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## Solving the Yitzhaki Paradox

Income Tax Evasion and Reference Dependence under Prospect Theory

Gwenola Trotin

# Solving the Yitzhaki paradox: Income tax evasion and reference dependence under prospect theory

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## Abstract

This paper examines the determinants of tax evasion under prospect theory. For prospect theory, reference dependence is a fundamental element (the utility function depends on gains and losses relative to a reference point and not on final wealths as in expected utility theory). In order to identify the determinants of the income tax evasion decision, a general reference income is used. We show that results obtained under expected utility theory are not robust. In particular, tax evasion is increasing in the tax rate as soon as a suitable relative risk aversion measure is larger with auditing, than without. With this simple and testable condition, prospect theory provides a general framework consistent with empirical evidence for the tax evasion behaviour problem.

*JEL classification:* D81; H26; K42

*Keywords:* Tax evasion; Prospect theory; Reference dependence; Decision weights

## 1. Introduction

Concerns about tax enforcement policies have led most governments to set up a large tax evasion fighting system with audits and fines. The first step to optimize this system is to make best knowledge of evasion decisions of taxpayers. A substantial literature has already studied this issue, most often within an expected utility theory framework, the seminal applications of this theory to tax evasion problem being the ones by Allingham and Sandmo (1972), where the fine is imposed on the undeclared income and Yitzhaki (1974), where the fine is imposed on the evaded tax. The second case is the most frequently seen, like for example in the US and in France.

The model of Yitzhaki has been extended to include many alternative assumptions, see for example Franzoni (2008). However, the expected utility theory has been criticized a lot these last years. Many empirical studies have emphasized its disability to describe the observed behavior patterns in an adequate way, see for example Skinner and Slemrod (1985), Alm et al. (1992), Andreoni et al. (1998) or Slemrod and Yitzhaki (2002). In particular, with a reasonable degree of risk aversion, it predicts a too large extent of tax evasion. Additionally, under the assumption of decreasing absolute risk aversion, it predicts that an increase in the tax rate leads to a decrease in tax evasion.

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Therefore, a number of works have developed alternatives to expected utility theory to account for the behavior patterns observed in experiments. Among them, rank dependent expected utility theory applies transformations to the cumulative probability distribution function in order to overweight unlikely extreme outcomes. This theory was designed in particular to explain the behaviour observed in the Allais paradox. Prospect theory provides differential treatments of gains and losses with respect to a reference point and applies transformations to individual probabilities in order to overweight all unlikely events. Prospect theory was introduced by the works of Kahneman and Tversky (1979). Building upon prospect theory and the works of Starmer and Sugden (1989) and Tversky and Kahneman (1992), cumulative prospect theory is a variant of prospect theory, weighting being applied to the cumulative probability distribution function, as in rank-dependent expected utility theory, rather than to the probabilities of individual outcomes.

Prospect theory and cumulative prospect theory have become two of the most prominent alternatives to expected utility. They are widely used in empirical research. The carriers of utility are not final levels of income any more but differences between final levels of income and a determined reference income. It expresses the *framing effect* phenomenon. The utility function convex for gains and concave for losses expresses the *loss aversion* phenomenon: individuals care generally more about potential losses than potential gains. There is risk-averse behaviour in case of gains and risk-seeking behaviour in case of losses. Furthermore, individuals tend to overweight unlikely events but underweight average and likely ones. The collection of papers in Kahneman and Tversky (2000), for instance, provides empirical confirmation of these properties.

The tax evasion problem has already been dealt with the literature on prospect theory. Alm et al. (1992) provides an experimental study. Among others, Yaniv (1999) analyzes the influence of obligatory advance tax payments on the taxpayer's evasion decision. He applies prospect theory to a simple model of tax evasion, using the income after the payment of the tax advance and prior to the filing of a return for the reference income, and demonstrates that advance tax payments may substitute for costly detection efforts in enhancing compliance. Bernasconi and Zanardi (2004) use cumulative prospect theory with a general reference point but with particular probability weighting and utility functions. Dhimi and al-Nowaihi (2007) also apply cumulative prospect theory to tax evasion. They consider the legal after-tax income to be the reference point because it is the only one with which the taxpayer is in the domain of gains if not caught and in the domain of losses if caught. Following Eide (2001), they argue that it is the only case in which the paradoxical comparative static results of the Allingham and Sandmo - Yitzhaki model do not carry over to rank dependent expected utility theory. They use a probability of detection which depends on the amount of income evaded and introduce stigma costs of evasion. Using the power utility function of Tversky and Kahneman (1992), they show that the predictions of prospect theory are consistent with the evidence. Using parameters estimated by the experimental literature and the weighting probability function of Prelec (1998), they show that relative to expected utility theory, prospect theory provides a much better explanation of tax evasion.

The present paper provides comparisons between expected utility theory and prospect theory. It is unique in providing the study for the tax evasion problem in a such general way. It provides the first very general use of prospect theory in tax evasion problem, with a general probability weighting function, a general utility function and a general reference point at the same time. It highlights a simple and testable condition under which prospect theory appears to be a general theoretical framework consistent with experimental evidence. It differs from previous papers in several respects. The main are the use of a general reference income and a general utility function. Arguing that it can be possible for the taxpayer to be in the domain of gains or in the domain of losses with or

without auditing, we show that the level and the expression of the reference income is an essential point in the use of prospect theory and that the paradoxical comparative static results of the Yitzhaki model do not carry all over the support of income distribution. The use of a general utility function let us to highlight general intuitions concerning tax evasion problem under prospect theory. In particular, introducing a suitable relative risk aversion measure, we show with several expressions for the reference income, that tax evasion is increasing in the tax rate as soon as the relative risk aversion measure is larger with auditing, than without. It is because an increase in the tax rate causes the taxpayer to be richer in terms of outcomes (differences between final incomes and the reference income). With a decreasing relative risk aversion, he chooses to increase the fraction of his initial income in the risky alternative and tax evasion increases. With tax evasion problem, qualitative results with probability weighting functions does not differ from those provided without probability weighting functions.

With qualitative concerns, expected utility theory account of tax evasion contradicts the empirical evidence in both following main ways:

- With a positive expected return to tax evasion, expected utility theory predicts that all taxpayers should hide some income.
- Yitzhaki (1974) showed that using expected utility theory under the reasonable assumption of decreasing absolute risk aversion, an increase in the tax rate leads to a decrease in tax evasion.

The present article shows that prospect theory provides a framework where results are in accordance with the qualitative empirical evidence. In particular, the well-known Yitzhaki paradox is solved.

The article is organized as follows. The next section sets up the basic model. Section 3 studies the effect of the use of a probability weighting function on the tax evasion decision of the taxpayer in an expected utility theory framework. Section 4 studies the effect of the use of a reference point. Section 5 studies tax evasion decision in a general setting of prospect theory. Section 6 makes the concluding remarks.

## 2. The model

### 2.1. Final incomes

A taxpayer has an exogenous taxable income  $w > 0$  which is private information. He declares some amount  $x \in [0, w]$ . In particular, he can not declare a negative income or an income higher than the initial income.<sup>1</sup> The government levies a tax on declared income at the constant marginal rate  $t$ ,  $0 < t < 1$ . The tax administration audits with the exogenous probability  $p \in [0, 1]$ . If he is caught cheating, the taxpayer must pay the evaded tax  $t(w - x)$  and a fine  $ft(w - x)$ , where  $f > 0$  is the fine rate on evaded taxes.<sup>2</sup> Denote by  $Y$  and  $Z$ , respectively, the net income of the taxpayer without and with auditing:

$$Y = w - tx, \tag{1}$$

$$Z = w - tx - (1 + f)t(w - x). \tag{2}$$

Prospect theory differs from expected utility theory by the use of two main characteristics. A probability weighting function expresses that people tend to overreact to small probability events, but underreact to medium and large probabilities. A reference

<sup>1</sup>This rules out the possibility to get some gain from being more than honest.

<sup>2</sup>It is assumed that if an audit occurs, the actual income of the taxpayer is discovered without error.

point expresses that individuals tend to think of possible outcomes relative to a certain reference point rather than to the final status. To highlight the differences between the application of the two theories, every characteristic is introduced alone before to study results in a general prospect theory setting. The next subsections precise the expressions of the probability weighting function and of the reference income.

## 2.2. Probability weighting function

The probability weighting function expresses that people tend to overreact to small probability events, but underreact to medium and large probabilities. A large literature backs up this fact, see for example Tversky and Kahneman (1992) or Starmer (2000).

The probability weighting function  $\pi$  is a continuous function on  $[0, 1]$ , differentiable on  $]0, 1[$ , strictly increasing from  $[0, 1]$  onto  $[0, 1]$ , with  $\pi(0) = 0$  and  $\pi(1) = 1$ . There exists  $p_0$  and  $p_1 \in ]0, 1[$ , such that  $p_0 \leq p_1$ <sup>3</sup> and such that for all  $p \in ]0, p_0[$ ,  $\pi(p) \geq p$  and for all  $p \in ]p_1, 1[$ ,  $\pi(p) \leq p$ . Figure 1 represents a typical weighting function. It transforms objective probabilities into subjective probabilities.

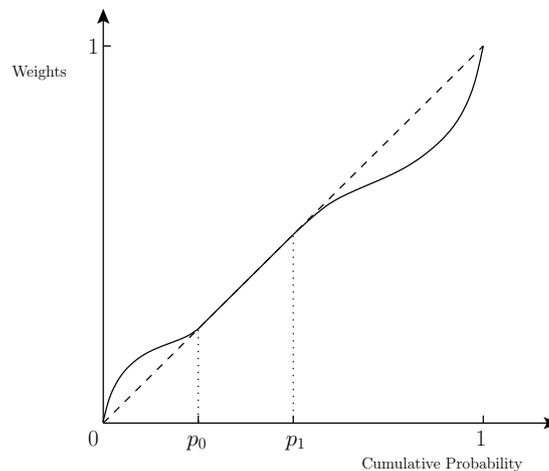


Figure 1: Probability weighting function

It can be observed that to consider  $\pi(p) = p$ , for all  $p \in [0, 1]$ , is equivalent to use objective probabilities, as in expected utility theory or in prospect theory.

The attitude towards risk depends on the curvature of the utility function and also on the shape of the probability weighting function. The random risk attitude of the taxpayer, determined by the shape of the probability weighting function, can be expressed by the following measure:

$$\Pi(p) = \frac{\pi(p)}{\pi(1-p)}, \text{ for } p \in [0, 1[, \Pi(1) = +\infty. \quad (3)$$

This measure is positive. For a fixed value for the audit probability, the more the taxpayer overweights small probabilities and underweights large probabilities, the higher it is. It measures the subjectivity of the taxpayer considering probabilities of events.

The weighting function of Prelec (1998) is consistent with much of the available empirical evidence. It will be useful to specify general results. It has the following form:

$$\pi(p) = e^{-(-\ln p)^\alpha}, \text{ with } 0 < \alpha < 1, \text{ for } 0 < p \leq 1 \text{ and } \pi(0) = 0. \quad (4)$$

<sup>3</sup>It is commonly supposed that  $p_0 = p_1 \simeq 0.3$ . See for example Prelec(1998).

The lower is  $\alpha$ , the higher is the degree of overweighting of small probabilities and of underweighting of large probabilities. As  $\alpha$  is close to 0, the probability function approximates a function flat everywhere except at the endpoints of the probability interval. As  $\alpha$  is close to 1, the probability function approximates the objective (linear) function. Figure 3 represents Prelec weighting function for different values for  $\alpha$ . Figures 3 illustrates examples of Prelec weighting functions with different values for  $\alpha$ .

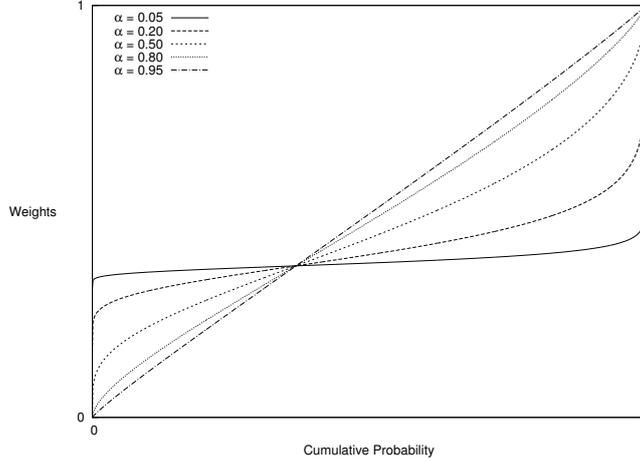


Figure 2: Prelec probability weighting functions with  $\alpha = 0.05, 0.2, 0.5, 0.8$  and  $0.95$ .

With this Prelec form, the random risk attitude measure has the following expression:

$$\Pi(p) = e^{(-\ln(1-p))^\alpha - (-\ln p)^\alpha}. \quad (5)$$

### 2.3. Reference income and utility of an outcome

Many empirical studies have shown that individuals tend to think of possible outcomes usually relative to a certain reference point rather than to the final status, see for example Kahneman and Tversky (2000). Following prospect theory, the taxpayer evaluates potential losses and gains. He sets a reference income and consider larger outcomes as gains and lower as losses. The income with which the taxpayer represents his final income includes at once what he considers to deserve (or the price he is willing to pay for public goods) – his initial income and the tax rate – and the characteristics of the cheating game to which he subjects himself by not declaring his entire income – the penalty rate and the probability of auditing:

$$R = R(w, t, f, p). \quad (6)$$

The incomes relative to the reference income without and with auditing are:

$$y = Y - R = w - tx - R, \quad (7)$$

$$z = Z - R = w - tx - (1 + f)t(w - x) - R. \quad (8)$$

The final income of the taxpayer is always non-negative.<sup>4</sup> The initial income is the final income obtained by the taxpayer if he does not declare any income and he is not

<sup>4</sup>The penalty rate is assumed to be not too high, to avoid that a taxpayer who declares an income equal to zero and is audited, pays more than his initial income. Formally, that is  $f \leq \frac{1-t}{t}$ . The tax administration can not use the strategy consisting of giving incentives to report honestly while the cost of auditing is minimized, by reducing the probability and imposing a huge fine.

audited ( $Y|_{x=0} = w$ ). It is then impossible, even if there is no auditing, that his final income is above his initial income. It is then quite natural to assume that the reference income chosen by the taxpayer is non-negative and below his initial income:

$$0 \leq R \leq w. \quad (9)$$

The legal after-tax income is the final income of the taxpayer if he is completely honest ( $Y|_{x=w} = Z|_{x=w} = (1-t)w$ ). It is the only value of the reference income for which for all levels of declared income, the taxpayer is in the domain of gains if not caught and in the domain of losses if caught<sup>5</sup> and from now on, it is denoted by:

$$\bar{R} = (1-t)w. \quad (10)$$

The relative incomes can be rewritten. From (7), (8) and (13), the incomes relative to the reference income without and with auditing are:

$$y = Y - R = t(w-x) + \bar{R} - R, \quad (11)$$

$$z = Z - R = -ft(w-x) + \bar{R} - R. \quad (12)$$

If the reference income is below the legal after-tax income ( $R \leq \bar{R}$ ), the outcome of the taxpayer without auditing is above the reference income, he is in the domain of gains ( $y \geq 0$ ), while his outcome with auditing may be above or below the reference income, he may be in the domain of gains as well as of losses ( $z \geq 0$  or  $z \leq 0$ ). In the same manner, if the reference income is above the legal after-tax income ( $R \geq \bar{R}$ ), with auditing the taxpayer is in the domain of losses ( $z \leq 0$ ) while without auditing he may be in the domain of losses as well as of gains ( $y \leq 0$  or  $y \geq 0$ ).

An other particular value for the reference income and which can be compared with the general one is the final income of the taxpayer if he does not declare any income and he is audited ( $Z|_{x=0} = w - (1+f)tw$ ). From now on, it is denoted by:

$$\tilde{R} = w - (1+f)tw. \quad (13)$$

This values for the reference income can be ordered in this manner:

$$0 \leq \tilde{R} < \bar{R} \leq w. \quad (14)$$

It is empirically well-established that individuals have different risk attitudes towards gains (outcomes above the reference point) and losses (outcomes below the reference point) and care generally more about potential losses than potential gains, see for example Rabin (2000) or Rabin and Thaler (2001). The taxpayer exhibits diminishing marginal sensitivity to increasing gains and losses: he is more sensitive to changes close to the reference point than to changes away from the reference point. He also exhibits loss aversion: he is more sensitive to losses than to gains, he prefers avoiding losses to making gains and he prefers risks that might possibly mitigate a loss.

The utility  $u$  associated with an outcome is thus assumed:

- (i) to be continuous on  $\mathbb{R}$ , twice continuously differentiable on  $\mathbb{R}^*$  and to vanish in zero:  $u(0) = 0$ ,
- (ii) to be increasing, convex for losses and concave for gains:  $u' > 0$  on  $\mathbb{R}^*$ ,  $u'' > 0$  on  $\mathbb{R}_-^*$  and  $u'' < 0$  on  $\mathbb{R}_+^*$  (*Diminishing marginal sensitivity*),
- (iii) to be steeper for losses than for gains:  $u'(-k) > u'(k)$  for  $k \in \mathbb{R}_+^*$  (*Loss aversion*).

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<sup>5</sup>See Dhimi and al-Nowaihi (2007).

The right-handed and left-handed limits of  $u'$  and  $u''$  thus exist in  $\mathbb{R} \cup \{-\infty, +\infty\}$  and are such that:  $u'(0_-) \geq u'(0_+) \geq 0$ ,  $u''(0_-) \geq 0$  and  $u''(0_+) \leq 0$ .

Figure 3 represents a typical utility function.

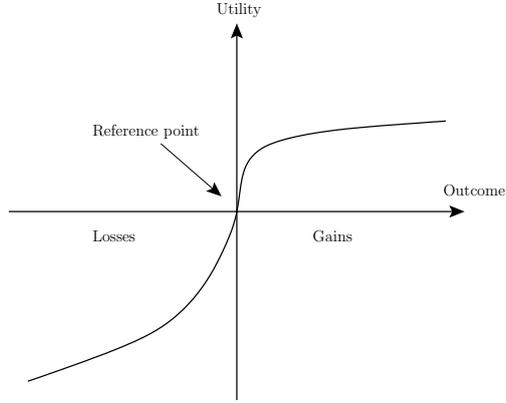


Figure 3: Utility of an outcome

It can be observed that to consider  $R = 0$ , is equivalent to use the classical increasing and concave utility function of expected utility theory. In a prospect theory setting, this corresponds to a taxpayer who is in the domain of gains whatever is his final income. He has an extremely low propensity to evaluate a tax payment as a loss.

The attitude towards risk (risk aversion and risk seeking) depends on the curvature of the utility function and on the shape of the probability weighting function. The monetary risk attitude of the taxpayer, determined by the curvature of the utility function, is expressed by the following risk aversion measure:

$$\text{Absolute risk aversion measure : } r_A(k) = -\frac{u''(k)}{u'(k)}, \text{ for all } k \in \mathbb{R}. \quad (15)$$

$$\text{Relative risk aversion measure : } r_R(k) = -k \frac{u''(k)}{u'(k)}, \text{ for all } k \in \mathbb{R}. \quad (16)$$

Contrary to the classical Arrow-Pratt risk aversion measures in expected utility theory, this monetary measures do not determine entirely the attitude towards risk. They determine the attitude of the taxpayer concerning the values of the outcomes. The taxpayer is monetary risk averse for gains (the utility function is concave and the absolute risk aversion measure is positive) and monetary risk seeker for losses (the utility function is convex and the absolute risk aversion measure is negative). The relative risk aversion measure is positive in both domains of losses and gains.

From experimental motives, Tversky and Kahneman (1992) states that the following power form is a satisfying form for the utility function to describe the behavior of individuals under risk:

$$u(k) = \begin{cases} k^\gamma & \text{if } k \geq 0, \\ -\mu(-k)^\gamma & \text{if } k < 0, \end{cases} \quad (17)$$

where  $0 < \gamma < 1$ , and  $\mu > 1$  because of loss aversion.<sup>6</sup> It will be useful to specify general results.

<sup>6</sup>More precisely, it suggests that empirically,  $\gamma = 0.88$  and  $\mu = 2.25$ .

With this power form, the monetary risk aversion measures have very simple expressions:

$$\text{Absolute risk aversion measure : } r_A(k) = \frac{1-\gamma}{k}, \text{ for all } k \in \mathbb{R}. \quad (18)$$

$$\text{Relative risk aversion measure : } r_R(k) = 1-\gamma, \text{ for all } k \in \mathbb{R}. \quad (19)$$

The absolute risk aversion measure is negative in the domain of losses (the taxpayer is risk seeker), positive in the domain of gains (the taxpayer is risk averse) and decreasing in outcome. The relative one is constant. They depend only on the power parameter  $\gamma$  and not on the loss aversion parameter  $\mu$ .  $1-\gamma$  measures the strength of the monetary risk aversion in both domains of losses and gains.

#### 2.4. Calibration assumptions

For sake of simplicity and because they are largely empirically verified, in all the following, this assumptions will be considered as true.

*Assumption 1.* The probability and penalty rates are such that:

$$f < \frac{1}{\Pi(p)}. \quad (20)$$

This is consistent with observed rates.<sup>7</sup> Actual values for the penalty rate range from 50% and 200% in most developed countries, while audit probabilities range from 1% to 3%. Assumption 1 is verified as soon as  $\alpha \geq 0.2$ , in the weighting function of Prelec (1998), which seems to be a condition rather realistic about the degree of weighting of probabilities by a taxpayer.

*Assumption 2.* The probability, the tax and penalty rates, the probability weighting and utility functions are such that:

$$f, f^2 < -\frac{1}{\Pi(p)} \frac{u''(tw)}{u''(-ftw)}. \quad (21)$$

This expression relies the parameters of the cheating game and the behavior functions of the taxpayer. It is consistent with observed rates. Using the weighting function of Prelec (1998) and the utility function of Tversky and Kahneman (1992), it is verified with very reasonable values for the parameters  $\alpha$ ,  $\gamma$  and  $\mu$ .<sup>8</sup>

Besides, the reference income of the taxpayer is modified by a change in the tax and penalty rates or the probability of auditing. Some assumptions about the direction of these modifications are natural:

*Assumption 3.* The modifications of the reference income are such that:

$$R_t \leq 0, R_f \leq 0 \text{ and } R_p \leq 0. \quad (22)$$

An increase of the taxable income (respectively, the tax and penalty rates or the probability of auditing) does not decrease (respectively, increase) the reference income

<sup>7</sup>They depend a priori on signals sent by the taxpayer to the tax authorities about his wealth, but in the great majority of cases they stay in a fixed range of values.

<sup>8</sup>This two first assumptions are based in particular on the low values of probability of being audited. This values can be higher for certain taxpayers considered by the tax administration as evading more likely. However, the effective probability of an audit is possibly no more than 5%, according to Slemrod and Yitzhaki (2002), while the assumptions are still verified with higher values, as 10% or 15%, taking reasonable values for others parameters.

which the taxpayer uses in assessing losses and gains. He assesses as being potentially richer (respectively, poorer).

Under expected utility theory, the Arrow-Pratt absolute risk aversion measure is decreasing with income. The same type of assumption can be made here:

*Assumption 4.* The monetary absolute risk aversion measure  $r_A$  is decreasing in outcome in both domains of losses and gains.

It means that the higher the losses are, the less the taxpayer is risk seeking. The lower the losses are, the more the taxpayer is risk seeking. In the same time, the higher the gains are, the less the taxpayer is risk averse. The lower the gains are, the more the taxpayer is risk averse. The power utility function of Tversky and Kahneman (1992) confirms this assumption.

### 3. Tax evasion decision with a probability weighting function

In this section, subjective probabilities represents the only difference with an expected utility theory setting. It is equivalent to apply prospect theory with zero for the reference income. The taxpayer maximizes the following utility of his declared income  $x$  :

$$U(x) = \pi(1-p)u(Y) + \pi(p)u(Z), \quad (23)$$

where  $Y$  and  $Z$  are the taxpayer's final incomes without and with auditing.

$U$  is a continuous function on the interval  $[0, w]$ . In the general case, such a function may reach its maximum at several points in the interval. We assume here, without the loss of too much of generality, that this maximum is reached at only one point, denoted by  $x^*$ . Tax evasion is thus measured by  $w - x^*$ . The first and second derivatives of the function  $U$  are respectively described by:

$$U'(x) = -\pi(1-p)tu'(Y) + \pi(p)ftu'(Z), \quad (24)$$

$$U''(x) = \pi(1-p)t^2u''(Y) + \pi(p)f^2t^2u''(Z) < 0. \quad (25)$$

The following proposition describes the tax evasion decision of the taxpayer.

**Proposition 1.** *When  $R = 0$ , the taxpayer does not declare all his income.*

*If  $0 < f < \frac{1}{\Pi(p)} \frac{u'(w)}{u'(\tilde{R})}$ , the taxpayer totally cheats ( $x^* = 0$ ).<sup>9</sup>*

*If  $\frac{1}{\Pi(p)} \frac{u'(w)}{u'(\tilde{R})} \leq f < \frac{1}{\Pi(p)}$ , the income declared by the taxpayer is interior ( $0 < x^* < w$ ).*

*Proof.* See the Appendix. □

The taxpayer declares less than his actual income because of Assumption 1. The penalty rate is not high enough to motivate the taxpayer to be honest. It generalizes to this setting, the condition obtained in an expected utility theory framework to ensure that the taxpayer evades. It means that the subjective expected payment on undeclared income underweights the subjective gain. The condition for an interior solution in the proposition generalizes also those obtained in expected utility theory.<sup>10</sup>

This proposition describes the change in the income declared by the taxpayer if the tax rate, the penalty rate or the probability of auditing increase.

<sup>9</sup>It is assumed in all the paper, that for equal values of utility, the taxpayer declares the higher level of income.

<sup>10</sup>See for example Yitzhaki (1974) for the linear case and Trannoy and Troitin (2010) for the non-linear case.

**Proposition 2.** *When  $R = 0$ , tax evasion is decreasing in the tax rate,  $t$ , the penalty rate,  $f$ , and the probability of auditing,  $p$ .*

*Proof.* See the Appendix. □

The results obtained in an expected utility framework are robust when a probability weighting function is introduced. The taxpayer staying in the domain of gains, the properties of the utility function are identical. Quantitative results about the level of tax evasion and the effects of changes in parameters would be slightly different, but they are qualitatively identical. Indeed, a change in the tax or penalty rate does not affect probabilities. In particular, the paradoxical result of Yitzhaki (1974) remains. An increase in the tax rate makes tax evasion more risky but reduces the income of the taxpayer, which leads to a reduction of tax evasion activity, with a decreasing risk aversion in the domain of gains. In addition, the probability weighting function being increasing, a change in the probability of auditing has the same positive effect than with objective probabilities.

#### 4. Tax evasion decision with a reference point

Subjective probabilities are not used here. The taxpayer is assumed to exactly weight the probability of events. The associated probability weighting function is the identical function ( $\pi(p) = p$ ). The tax evasion behavior is studied in a prospect theory framework, with a reference income and a utility function convex for losses and concave for gains. The taxpayer thinks of possible outcomes relative to a certain reference point, caring more about potential losses than potential gains. Compared to an expected utility theory framework, risk aversion is not computed from zero but on both sides of the reference income. The taxpayer now maximizes the following utility of his declared income  $x$  :

$$U(x) = (1 - p)u(y) + pu(z). \quad (26)$$

$U$  is a continuous function on  $[0, w]$ . It is assumed here that it reaches its maximum at only one point, denoted by  $x^*$ .  $w - x^*$  measures tax evasion. The first and second derivatives of  $U$  are respectively:

$$U'(x) = -(1 - p)tu'(y) + pftu'(z), \quad (27)$$

$$U''(x) = (1 - p)t^2u''(y) + pf^2t^2u''(z). \quad (28)$$

Before to study the tax evasion problem in the present prospect theory setting, it is interesting to understand the differences with an expected utility theory, where the utility function  $u$  is increasing and concave and the absolute risk aversion measure of Arrow-Pratt is decreasing, the taxpayer maximizing the following utility:

$$E(x) = (1 - p)u(Y) + pu(Z). \quad (29)$$

The following proposition explains the case where the results are similar.

**Proposition 3.** *When  $\pi(p) = p$ ,  $R \leq \tilde{R}$  and  $R$  does not include the tax and penalty rates and the probability of auditing (that is  $R_t, R_f$  and  $R_p = 0$ ), in both frameworks, the taxpayer does not declare all his income and the necessary and sufficient conditions under which he declares an interior income are similar:*

$$f \geq \frac{1 - p}{p} \frac{u'(w)}{u'(\tilde{R})}, \quad \text{in expected utility theory,}$$

$$f \geq \frac{1 - p}{p} \frac{u'(w - R)}{u'(\tilde{R} - R)}, \quad \text{in prospect theory.}$$

*In addition, in both frameworks, tax evasion decreases with the tax rate, the penalty rate and the probability of auditing.*

*Proof.* See the Appendix. □

More generally, results brought by prospect theory differ from those under expected utility theory as soon as the reference income is above  $\tilde{R}$ , because of the possible convexity of  $u$ .<sup>11</sup> In particular, the utility function  $U$  is not concave everywhere and the second order conditions is not so easily verified. Comparative static results differ also as soon as the reference income includes the considered parameter ( $t$ ,  $f$  or  $p$ ) because its derivatives intercede. Concerning the present tax evasion problem, prospect theory differs from expected utility theory through two elements: the level of the reference income and its dependence in the parameters.

The income declared by the taxpayer thus depends on the expression of his reference income. As mentioned above,  $\tilde{R}$ ,  $\bar{R}$  and  $w$  are interesting because their values are equal to final incomes in extreme cases. They represent boundaries from which the behavior of the taxpayer changes. They are thus considered here. To interpret conditions more easily, it will be interesting to outline results using the probability weighting function of Prelec (1998) and the utility function of Tversky and Kahneman (1992).

*First case:  $R = \tilde{R}$*

In this case, the taxpayer is in the domain of gains whatever is his final income. He has an extremely low propensity to evaluate a tax payment as a loss. It is linked for example to a high preference level for public goods. The outcomes without and with auditing are:

$$y = (1 + f)tw - tx > 0_+, \quad (30)$$

$$z = ftx \geq 0_+. \quad (31)$$

The following proposition describes his tax evasion behavior.

**Proposition 4.** *When  $\pi(p) = p$  and  $R = \tilde{R}$ , the taxpayer does not declare all his income.*

*If  $0 < f < \frac{1 - p u'(w - \tilde{R})}{p u'(0_+)}$ , the taxpayer totally cheats ( $x^* = 0$ ).*

*If  $\frac{1 - p u'(w - \tilde{R})}{p u'(0_+)} \leq f < \frac{1 - p}{p}$ , the income declared by the taxpayer is interior ( $0 < x^* < w$ ).*

*Proof.* The proof, similar to the one for Proposition 1, is available from the author, upon request. □

As in the previous section, this condition are equivalent to those obtained in expected utility theory. With a so low reference income, properties about the utility functions are similar, the scheme of the tax evasion decision is then similar too.

**Corollary 5.** *When the utility function has the Tversky and Kahneman form and when  $\pi(p) = p$  and  $R = \tilde{R}$ , the income declared by the taxpayer is interior ( $0 < x^* < w$ ).*

*Proof.* See the Appendix. □

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<sup>11</sup>As mentioned above,  $\tilde{R}$  is the lower level of final income that the taxpayer can obtained because it corresponds to the case where he totally cheats and is audited.

The utility  $u$  rises infinitely from the income reached when there is total evasion and auditing,  $\tilde{R}$ .<sup>12</sup> This motivates the taxpayer to declare. Indeed, the benefit obtained from the first declared dollar, in case of auditing, infinitely outweighs the cost imposed by the first declared dollar without auditing.

Effects on tax evasion of changes in parameters, are described by the following proposition.

**Proposition 6.** *When  $\pi(p) = p$  and  $R = \tilde{R}$ ,*

- i. if  $r_R(z) > r_R(y)$ , tax evasion is increasing in the tax rate,  $t$ ,  
if  $r_R(z) \leq r_R(y)$ , tax evasion is non-increasing in the tax rate,  $t$ ,*
- ii. if  $f > \frac{r_R(z) - r_R(y)}{t} + (w - x^*)r_A(y)$ , tax evasion is decreasing in the penalty rate,  $f$ ,  
if  $f \leq \frac{r_R(z) - r_R(y)}{t} + (w - x^*)r_A(y)$ , tax evasion is non-decreasing in the penalty rate,  $f$ ,*
- iii. tax evasion decreases with the probability of auditing,  $p$ .*

*Proof.* See the Appendix. □

Many experimental and econometric studies have emphasized that a rise in the tax rate increases tax evasion.<sup>13</sup> Yitzhaki (1974) showed that, using expected utility theory under the assumption of decreasing absolute risk aversion of Arrow-Pratt, a rise in the tax rate leads to a decrease in tax evasion. With the present framework, with a so low reference income, a rise in the tax rate increases tax evasion if the relative risk aversion, as defined in (16), is larger with, than without auditing. It is verified, for instance, if the relative risk aversion is decreasing in outcome in the domain of gains.<sup>14</sup> The intuition is that an increase in the tax rate causes the taxpayer to be richer in terms of outcomes ( $y$  and  $z$  are increasing in  $t$ ). With a decreasing relative risk aversion, he chooses then to increase the fraction of his initial income in the risky alternative and tax evasion increases. The use of the reference income implies that the outcomes can be increasing in the tax rate, contrary to incomes which are decreasing under the standard expected utility theory setting. The intuition behind the present relative risk aversion is similar to that behind the standard relative risk aversion of Arrow-Pratt. In addition, empirical studies show that a rise in the penalty rate decreases tax evasion. It is verified only if the penalty rate is sufficiently high. The empirically correct result is predicted when the probability of auditing increases, this leads to a decrease in tax evasion.

**Corollary 7.** *When the utility function has the Tversky and Kahneman form and when  $\pi(p) = p$  and  $R = \tilde{R}$ ,*

- i. tax evasion is not modified by a change in the tax rate,  $t$ ,*
- ii. if  $f(1 + f) > \frac{1 - \gamma}{t}$ , tax evasion is decreasing in the penalty rate,  $f$ .*

*Proof.* See the appendix. □

The relative risk aversion, defined in (19), is constant. This implies that a change in the tax rate does not modify tax evasion. In addition, tax evasion decreases with the penalty rate if it is sufficiently high. It is verified with observed tax and penalty rates.<sup>15</sup>

<sup>12</sup>It is because  $\tilde{R}$  is the reference income and  $u'(0_+) = +\infty$  with the utility function of Tversky and Kahneman (1992).

<sup>13</sup>See for example Friedland et al. (1978), Clotfelter (1983) and Pudney et al. (2000).

<sup>14</sup>As with the Arrow-Pratt measures, the decrease of the relative risk aversion is a stronger assumption than the decrease of the absolute one.

<sup>15</sup>For example, with  $\gamma = 0.88$  and  $f = 50\%$ , it is verified as soon as  $t > 16\%$ . With  $\gamma = 0.88$  and  $f = 150\%$ , it is verified as soon as  $t > 3,2\%$ .

All the results would be very similar for a taxpayer whose reference income is below  $\tilde{R}$ , ( $0 \leq R \leq \tilde{R}$ ), up to minor changes caused by different values for  $R_t$ ,  $R_f$  and  $R_p$ . Indeed, in the same manner, he would be always in the domain of gains.

*Second case:  $R = \bar{R}$*

In this case, the taxpayer is in the domain of losses as soon as he is audited and in the domain of gains as soon as he is not audited, whatever is his declared income. With this reference income, if he is or not discovered cheating by the tax administration, is the central element for the taxpayer to evaluate his outcome. The outcomes without and with auditing are:

$$y = t(w - x) \geq 0_+, \quad (32)$$

$$z = -ft(w - x) \leq 0_-. \quad (33)$$

It is not possible to anticipate the tax evasion decision with this reference income as with others, because the incomes with and without auditing are always non-equal, even if the taxpayer is completely honest.<sup>16</sup> Signs of the maximized utility function  $U$  and its marginal functions are not easily computed. In particular,  $U$  is not concave everywhere. Conditions generalizing those obtained in expected utility theory, to ensure an interior solution, can not be highlighted here.

Effects on tax evasion of changes in parameters, are described by the following proposition.

**Proposition 8.** *When  $\pi(p) = p$  and  $R = \bar{R}$ ,*

- i. when the declared income is interior,*
  - if  $r_R(z) > r_R(y)$ , tax evasion is increasing in the tax rate,  $t$ ,*
  - if  $r_R(z) \leq r_R(y)$ , tax evasion is non-increasing in the tax rate,  $t$ ,*
  - when there is total evasion, it is decreasing in the tax rate,  $t$ ,*
  - when there is no evasion, it is not modified by a change in the tax rate,  $t$ ,*
- ii. when the declared income is interior,*
  - if  $r_R(z) < t$ , tax evasion is decreasing in the penalty rate,  $f$ ,*
  - if  $r_R(z) \geq t$ , tax evasion is non-decreasing in the penalty rate,  $f$ ,*
  - when there is total evasion,*
    - if  $r_A(-ftw) > -\frac{1}{w}$ , tax evasion is decreasing in the penalty rate,  $f$ ,*
    - if  $r_A(-ftw) \leq -\frac{1}{w}$ , tax evasion is not modified by a change in the penalty rate,  $f$ ,*
  - when there is no evasion, it is not modified by a change in the penalty rate,  $f$ ,*
- iii. when there is no evasion, it is not modified by a change in the probability of auditing,  $p$ ,*
- otherwise, tax evasion is decreasing in the probability of auditing,  $p$ .*

*Proof.* See the Appendix. □

As in the previous case, when the declared income is interior, tax evasion is increasing in the tax rate if the relative risk aversion is larger with auditing than without. In the present case, levels of the relative risk aversion measure in the two domains are compared. If the relative risk aversion is larger in the domain of losses than in that of gains, a rise in the tax rate increases tax evasion. Again, it is verified if the relative risk

<sup>16</sup>More precisely, the income with auditing  $z$  is always negative and the income without auditing  $y$  is always positive. Even if the taxpayer totally declares,  $y(w) = 0_+$  and  $z(w) = 0_-$ , and a priori, the values of  $u$ ,  $u'$  and  $u''$  are non-equal.  $\bar{R}$  is the only reference income with which  $U'(w)$  can be non-negative.

aversion is decreasing on both domains together. The intuition is similar because an increase in the tax rate causes the taxpayer to be richer in terms of outcomes. Indeed, the expected outcome,  $(1 - p)y + pz$ , is increasing in the tax rate. In addition, tax evasion is decreasing in the penalty rate if the relative risk aversion with auditing is lower than the tax rate. Under the condition  $r_R(y) < r_R(z) < t$ , results are then consistent with empirical evidence, because the empirically correct result is predicted when the probability of auditing increases.

**Corollary 9.** *When the utility function has the Tversky and Kahneman form and when  $\pi(p) = p$  and  $R = \bar{R}$ ,*

- i. when there is total evasion, it is decreasing in the tax rate,  $t$ , otherwise, tax evasion is not modified by a change in the tax rate,  $t$ ,*
- ii. when the declared income is interior,*
  - if  $t > 1 - \gamma$ , tax evasion is decreasing in the penalty rate,  $f$ ,*
  - if  $t \leq 1 - \gamma$ , tax evasion is non-decreasing in the penalty rate,  $f$ ,*
  - when there is total evasion,*
    - if  $ft > 1 - \gamma$ , tax evasion is decreasing in the penalty rate,  $f$ ,*
    - if  $ft \leq 1 - \gamma$ , tax evasion is not modified by a change in the penalty rate,  $f$ ,*
    - when there is no evasion, it is not modified by a change in the penalty rate,  $f$ .*

Tax evasion is not modified by a change in the tax rate. It is due to the specific form of the risk aversion measure, which is due to the symmetric form of the utility function in (17). In particular, the same power parameter,  $\gamma$ , is used. To find that a rise in the tax rate increases tax evasion, the power parameter in the domain of losses should be lower than in the domain of gains, because risk aversion should be stronger in the domain of losses than in the domain of gains,<sup>17</sup> as for instance:

$$u(k) = \begin{cases} k^\gamma & \text{if } k \geq 0, \\ -\mu(-k)^\rho & \text{if } k < 0, \end{cases} \quad (34)$$

where  $0 < \rho < \gamma < 1$  and  $\mu > 1$ .

With a reference income just above  $\tilde{R}$ , the taxpayer considers the most of possible outcomes as gains. Remaining below the legal income  $\bar{R}$ , the outcome without auditing is always considered as a gain, while the higher the reference income is, the more probably the outcome with auditing is a loss. What we can observe is that, with a reference income such that  $\tilde{R} < R < \bar{R}$ , the result about the tax evasion decision would be very similar to that with  $\tilde{R}$ . Up to minor changes caused by different values for  $R_t$ ,  $R_f$  and  $R_p$ , the schemes of results about changes in parameters would be very close to that with  $\tilde{R}$  and  $\bar{R}$ .<sup>18</sup>

With a reference income higher than the legal income ( $\bar{R} < R < w$ ), the taxpayer is always in the domain of losses when he is audited and also when he is not audited but declares a too high income. He has a high propensity to evaluate a tax payment as a loss. A priori, this is the most usual case in practice because it corresponds to a case where the taxpayer regards positively the situation where his final income exceeds the legal income while staying below his initial income. It is interesting to notice that with such a reference income, the taxpayer does not declare an income close to the legal one. At this level, the higher is the reference income, the lower is the maximum declaration of

<sup>17</sup>Dhami and al-Nowaihi (2007) show that when the declared income is interior, an increase in the tax rate increases tax evasion, because of the introduction of a stigma proportional to the evaded income.

<sup>18</sup>Between  $\tilde{R}$  and  $\bar{R}$ , results differs essentially at corner declarations. In fact, with  $\tilde{R} < R < \bar{R}$ , the taxpayer would not declare all his income, as with  $\tilde{R}$ . However, in case of total evasion, results would be similar to that with  $\bar{R}$ .

the taxpayer.<sup>19</sup> As mentioned before,  $\bar{R}$  is the only one with which the taxpayer can be honest because it is the only one with which the income with and without auditing are never equal, even if the taxpayer is honest. In addition, up to minor changes caused by different values for  $R_t$ ,  $R_f$  and  $R_p$ , the schemes of results about changes in parameters would be very close to that with  $\bar{R}$ .

*Third case:  $R = w$*

This extreme case corresponds to an extremely tax-averse taxpayer. Any payment to the tax administration is considered a loss. The outcomes without and with auditing are:

$$y = -tx \leq 0_-, \quad (35)$$

$$z = -tx - (1 + f)t(w - x) < 0_-. \quad (36)$$

The following proposition describes the tax evasion behavior, as well as the effects on tax evasion of changes in parameters.

**Proposition 10.** *When  $\pi(p) = p$  and  $R = w$ , the taxpayer evades all his income ( $x^* = 0$ ).*

*Proof.* See the Appendix. □

With a so high reference income, the taxpayer is completely dishonest. Formally, it is due to the convexity of the utility function  $U$ , linked to the convexity of  $u$ , the taxpayer staying in the domain of losses. He is a complete risk taker.

In all this section, results differs significantly from those given in the previous one. This highlights that results given in an expected utility theory framework are not robust to the use of a reference income, whatever is the value of the reference income.

## 5. Tax evasion decision under prospect theory

The general setting of prospect theory is now used to study the tax evasion decision of the taxpayer. In fact, even when a change of the probability of auditing is studied, results do not really differ to those highlighted in the previous section. In particular, intuitions are exactly the same. This shows that the presence of a probability weighting function in this tax evasion problem does not transform the framework of expected utility theory as does the use of a reference income. It is because there are only two kind of outcomes, with or without auditing. Results differs only quantitatively from those without probability weighting function highlighted in the previous section.<sup>20</sup>

The taxpayer maximizes the following utility of his declared income  $x$  :

$$U(x) = \pi(1 - p)u(y) + \pi(p)u(z). \quad (37)$$

$U$  is a continuous function on  $[0, w]$ . It is assumed that it reaches its maximum at only one point, denoted by  $x^*$ .  $w - x^*$  measures tax evasion. The first and second derivatives of  $U$  are respectively:

$$U'(x) = -\pi(1 - p)tu'(y) + \pi(p)ftu'(z), \quad (38)$$

$$U''(x) = \pi(1 - p)t^2u''(y) + \pi(p)f^2t^2u''(z). \quad (39)$$

<sup>19</sup>Formally, the income declared by the taxpayer stays below  $\frac{w-R}{t}$ .

<sup>20</sup>Certain parts of proof, very similar than those in the previous section, are not given in this section.

As in the previous section, the income declared by the taxpayer depends on the expression of his reference income.

*First case:  $R = \tilde{R}$*

**Proposition 11.** *When  $R = \tilde{R}$ , the taxpayer does not declare all his income.*

*If  $0 < f < \frac{1}{\Pi(p)} \frac{u'(w - \tilde{R})}{u'(0_+)}$ , the taxpayer totally cheats ( $x^* = 0$ ).*

*If  $\frac{1}{\Pi(p)} \frac{u'(w - \tilde{R})}{u'(0_+)} \leq f < \frac{1}{\Pi(p)}$ , the income declared by the taxpayer is interior ( $0 < x^* < w$ ).*

This conditions are the equivalent in a prospect theory framework, of those obtained in expected utility theory. Under Assumption 1, the penalty is not high enough to motivate the taxpayer to be completely honest.

**Proposition 12.** *When  $R = \tilde{R}$ ,*

- i. if  $r_R(z) \leq r_R(y)$ , tax evasion is non-increasing in the tax rate,  $t$ ,  
if  $r_R(z) > r_R(y)$ , tax evasion is increasing in the tax rate,  $t$ ,*
- ii. if  $x^* r_A(z) < w r_A(y) + 1$ , tax evasion is decreasing in the penalty rate,  $f$ ,  
if  $x^* r_A(z) \geq w r_A(y) + 1$ , tax evasion is non-decreasing in the penalty rate,  $f$ ,*
- iii. tax evasion decreases with the probability of auditing,  $p$ .*

*Proof.* See the Appendix. □

*Second case:  $R = \bar{R}$*

**Proposition 13.** *When  $R = \bar{R}$ ,*

- i. when the declared income is interior,  
if  $r_R(z) \leq r_R(y)$ , tax evasion is non-increasing in the tax rate,  $t$ ,  
if  $r_R(z) > r_R(y)$ , tax evasion is increasing in the tax rate,  $t$ ,  
when there is total evasion, it is decreasing in the tax rate,  $t$ ,  
when there is no evasion, it is not modified by a change in the tax rate,  $t$ ,*
- ii. when the declared income is interior,  
if  $r_R(z) < t$ , tax evasion is decreasing in the penalty rate,  $f$ ,  
if  $r_R(z) \geq t$ , tax evasion is non-decreasing in the penalty rate,  $f$ ,  
when there is total evasion,  
if  $r_A(-ftw) > -\frac{1}{w}$ , tax evasion is decreasing in the penalty rate,  $f$ ,  
if  $r_A(-ftw) \leq -\frac{1}{w}$ , tax evasion is not modified by a change in the penalty rate,  $f$ ,  
when there is no evasion, it is not modified by a change in the penalty rate,  $f$ ,*
- iii. when there is total evasion, tax evasion is decreasing in the probability of auditing,  $p$ ,  
otherwise, tax evasion is not modified by a change in the probability of auditing,  $p$ .*

*Proof.* See the Appendix. □

*Third case:  $R = w$*

**Proposition 14.** *When  $R = w$ , the taxpayer evades all his income.*

**Proposition 15.** *When  $R = w$ ,*

- i. tax evasion is not modified by a change in the tax rate,  $t$ ,
- ii. if  $r_R(z) < \frac{t(1+f)}{f}$ , tax evasion is decreasing in the penalty rate,  $f$ ,  
if  $r_R(z) \geq \frac{t(1+f)}{f}$ , tax evasion is not modified by a change in the penalty rate,  $f$ ,
- iii. tax evasion is decreasing in the probability of auditing,  $p$ .

*Proof.* See the Appendix. □

## 6. Concluding remarks

This paper characterizes tax evasion decision under a few or all properties of prospect theory. It shows that with the testable condition: a suitable relative risk aversion measure is larger with auditing than without, prospect theory provides a general setting for the tax evasion problem which is consistent with empirical evidence.

## Appendix A

*Proof of Proposition 1.*  $U'' < 0$ , and  $U'(w) < 0$ , then the taxpayer totally cheats if and only if  $U'(0) < 0$  and his declared income is interior if and only if  $U'(0) \geq 0$  and  $U'(w) < 0$ . □

*Proof of Proposition 2.* Denote  $\Phi_p(x, t) = U'(x)$ , when the declared income is interior,  $\Phi_p(x^*, t) = 0$  and  $\frac{\partial \Phi_p}{\partial x}(x^*, t) < 0$ . Thus, by applying the Implicit Function Theorem (IFT), we obtain that the sign of  $\frac{\partial x}{\partial t}|_{x=x^*}$  is the same as that of  $\frac{\partial \Phi_p}{\partial t}(x^*, t)$ . After computing we find:

$$\frac{\partial \Phi_p}{\partial t}(x^*, t) = \pi(1-p)tu'(Y) [x^* [r_A(Z) - r_A(Y)] + (1+f)(w-x^*)r_A(Z)] > 0.$$

When the taxpayer totally cheats,  $U'(0) \leq 0$ . By applying the IFT to  $\Phi_p^0(x, t) = U'(x) - U'(0)$ , we obtain that the sign of  $\frac{\partial x}{\partial t}|_{x=0}$  is the same as that of  $\frac{\partial \Phi_p^0}{\partial t}(0, t)$ , with:

$$\frac{\partial \Phi_p^0}{\partial t}(0, t) = -\pi(1-p)t\Pi(p)u''(\tilde{R})(1+f)w > 0.$$

Denote  $\Psi_p(x, f) = U'(x)$ . By applying the IFT, we obtain that the sign of  $\frac{\partial x}{\partial f}|_{x=x^*}$  is the same as that of  $\frac{\partial \Psi_p}{\partial f}(x^*, f)$ , with:

$$\frac{\partial \Psi_p}{\partial f}(x^*, f) = \pi(1-p)t^2u'(Y) \left[ (w-x^*)r_A(Z) + \frac{1}{f} \right] > 0.$$

When  $x^* = 0$ , by applying the IFT to  $\Psi_p^0(x, f) = U'(x) - U'(0)$ , we obtain that the sign of  $\frac{\partial x}{\partial f}|_{x=0}$  is the same as that of  $\frac{\partial \Psi_p^0}{\partial f}(0, f)$ , with:

$$\frac{\partial \Psi_p^0}{\partial f}(0, f) = \pi(1-p)t^2u'(w) \left[ wr_A(\tilde{R}) + \frac{1}{f} \right] > 0.$$

Denote  $\Gamma_p(x, p) = U'(x)$ . By applying the IFT, we obtain that the sign of  $\frac{\partial x}{\partial p}|_{x=x^*}$  is the same as that of  $\frac{\partial \Gamma_p}{\partial p}(x^*, p)$ , with:

$$\frac{\partial \Gamma_p}{\partial p}(x^*, p) = \pi'(1-p)tu'(Y) + \pi'(p)ftu'(Z) > 0.$$

When  $x^* = 0$ , by applying the IFT to  $\Gamma_p^0(x, p) = U'(x) - U'(0)$ , we obtain that the sign of  $\frac{\partial x}{\partial p} \Big|_{x=0}$  is the same as that of  $\frac{\partial \Gamma_p^0}{\partial p}(0, p)$ , with:

$$\frac{\partial \Gamma_p^0}{\partial p}(0, p) = \pi'(1-p)tu'(w) + \pi'(p)ftu'(\tilde{R}) > 0.$$

□

*Proof of Proposition 3.*  $U$  and  $E$  are concave,  $U'(w)$  and  $E'(w) < 0$ , and  $U'(0) = -(1-p)tu'(w-R) + pftu'(\tilde{R}-R)$  and  $E'(0) = -(1-p)tu'(w) + pftu'(\tilde{R})$ . In addition, under prospect theory, the sign of  $\frac{\partial x}{\partial t} \Big|_{x=x^*}$  is the same as that of  $\frac{\partial \Phi}{\partial t}(x^*, t)$ , with:

$$\frac{\partial \Phi}{\partial t}(x^*, t) = (1-p)tu'(Y-R)[-xr_A(Y-R) + (x + (1+f)(w-x))r_A(Z-R)] > 0,$$

with  $0 < r_A(Y-R) < r_A(Z-R)$ , because  $R < Z < Y$ , and under expected utility theory, the sign of  $\frac{\partial x}{\partial t} \Big|_{x=x^*}$  is the same as that of  $\frac{\partial \Delta}{\partial t}(x^*, t)$ , with:

$$\frac{\partial \Delta}{\partial t}(x^*, t) = (1-p)tu'(Y)[-xr_A(Y) + (x + (1+f)(w-x))r_A(Z)] > 0.$$

The proof is similar with  $f$  and  $p$ .

□

*Proof of Corollary 5.* When  $u$  has the power form as described in (17),  $u'(0_+) = +\infty$  and  $u'(w-\tilde{R}) = \frac{\gamma}{(w-\tilde{R})^{1-\gamma}} > 0$ , then  $\frac{1}{\Pi(p)} \frac{u'(w-\tilde{R})}{u'(0_+)} = 0$ .

□

*Proof of Proposition 6.* The sign of  $\frac{\partial x}{\partial t} \Big|_{x=x^*}$  is the same as that of  $\frac{\partial \Phi_{\tilde{R}}}{\partial t}(x^*, t)$ , with:

$$\frac{\partial \Phi_{\tilde{R}}}{\partial t}(x^*, t) = (1-p)u'(y)[r_R(y) - r_R(z)].$$

When  $x^* = 0$ , the principle of proof is the same as that for Proposition 2.

The sign of  $\frac{\partial x}{\partial f} \Big|_{x=x^*}$  is the same as that of  $\frac{\partial \Psi_{\tilde{R}}}{\partial f}(x^*, f)$ , with:

$$\frac{\partial \Psi_{\tilde{R}}}{\partial f}(x^*, f) = (1-p)\frac{t^2}{f}u'(y)[w r_A(y) - x^* r_A(z) + 1].$$

The sign of  $\frac{\partial x}{\partial p} \Big|_{x=x^*}$  is the same as that of  $\frac{\partial \Gamma_{\tilde{R}}}{\partial p}(x^*, p)$ , with:

$$\frac{\partial \Gamma_{\tilde{R}}}{\partial p}(x^*, p) = tu'(y) + ftu'(z) > 0.$$

□

*Proof of Corollary 7.* If  $f > (w-x^*)\frac{1-\gamma}{y}$ , tax evasion decreases with  $f$ .  $(w-x^*)\frac{1-\gamma}{y}$  is decreasing in  $x^*$ , therefore, as soon as  $f > w\frac{1-\gamma}{y(0)}$ , tax evasion decreases with  $f$ .

□

*Proof of Proposition 8.* The sign of  $\frac{\partial x}{\partial t} \Big|_{x=x^*}$  is the same as that of  $\frac{\partial \Phi_{\tilde{R}}}{\partial t}(x^*, t)$ , with:

$$\frac{\partial \Phi_{\tilde{R}}}{\partial t}(x^*, t) = (1-p)yu'(y)[fr_A(z) + r_A(y)].$$

When  $x^* = 0$ , the sign of  $\frac{\partial x}{\partial t} \Big|_{x=0}$  is the same as that of  $\frac{\partial \Phi_{\tilde{R}}^0}{\partial t}(0, t)$ , with:

$$\frac{\partial \Phi_{\tilde{R}}^0}{\partial t}(0, t) = -tw \left[ (1-p)u''(tw) + fp u''(-ftw) \right].$$

When  $x^* = w$ , the sign of  $\frac{\partial x}{\partial t}\Big|_{x=w}$  is the same as that of  $\frac{\partial \Phi_{\bar{R}}^w}{\partial t}(w, t) = 0$ .  
The sign of  $\frac{\partial x}{\partial f}\Big|_{x=x^*}$  is the same as that of  $\frac{\partial \Psi_{\bar{R}}}{\partial f}(x^*, f)$ , with:

$$\frac{\partial \Psi_{\bar{R}}}{\partial f}(x^*, f) = (1-p) \frac{t}{f} u'(y) [-r_{\bar{R}}(z) + t].$$

When  $x^* = 0$ , the sign of  $\frac{\partial x}{\partial f}\Big|_{x=0}$  is the same as that of  $\frac{\partial \Psi_{\bar{R}}^0}{\partial f}(0, f)$ , with:

$$\frac{\partial \Psi_{\bar{R}}^0}{\partial f}(0, f) = (1-p) \frac{t^2}{f} u'(tw) [w r_A(-ftw) + 1].$$

When  $x^* = w$ , the sign of  $\frac{\partial x}{\partial f}\Big|_{x=w}$  is the same as that of  $\frac{\partial \Psi_{\bar{R}}^w}{\partial f}(w, f)$ , with:

$$\frac{\partial \Psi_{\bar{R}}^w}{\partial f}(w, f) = (1-p) u'(0_+) \frac{t^2}{f}.$$

The sign of  $\frac{\partial x}{\partial p}\Big|_{x=x^*}$  is the same as that of  $\frac{\partial \Gamma_{\bar{R}}}{\partial p}(x^*, p)$ , with:

$$\frac{\partial \Gamma_{\bar{R}}}{\partial p}(x^*, p) = tu'(y) + ftu'(z) > 0.$$

When  $x^* = 0$ , the sign of  $\frac{\partial x}{\partial p}\Big|_{x=0}$  is the same as that of  $\frac{\partial \Gamma_{\bar{R}}^0}{\partial p}(0, p)$ , with:

$$\frac{\partial \Gamma_{\bar{R}}^0}{\partial p}(0, p) = tu'(tw) + ftu'(-ftw) > 0.$$

When  $x^* = w$ , the sign of  $\frac{\partial x}{\partial p}\Big|_{x=w}$  is the same as that of  $\frac{\partial \Gamma_{\bar{R}}^w}{\partial p}(w, p)$ , with:

$$\frac{\partial \Gamma_{\bar{R}}^w}{\partial p}(w, p) = t(1+f)u'(0_+) > 0.$$

□

*Proof of Proposition 10.* With  $R = w$ ,  $z \leq y \leq 0$ ,  $u$  is thus increasing and convex and  $U$  is decreasing.

□

*Proof of Proposition 15.*  $x^* = 0$ , the sign of  $\frac{\partial x}{\partial t}\Big|_{x=0}$  is then the same as that of  $\frac{\partial \Phi_w^0}{\partial t}(0, t)$  with:

$$\frac{\partial \Phi_w^0}{\partial t}(0, t) = -(1+f)twpu''(-1+f)tw < 0.$$

$x^* = 0$ , the sign of  $\frac{\partial x}{\partial f}\Big|_{x=0}$  is then the same as that of  $\frac{\partial \Psi_w^0}{\partial f}(0, f)$  with:

$$\frac{\partial \Psi_w^0}{\partial f}(0, f) = t^2(1-p)u'(0_+) \left[ w r_A(-(1+f)tw) + \frac{1}{f} \right].$$

$x^* = 0$ , the sign of  $\frac{\partial x}{\partial p}\Big|_{x=0}$  is then the same as that of  $\frac{\partial \Gamma_w^0}{\partial p}(0, p)$  with:

$$\frac{\partial \Gamma_w^0}{\partial p}(0, p) = tu'(0_+) + ftu'(-(1+f)tw) > 0.$$

□

*Proof of Propositions 12, 13 and 15.* The sign of  $\frac{\partial x}{\partial p}\Big|_{x=x^*}$  is the same as that of  $\frac{\partial \Gamma_c}{\partial p}(x^*, p)$ , with:

$$\frac{\partial \Gamma_c}{\partial p}(x^*, p) = \pi'(1-p)tu'(y) + \pi'(p)ftu'(z) > 0.$$

□

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