $K$  is the global coefficient of wear between the two bodies,  $s$  the sliding distance and  $F$  the applied load. While insisting on the nonlinearity of the results and with the aim of explaining directional effects, a nonlinear law of wear is proposed, of the form:

$$W = K(s)F$$

Given that the abrasive potential of materials in contact is a function of the cumulative sliding distance undergone by surface in contact, it is advisable to specify the variable  $s$ . To do so, the following observation is used. One considers a rectilinear form of slip between two bodies  $S1$ , of length equal to  $d1$ , and  $S2$ , of length equal to  $d2$ . (see figure 4).

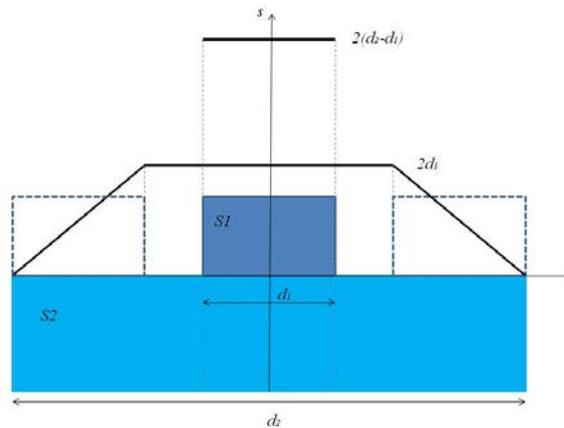


Fig. 4 Sliding distance

For one cycle, each particle of  $S1$  rubs over a distance equal to  $2(d2 - d1)$  whereas the particles of  $S2$  rub over a distance equal to a maximum of  $2d1$ . For  $n$  cycles, one define  $s1 = 2n(d2 - d1)$ ,  $s2 = 2nd1$  and the preceding nonlinear law is enriched by

$$W = (K1(s2) + K2(s1))F \quad (1)$$

where  $K1$  is the coefficient of wear for material associated with body  $S1$  and  $K2$  that associated with  $S2$ . Thus, the coefficient of wear  $K$  is a combination of coefficients depending on the cumulative distances from opposite surfaces. The volume  $K2(s1)F$  is that lost by body  $S1$ , and  $K1(s2)F$  is that lost by body  $S2$ . The coefficients  $K1$  and  $K2$  can be given by measuring the volume of wear lost by each solid. This law of wear (1) allows a total

volume of wear to be predicted, but does not allow the wear on each body to be located. To obtain this local information, a local model of wear is drawn from this law.

#### 4. LOCAL LAW OF WEAR

The model presented in this paragraph takes as a starting point the preceding works [9], [10] and the nonlinear law (1).

Let  $\Omega_0$  be the domain at time  $t = 0$  and  $X$  be the initial position of a particle of the body, then the transformation operator  $\varphi$  gives the position  $x$  of the particle at time  $t$ , as follows

$$x = \varphi(X, t) \quad (2)$$

The transformation operator  $\varphi$  is split in two parts

$$\varphi(X, t) = \varphi^e(X, t) + \varphi^w(X, t) \quad (3)$$

where  $\varphi^w$  is transformation due to wear and  $\varphi^e$  is the complementary part (reduced in this study to the elastic contribution). Transformation  $\varphi^w$  is assumed to be nil beyond the contact zone. Before developing the evolution of transformation due to wear in the contact zone, it is necessary to introduce some notations relative to contact conditions. Two bodies are considered as contact candidates. Vectors  $T_1$  and  $T_2$  of the local frame  $(T_1, T_2, N)$  are chosen in the common tangent plane of the two bodies. Vector  $N$  is the normal vector at the tangent plane such that  $(T_1, T_2, N)$  is orthonormal. Let  $\varphi_1$  be the transformation operator of the first body, and  $\varphi_2$  the transformation operator of the second body. The particle  $P_1$  of body  $S_1$ , of initial position  $X_1$  and the particle  $P_2$  of body  $S_2$ , of initial position  $X_2$  are in contact at time  $t$  and then

$$x = \varphi_1(X_1, t) = \varphi_2(X_2, t)$$

The relative velocity  $v$  is defined as

$$v = \frac{d\varphi_1}{dt}(X_1, t) - \frac{d\varphi_2}{dt}(X_2, t) \quad (4)$$

and the tangential relative velocity  $v_T$  is the projection of  $v$  on the common tangent plane. Let  $r = (r_{T_1}, r_{T_2}, r_N)$  be the reaction force exerted on the first body, expressed in the local frame, so the normal component of velocity due to wear, is assumed to be governed by the following relationships:

$$\begin{aligned} \frac{d(\varphi_1^w \cdot N)}{dt} &= k_1^w(P_1, s_2)r_N |v_T| \\ \frac{d(\varphi_2^w \cdot N)}{dt} &= -k_2^w(P_2, s_1)r_N |v_T| \end{aligned} \quad (5)$$

The tangential component is assumed to be nil:

$$\varphi_i^w \cdot T_j = 0, \quad \forall i, j = 1, 2 \quad (6)$$

The local wear factor  $k_i^w$  of the body  $i = 1, 2$  is defined as

$$k_i^w = \frac{1}{A(P_i)} \frac{\partial K_i(s_j)}{\partial s_j}, \quad i \neq j$$

where  $A(P_i)$  is the elementary surface associated with the particle  $P_i$  of position  $x$  at time  $t$ . Then, for particle  $P_i$  of body  $S_i$ , the local wear factor  $k_i^w(P_i, s_j)$ ,  $i \neq j$ , depends on the cumulative distance  $s_j$  of the opposite particle  $P_j$  of body  $S_j$ . This distance is defined as

$$s_j(t) = \int_0^t |v_T(X_j, \tau)| d\tau \quad (7)$$

This wear law has to be considered in the context of frictional contact laws. Unilateral Signiorini's conditions and the Coulomb's dry friction law were chosen for this study. If  $g_N = (\varphi_1 - \varphi_2) \cdot N$  denotes the gap between the two bodies, unilateral Signiorini's conditions can be expressed in the following variational equation form

$$r_N = \text{proj}_{R^+}(r_N - \rho_N g_N) \quad (8)$$

where  $\text{proj}_{R^+}(\cdot)$  denotes the orthogonal projection on the positive real set  $R^+$  and  $\rho_N$  is a positive definite real number. Let  $r_T$  be the tangential part of the reaction force, then Coulomb's law is governed by the following equation

$$r_T = \text{proj}_{C(r_N)}(r_T - \rho_T v_T) \quad (9)$$

where  $C(r_N)$  is the section of the Coulomb cone  $\{r_T \in R^2 \text{ such that } |r_T| \leq \mu r_N\}$ ,  $\mu$  is the friction coefficient and  $\rho_T$  is a positive definite real number. The parameters  $\rho_N$  and  $\rho_T$  are neither Lagrange multipliers, nor penalty coefficients. Their values are not necessarily large and do not have physical significances. However, for numerical reasons, they have optimal values.

## 5. NUMERICAL RESULTS

The numerical method is not developed in this paper. Details could be found in papers [9], [10] and [13].

### 5.1. Data set

The numerical simulations carried out in this study were built to reproduce the experimental tests proposed by Turell et al.[3]. The chromium-cobalt plate S2 was assumed to be rigid. The upper surface of the plate of  $20 \times 20 \text{ mm}^2$  was discretized by a regular grid constituted by 40.000 quadrangular elements. Every element represents a friction zone in which each particle has a same cumulative sliding distance. The wear factor  $K_2$  was neglected and  $K_1(s_2) = K(s_2)$  was given by the law of figure 5. The shape of this law was inspired by the results of Shi [4]. Wear values were fitted to be agree with experimental results obtained by Turell et al.[3].

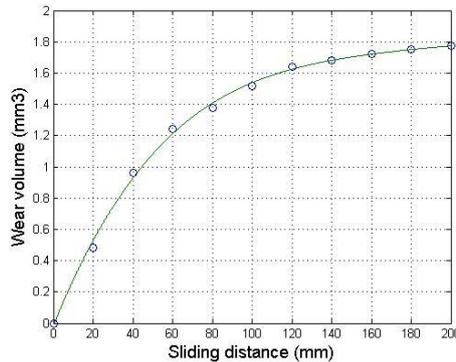


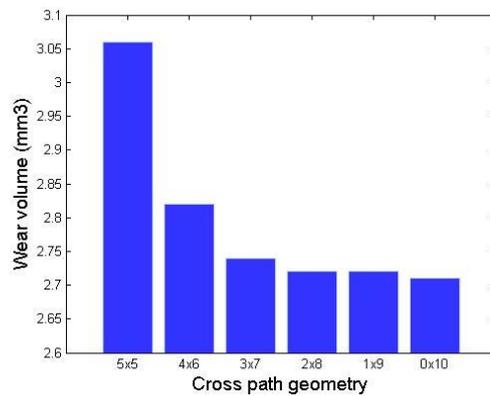
Fig. 5 Wear law for computations: wear volume loss (mm<sup>3</sup>) as a function of the sliding distance (mm)

The cylinder S1 was discretized by 540 tetrahedral elements. The material characteristics of the UHMWPE were: a Young modulus of 1016 MPa, a Poisson rate of 0.46 and a mass

density of 938 Kg/m<sup>3</sup>. The friction coefficient was 0.07. All wear simulation tests were conducted with a constant applied load of 200 N.

## 5.2. Results

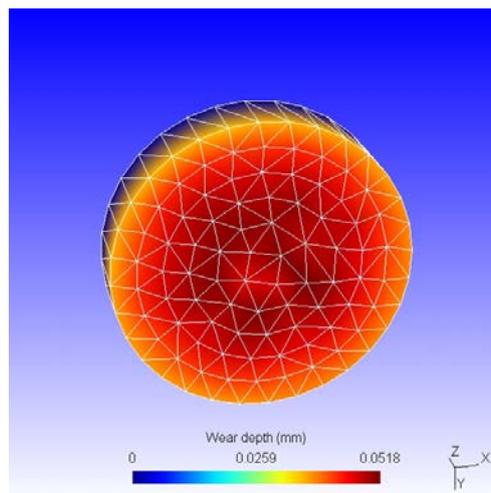
In the first series of tests, as in Turell et al.[3], six motion path patterns were tested: 5mm×5mm, 4mm×6mm, 3mm×7mm, 2mm×8mm, 1mm×9mm, 0mm×10mm. Wear volume losses are gathered in figure 6.



**Fig. 6** Wear volume loss (mm<sup>3</sup>) versus motion path pattern (mm×mm)

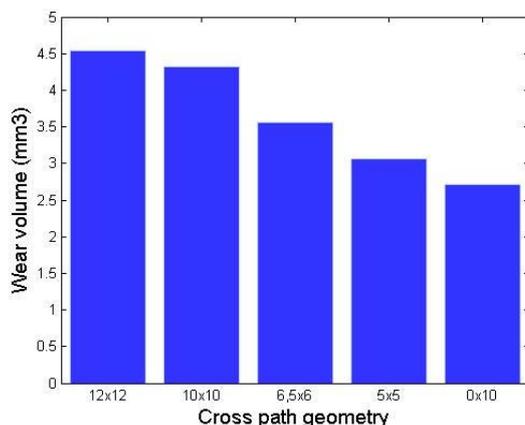
The results of simulations in figure 6 show an increase in the volume of wear according to the type of path pattern. These results were obtained by the approach developed in the preceding paragraph, with a nonlinear wear rate resulting from the law of wear (figure 5). One observes approximately 13% of increase between the rectangular path pattern 5mm×5mm and the rectilinear path pattern 0mm×10mm.

Figure 7 shows that the distribution of wear is relatively homogeneous at the end of the calculation. This simulation resulted from the rectangular path pattern 5mm×5mm.



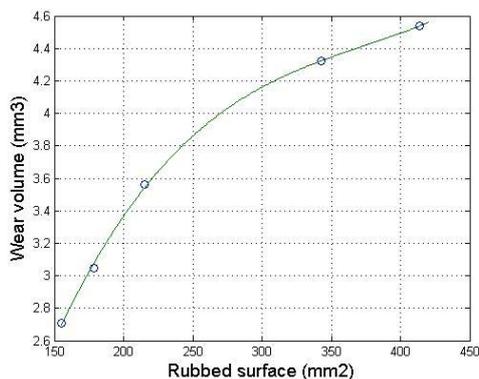
**Fig. 7** Wear depth on pin (mm)

Further simulations with rectangular path patterns of types 6.5mmx6mm, 10mmx10mm, 12mmx12mm, preserving the same traversed total distance (just the number of cycles is changing), produced the volumes of wear represented in figure 8.



**Fig. 8** Wear volume loss (mm<sup>3</sup>) versus motion path pattern (mm×mm) (serie 2)

The results show that volumes of wear increase according to the surface of the rectangle. The volume of wear is maximum for path pattern 12mmx12mm. This behavior confirms the observations of the first series of numerical tests (figure 6). There seems to be a correlation between the surface of the rectangle and the volume of wear. To confirm this, let us continue by analyzing the friction surfaces. It is observed that the rectangular path pattern 10mmx10mm admits maximum rubbed surface, but that the cumulative sliding distance  $s$  is minimum. According to the theory developed in this paper, rubbed surfaces are thus more abrasive; therefore the volume of wear is greater. The correlation between the volume of wear and rubbed surface is illustrated in figure 9.



**Fig. 9** Wear volume versus rubbed surface

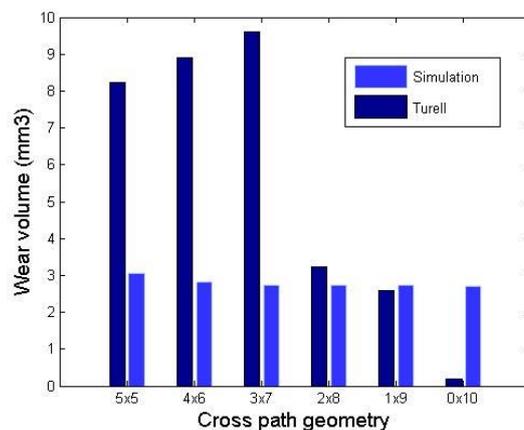
Rubbed surface was calculated by multiplying the total number of quadrangular elements where  $s$  is no null by the surface of one element. This graph shows that the relation between these two parameters is not linear. The slope of the curve tends to decrease when the surface of friction increases.

### 5.3. Discussion

The numerical results show that the approach adopted in this article complies with the measurements observed by Turell et al.[3] and Laurent et al.[2]. The rectangular path patterns increase volumes of wear compared to a rectilinear path. A factor 1.7 can be observed between the minimum (0mmx10mm) and the maximum (12mmx12mm). This shows that the approach adopted in this study is relevant.

The numerical results show that there is a correlation between the entire friction surface on the plate and the volume of wear. The greater the surface is the greater the volume of wear is. This remains faithful to the model developed in this article. If the friction surface is wider, its abrasive capacity is stronger. However, the abrasive potential tends to decrease beyond a certain friction surface.

However the increases in wear revealed by simulations were not as spectacular as the experimental results. These numerical results were on average very far removed from Turell et al.[3]'s measurements (see figure 10).



**Fig. 10** Wear volume loss (mm<sup>3</sup>) versus motion path pattern (mm×mm) (comparisons between simulations and experimentations)

Moreover, simulations did not seem to reproduce the drop in volume of wear observed in experiments after the path pattern 3mmx7mm. This decrease in wear volume was not only observed by Turell et al.[3], but also by Laurent et al.[2]. The model proposed in this paper cannot model this phenomenon.

In addition and to supplement this study, is necessary to note that for a constant wear rate, i.e. for a linear law of wear, the volume of wear calculated is of 1,8 mm<sup>3</sup> for a rectilinear path pattern. This result is to be compared with the volume of wear 2,71 mm<sup>3</sup> obtained by the nonlinear approach. In this case, the increase between the rectilinear path pattern with the linear approach and the rectangular path pattern 5mmx5mm with the nonlinear approach is approximately 70%. It is significant and justifies the approach adopted in this article. In other words, a prediction of wear using the method adopted in [9] is computed with a relative error of 70% for a rectangular path pattern 5mmx5mm.

## 6. CONCLUSION

The model of wear developed in this article is based on experimental observations collected in the literature. These highlight wear volume loss as a nonlinear function of the number of cycles. From this report a numerical wear model was developed. It was built on a local law of wear inspired by Archard's law, but with a nonconstant wear rate. This wear rate

was assumed to be a function of a cumulated variable of sliding distance. Simulations show that part of the increase in multi-directional wear can be explained by the nonlinearity of wear versus the sliding distance. However, this nonlinearity does not explain all the experimental observations. It can justify the increase of wear up to factor two or even three, but not beyond. Although it is not insignificant, it is not sufficient. Different phenomena may be able to justify an increase up to a factor of 10 even 50. Some of them could be the molecular orientation of the PolyEthylene material and the orientation softening effect.

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