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Attenuation correction in a new modality of Compton Scattering Tomography

Omar Alejandro OLVERA GUERRERO[1], Gäel RIGAUD [2], Rémi Régnier [1] and Maï K. NGUYEN[1]

[1]ETIS Laboratory - ENSEA/Univ.Cergy-Pontoise/CNRS, 95000 Cergy-Pontoise, France
[2]Institute for Applied Mathematics, Saarland University, Saarbruüken, Germany

Abstract

Compton scattering tomography (CST) is an alternative imaging process which reconstructs, slice by slice, the electron density of an object by collecting radiation emitted by an external source and scattered throughout the object. A new CST modality, has been proposed based on a Radon Transform over a family of circular arcs but no attenuation factor was taken into account. Reconstruction of the unknown electron density is achieved by the inversion of the corresponding circular-arcs Radon transform (CART). In this work, medium attenuation factor is considered in the modelling and we propose an iterative correction algorithm to compensate this issue.Thus the feasibility of this new CST modality under realistic working conditions is demonstrated.

I. Introduction

Compton scattering tomography(CST) is an original way to explore the inner parts of an object as it relies on scattered radiation data instead of primary radiation data of conventional tomography. However only one practical modality, based on a Radon transform on circles has been proposed by S.J. Norton in 1994 [1], nevertheless the architecture of this modality appears to be more suited for non-destructive testing than for medical applications.

Recently a new modality, whereby the radiation source and the detector are placed on a diameter of a fixed circle has been proposed in 2010 in view of medical imaging [2]. This new modality is based on the inversion of a Radon transform on circular arcs subtending a chord of fixed length which rotates around its middle point. In this setting, radiation beam spreading due to photometric effects can be nicely taken into account in the inversion process or equivalently image reconstruction.

However, the beam attenuation cannot be considered in the inversion process of this Radon transform, because attenuation occurs along two straight lines and not along the circular arc of integration. So this is fundamentally different from the attenuation problem of the classical Radon transform, already solved by R. Novikov in 2002 [3].

To face up this challenge, an attenuation correction procedure is proposed in the studied case. In this work, instead of using standard attenuation correction methods such as the Generalized Chang Correction (GCC) which corrects the reconstructed function or the Iterative Pre Correction (IPC) which corrects data (see [4]), we propose an alternative algorithm in which the attenuation map is obtained iteratively from the electron density making the assumption that we know the kind of medium and so the corresponding total cross section.

Attenuation is then dominantly due to Compton scattering. Under these assumptions, an attenuation correction scheme uniquely based on the determination of the electron density can be conceived, since the linear attenuation function depends on the electron density. We shall proceed by iterations using the properties of the analytic inverse operator of the circular arc Radon transform and the characteristics of the medium.

In section II, this new CST approach is introduced followed by the inclusion of the attenuating beam in section III. The proposed attenuation correction technique is given in section IV, its results on two medical toys are given in section V and our Conclusions are in section VI.

Rémi Régnier is supported by DGA
omar-alejandro.olvera-guerrero@ensea.fr
gael.rigaud@ensea.fr
remi.regnier@ensea.fr
mai.nguyen-verger@u-cergy.fr

II. A new modality in CST

Recently we have suggested a new modality for CST and presented some results on its performance [5]. The next figure 1 shows how this novel modality of CST works. An emitting radiation point source **S** is placed at a distance $2p$ from a point detector **D**.

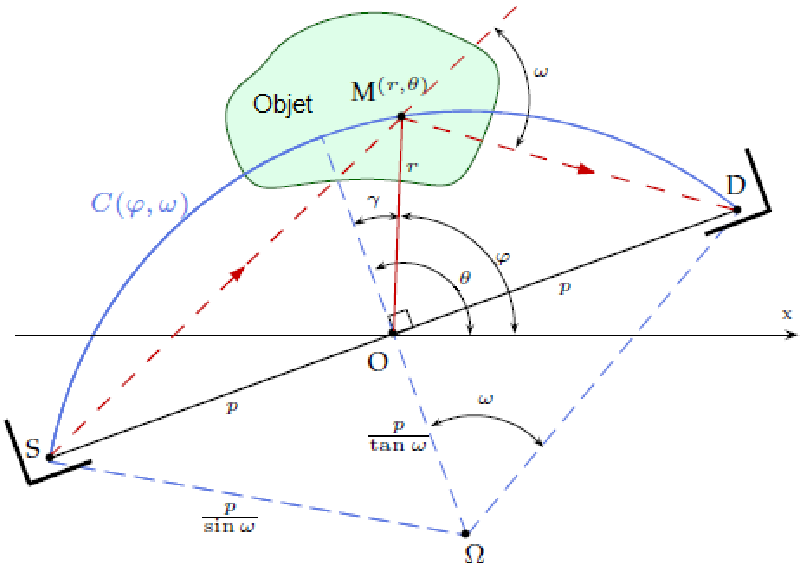

Fig. 1. Principle of the CAR transform.

The segment SD joining them rotates around its middle point **O**. At site **D** the single-scattered radiation flux density is collected from the scanned object for a given angular position of the line SD and at a given scattering energy E, (or equivalently at scattering angle ω). Thus, thanks to the physics of the Compton effect, the detected radiation flux density $g(\varphi,\omega)$ is the integral of the electron density $f(\mathbf{M})$ on this class of circular arc, where $\tau = cot\ \omega$ and φ is the rotation angle made by the mediator line of segment SD with a fixed reference axis, see figure 1 .

A. Data acquisition

The detected scattered flux density $g(\varphi,\omega)$ is proportional to the integral of the function $f(M)$ for $M \in C(\varphi,\omega)$,

$$g(\varphi,\omega) = \int_{(r,\theta)\in C(\varphi,\omega)} f(r,\theta)\ ds, \tag{1}$$

in which ds represents the circular arc line element. For its calculation, we start from the circular arc equation:

$$r = p(\sqrt{1+\tau^2 cos^2\ \gamma} - \tau cos\ \gamma), \tag{2}$$

where $\tau = cot\ \omega$ and $\gamma = \varphi - \theta$. We state that the case $\omega = 0$ (primary radiation) is a limit of our model.

By taking τ as a parameter, we can thus rewrite the measurements as the generalization of the Radon transform over circular arcs (CART):

$$g(\varphi,\tau) = \int_{-\pi/2}^{\pi/2} f(r(\gamma),\gamma+\varphi) r(\gamma) \frac{\sqrt{1+\tau^2}}{\sqrt{1+\tau^2 cos^2(\gamma)}} d\gamma. \tag{3}$$

B. Image Reconstruction

The formula (3) has a rotational invariance around **O**. Under these circumstances, the circular harmonic decomposition (CHC) is an appropriated tool to establish its analytic inverse formula. Thus, we define the CHC $f_l(r)$ of a function $f(r,\theta)$ as:

$$\begin{cases} f(r,\theta) = \sum_{l\in\mathbb{Z}} f_l(r) e^{il\theta} \\ \quad with \\ f_l(r) = (1/2\pi)\int_0^{2\pi} f(r,\theta) e^{-il\theta} d\theta. \end{cases} \tag{4}$$

We obtain the circular harmonic components of (3) and we consider the symmetry of γ to come up with the next expression:

$$g_l(\tau) = 2\int_0^{\pi/2} r(\tau)\frac{\sqrt{1+\tau^2}}{\sqrt{1+\tau^2 cos^2(\gamma)}} f_l(\gamma) cos(l\gamma) d\gamma. \tag{5}$$

Using the Cormacks inversion method [6-7] we are able to define the inverse formula $CART^{-1}$ with the next integral:

$$f_l(r) = (-)\frac{2p(p^2+r^2)}{\pi(p^2-r^2)^2}\left[\int_t^{\infty} \frac{\cosh(l\cosh^{-1}(q/t))}{\sqrt{q^2-t^2}} \frac{d}{dq}\left(\frac{Cf_l(1/q)}{\sqrt{1+q^2}}\right) dq\right]_{t=2pr/(p^2-r^2)}. \tag{6}$$

By rewriting the $f_l(r)$ in the spatial domain, the function $f(r,\theta)$ is reconstructed. For further details about the derivations of our formulas (3) and (6), refer to [2].

Within the mathematical modeling of this new CST modality, the phenomenon of attenuation is not yet taken into account. In the next chapter, we establish a new forward formula $CART$ with the inclusion of the attenuation factor. The inverse formula of this new approach is not yet known, therefore we propose a suited iterative attenuation correction algorithm.

III. Attenuated direct CART transform and attenuation correction

A. Attenuation factor

Under realistic working conditions, traveling radiation is affected by medium attenuation and by dispersion due to photometric propagation effects. A standard way to take into account for these effects is to put in the integrand of the CART transform (3) the following factors:

$$\frac{e^{-\int_{SM}\mu_o(x,y)dl}}{\bar{SM}^2} \quad \text{and} \quad \frac{e^{-\int_{MD}\mu(x,y)dl}}{\bar{MD}^2}, \tag{7}$$

where μ is the attenuation map. This value has to be considered in the simulations in order to obtain a more realistic value of measured photons.

In addition we have:

$$\overline{SM}^2\ \overline{MD}^2 = (p^2-r^2)^2(1+\tau^{-2}). \tag{8}$$

B. Klein-Nishina probability

To go further in the mathematical modeling of the scattering phenomenon, we consider a photon flux density instead of mere points. The collision cross section σ^c is the probability that a photon undergoes Compton scattering, then the differential cross section is given by

$$\left.\frac{d\sigma}{d\Omega}\right|_{\omega} = r_e^2 P(\omega) = \frac{r_e^2}{2}\left[\frac{1}{[1+k(1-sin\ \omega)]^2}\left(1+sin^2\ \omega + \frac{k^2(1-sin\ \omega)^2}{1+k(1-sin\ \omega)}\right)\right] \tag{9}$$

where $r_e = e^2/mc^2$ is the electron radius and $P(\omega)$ is the Klein-Nishina probability for a given diffusion angle ω. After considering the physical phenomenons brought into play, the forward transform CART T^{ϕ} becomes :

$$T^{\phi}(\varphi,\tau) = \frac{P(tan^{-1}(\frac{1}{\tau}))}{1+\tau^{-2}}\int_{-\pi/2}^{\pi/2} W_{att}(\varphi,\tau,\gamma)\ \frac{r(\gamma)\ f(r(\gamma),\gamma+\varphi)\ \sqrt{1+\tau^2}\ d\gamma}{(p^2-r(\gamma)^2)^2\ \sqrt{1+\tau^2 cos^2(\gamma)}} \tag{10}$$

where the attenuation factor is given by:

$$W_{att}(\varphi,\tau,\gamma) = exp\left(-\int_{(x,y)\in SM}\mu_0(x,y)dl\right)\ exp\left(-\int_{(x,y)\in MD}\mu_w(x,y)dl\right) \tag{11}$$

Unfortunately, we don't know yet an inversion technique for (10) with the consideration of the attenuating kernel $W_{att}(\varphi,\tau,\gamma)$. In the next section we introduce an iterative correction algorithm to corret the negative effects of $W_{att}(.)$ in the reconstructed image.

IV. Attenuation correction algorithm

A. General context

The chosen attenuation correction technique was proposed in [8]. Its principle is to consider two measurable spaces $(\mathcal{X},\mathcal{Y}) \subset \Re^4$. Let $\mathcal{L}^2_+(\mathcal{X})$ (resp. $\mathcal{L}^2_+(\mathcal{Y})$) be the space of square-summable functions defined on $\mathcal{X}$ (resp. $\mathcal{Y}$) such that

$$\forall f \in \mathcal{L}^2_+(\mathcal{X}) \quad et \quad \forall X \in \mathcal{X}, \quad f(X) \geq 0$$

Let T : $\mathcal{L}^2_+(\mathcal{X}) \longrightarrow \mathcal{L}^2_+(\mathcal{Y})$ be an integral operator defined by its integral kernel

$$(Tf)(Y) = \int_{X\in\mathcal{X}} K_T(Y,X) f(X) dX \tag{12}$$

with

$$\forall (X,Y) \in \mathcal{X}\times\mathcal{Y}, \quad K_T(X,Y) \geq 0 \ ,$$

then T is a positive linear operator. The kernel of the operator T is submitted to a positive distorsion $D(Y,X) \in \mathcal{L}^2(\mathcal{X}\times\mathcal{Y})$ such that

$$\exists\ D_m \in \Re^+ < \infty \quad s.t. \quad \forall (X,Y) \in \mathcal{X}\times\mathcal{Y}, \ D(Y,X) \leq D_m$$

Thus we can define a "distorted" operator $T^\Phi : \mathcal{F}_1 \subset \mathcal{L}^2_+(\mathcal{X}) \longrightarrow \mathcal{F}_2 \subset \mathcal{L}^2_+(\mathcal{Y})$,

$$\left(T^\phi \ f\right) = \frac{1}{D_m} \int_{X\in\mathcal{X}} D(Y,X) K_T(Y,X) f(X) dX \tag{13}$$

Generally the addition of D doesn't allow an inversion of T^ϕ (or the inversion procedure gets devious). Therefore a correction of its effects has to be applied.

B. Suited attenuation/electron density correction procedure for the CART transform

Numerous methods for correcting the attenuation factor were proposed : for example, the Generalized Chang Correction (GCC) which corrects the reconstructed function or the iterative Pre correction (IPC) which corrects data, see [4]. Nevertheless these algorithms need the attenuation map as prior information. Here we cannot have this information since we want to recover the attenuation map from the electron density.

To avoid such assumptions we propose an alternative IPC algorithm in which the attenuation map is obtained iteratively from the electron density making the assumption that we know different kinds of matters in the studied medium and so the corresponding total cross section. Thus an approximated attenuation map is deduced from the approximated electron density and is used to correct the data. We iterate until convergence is reached.

We denote by T the CART operator T^Φ (equation (10)) and by $(\mathcal{X},\mathcal{Y})$, the measurable space $(\Re^+\times[0,\pi], \Re^+\times[0,2\pi[)$ (resp. $(\Re^+\times\Re^+, \Re^+\times[0,\pi])$).

Assuming that the studied image, f, can be reconstructed from its measurements, Tf, the following recurrence relation converges towards f

$$f^{n+1} = f^n + T^{-1} \circ T^\phi (f - f^n) \quad with \quad f^0 = 0 \tag{14}$$

See [8] for the convergence proof of (14). This algorithm requires an analytical inversion of the mathematical modeling of our CART (equation (6)). The algorithm diagram is given below, see figure 2 .

To assess the quality of the reconstructions, we define the Normalized Mean Squared Error (NMSE) by

$$NMSE = \frac{1}{N^2} \frac{\sum\limits_{(i,j)\in[1,N]^2} |\mathcal{I}_r(i,j) - \mathcal{I}_o(i,j)|^2}{\max\limits_{(i,j)\in[1,N]^2} \{\mathcal{I}_o(i,j)\}^2} \tag{15}$$

where $\mathcal{I}_r$ is the reconstructed image and $\mathcal{I}_o$ is the original image. This value represents a good criterion for convergence of our iterative algorithm. Thus we stop either by number of iterations or by an NMSE error smaller than a given threshold ϵ defined by the user.

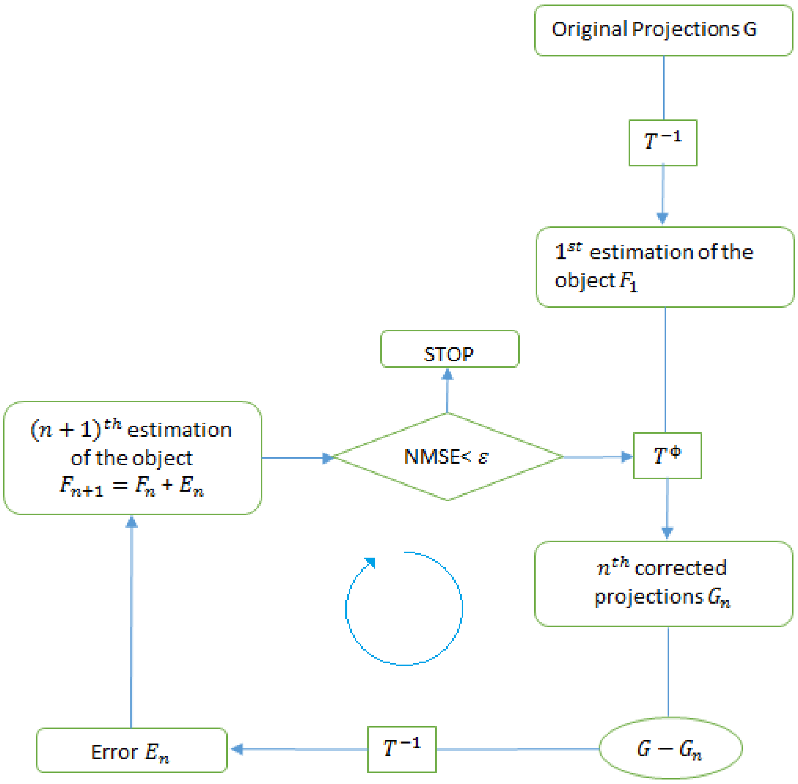

Fig. 2. Proposed attenuation/Electron density correction algorithm for the CART transform.

V. Simulation Results

We have performed numerical simulations on two medical toys: the Shepp-Logan phantom and the Zubal phantom [9] which represents the thorax. The iterations are stopped when the Normalized Mean Square Error (NMSE) is small enough.

We use the Housenfield scale to deduce from water the electron density and the attenuation coefficient of a given matter at a given energy. We consider 7 different substances : air, water , lung, bone(for organ categories 5 and 11), muscle, (for organ categories 6 and 8), blood and tissue (for organ categories 2 and 9).

The scattering medium is discretized with 128×128 pixels. We consider the number of detector positions N_φ and the number of energy levels N_ω. In order to have a "well-conditioned" problem, the number of projections ($N_\varphi \times N_\omega$) must be larger than or equal to the number of image pixels (N^2). This is why we take $N_\omega = N_\varphi = 4N$. Moreover let $p = N$ and $min\{w\} = d\omega$.

Afterwards, we follow the diagram presented in figure 2 for our CART. So we first reconstruct the electron density(n_e), then the attenuation map is deduced from the first approximation of n_e and iterations are done until convergence is reached to obtain a final corrected n_e.

A. The Shepp-Logan P(SLP)

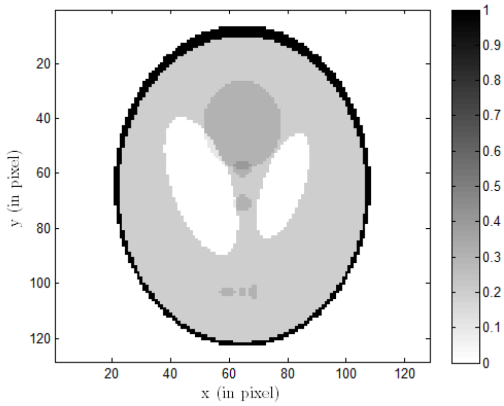

Fig. 3. Original n_e of the Shepp-Logan Phantom

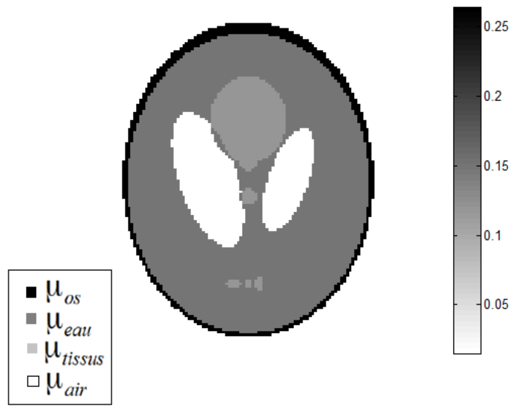

Fig. 4. Attenuation map of Fig.(3)

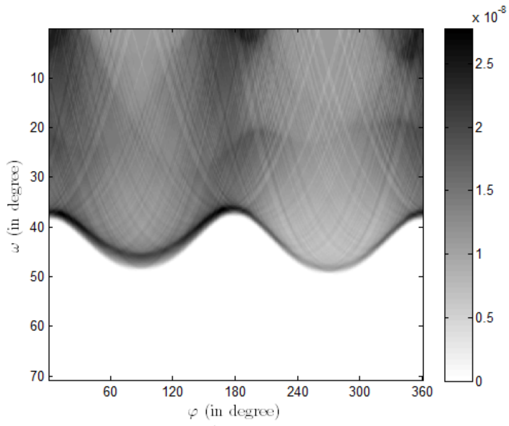

Fig. 5. CART of Fig.(3)

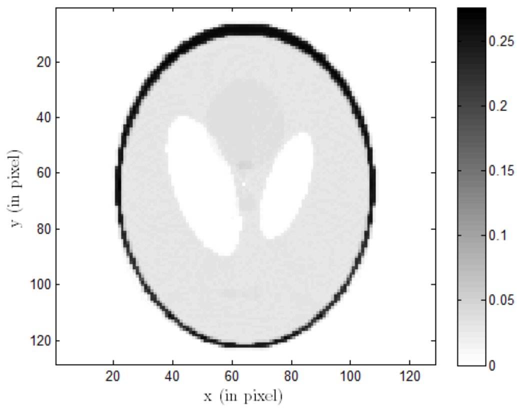

Fig. 6. Reconstruction without attenuation correction of n_e of Fig.(3)

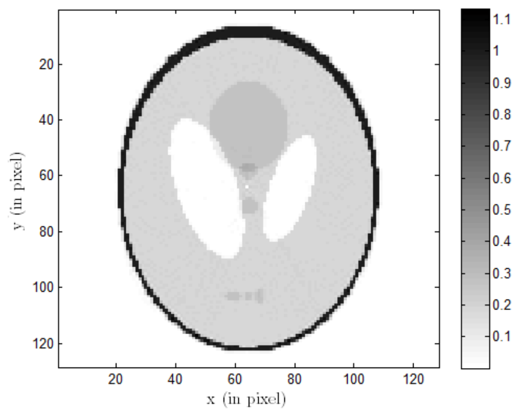

Fig. 7. n_e of the Shepp-logan phantom with attenuation correction

B. The Zubal Phantom (ZP)

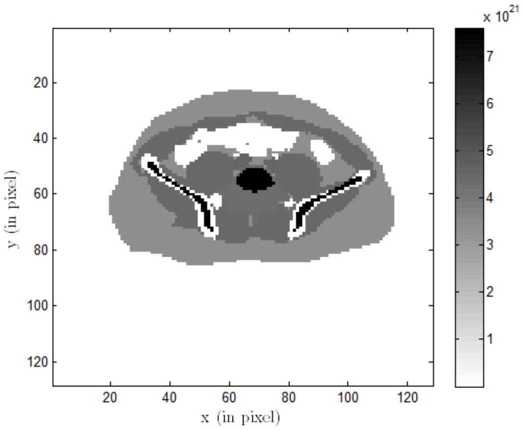

Fig. 8. Original n_e of the Zubal Phantom

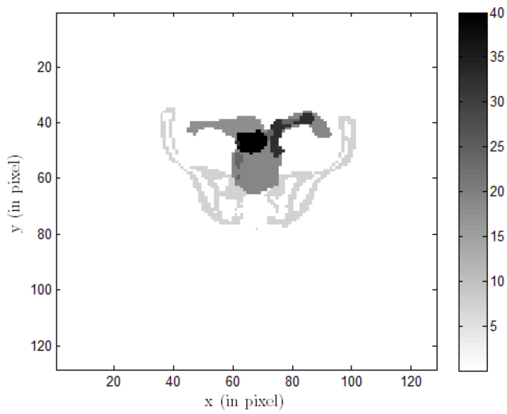

Fig. 9. Attenuation map of Fig.(8)

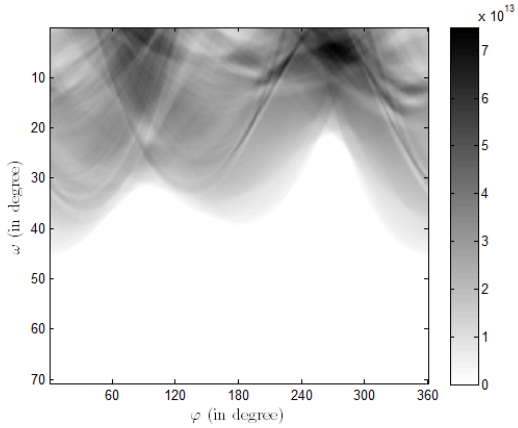

Fig. 10. CART of Fig.(8)

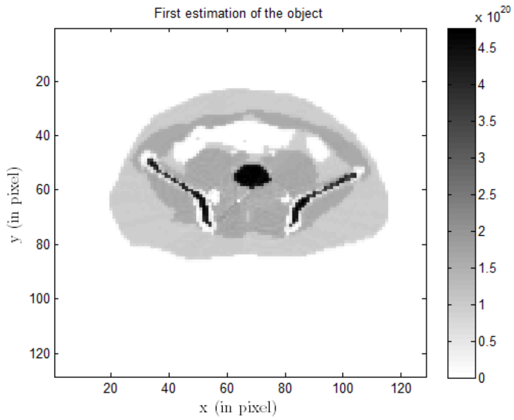

Fig. 11. Reconstruction without attenuation correction of n_e of Fig.(8)

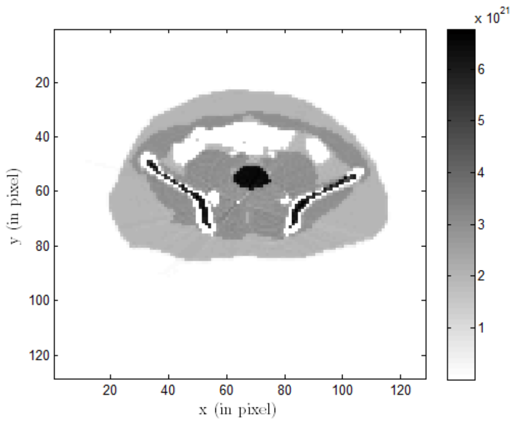

Fig. 12. n_e of the Zubal phantom with attenuation correction

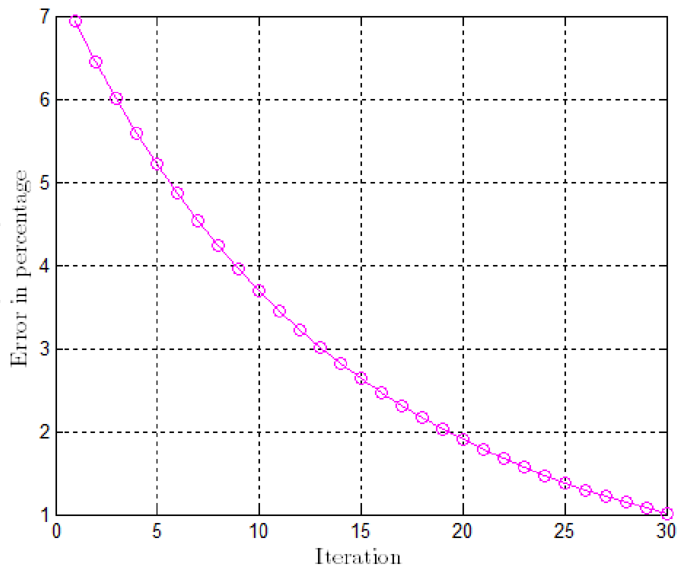

Fig. 13. NMSE in terms of iterations for the correction of the electron density of the Zubal Phantom of Fig.(a)

To illustrate the convergence of the algorithm towards the original Zubal, we show the plot of the NMSE error in terms of iterations. We can see the interest of our method in terms of quality of reconstruction, the final read NMSE error after 30 iterations is 1.001%. The are still improvements that need to be done, such as Image contrast, but all these enhancements will take place in future works.

VI. Conclusion

The new CST approach proposes an alternative to current tomographic imaging techniques by keeping almost the same quality of reconstructions. This modality characterizes the studied material directly by its electron density (scattering sites) and permits reconstructing an almost accurate attenuation map.

We observe that the attenuation correction allows to recover the more accurate electron density (in comparison of maximal values (of 1 for the Shepp-Logan phantom and of 10^{21} for the Zubal phantom) between reconstructions with and without attenuation correction).

The simulation results illustrate the functioning of this new Compton Scatter Tomography in the presence of attenuation. The iterative attenuation correction scheme converges rapidly. Of course there is room for improvement but no doubt that the algorithm for our CART works efficiently.

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