The application of the factor analysis to the studies of the nonlinear dynamic elasticity

Valery Tsaplev, Paul Grushevsky

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V. Tsaplev and P. Grushevsky

North-West State Technical University, 5, Millionnaya Street, Department of Physics, 191186 Saint-Petersburg, Russian Federation
valery@convergences-fr.ru

The application of the factor analysis to the studies of the nonlinear elasticity of the piezoceramic materials is presented. All physical properties of these materials are strongly nonlinear – dielectric, elastic and unelastic. Moreover, all these properties manifest creep. Simultaneous measurements of all these characteristics are very labor- and time-consuming and often even impossible, especially concerning the second- or third-order elastic moduli. The usage of the factor analysis methods helps to optimize the process. The results of the application of these methods to the study of the titanate-barium and PZT piezoceramics elastic moduli, internal friction and non-stationary creep are presented.

1 Introduction

Piezoceramic materials, that are widely used for electromechanical transducers show very strong nonlinear properties. Elastic moduli and compliances, internal friction and dielectric constants manifest strong dependences on mechanical stresses, electric strength, frequency and temperature below the Curie point. It was found [1], that the defect of the dynamic Young's modulus (i.e. it's relative change) under the uniaxial constant mechanical stress may reach 25-30% below the Curie point (fig.1), while it does not exceed 1-2% above the Curie point. One may also add the strong frequency dispersion and creep under maintained stress [2,3].

The elastic properties of piezoceramic materials was studied in a great number of papers, but the nonlinear properties are still weakly examined, due to the experimental problems. The full set of the first-order elastic and dielectric constants of the titanate-barium ceramics was studied by V. Yakovlev [4], who did not find any frequency dispersion. But under stress there appear second- and third-order constants, appears dispersion and creep, and the measurement of the full set of constants becomes extremely complicate, maybe even impossible.

However, just the knowledge of the nonlinearity of the elastic constants is necessary to design piezoelectric transducers, that work under high static, dynamic and shock load. This information provides also the possibility of designing the new type of transducers – parametric transducers, using the nonlinear effect.

The present paper shows the new approach – the application of the factor analysis to the ultrasonic studies of the nonlinear elasticity and designing an experiment using the system analysis methods and the package STATGRAPHICS *Plus for Windows.*

2 Planning of experiment

The estimation results are performed using the full factor tests and constructing the full prognostic models using the experimental dependences of the Young's modulus defect on the uniaxial stress (Figure 1) [1].

The experiment was planned using the active approach, with simultaneous variations of values of ensemble factors $(X_1, X_2, \ldots, X_K)$ according to some rational program - design matrix. The planning includes: the definition of the space of factors, the choice of strategy of test. The ensemble of factors must include all essential factors, and every factor must satisfy requirements of unambiguity, controllability, independence and compatibility with all other factors. Here we consider the following factors: temperature, frequency, mechanical stress. These factors completely satisfy these requirements.

Choosing the strategy we state three main steps:

1. Design and test operation in some bounded region for the purpose of finding out the gradient direction (the direction, where the slope angle of the response function is ultimate. The problem may be solved using the linear regression equations.
2. Successive movement in the line of gradient direction (path-of-steepest-ascent method). Knowing the line of gradient direction, one chooses another bounded region in the space factor and implement new test series. The steepest-ascent is repeated until "the nearly stationary region" is reached, where factor variations weakly affect the output parameters
3. Design and test operation in "the nearly stationary region", where finally the set of factor values, leading to the necessary value of y is determined. In this stage the nonlinear character of dependences between x_i and y is considered.

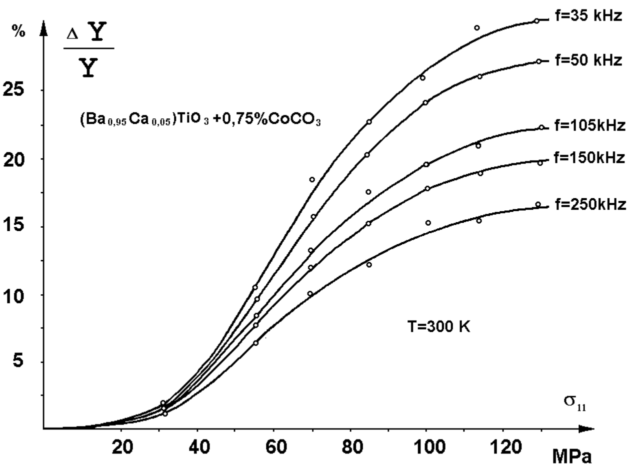


Figure 1: The Young's modulus defect on stress.

When planning the multiple-factor studies of piezoceramics by active methods it is necessary to divide plans to *full factorial design* and *fractional factorial design*. To compose the full plan one must know the full set of already known data about the nonlinear properties of this material. The quality forecast of creep helps to more accurate measurement of Young's modulus defect, or Q-factor depending on temperature, frequency or stress.

The full factorial design includes the ranges of factor values: mechanical stress - 10-130 MPa; frequency - 50-250 kHz; temperature – 280-300 K. We consider the Young's modulus defect as the response function.

This range of factor values is rather wide and it seemed to be useful to divide it to two parts and make separate experiment for each part. As the data were not enough for full factorial design (data for Young's modulus defect from stress and temperature), the experiment was made separately for factors of temperature and frequency.

3 Results of multi-factor analysis

Two factors were used for the first experiment:
1. Factor *A*: stress *G* (10-90 MPa);
2. Factor *B*: frequency *f* (50-250 kHz).
Factor *A* and factor *B* vary on 5 levels.

For the second experiment were used also 2 factors:
1. Factor *A*: stress *G* (70-130 MPa) ;
2. Factor *B*: frequency *f* (50-250 kHz).
Factor *A* varies on 4 levels, and factor *B* - on 5 levels.

For the third experiment also were used 2 factors:
1. Factor *A*: frequency *f* (50-250 kHz);
2. Factor *B*: temperature *T* (280-370 K).
Factor *A* varies on 5 levels, factor *B* - on 5 levels.

The initial data were obtained from the natural experiment [2]. On the base of these data the mathematical models of the piezoceramic properties and behavior were constructed. The range bounds for all factors were estimated using a priori information. Within these range bounds for every factor the ground level and the variability interval were chosen. These parameters were used for the experiment design. Tables 1, 2 and 3 show the full experimental plans for the first, second and third models (factors of stress, frequency and temperature) correspondingly. Here *Y* – is the response function relatively to stress.

Table 1: Full experimental plan for the first model

Number of degrees of freedom	*G*	*f*	*G*
1	-1	-1	0,07
1	-0,5	-1	1,25
1	0	-1	7,38
1	0,5	-1	15,61
1	1	-1	21,75
1	-1	-0,5	0,07
1	-0,5	-0,5	1,25
1	0	-0,5	6,87
1	0,5	-0,5	13,75
1	1	-0,5	18,7
1	-1	0	0
1	-0,5	0	1,23
1	0	0	6,05
1	0,5	0	11,87
1	1	0	15,4
1	-1	0,5	0
1	-0,5	0,5	1,22
1	0	0,5	5,2
1	0,5	0,5	10,9
1	1	0,5	14,8
1	-1	1	0
1	-0,5	1	0,82
1	0	1	4,98
1	0,5	1	9,99
1	1	1	13,2

Table 2: Full experimental plan for the second model

Number of degrees of freedom	*G*	*f*	*Y*
1	-0,333333	0,5	14,8
1	1	1	16,35
1	0,333333	-0,5	21,3
1	-1	0,5	10,9
1	0,333333	-1	25,1
1	0,333333	1	15,1
1	0,333333	0	18,35
1	0,333333	0,5	16,66
1	-1	0	11,87
1	1	-1	27,45
1	-1	-1	15,61
1	-0,333333	1	13,2
1	1	0	19,9
1	-0,333333	-0,5	18,7
1	-0,333333	-1	21,75
1	-0,333333	0	15,4
1	-1	-0,5	13,75
1	1	-0,5	22,6
1	-1	1	9,99
1	1	0,5	17,6

Table 3: Full experimental plan for the third model

Number of degrees of freedom	*f*	*T*	*Y*
1	-1,0	-1,0	14,4
1	-0,5	-1,0	11,8
1	0,0	-1,0	10,5
1	0,5	-1,0	9,2
1	1,0	-1,0	9,99
1	-1,0	-0,5	14
1	-0,5	-0,5	11,5
1	0,0	-0,5	10,2
1	0,5	-0,5	9
1	1,0	-0,5	8
1	-1,0	0,0	14,6
1	-0,5	0,0	12
1	0,0	0,0	10,7
1	0,5	0,0	9,3
1	1,0	0,0	7,9
1	-1,0	0,5	18,5
1	-0,5	0,5	15,5
1	0,0	0,5	13,5
1	0,5	0,5	11,8
1	1,0	0,5	10,3
1	-1,0	1,0	9,8
1	-0,5	1,0	8
1	0,0	1,0	7,2
1	0,5	1,0	6,3
1	1,0	1,0	5,3

The full factor experiment on regression analysis for all three cases was made using package STATGRAPHICS *Plus for Windows*. The results one can see in table 4.

Table 4: Results of experiment

№ exp.	Model constants	Values of constants	Number of degrees of freedom
1	2	3	4
1	Constant	5,94389	
	A:G	8,9496	1
	B:f	-1,7052	1
	AA	2,23429	1
	AB	-2,1924	1
	BB	0,469143	1
2	Constant	17,4385	
	A:G	4,14	1
	B:f	-4,3465	1
	AA	-1,61325	1
	AB	-1,2891	1
	BB	1,55357	1
3	Constant		
	A:f	11,6099	1
	B:T	-2,9128	1
	AA	-0,8672	1
	AB	0,970286	1
	BB	-0,1664	1
		-2,64686	

The final step of experiment on full factorial design or fractional factorial design was the statistical analysis of obtained results. This analysis include: estimation of reproducibility of test results; estimation of significance of regression coefficients; improvement of the model.

It was necessary to find out the correlation between the factors that have an effect on the nonlinear elasticity. For this purpose in accordance with the full factorial plan and with the help of STATGRAPHICS the multiple regression equations were composed (table 5)

Table 5: Confidence coefficients of models

№ exp.	Fact.	Model	quality factor (%)
1	A: G B: f	$Y = 5{,}94389 + 8{,}9496 \cdot G - 1{,}7052 \cdot f + 2{,}23429 \cdot G^2 - 2{,}1924 \cdot G \cdot f + 0{,}469143 \cdot f^2$	98,07
2	A: G B: f	$Y = 17{,}4385 + 4{,}14 \cdot G - 4{,}3465 \cdot f - 1{,}61325 \cdot G^2 - 1{,}2891 \cdot G \cdot f + 1{,}55357 \cdot f^2$	99,39
3	A: f B: f	$Y = 11{,}6099 - 2{,}9128 \cdot f - 0{,}8672 \cdot T + 0{,}970286 \cdot f^2 - 0{,}1664 \cdot f \cdot T - 2{,}64686 \cdot T^2$	76,9

One can see, that the confidence coefficients of models are rather high (98%,99%). The second model seems to be more appropriate, due to the higher quality factor. The third model is also not bad. Figures 2 and 3 show the influence of coefficients on the response function in the first and the second models for defect Young's modulus dependences from stress and frequency. The response function surfaces are also shown.

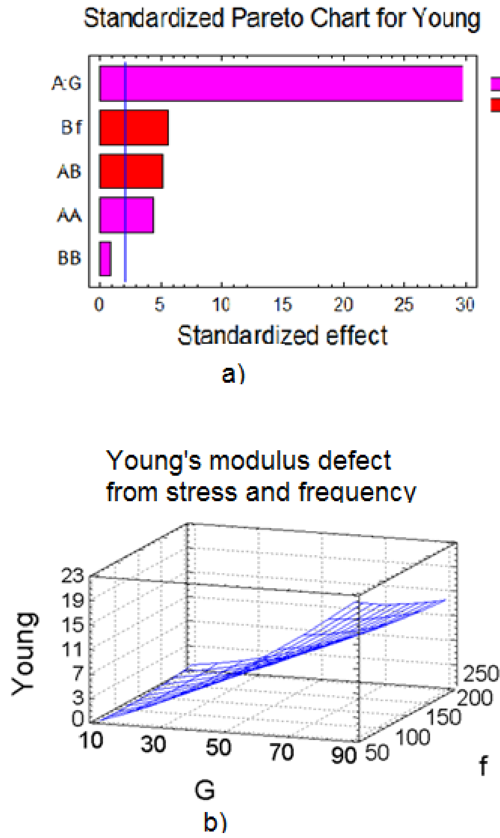


Figure 2: a) The effect of coefficients on the response function in the first model for defect Young's modulus dependences from stress and frequency; b) Response function surface for the first model.

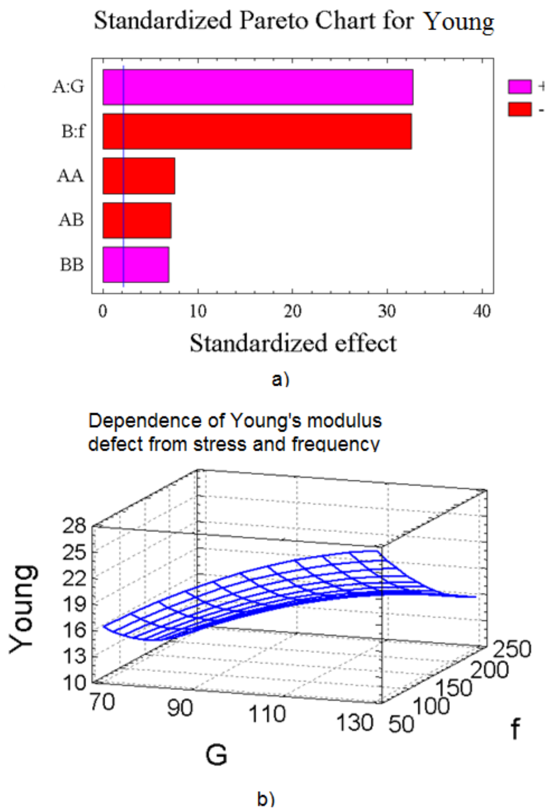
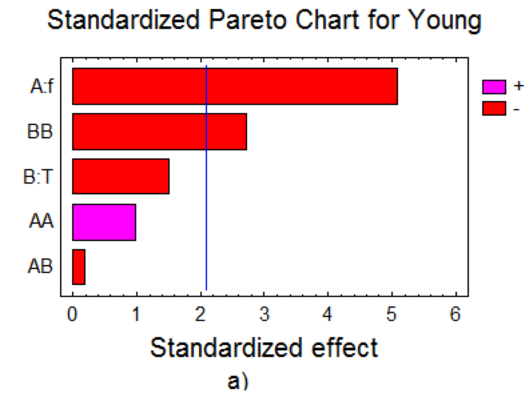


Figure 3: a) The effect of coefficients on the response function in the second model for defect Young's modulus dependences from stress and frequency; b) Response function surface for the second model.



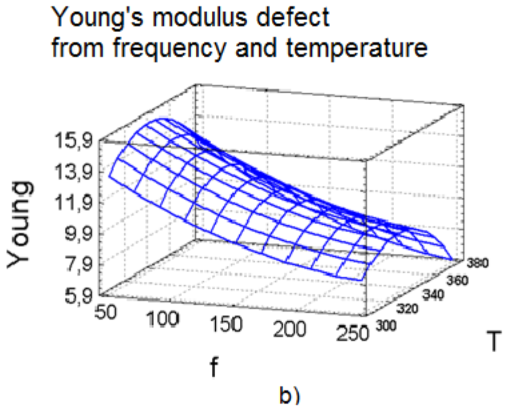


Figure 4: a) The effect of coefficients on the response function in the third model for defect Young's modulus dependences from frequency and temperature; b) Response function surface for the second model.

It can be seen from figure 2,a, that the factor f has the most negative influence on the response function. The factor G affects more on the response factor, but positively. Combination of two factors (A and B) has a negative influence.

Figure 2,b shows, that the most negative influence on the response function has the factor f, while the factor T affects less, but still in the negative. The joint influence of two factors (A и B) is negative. Reduction of Y causes the decrease of the Young's modulus defect. To obtain this effect one must enhance factors f and T, while the increment of Y can be achieved by simultaneous change of both factors, but in different directions. The computing experiment was set according to designed plans. Models of evaluation of Young's modulus defect dependence on stress, frequency and temperature were obtained as quadratic equations of multiple regression. These models have rather high quality factors (table 5).

The obtained regression models proved to be very useful to minimize the time and quantity of experiments on measuring the elastic, unelastic and piezoelectric constants of higher orders and their dependences on different external factors.

5 Conclusion

Experimental studies of anisotropic nonlinear materials, such as piezoelectric ceramics are very labour-intensive and take a lot of time. Measurements of full set of elastic and piezoelectric constants and these constants of higher orders and their dependences on external factors seem to be very difficult and even impossible because of the necessary time and a huge number of specimen. The usage of computer design of experiment helps to minimize these efforts and forecast the behavior of material under extreme conditions. The statistical interpretation of the computer modeling results helps to choose necessary models for further measurements and estimations of their results. The mathematical models for estimation of dependences of nonlinear Young's modulus defect from stress, frequency and temperature are based on preliminary experimental data. Prediction of nonlinear elasticity was made using the STATGRAPHICS *Plus for Windows* package. The equations of multiple regression were obtained and it helped to find the quadratic relationships between the main piezoceramic constants.

The method is useful to simulate not only piezoceramics, but a number of other nonlinear materials, that makes this method promising.

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