Spectral enrichment and wall losses in trombones played at high dynamic levels

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The characteristic spectral enrichment in the radiated sound of a brass instrument played at high dynamic levels is primarily a result of two closely coupled effects: the nonlinearity at the input to the tube, ie the lip aperture/mouthpiece effect; and the gradual nonlinear distortion of the wave form as it travels along the length of the instrument. In an instrument such as a trombone, which is largely comprised of cylindrical tube, the nonlinear distortion of the wave form can be dramatic, to the point of developing into a shock wave. The higher frequency components of the sound wave suffer more from viscothermal wall losses than do the lower frequency components. These losses act to damp the high frequency components to some extent. Experiments using cylindrical tubes of length and diameter representative of those of a trombone, have been carried out. The results of these experiments are compared with numerical simulations based on weakly nonlinear shock theory in an attempt to better understand the relationship between nonlinear distortion and the effect of the viscothermal losses.

1 Introduction

The bore profile of a brass instrument is one of the major factors in determining the playing characteristics of a particular instrument. Different instruments that are pitched in the same sounding key can have very different characteristic timbres and this is mainly due to the differences in bore profile [1, 2]. The different timbres of these instruments can be less apparent at low dynamic levels, with instruments in the same sounding key sounding similar. When played at higher dynamic levels, energy is transferred into the upper harmonics which leads to a 'brassy' sound. The spectral enrichment is caused as the pressure wave travels along the instrument bore and nonlinear wave steepening occurs because of the high dynamic levels, which alters the harmonic content of the sound [3, 4, 5]. However, the extent of spectral enrichment can vary with bore profile, hence causing instruments to have significantly different characteristic timbres. Instruments, such as trumpets and trombones, which have a large proportion of cylindrical tubing show more spectral enrichment than instruments such as euphoniums and flugelhorns whose bore profile expands steadily throughout a significant amount of the tubing.

Nonlinear wave steepening is not the only factor affecting the harmonic content in brass instruments while being played. Viscothermal losses in the boundary layer at the tube wall act to dissipate the high frequency energy that is the result of nonlinear wave steepening. The boundary layer thickness is typically in the region of 0.1mm which will have a different effect in different diameter tubes. The viscothermal wall losses are likely to have a larger effect in small diameter tubes than in larger diameter tubes because the boundary layer occupies a larger percentage of the internal area of the tube.

This paper will explore the relative significance of the effects of nonlinear wave steepening and viscothermal wall losses in brass instruments using the simplified case of cylindrical tubes. Using a forced oscillation input and cylindrical tubes of varying diameter, comparable to those of trombones (and many other brass instruments), experimental results will be compared with those produced using a computational simulation based on weakly nonlinear propagation theory.

2 Theoretical Background

For the weakly non-linear case, the local sound propagation speed c is given by [6]

$$c = c_0 + \left(\frac{\gamma + 1}{2}\right) v \tag{1}$$

where c_0 is the small-signal value of the speed of sound in air, γ is the ratio of specific heats at constant volume and constant pressure, and v is the acoustic particle velocity. It can be seen from equation 1 that for part of the acoustic cycle where v is positive, the speed of propagation is greater than c_0, and for the part of the cycle where v is negative the propagation speed is less than c_0. This can give rise to a shock wave as the higher amplitude parts of the pressure wave "catch up" with the lower amplitude parts, steepening the rising wave-front more and more until a shock wave is formed.

The distance X_s needed for a shock wave to be generated in a cylindrical tube, using a lossless approximation, is given by [5]

$$X_s \approx \frac{2\gamma P_{atm} c}{(\gamma + 1)(\delta p(in)/\delta t)_{max}} \tag{2}$$

where $p(in)$ denotes the input pressure, P_{atm} atmospheric pressure, c the speed of sound in air and γ the ratio of specific heats for air. Equation 2 shows that X_s does not depend on D, the internal diameter of the tube; therefore tubes with differing internal diameters, but the same total length, should have the same rate of nonlinear distortion if they are excited with the same input pressure waveform.

A brassiness potential parameter B has been defined by taking into account the reduction in the mean pressure amplitude as a sound wave propagates down the expanding bore of a typical brass instrument. This parameter B can be used to rank instruments in terms of the degree of spectral enrichment likely to occur at a given dynamic level as a result of nonlinear wave distortion [8]. The brassiness potential parameter is based on the lossless approximation and therefore insensitive to absolute radial bore dimensions. However, musical experience confirms that radial bore scale does have an important effect on timbre. The influence of radial scale is related to the dependence of the transfer function of the instrument on its cross-sectional area [8].

For a tube whose radius a is more than 10 times the boundary layer thickness, the effect of linear viscothermal losses on a wave of frequency f can be approximated by introducing a complex wave number [7]:

$$k' \approx 2\pi f/c - j\alpha \tag{3}$$

with loss parameter

$$\alpha \approx \frac{3 \times 10^{-5} f^{1/2}}{a}. \tag{4}$$

These frequency dependent losses can have two different and opposing effects which are musically significant.

The first of these effects is a preferential damping of the upper harmonics which have been generated through nonlinear distortion, therefore reducing the degree of spectral enrichment. The second effect is that $T14$ - the notation used in this paper for the transfer function from the mouthpiece to the bell - is diminished. Thus, the player is required to generate a larger amplitude wave in the mouthpiece to achieve a given radiated amplitude. From equation 2 it can be seen that this increase in input pressure amplitude produced by the player will result in an increase in nonlinear distortion and spectral enrichment itself. Therefore these two effects of the losses act to oppose each other and this paper will attempt to explore which is dominant in cylindrical tubes of different diameter.

3 Experimental Procedure

The experimental set-up used for this work examining the interaction between nonlinear wave steepening and viscothermal losses in cylindrical tubes is shown in Figure 1.

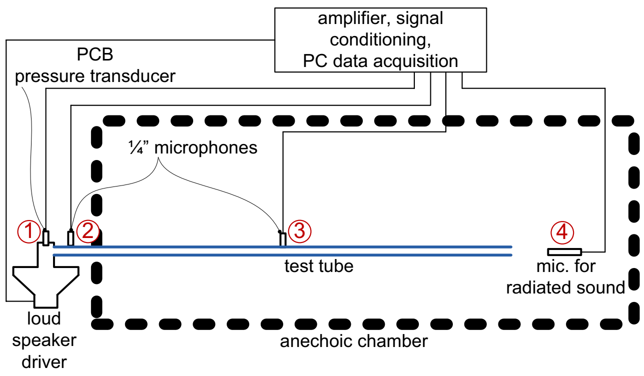

Figure 1: Schematic of experimental setup. Pressure measured at the four labelled locations 1-4.

Five tubes were used, each with length 2.96m and bore diameters of 6mm, 8mm, 10mm, 12mm, and 14mm. These diameters were chosen as 14mm is a typical slide bore diameter for a modern orchestral tenor trombone, and 10mm slide diameters can be found in some nineteenth-century French trombones. While the 6mm diameter tube is unrealistically narrow for a trombone, it was chosen to explore the effect of viscothermal losses in a tube where they are very significant. All five tubes were stainless steel with a wall thickness of 2mm, manufactured to ASTM A269/A213, and with uniform, semi-polished surface finish on the inside wall.

A 106B PCB Piezotronics dynamic pressure transducer was used to measure the pressure at the entrance to the tube. To measure the pressure within the tube, Bruël and Kjær 4938 1/4-inch pressure-field microphones were used; these were located a short distance from the entrance - 0.05m in, and close to the mid point of the tube - 1.47m in. The radiated sound was measured using a Bruël and Kjær 4145 1-inch free-field microphone at a distance of 0.2m on axis from the exit of the tube.

The 1/4-inch microphones, measuring the pressure signals within the tube, were housed in brass couplers. These couplers allow the microphones to be inserted so that the diaphragm is nominally flush with the inner tube wall and causes as little perturbation to the wall surface as possible.

The tubes were excited by a JBL 2446H compression driver producing a sine wave input signal. The input sine wave pressure amplitude was increased from zero to a maximum value, determined by the limit at which the microphone measuring the input signal recorded distortion by the loudspeaker.

The frequency at which the experiments were carried out was chosen for each tube independently. For each tube the frequency of a corresponding minima of $T14$ (the ratio between the rms pressure measured by microphone 4, in the radiated field 20cm from the tube exit on axis, and the rms pressure measured by microphone 1 at the tube entrance) was found and henceforth used for all measurements. A minimum in $T14$ occurs when the wave reflected from the open end arrives at the input in phase with the forward going wave and hence lies approximately on a resonant frequency of the tube - the 13th resonant mode was chosen since this was at a pitch which was (just) playable on a trombone, but sufficiently high for the speaker driver sound at a sufficiently high dynamic level. Due to the tubes having different end correction conditions the frequency of the 13th resonant mode is slightly different for each tube.

Table 1: Transfer function minima frequencies used.

Tube Diameter	Frequency (Hz)	
	Experiment	Simulation
6mm	762	760
8mm	767	762
10mm	765	764
12mm	770	766
14mm	770	766

4 Results

All experimental results presented here are compared with simulations based on weakly nonlinear propagation theory [9]. This theory takes into account the effects of nonlinear distortion but assumes that the viscothermal losses are linear.

4.1 Transfer Function

Figure 2 shows experiment and simulation results, for the five tube diameters used, of the transfer function $T14$.

There is a clear increase in $T14$ with bore diameter increase from the 6mm to 14mm. This increase with diameter is in line with expectations, as the increase in tube diameter increases the surface area at the end of the tube and hence the volume velocity and associated radiated pressure will be larger. We also know that the viscothermal wall losses become less significant in the larger tubes, and therefore in the smaller diameter tubes there will be more dissipation of energy as the sound wave propagates along the tube before some of this wave is radiated at the end of tube. The smallest diameter tube, 6mm, experiences almost no change in transfer function $T14$ with amplitude, which is what we would

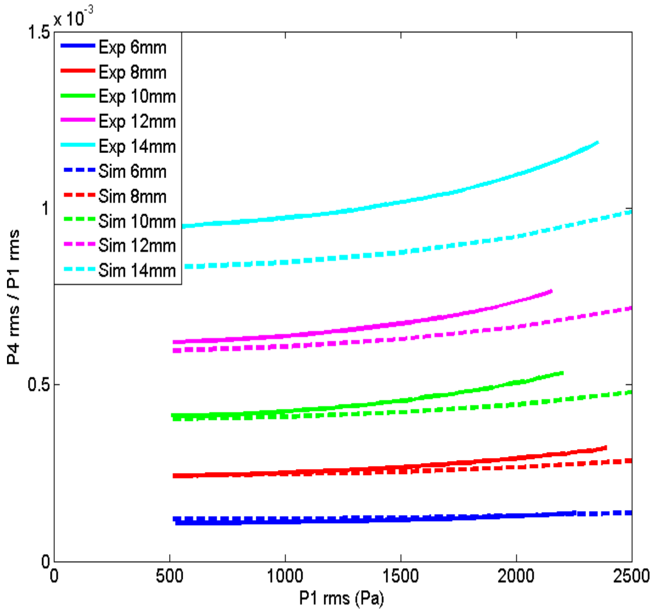

Figure 2: Transfer function T14 during an amplitude sweep at frequencies corresponding to a minima of the transfer function. Five tubes with diameters 6mm, 8mm, 10mm, 12mm, and 14mm.

expect if we were to assume the viscothermal losses were independent of amplitude and that there was no nonlinear distortion. As the diameter of the tubes increase, $T14$ increases more with increasing pressure amplitude, with this being most evident for the 14mm diameter tube. This behaviour of $T14$ can be partly understood as a consequence of the nonlinear steepening that has occurred within the tubes and the smaller viscothermal loss coefficient for the larger diameter tubes. At the highest input pressure amplitudes it can be seen that the experimental results deviate more from the simulation results; because the transfer function is the ratio of two pressure signals this shows that the radiated pressure $P4$ has a smaller value in the simulation. The simulation results are qualitatively similar to the experimental results but are not entirely quantitatively similar across all five tubes. The experimental $T14$ results are all larger than the simulated results, with the exception of the 6mm diameter tube, and the discrepancy increases with tube diameter. The quantitative difference between simulation and experiment is most marked for the 14mm diameter tube with the simulation results being around 10% lower than experiment. This difference could perhaps be attributed to the monopole radiation approximation made in the numerical simulation being an oversimplified approximation of the realistic radiation from the tubes.

The placement of microphones 2 and 3 within the tube length allowed the evolution of the nonlinear distortion and viscothermal losses to be explored at different points along the wave propagation. The weakly nonlinear propagation theory assumes that there is no local nonlinear interaction between the forward and backward going waves, but that the pressures measured by the microphones are the sums of these two waves.

Figure 3 shows measured and simulated results for $T23$ (the ratio between the rms pressure measured by microphone 3, roughly in the middle of the tube - 147cm from the tube entrance, and the rms pressure measured by microphone 2, 5cm from the tube entrance). The $T23$ results are essentially

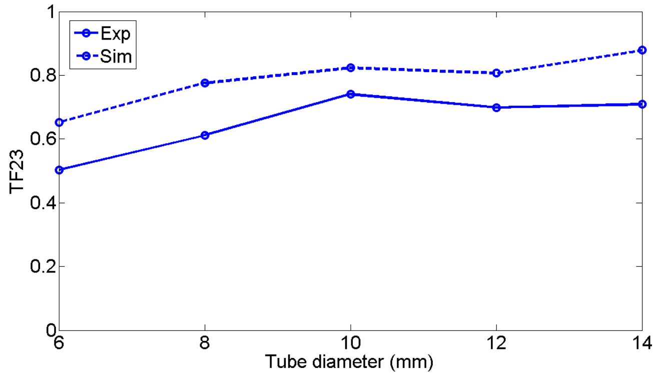

Figure 3: Transfer function T23 at 1000Pa rms input pressure at frequencies corresponding to a minima of the transfer function plotted against the five tube diameters 6mm, 8mm, 10mm, 12mm, and 14mm.

independent of input pressure amplitude so are plotted here against tube diameter. The absolute values of the internal transfer function $T23$ depend on the relative phases of the forward and backward going waves at the microphone position. The evaluation of $T23$ allows us to examine purely the effects of the nonlinear wave steepening and the viscothermal losses without the added complication of the radiation from the end of the tube. So from these results we can see that excluding the slightly crude radiation approximation in the simulation the results are broadly in accord with the experimental results but still between 15-30% higher. This difference between simulation and experiment suggests that the treatment of the losses in the simulation is not fully in accord with what is observed experimentally; the simulation appears to be under predicting the amount of losses in the system. The trends, with regards to tube diameter, are slightly different between the two curves - the simulation results increase with tube diameter almost steadily, with the exception of the 12mm diameter tube showing a slightly lower value for $T23$. However, the experimental results increase with a peak at the 10mm diameter tube and then decrease slightly with the two largest diameter tubes.

4.2 Spectral Centroid

The spectral centroid (as defined in [3]) is calculated using Equation 5

$$SC = \frac{\sum_n nP_n}{\sum_n P_n} \tag{5}$$

where P_n is the amplitude of the n^{th} harmonic of the signal.

It was found in the experimental results, at the highest amplitudes, that the input signal was not a perfectly sinusoidal signal and that there were very small, yet significant, 2nd, 3rd and 4th harmonic components present. These small harmonic components have been incorporated in the simulation input to give a true comparison of spectral centroid results. When calculating the spectral centroid for both experimental and simulated data only 15 harmonics are taken into account.

Figure 4 shows simulated results of the spectral centroid calculated in the radiation field, 20cm from the tube exit on axis, assuming monopole radiation. Here we have chosen a larger range of tube diameters than measured experimentally to explore the overall effect on spectral centroid of varying diameter. This plot shows the results of a pure sinusoidal input signal at a constant frequency of 764Hz.

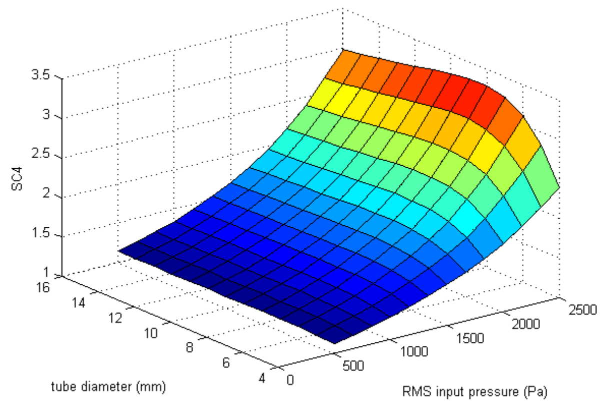

Figure 4: Simulated far field spectral centroid of tubes ranging in diameter from 4-16mm during an amplitude sweep at 764Hz.

It can be seen here that for high amplitude input signals there is a small but significant increase in the value of the spectral centroid with decreasing tube diameter, to a maximum point at 9 - 10mm diameter. The spectral centroid then falls off rapidly as the tube diameter decreased further. We might expect that for the same pressure amplitude at the input of the tube, and neglecting losses, the nonlinear distortion should be independent of tube diameter and hence produce similar spectral centroid values. However, this is not what is seen here.

This might be explained by the fact that, for larger diameter tubes, where wall losses are very small, for a constant input pressure at a transfer function minima, the forward going wave and backward going wave will have similar amplitudes (and we note that the measured input pressure, *P1*, is the sum of the forward and backward going wave). As the tube diameter decreases, the wall losses increase and thus the contribution of the backward going wave to the input pressure is reduced. An increase in the amplitude of the forward going wave is then required to achieve a given value of *P1*, increasing the level of nonlinear distortion and hence increasing spectral centroid seen in 4, reaching a maximum for the 9mm diameter tube. As tube size continues to decrease, the effect of viscothermal losses becomes the dominant effect and their damping of high frequency components is more prominent than the addition of these components due to nonlinear distortion.

Figure 4 does not however take into account the small phase differences that occur in the experimental results from having tuned the measurement frequencies to be at transfer function minima. Figure 5 shows both the experimental and simulation results at the transfer function minima frequencies for the far field spectral centroid. Figure 5 shows that there is broadly good agreement between the experimental and simulated results, with the highest measured spectral centroid value, of approximately 3, being that of the 10mm diameter tube. There is a significant difference between measured and simulated results with absolute values of the simulated spectral centroid being lower by as much as 40%. The difference between simulated and experimental results might be

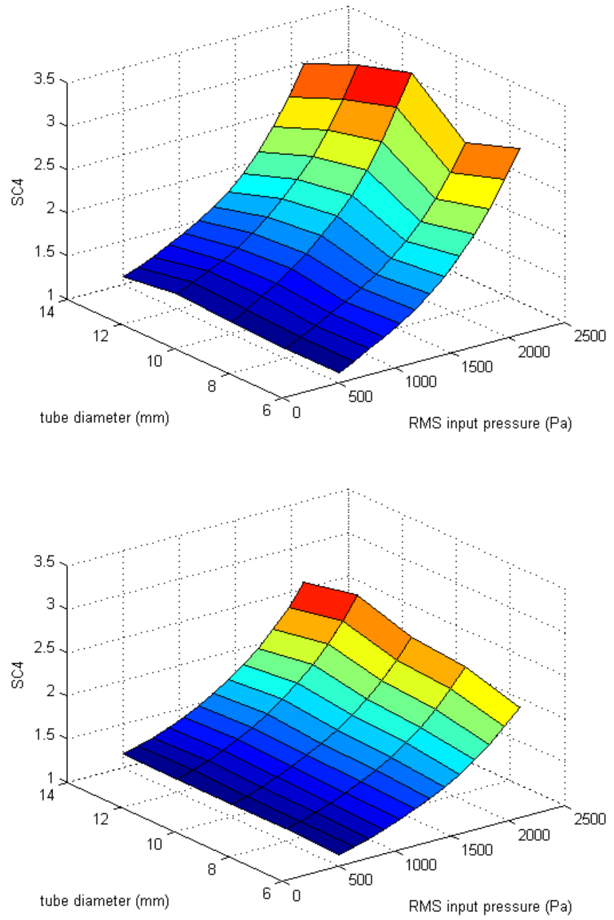

Figure 5: Far field spectral centroid of the five tubes (6mm, 8mm, 10mm, 12mm, and 14mm diameter) during an amplitude sweep at frequencies corresponding to a minima of the transfer function. **Top**: Experiment **Bottom**: Simulation

attributed, at least in part, to the fact that for larger tube diameters the assumption of a monopole radiation source at the tube exit becomes less applicable for the higher frequency components.

To eliminate possible effects resulting from the monopole radiation assumption, the spectral centroid at near the mid point of the tube (microphone 3) was also investigated and is shown in Figure 6. There is evidence of some variation. However, the maximum spectral centroid values for the internal results are comparable and on average very similar, at around 1.3.

It is worth noting that the spectral centroid scale (y-axis) for Figure 6, is smaller than that of Figure 5. This is unsurprising since we would not expect there to be as much nonlinear distortion halfway along the tube as in the far field.

5 Conclusions

The results presented here only relate to cylindrical tubes, and not real brass instruments, but have enabled us to explore the effects of nonlinear wave steepening, viscothermal losses and the way in which they combine to create the characteristic timbre of a brass instrument during a crescendo. It is clear why instruments with bore diameters as small as 6mm are relatively rare - the loss shown in the transfer function due

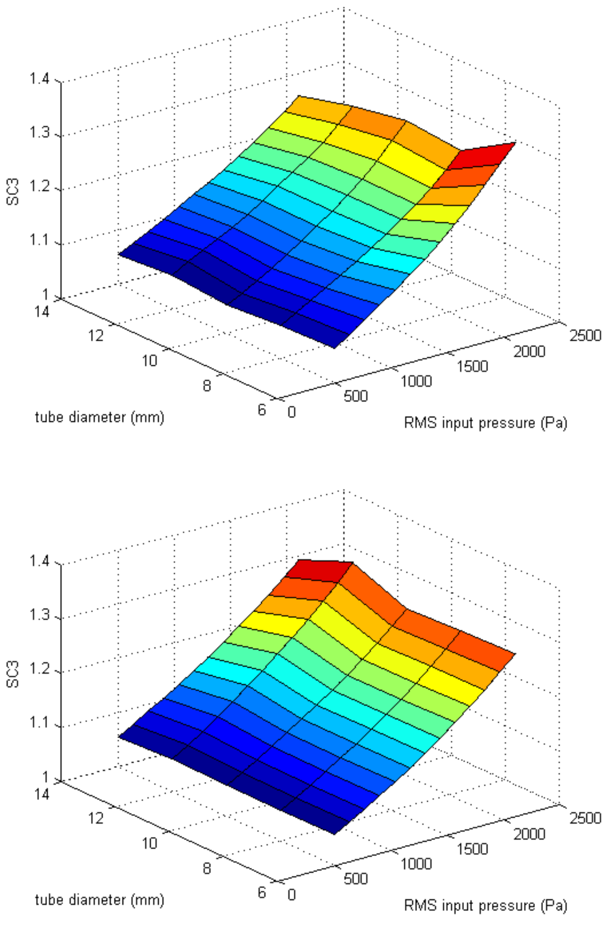

Figure 6: Internal spectral centroid of the five tubes (6mm, 8mm, 10mm, 12mm, and 14mm diameter) during an amplitude sweep at frequencies corresponding to a minima of the transfer function. **Top**: Experiment **Bottom**: Simulation

to the reduction in cross-sectional area is greatly increased by the dominance of viscothermal losses.

The simulation program has been shown to successfully predict the variation of both the transfer function and spectral centroid and is broadly in agreement with experimentally measured results. As the tube diameter increases the simulation does however seem to under predict the transfer function $T14$ results; this could be due the the realistic radiation field having a higher concentration of energy on axis, where the microphone is placed. The internal transfer function $T23$ results are shown to be slightly higher than the measured results suggesting that the simulation maybe under predicting the viscothermal losses within the tube. The spectral centroid results have equally been slightly under predicted by the simulation - again this could be because the higher frequencies are more efficiently radiated in the central lobe of the radiation field and the monopole approximation therefore breaking down more rapidly for the higher frequencies.

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