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On the loss of stability of von Mises truss with the effect of pseudo-elasticity

Victor A. Eremeyev

Leonid P. Lebedev

Abstract

The stability of von Mises truss is investigated when the stress–strain diagram of the material constituting the rods displays hysteresis. This behavior, known as pseudo-elasticity, is common in such materials as filled rubbers and shape memory alloys. Loading diagrams are presented; these show that the upper and lower critical loads depend on the properties of the hysteresis loop for the truss material. Examples for trusses made of shape memory alloys are presented.

Introduction

In recent years, materials having complex rheologic properties have begun to attract interest. Certain materials, in particular, demonstrate the existence of hysteresis loops on the stress–strain diagram. This is typical for many filled rubbers [1, 2, 3] and shape memory alloys [4, 5]. A vast amount of experimental material for shape memory alloys is collected in [6].

The shape memory effect in rods, plates, and films is used in various engineering structures [6]. Such structures can experience a loss of stability in their thin-walled elements under compression or realistic temperature variations. So the problem of the loss of stability of thin-walled structures with phase transitions of martensitic type is quite relevant.

In the theory of stability of elastic and viscoelastic structures, an important part is played by the von Mises truss [7, 8], which is a touchstone for various approaches to the study of more complex thin-walled structures. The equations for the von Mises truss are quite simple; however, the corresponding solutions allow us to understand the nature of the loss of stability for more complex structures, like plates and shells, for which results are still unavailable. This work does present a qualitative picture of the loading process for these more complex structures. We mention only a few works on the von Mises truss [9]– [12].

We will study the symmetric deformation of the von Mises truss (see Figure 1). The truss consists of two rods undergoing martensitic type phase transitions, and an elastic spring of rigidity C . We find the bifurcation points corresponding to the static loss of stability in the truss. We should mention

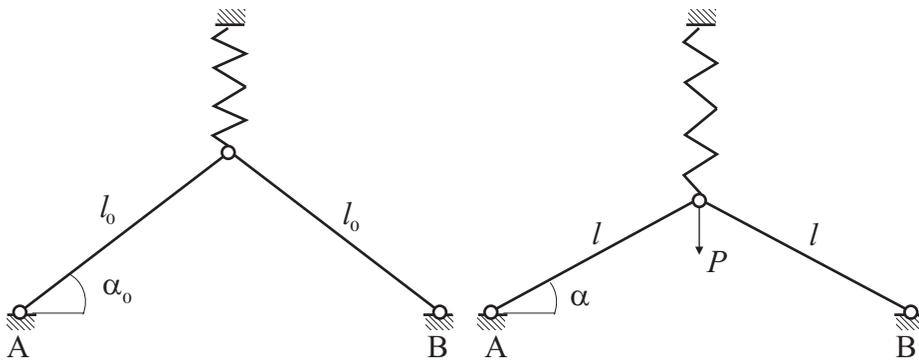


Figure 1: A von Mises truss before and after deformation.

the earlier treatments of stability of elastic bodies with phase transitions in the framework of spatial elasticity [13]–[15], and the buckling analysis of one-dimensional structures made from memory shape alloys [16, 17]. There it is shown that the presence of phase transitions has a significant effect on the loss of stability of the structure. In particular, we encounter new bifurcation points that are absent in corresponding systems made from materials without phase transitions [14, 15].

1 Von Mises truss: linear elastic material

Following [10, 12], we will consider the equilibrium problem for a von Mises truss made of linear elastic material. Let l_0 be the length of the rods before deformation, and α_0 the angle between the rods and the base AB .

Under the force P the lengths and the angles become l and α , respectively (Figure 1). The dependence of P on the vertical displacement u of the node N in equilibrium is given by

$$P = 2\sigma F \sin \alpha + Cu, \quad (1)$$

where σ is the tension and F is the cross-sectional area of the rod. If the rods obey Hooke's law, the dependence of tension on deformation is $\sigma = E\varepsilon$, where E is Young's modulus and the deformation ε is determined through α by the formula

$$\varepsilon = \frac{l_0 - l}{l_0} = 1 - \frac{\cos \alpha_0}{\cos \alpha}. \quad (2)$$

Expressing α in terms of the displacement u by the relation

$$u = l_0 \cos \alpha_0 (\tan \alpha_0 - \tan \alpha),$$

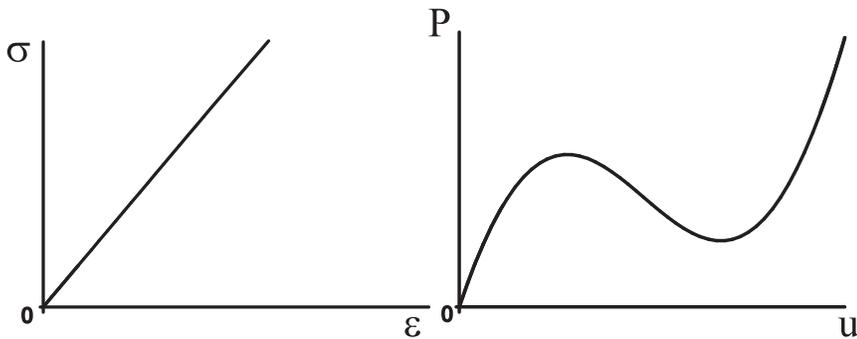


Figure 2: σ - ε diagram for a rod, and the corresponding load (P - u) diagram of a von Mises truss when $C = 0.1EF$ for linearly elastic rods.

we obtain a formula that differs from its counterpart in [9, 10] by a term representing the spring reaction:

$$P = 2EF (l_0 \sin \alpha_0 - u) \left(\frac{1}{\sqrt{a^2 + (l_0 \sin \alpha_0 - u)^2}} - \frac{1}{l_0} \right) + Cu \quad (3)$$

For linear elastic constitutive equations, a typical deformation diagram is given in Figure 2. The loads corresponding to the minimum and maximum points on the graph are the lower and upper critical forces. The decreasing part of the diagram for the truss under load corresponds to the unstable state, but it can be realized under a given displacement u . Here we use the dimensionless values $\alpha = 30^\circ$ and $C = 0.1EF$.

2 Von Mises truss: idealized pseudo-elastic material

Let us consider first the idealized model of a tension-extension diagram of martensitic or rubber-like material. For rods composed of memory shape alloys, Hooke's law is not appropriate. At temperatures near phase transition, the σ - ε diagram has a complex form with one or more hysteresis loops [6].

As an example, we present the stress-strain diagrams for the alloy Au-47,5%Cd at 357 K [5] and for the alloy Cu-15,1%Al-4,2%Ni at 198 K [4]. See Figure 3. It is seen that after the strain reaches some value, the shape of the stress-strain diagram changes significantly. For memory shape alloys this relates to the fact that at this value of strain in the material, the martensitic transformation begins. For polymeric materials and rubbers the mechanisms which imply existence of the hysteresis loop can be related with structural transformations as well. After the end of the martensitic transformation, the stress-strain curve has the form of the straight line corresponding to Hooke's law. When unloading, the inverse martensitic transformation starts

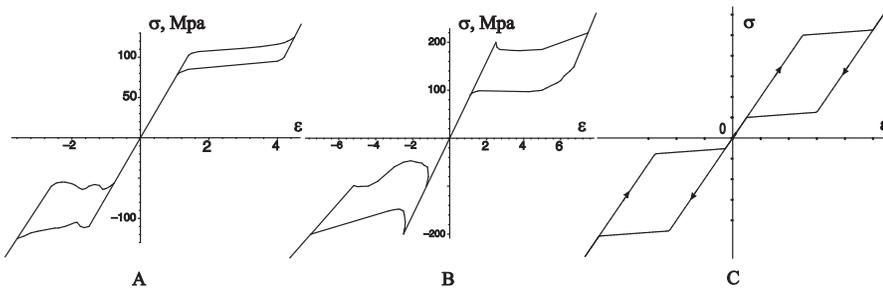


Figure 3: Some σ - ε diagrams: (A) for the alloy Au-47,5%Cd at 357 K; (B) for the alloy Cu-15,1%Al-4,2%Ni at 198 K; (C) an idealized material.

at another value of the strain. This implies the existence of a hysteresis loop on the stress–strain diagram. The shape of this loop and of the whole stress-strain diagram depends significantly on the temperature and the type of stressed state.

Without loss of generality, we may represent the hysteresis loop using a parallelogram as in Figure 3, part (C). Here the arrows show the direction of deformation change. The choice of the form of parallelogram that most closely describes the shape of the hysteresis loop can be made in a few different ways. In particular, it can be based on the energy criterion when we take equal areas of the loops for the idealized and real material.

Using MapleTM software, we generated the σ - ε diagrams in Figure 4 (parts I–III, along the right-hand side). A monotonic increase in u was assumed. For comparison, we also present diagrams for the rods without phase transition. It is seen that an increase in the width of the hysteresis loop significantly affects the $P - u$ diagram for the von Mises truss. In particular, it decreases the upper critical load and increases the lower critical load.

The influence of the spring constant on truss behavior is also seen in Figure 4. A relatively high degree of stiffness inhibits the effects of the phase transitions. We would like to note that we found cases corresponding to the appearance of the third bifurcation point of the diagram, and so of two decreasing portions (the instability domains). An example appears in the second row, first column. Here a loss of stability in two bucklings becomes possible for the von Mises truss. The existence of a hysteresis loop on the σ - ε diagram results in two loops on the $P - u$ diagram (Figure 5). Here the arrows show the direction of change of u . The spring rigidity takes values $C = 0, 0.05 EF$, and $0.5 EF$.

When the spring is absent (i.e., $C = 0$), the $P - u$ diagram (Figure 5) is symmetric. Note that the presence of the spring destroys the symmetry. With an increase in C , the hysteresis loops decrease as seen in Figure 5 where $C = 0.5 EF$.

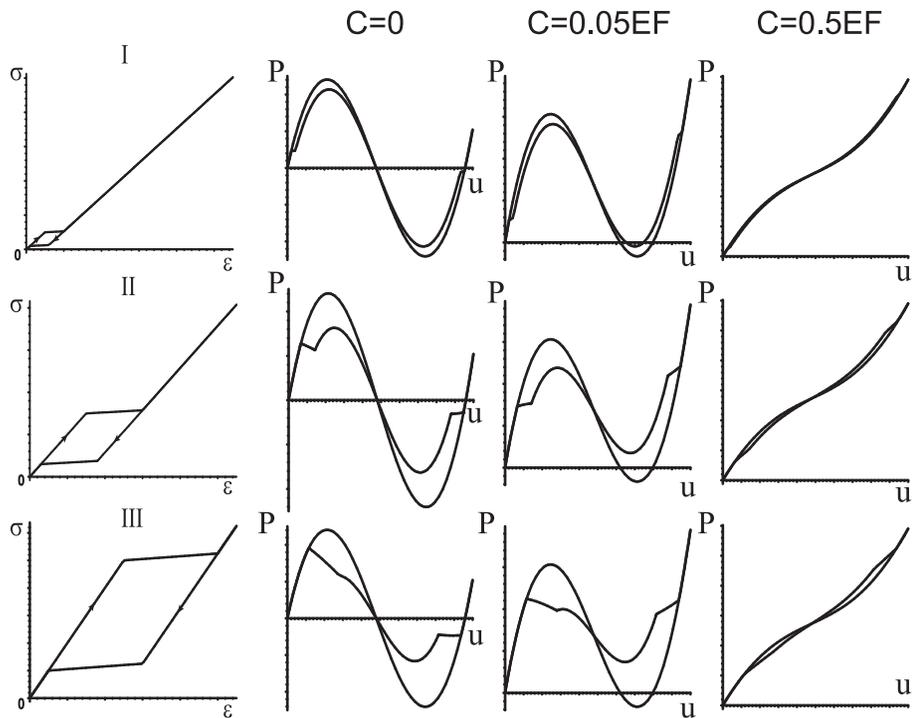


Figure 4: Various σ - ε diagrams for a rod, and the corresponding load (P - u) diagram of von Mises truss. The graphs are for pseudo-elastic materials exhibiting hysteresis loops of various magnitudes.

3 Von Mises truss: real pseudo-elastic material

It is worth noting that the use of the stress-strain diagram for a real material involves technical difficulties. In Figure 6 we present the hysteresis loops on the P - u diagram for a von Mises truss made of CuAlNi shape memory alloy with $C = 0$ (see Figure 3(B).) It is evident that Figure 6 differs only in certain details from the results obtained by using the idealized model.

Conclusions

In this paper we study axisymmetrical deformation of the von Mises truss under load. The truss consists of two deformable rods, whose material displays phase transitions of martensitic type, and an elastic spring. The study of the stability of structures involving such materials is done for the first time. It demonstrates the qualitative behavior at the loss of stability for more complex structures, such as plates and shells, for which results are absent. We find bifurcation points on the static load diagram for the truss. The numerical results demonstrate that the existence of phase transitions of the material

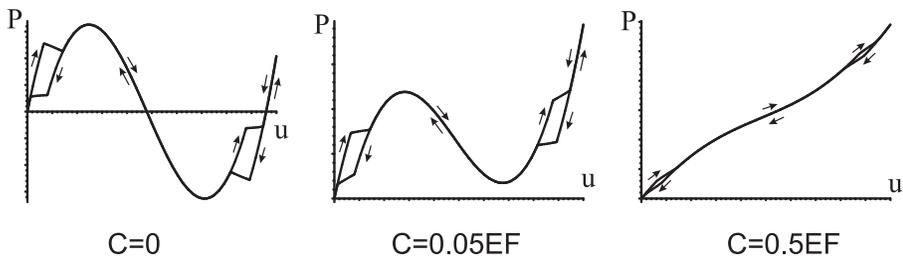


Figure 5: Hysteresis loops on P - u diagram for von Mises truss: (a) $C = 0$, (b) $C = 0.05 EF$, and (c) $C = 0.5 EF$.

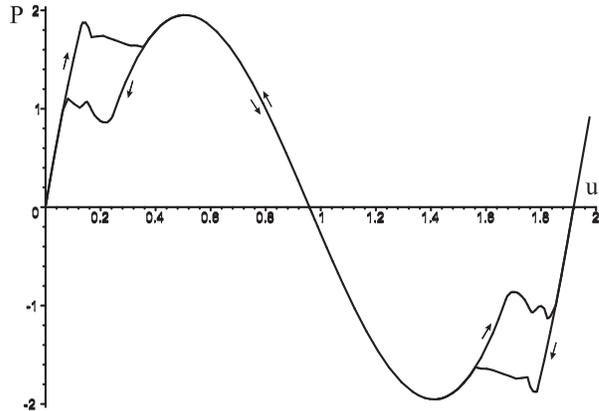


Figure 6: P - u diagram for von Mises truss made of the alloy Cu-15,1%Al-4,2%Ni, $C = 0$.

decreases the upper critical load and increases the lower one; it also results in additional bifurcation points, which means that buckling of the truss can be realized in two steps. We consider an idealized model of a material with shape memory for which the hysteresis loop on the stress-strain diagram has the form of a parallelogram. We consider stress-strain diagrams of more complex forms for real alloys with memory shape as well.

Here we have not discussed the problem of best approximation of the diagrams for a real material by the idealized model. We expect to pursue such a study in our future work.

The equations describing the von Mises truss for the one-dimensional idealized model of a pseudo-elastic material are relatively simple. Nonetheless, they indicate the nature of the loss of stability for more complex elastic structures like plates and shells made of materials that happen to exhibit phase transition properties like those of memory shape alloys. The qualitative picture of the loss of stability for structures made of such materials is new.

The analysis of numerical results allows us to conclude in particular that

1. *phase transition in the material significantly affects the critical loads of the system, up to the appearance of new bifurcation points;*
2. *the presence of a hysteresis loop in the σ - ε diagram of the rod material, which is related to a phase transition of martensitic type, is inherent in a structure containing elements made from the material and the whole structure demonstrates hysteresis behavior of a more complex nature;*
3. *hysteresis loops on the σ - ε diagram have the effect of decreasing the upper critical loads and increasing the lower ones – this was seen for the first time.*

In a similar way, it is possible to study the loss of stability for a more complex rod structure whose material exhibits phase transitions of martensitic type or a hysteresis loop.

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