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# Trip timing and scheduling preferences

Mogens Fosgerau                      André de Palma  
Technical University of Denmark      Ecole Normale Supérieure

Anders Karlstrom  
Royal Institute of Technology, Sweden

Ken Small  
University of California, Irvine, US

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# 1 Introduction

The congestion of the morning rush hour is a prominent feature of urban life. Aggregate congestion delay is a significant burden on industrialized economies and much attention has been given to policy measures, notably pricing, that can reduce congestion.

This note summarizes the results from the project SURPRICE: Trip timing and scheduling preferences. The general emphasis of this project is the importance of trip timing as a cause of congestion. It is important to recognize that departure time is a choice of travellers and that congestion arises because many travellers choose to travel at the same time. The design and evaluation of pricing schemes should explicitly take changes in departure time patterns into account, in particular with time-varying charges. Failure to take trip timing into account will lead to failure in identifying important benefits and will lead to less efficient pricing schemes.

Most current traffic models are notoriously poor in incorporating trip timing. Much remains to be done in making models both sufficiently realistic in this respect and at the same time sufficiently simple for application. There are furthermore many remaining challenges at the conceptual level.

This project has addressed four fundamental questions of vital interest to the design and evaluation of road user charging (RUC) schemes.

- The interaction of urban structure with trip timing
- Alternatives to pricing - mechanisms for allocating capacity
- The nature of scheduling preferences
- Models for dynamic assignment and nature of equilibrium

It is first important to understand and to be able to model trip timing in an urban context. The challenge is that travellers, located at various distances from their destination, face different scheduling constraints. This has an impact on their choices and hence on the resulting equilibrium, such that the shape of the rush hour depends on the spatial structure of a city. Hence spatial structure must be taken into account when designing tolls. This project is the first to achieve this goal in an analytical model.

Second, the view of congestion that recognizes equilibrium in the timing of trips opens for alternatives to pricing. These could be a further development of the high occupancy/toll (HOT) lanes and high-occupancy vehicle (HOV) lanes that have been implemented in the US. Various ways can be devised of assigning a share of capacity to designated groups of vehicles for some period of time

during the peak. Such mechanisms have the potential to reduce congestion without pricing and without capacity expansion. Under bottleneck queueing they can be designed such that no traveller is worse off than before the scheme. This is important for acceptability.

Third, in order to account for trip timing choices of travellers, it is necessary to improve our current understanding of scheduling preferences. In particular, while the state-of-the-art so far has treated scheduling preferences as exogenous, it is reasonable to think that they are in fact endogenous: a commuter's preferred arrival time at work depends essentially on when everybody else arrive at work. Taking this into account may change forecasts and assessment of the optimal toll.

Fourth, for applications it is important to understand the nature of equilibrium in dynamic assignment models. In this project we will address the theory based on concepts of equilibrium, and its relation to applied dynamic assignment models, used in forecasting.

This project is the first to address these questions. The project has been carried out in collaboration between leading researchers within transport economics from Denmark, France, Sweden, and the US.

## **2 The interaction of urban structure with trip timing**

The research that is summarized in this section has been published in *Journal of Urban Economics* in a paper with the title "Congestion in a city with a central bottleneck" ([Fosgerau and de Palma, 2012](#)). This paper presents a model that integrates two prominent features of urban congestion, focusing on the exemplary case of the morning commute. The first feature is that congestion is a dynamic phenomenon in the sense that congestion at one time of day affects conditions later in the day through the persistence of queues. The second feature is that trip origins are spatially distributed. We analyze how these features interact in a city with a central bottleneck and provide results concerning optimal pricing.

### **2.1 Background**

The dynamics of congestion were analyzed in the seminal [Vickrey \(1969\)](#) bottleneck model (see also [Arnott et al., 1993](#)), which captures the essence of congestion dynamics in a simple and tractable way. Travellers are viewed as having scheduling preferences concerning the timing of trips that have to pass the bottleneck. The analysis concerns equilibrium in the traveller choice of departure time.

The [Vickrey \(1969\)](#) analysis of congestion, however, essentially ignores space.

Using the notation of the current paper, travellers are depicted as travelling some distance  $c$  (measured in time units) until they reach a bottleneck at time  $a$ . They exit the bottleneck to arrive at the destination at time  $t$ . They have scheduling preferences, always preferring to depart later and always preferring to arrive earlier. The Vickrey (1969) scheduling preferences can be expressed by the scheduling cost  $\alpha \cdot c + \alpha \cdot (t - a) + D(t)$ , where  $\alpha$  is the value of travel time,  $t - a$  is the time spent in the bottleneck and  $D(t) = \beta \cdot \max(0, t^* - t) + \gamma \cdot \max(0, t - t^*)$  is a convex function capturing the cost of being early or late relative to some preferred arrival time  $t^*$ . This model is often called the " $\alpha - \beta - \gamma$ " model and the preferences " $\alpha - \beta - \gamma$ " preferences. The Vickrey formulation of scheduling preferences is additively separable in trip duration and arrival time and it is linear in trip duration. So it is clear that the distance  $c$  to the bottleneck does not matter for the Vickrey analysis of how travellers time their arrival at the bottleneck and the ensuing congestion.<sup>1</sup>

It is not generally true that the distance from trip origins to the destination is irrelevant for the timing of trips. Consider a traveller who always prefers to depart later and always prefers to arrive earlier. Faced by a fixed trip duration that is independent of the departure time, such a traveller will optimally time his trip such that his marginal utility of being at the origin at the departure time equals his marginal utility of being at the destination at the arrival time. If the marginal utility at the origin is decreasing and his marginal utility of being at the destination is increasing, then an increase in trip duration will cause him to depart earlier and to arrive later. In this way the distance can matter for the timing of trips. This paper concerns travellers with such scheduling preferences.

Congestion can arise when there is a bottleneck and many individuals who want to pass the bottleneck at the same time. It is not a sufficient condition for congestion to arise that travellers have similar scheduling preferences. Trip origins must also be located with similar distances to the bottleneck. If trip origins are sufficiently dispersed, then congestion does not arise as there is no overlap in the times when travellers want to pass the bottleneck. Hence it is clear that the spatial distribution of travel demand is a fundamental determinant of urban congestion. This observation stands in contrast to the standard urban model, where congestion increases with population dispersion.

This paper is the first to allow for spatial heterogeneity in the bottleneck model in a meaningful way. In our model, heterogeneity is induced by the structure of the city. A number of earlier contributions have considered preference heterogeneity

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<sup>1</sup>The analysis of the bottleneck model has been developed and extended in various directions by Arnott, de Palma and Lindsey in a series of papers; notably Arnott et al. (1993). These authors use the above  $\alpha - \beta - \gamma$  preferences or a version where the function  $D(t)$  has a more general form. They always maintain linearity and additive separability of travel time and are hence unable to analyse the consequences of distance for congestion.

in the context of the bottleneck model (e.g., [Vickrey, 1973](#); [Arnott et al., 1994](#); [van den Berg and Verhoef, 2011](#)). These papers work in the context of linear separable [Vickrey \(1969\)](#) scheduling preferences and heterogeneity is introduced by varying  $\alpha - \beta - \gamma$ , while maintaining the ratio  $\beta/\gamma$  fixed for reasons of analytical convenience. Generally speaking, this sort of heterogeneity can induce travellers to sort according to the degree of closeness to the center of the congestion peak; in a two group case, sorting has the form that one group occupies a central time interval while the other group occupies the early and late shoulders. In contrast, this paper finds that travelers sort according to their distance to the bottleneck; this occurs both under no tolling and under optimal tolling, and the result is derived under quite general assumptions concerning scheduling preferences.<sup>2</sup> [Hendrickson and Kocur \(1981\)](#), [Smith \(1984\)](#), [Newell \(1987\)](#), and [Arnott et al. \(1994\)](#) consider the case of travellers with scheduling preferences, such as  $\alpha - \beta - \gamma$ , that are additively separable in trip duration and arrival time and who differ in their preferred arrival time. In that case, travellers sort according to their preferred arrival time, which is similar to what we obtain here. [Kuwahara \(1990\)](#) extends this to a geometry consisting of two residential areas and a CBD with bottlenecks in between. Travellers within each group then still sort according to their preferred arrival time, but a strict sequence does not hold for the two groups together. The present case is more involved, as travellers have different distances to the CBD as well as strictly concave and non-separable scheduling preferences. We show that the optimal arrival time  $a_*$ , in the absence of congestion, is increasing in distance  $c$ , such that also here travellers sort according to their preferred arrival time.

[Daganzo \(2007\)](#) and [Geroliminis and Daganzo \(2008\)](#) show that several aspects of congestion in an urban area can be described as a form bottleneck congestion. A space average of traffic measurements show that the trip completion rate is a stable inverse u-shaped function of the number of vehicles present in the system. Cars that have not yet completed their trips remain in the urban area, such that it is possible to think of the system as a generalized sort of queue. See [Geroliminis and Levinson \(2009\)](#). The bottleneck model supposes a constant trip completion rate and a queueing system that maintains a first-in-first-out queue.

## 2.2 Findings

This paper has introduced spatial heterogeneity into the bottleneck model such that it can be used to represent a city with a central bottleneck.

Our analysis first shows that under *laissez-faire*, travellers sort according to their distance to the bottleneck such that those who are closest to the bottleneck reach the destination first. However, in general there is not a monotonous relation-

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<sup>2</sup>[Lindsey \(2004\)](#) considers more general heterogeneity with a finite number of user classes.

ship between distance and departure time; it is not necessarily the case that those who are located further away will depart earlier.

The paper goes on to consider equilibrium under socially optimal tolling at the bottleneck. The toll can be taken to be zero for the first and last travellers and strictly positive for everybody else. The optimal toll exactly removes queuing. The sequence of arrivals at the destination is preserved from the laissez-faire equilibrium.<sup>3</sup> However, in contrast to the Vickrey analysis with homogenous travellers, arrivals at the destination occur earlier in social optimum than under laissez-faire. When the use of toll revenues does not affect the utility of travellers, then the toll just represents a loss for them. This is compensated to some extent by a gain in utility. Comparing social optimum to laissez-faire reveals that those who are located furthest away from the bottleneck will experience a net gain, while those who are located near the bottleneck will experience a net loss.

A number of new insights are generated from the model. Perhaps the most important insight is that travellers located near the bottleneck will tend to lose from optimal tolling, while those located far away will tend to gain, when the use of toll revenues is not accounted for. The paper also shows that a reason for the congested demand peaks to be uni-modal can be found in the properties of equilibrium in combination with our general specification of scheduling preferences.

## 2.3 Scientific perspectives

The spatial distribution of travellers is a source of heterogeneity in the model. It would be of interest to introduce other sources of heterogeneity into the model. One issue would be the robustness of the sorting property. Another kind of extension would be to introduce risk into the model, for example in the form of random capacity (Arnott et al., 1999) or random queue sorting (de Palma and Fosgerau, 2011).

Perhaps the most interesting extension would be to make the location of individuals endogenous as in the Mirrlees (1972) standard urban model. This would tie together congestion dynamics and urban economic models. For example Arnott (1998) combines a model of urban spatial structure with the  $\alpha - \beta - \gamma$  bottleneck model; optimal tolling does not change transport costs for travellers so when the revenues are not returned, optimal tolling will have no effect on urban structure. As Arnott (1998) notes, this is in contrast to urban economic models with static congestion. However, the Arnott (1998) result is a consequence of space essen-

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<sup>3</sup>Arnott et al. (1991) consider a variant of the standard bottleneck model in which drivers have to park, and parking spots are located at varying distances from the CBD. In the laissez-faire user equilibrium, drivers park in order of increasing distance from CBD. The optimal location-dependent parking fee reverses this pattern. This contrasts with the present setting where optimal tolling does not change the order of arrivals.

tially being assumed away in the specification of preferences as described in the paper's literature review.

### 3 Alternatives to pricing

The research that is summarized in this section has been published in Transportation Research Part B in a paper entitled "How a fast lane may replace a congestion toll" (Fosgerau, 2011). This paper considers a fast lane scheme as a means to regulate congestion in a regularly occurring demand peak.

#### 3.1 Background

The fast lane scheme plays explicitly on the dynamics of congestion, which makes the Vickrey (1969) bottleneck model an appropriate framework. The elements of the basic bottleneck model are a description of the queueing technology in the bottleneck, a continuum of identical travellers with scheduling preferences who have to pass the bottleneck, and the concept of Nash equilibrium in arrival times at the bottleneck.

The fast lane scheme allocates the bottleneck capacity to different classes of travellers. The scheme is the following.

*A set of travellers is assigned to a priority group. Not all travellers can be given priority. A more than proportional share of capacity is reserved for the priority group. When the reserved capacity is not used, it is available for the nonprioritized travellers.*

This is similar to, e.g., the check-in in airports with separate queues and servers for economy and business class passengers. Whenever the business class server is idle, it may serve passengers from the economy class queue. Another example is the HOV or HOT lanes as found on US motorways. Yet another example is the flows at different motorway on-ramps that could be given different priority (Shen and Zhang (2010) consider such a scheme). Even though such a scheme is called a fast lane scheme in this paper, the definition encompasses many other policies that do not involve the allocation of road lanes for different classes of vehicles; it is more general than allocation of lanes.

The paper compares the fast lane scheme to tolling. Like Arnott et al. (1990), this paper considers a coarse toll, which is a constant toll that applies only during part of the peak.<sup>4</sup> Arnott et al. (1990) found that Nash equilibrium under a coarse toll comprises a point mass in the arrival schedule at the time when the toll is lifted. This is an undesirable feature of their model as such point masses are

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<sup>4</sup>Arnott et al. (1990) applied also a base toll level. This paper considers the coarse toll only under inelastic demand and so the base toll level does not matter.

physically implausible. The problem is avoided in this paper by a reformulation of the queueing technology. In this paper, the congestion technology is such that travellers who choose not to pay the toll can queue at the same time as travellers who are paying the toll pass the bottleneck. This is also true of the examples of fast lanes given above. In this case, a point mass in the arrival schedule does not arise. The analysis below uses the reformulated queueing technology and repeats the [Arnott et al. \(1990\)](#) analysis of a coarse toll under this assumption.

### 3.2 Findings

The first main result of this paper is that the fast lane scheme is always Pareto improving when demand is not price sensitive. There are no restrictions on how large the group of prioritized travellers should be as long as it is fixed exogenously. Prioritized travellers experience a strict utility gain while the properties of Nash equilibrium imply that nonprioritized travellers do not lose. It is significant that this occurs even when travellers are homogenous and there are no toll payments. This robustness is very desirable since it means that a regulator needs little information to implement the scheme and be certain to achieve a welfare improvement. In fact, the regulator can monitor traffic in real time and assign capacity accordingly. This is consistent with the way the fast lane scheme is formulated in the model. With price sensitive demand, the fast lane scheme is still welfare improving if the price elasticity of demand is not too high and the share of prioritized travellers is not too large.

The second main result of this paper is that the fast lane scheme can reproduce the equilibrium arrival pattern of the optimal coarse toll when demand is not price sensitive.<sup>5</sup> In fact, the scheme can reproduce the equilibrium arrival pattern of any coarse toll, provided that the tolling interval is the same as the arrival interval that a prioritized group would endogenously select. This is significant since the fast lane scheme has a number of advantages over tolling. First, the fast lane scheme is always welfare improving and can be adjusted in real time. In order to set the right coarse toll it is necessary to know exactly when to start and when to end the tolling interval. Mistakes will reduce the welfare gain from tolling and can even lead to a welfare loss. Second, it is plausible that system costs can be a lot lower for a fast lane scheme than for a toll as the fast lane does not involve any payment. Finally, as there is no payment, a fast lane scheme may be more acceptable to travellers than tolling. Within the simple theoretical model

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<sup>5</sup>Using  $\alpha - \beta - \gamma$  scheduling preferences, [Knockaert et al. \(2010\)](#) show that the coarse charge user equilibrium can be obtained by barring a certain group from travelling during the charging period and letting the remainder of drivers travel in that period without paying any charge. The present fast lane scheme does not have to designate specific time periods for specific groups of travellers. Moreover, the result is shown to hold for quite general scheduling preferences.

presented here, prioritized travellers would be strictly better off under the fast lane scheme than under no policy, while the remaining travellers would be indifferent. In contrast, all travellers would be indifferent between tolling and no policy as toll revenues are not returned to travellers. This property of fast lanes may explain why fast lanes have been introduced while there is generally a reluctance to introduce tolls.

A notable feature of the present paper is the formulation of scheduling utility which generalizes those employed by [Vickrey \(1969, 1973\)](#), [Arnott et al. \(1990, 1993\)](#), and many others. Here, scheduling utility is taken to be a strictly concave function of times at which the trip starts and ends. Travellers prefer to depart later and to arrive earlier. For any fixed travel time there is a unique preferred departure time. These assumptions are sufficient for the results of this paper. The paper establishes that the socially optimal fast lane scheme achieves more than half the welfare gain of the socially optimal continuously time varying toll. This generalizes the parallel result by [Arnott et al. \(1990\)](#) for the coarse toll under their first-in-first-out congestion technology to the present formulation of scheduling utility combined with parallel queueing.

### 3.3 Scientific perspectives

It is straightforward but tedious to generalize the results of this paper to tolls with more steps and fast lane schemes with more user classes.<sup>6</sup> The general conclusion remains that fast lanes can achieve the same benefits as step tolls when demand is not price sensitive. It is also straightforward to see that a sequence of step tolls, and hence a sequence of fast lane schemes, can be constructed that approach the optimal time varying toll. In the limit, the step toll would become the optimal continuously varying toll while the fast lane scheme would become equivalent to allocating a specific time slot to every traveler.

A potentially useful feature of the fast lane scheme is its robustness. As long as demand is not too elastic, or as long as the share of prioritized travellers is not too large, then any fast lane scheme satisfying the conditions set up in the paper is welfare improving. If demand is not price sensitive, then any such fast lane scheme is Pareto improving. An interesting direction for further inquiry is how this robustness can be utilized. Is it the case that the fast lane scheme retains its favorable properties when some element of stochasticity is introduced into the model?

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<sup>6</sup>[Laih \(1994\)](#) showed that it is straightforward to extend the coarse toll to a multistep toll. It is similarly straightforward to extend a fast lane scheme in this way. [Laih \(1994\)](#) did not recognize that it was necessary to reformulate the queueing technology in order to obtain his results. This was rectified in [Laih \(2004\)](#).

## 4 The nature of scheduling preferences

This research appears so far in a working paper "Endogenous scheduling preferences and congestion". It concerns a version of the model in which travellers do not have exogenous scheduling preferences. Instead they care about leisure and consumption. Both (effective) leisure and consumption are produced under increasing returns to scale. This, in combination with bottleneck congestion, leads to scheduling preferences arising endogenously in equilibrium. The importance for policy of this insight lies in the possibility that policies that affect congestion also affect scheduling preferences. This makes the effect of policies harder to assess *ex ante*.

### 4.1 Background

Static models of congestion use a static representation of demand in combination with a technology that relates travel cost to the number of travelers (see, e.g., [Small and Verhoef, 2007](#)). In essence, these models are based on the view that congestion occurs simply because many people want to travel to the same place, using the same infrastructure.

More recently, dynamic models of congestion take into account that congestion varies continuously over time and that conditions at one time affect congestion at later times. These models essentially view congestion as arising because many people want to go to the same place at the same specific time. For example, in the well-known [Vickrey \(1969\)](#) bottleneck model, this desire is hard-wired in individuals' utility functions through the concept of scheduling preferences for being at specific places at specific times; it is those preferences that explain the occurrence of congestion.

However, it seems plausible that the preferences for starting work at certain time are not innate, but arise for a reason. Thus [Henderson \(1981\)](#) posits agglomeration forces at the workplace to explain them, and [Hall \(1989\)](#) discusses rather generally how thick-market efficiencies lead to temporal agglomeration at various time scales. Such effects may explain the strong tendency for production to concentrate in the hours 9-12 a.m. and 1-5 p.m. We use this intuition to motivate an assumption that worker productivity increases in the number of people at work.

We apply the same reasoning to time off work, assuming that productivity in the production of effective leisure increases in the number of people off work at any given time. This is a reasonable assumption considering that many leisure activities are social, and others involve family members caring for each other.

In this paper we take workers simply to have preferences defined over leisure and consumption. We consider the morning commute with bottleneck congestion between home and work. Agglomeration economies at home and at work lead to

temporal agglomeration which in turn entails congestion. Thus, we present a view in which congestion occurs because people want to be at a place at the same time as other people are there.

## 4.2 Findings

It turns out that scheduling preferences of the kind assumed by Vickrey arise endogenously in equilibrium. That is, an individual taking equilibrium as given will appear to have scheduling preferences in the form of a utility function that depends on when the commute starts and ends. These scheduling preferences belong to a general class that, as far as we are aware, comprises all those specifications that have been considered by Vickrey and later authors in the context of the bottleneck model. We derive some properties of Nash equilibrium for this general class, in order to compare its predictions to those of our model, where the scheduling preferences arise endogenously. This allows us to evaluate the errors that result if policies aimed at regulating congestion are developed assuming incorrectly that scheduling preferences are exogenous.

We find that the assumption of exogenous scheduling preferences would lead an analyst to underestimate the benefit of congestion tolling. If the use of toll revenues does not affect workers, then an analyst relying on a Vickrey-like model would find workers to be indifferent between the situations with no or optimal tolling. This sort of conclusion is not available in the model with endogenous scheduling preferences, where travellers may either gain or lose from optimal compared to no tolling.

The results concerning optimal capacity provision and the marginal external cost of congestion are also ambiguous in general concerning the relative sizes of these under endogenous and exogenous scheduling preferences. We examine these and other properties of the two models using numerical simulations with a Cobb-Douglas utility function and simple power functions to describe agglomeration.

A few previous contributions analyze congestion and agglomeration economies in combination. [Henderson \(1981\)](#) analyses scheduling of work hours and work trips, based on the effect of agglomeration economies at work on wages and exogenous preferences concerning the timing of leisure. [Wilson \(1988\)](#) finds empirical evidence supporting the idea behind Henderson's model, namely that agglomeration economies at work cause workers to earn more if they start work during peak hours. [Arnott \(2007\)](#) reviews these papers and further applications, while adding his own innovation (still within a static framework) by allowing aggregate labor supplied to be affected by congestion tolls via a reduction in their net wage.

We employ a more general specification of agglomeration economies at work and introduce agglomeration economies in the production of leisure. Furthermore,

we substitute bottleneck queuing for Henderson's model of flow congestion; we thereby bypass certain inconsistencies that can arise between flow congestion in a dynamic setting and traffic dynamics (Chu, 1995), and take advantage of analytical advances that have accumulated in the many papers applying bottleneck queuing to analyze equilibrium scheduling (e.g., Vickrey, 1969, 1973; Newell, 1987; Fargier, 1983; Arnott et al., 1993).

This paper has presented a dynamic model of traffic congestion in which scheduling preferences arise endogenously. A naive analyst - observing equilibrium and assuming scheduling preferences to be exogenously given - would then make errors in predicting the effect of policies such as capacity expansion and tolling. The naive analyst would fail to identify one cost of queueing, namely the decrease in productivity of work and leisure that follows when some are stuck in traffic. Hence such an analyst would underestimate the benefit of a toll that removes queueing. Also, for some parameter sets, such an analyst would apply a toll schedule and/or aim for a departure pattern that is quite far removed from the optimal one. So a take-away of this paper for policy is that a gradual approach to introducing a policy such as road pricing is advisable, since that allows the consequences to be observed as one goes along.

The model with endogenous scheduling preferences generates an equilibrium that is indistinguishable from a model with exogenous scheduling preferences. It is hence not possible to falsify the latter model using only observation of individual choices in a single equilibrium; rather, in order to identify endogeneity, it is necessary to compare different equilibria. It may be possible to employ such an identification strategy empirically, for example by using capacity expansion or the introduction of a road pricing scheme as an exogenous instrument in an empirical investigation explaining variations in the temporal shape of the morning peak.

Humans are social animals and so it is entirely natural that the scheduling preferences of one individual should depend on the scheduling choices of others. We have shown that it is possible to model such a situation and that this interdependence affects transportation policy.

### 4.3 Scientific perspectives

There is a literature on social interactions (Manski, 2000), and traffic congestion may be viewed as an example of a social interaction. The social interactions in our model may be interpreted as occurring roughly at the level of a city. However, it seems likely that smaller scale interactions are also relevant. It would be interesting to develop, both theoretically and empirically, models of such interactions down to the scale of appointments between small groups of travelers.

## 5 Models for dynamic assignment and the nature of traffic equilibrium

An outstanding problem in transportation economics is the notion of traffic equilibrium. Consider the case where a number of individuals each morning choose departure time and route when going to work on a congested road network. This situation may be considered as a game, where each individual's utility (generalized cost, including time, monetary cost etc.) is affected by the strategies (departure time and route given information at hand) of the other individuals travelling in the morning commute.

A Nash equilibrium occurs when no commuter can be better off by unilaterally changing departure time and route, taking the strategies of all other commuters as granted. The Nash equilibrium is an extremely natural concept. It is also very useful since it allows us to predict the aggregate response to changes in policy. So it is very relevant to ask whether Nash equilibrium is a realistic description of actual behaviour. That requires that there is some process that can lead actual travellers to the equilibrium.

We can imagine that commuters are able to observe their own utility associated with the choice of departure time and route they made yesterday, under the conditions that prevailed yesterday. They may also be assumed to be able to observe the utility they would have achieved under alternative choices. Based on information of this kind, they make a new choice of departure time and route today. All commuters update their choices in this way and so the aggregate morning commute changes from day to day. The question is then whether such a process reaches a stable situation. Does simple heuristic learning rules exist that individual commuters can use to update their choice such that the aggregate converges towards an equilibrium? If not, then we (as researchers) have to reconsider our understanding of what it is that we observe and find new ways of making predictions.

Let us here briefly summarize what is known about the existence of convergent algorithms and learning equilibrium in this class of games. First, consider a static routing game in a network where players choose a route from origin to destination which is fixed (no en-route adaptation) and where all individuals travelling a particular link affect each other symmetrically. The timing of events is not important in this situation. This is a standard congestion game which has a number of important and nice features. For this class of games there exists a very intuitive and appealing learning rule. As individual simply update their path towards better paths, this process will converge towards a Nash equilibrium from any starting point. This follows from the fact that the game is a potential game, and the learning process will move towards a local optimum of the potential function. The potential function is a macro concept: it captures the state of the game

on a macroscopic level as individuals take decisions at the micro level. On a fundamental level, the potential function can be found by taking the limit of the dynamic process at the micro level. Mathematically, this limit between micro and macro is similar to the limit between quantum mechanics and classical mechanics in physics. In physics, it is known as the correspondence principle: we are able to analyze the behavior of the system on the macro scale (equilibrium) without have to worry about the details of the behavior on the micro scale (individual behavior). This is a very useful property when analyzing proposed policy measures (Karlstrom, 2012).

Second, consider the departure time game, where individuals only choose departure time. This game is not a potential game, but it still has some interesting properties. As has been shown in Hu and Fosgerau (2012), this game can be formulated as a stable game. This means that processes exist that converge to an equilibrium but, as far as we know, it is not the Nash equilibrium discussed above, but a noisy, probabilistic equilibrium.

Third, in a recent paper Young and Pradelski (2010) show that a particular heuristic simple learning mechanism will converge to a socially optimal Nash equilibrium in any game that exhibits at least one pure strategy Nash equilibrium. This shows that it may be possible to devise algorithms that calculate the social optimal Nash equilibrium in the static congestion game above, which does exhibit pure strategy Nash equilibria.

Finally, consider a dynamic routing game where the timing of entering links determines how different players asymmetrically affect the travel time of other players. For instance, one may assume that cars behind do not affect the travel time of cars further ahead.<sup>7</sup> This game is not a potential game, and much less is known about convergent algorithms or learning mechanisms that converge towards an equilibrium. In this dynamic routing game, it is unknown whether there exists a pure strategy equilibrium. It is also unknown whether it is a stable game. Likewise, less is known when introducing departure time into the dynamic routing game.

In summary, policy analysis so far has simply taken for granted that Nash equilibrium is a good description of what we observe in reality and has used that concept to predict the effect of policies. The strand of research discussed in this section investigates whether Nash equilibrium is the relevant equilibrium concept. The insights regarding convergence of learning mechanisms is also useful for devising algorithms that compute equilibrium in traffic simulation models.

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<sup>7</sup>This is not universally true in all networks.

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