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Joris van der Hoeven

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# FAST MULTIPLICATION OF INTEGER MATRICES

## Withdrawn paper

*Joris van der Hoeven*

Laboratoire d'informatique  
UMR 7161 CNRS  
École polytechnique  
91128 Palaiseau Cedex  
France

*Email:* [vdhoeven@lix.polytechnique.fr](mailto:vdhoeven@lix.polytechnique.fr)

*Web:* <http://www.lix.polytechnique.fr/~vdhoeven>

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In this paper we will show that dense  $n \times n$  matrices with integer coefficients of bit sizes  $\leq b$  can be multiplied in quasi-optimal time. This shows that the exponent  $\omega_{\mathbb{Z}}$  for matrix multiplication over  $\mathbb{Z}$  is equal to two. Moreover, there is hope that the exponent can be observed in practice for a sufficiently good implementation.

KEYWORDS: matrix multiplication, FFT, skew polynomials

A.M.S. SUBJECT CLASSIFICATION: 15-04, 68Q25, 68W30

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**Erratum.** Eric SCHOOST discovered a bug in this paper: by construction, formula (7) does not hold, since the matrix  $V_{a,n}$  is not invertible whenever  $l \geq 2$ . Unfortunately, we do not see how to repair this bug: not only  $V_{a,n}$  is not invertible, but the rank of  $V_{a,n}$  is actually very small (and equal to  $\max\{q_1, \dots, q_l\}$ ). This makes the proof collapse.

We have also tried a few other things, such as fixing  $a = 2$  and trying to take many  $q_i$  for which  $a$  is a primitive root of unity of a small order. This also does not work, since the product of two operators in  $\mathbb{Z}[X, Q]_2$  makes the coefficient sizes increase by  $n^2$  (and not merely  $n$ ) bits. Going until “small order  $n$ ”, we must have  $q_1 \cdots q_l \mid \text{lcm}(2^1 - 1, 2^2 - 1, \dots, 2^n - 1)$ . This lcm is of the same order as  $2^{n^2}$ , but strictly smaller than  $2^{n^2}$ .

## 1. INTRODUCTION

### 1.1. Main results

Let  $\mathbb{A}$  be an effective ring, which means that there exist algorithms for performing all ring operations. It is classical that there exist quasi-optimal algorithms for many kinds of computations with polynomials over  $\mathbb{A}$ . For instance, two polynomials of degrees  $< n$  can be multiplied using  $M_{\mathbb{A}}(n) = \mathcal{O}_{\mathbb{A}}(n \log n \log \log n)$  operations in  $\mathbb{A}$  [5, 17, 6, 13]. If  $\mathbb{A}$  admits primitive  $2^p$ -th roots of unity for any  $p$ , then we even have  $M_{\mathbb{A}}(n) = \mathcal{O}_{\mathbb{A}}(n \log n)$ . From the complexity point of view, multiplication is the central operation: good bounds for the complexities of division, g.c.d., multipoint evaluation, etc. are known in terms of the complexity of multiplication.





























