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Abstract

This paper highlights the energy errors made for the design of supercapacitors used as a main energy source. First of all, the paper presents the two definitions of capacitance of a capacitance-voltage dependent material. The number of supercapacitors is important for the application purchasing cost. That is why the paper introduces an analytical model and an electrical model along with an identification method for the capacitance variation. This variation is presented and compared to the manufacturer value in order to underscore the energy error between the manufacturer, the constant approximation and the first order approximation which interests us here. This paper also presents the sizing aspect considering the losses associated to a constant-current charging mode in order to minimize the electrical losses. At last, an application case compares the number of supercapacitors for different approximations of the capacitance.

Introduction

The supercapacitor has a high power density of 1000W/kg, a high specific power of 1000W/l and a low specific energy of 10Wh/kg. These power characteristics make this energy storage unit suitable for peak power applications such as the voltage compensation into the electrical network [1], for recovery of braking energy [4] and for hybridization with other electrical systems with low specific power and low power density. This energy storage system is more and more proposed into transportation applications thanks of its ability to save regenerating peak power energy into a full-electric or a hybrid electric drive train.

Today, most of full-electrical transportation drive trains use batteries as main energy storage unit. Indeed, the specific energy of battery is high (hundred of Wh/kg for lithium batteries) and it allows a medium autonomy for a reasonable size. While the replacement price is high because the lifetime of this electrochemical energy storage unit is only around few 1000 cycles regarding the best available technology such as lithium-ion battery.

In hybrid transportation systems and in full-electric transportation applications (lifetime > 15 years), the lifetime cycle of the energy storage is highly important for the whole-life cost of the transportation application. In these particular cases [6][9] where the user does not want to change often the main energy unit, the supercapacitor is highly suitable because its lifetime is around 100 000 cycles. Even if its specific energy cost is higher compared to a lithium-ion battery, the characteristics of supercapacitors can make it preferable versus batteries as a main on-board energy source. This kind of use of supercapacitors is a challenge because the behaviour of the supercapacitors is different from the existing electrochemical batteries. Moreover regarding the supercapacitor price case, the supercapacitor bank has to be sharply designed to stay more competitive against a battery design.

Usually, the manufacturer gives an equivalent capacitance value which is for a power peak application [3]. To use a supercapacitor as an energy storage system, it is important to know the real available energy which do not correspond to the formulas [3] because of the capacitance variation along with the capacitance voltage [4-5][7-8]. This variation makes the design more stringent for energy applications such as highlight in the papers [4] and [7] and the constant capacitance design presents some shortcoming. In fact, the stored energy in the capacitance depends on the voltage range, the lifetime of the vehicular applications and the depth of the voltage discharge. Furthermore, the usual analytical relations have to be corrected because of the variation of the capacitance according to its voltage. Indeed few papers on the design of the supercapacitor bank have taken into account the capacitance variation. For the other values of depth of discharge and current, this paper shows the error done and how to avoid it.

In the first part of this paper, the modified analytical model of the energy for a variable capacitance and the bench used for the identification of the supercapacitors are presented. We recall the two useful electrical capacitance definitions and we introduce a capacitance identification method. In the second part, the energy profiles obtained with two approximations of the capacitance variation are described and compared to the manufacturer value. At last, this paper presents the total losses for a constant-current charging method. In the last part, a numerical application on a full-electrical vehicle is to highlight the importance of taking the capacitance variation into account for an accurate sizing.

Analytical model developed for the sizing method

We built an analytical model which maintains the particularity of the dynamical equation to charge and discharge a capacitance. The voltage and current equations have to be modified for the non-water soluble supercapacitor. Indeed the capacitance value varies along its voltage range [4-5][7-8] and so the voltage-time parameter $u_c(t)$ has no exact-analytical solution whatsoever the kind of charge and discharge. These voltage capacitances have two main definitions which conduct to the same energy results.

Definitions of electrical capacitance

Because of the variation of the capacitance, we have to define what we call the capacitance of the supercapacitor because the equation of the energy stored is not as a constant capacitance. There are two mains definitions of a nonlinear capacitance material. The first capacitance definition introduces the total electrical charge as a voltage-dependent function $q(u)$. This function is defined at a voltage state to give a constant capacitance C_T [11]. This equivalent constant capacitance of the equation (1) is the total capacitance for the state-voltage. If we suggest the decomposition of the electrical charge as the product of a capacitance $C_T(u)$ and the state voltage u, therefore we can write the equation (1) of the total charge Q.

$$dQ = d\left(C_T(u)\cdot u\right) \tag{1}$$

With this capacitance definition, the equation (2) gives the incremental charge which flows into the electrical double layer.

$$i(t) = \frac{dQ}{dt} = \frac{d\left(C_T(u)\cdot u\right)}{dt} = \frac{C_T(u)\cdot du + d\left(C_T(u)\right)\cdot u}{dt} \tag{2}$$

The second definition uses the local definition of the relation between the local electrical charge dQ and the incremental voltage du such as presented in the equation (3). The capacitance is the coefficient which links the electrical charge to the differential electric potential. This coefficient can be a voltage-dependent coefficient or a polynomial function $C(u)$.

$$dQ = C(u)\cdot du \tag{3}$$

This capacitance, called the Rowe incremental capacitance [10-11], can be directly measured from voltage and current parameters if we use the equation (4) between the electrical charge per time-unit and the current flow.

$$i(t) = \frac{dQ}{dt} = C(u) \cdot \frac{du}{dt} \tag{4}$$

In this paper, we use the Rowe definition to identify the electrical capacitance because it is very simple to associate the current into the capacitance and the voltage increment per time-unit. It is well-suited for our identification process which consists to record the voltage variation for a constant step current I_{SC0}. We also introduce an accurate capacitance variation method with multiple signal steps with constant current likewise call MuSSiCC.

Electrical model and identification method MuSSiCC

The figure 1 presents the branch analytical model based on the approach of the supercapacitor as energy system with its transfer losses r_s, balancing and/or self-discharge losses r_p. All balancing circuits which commonly go with supercapacitor modules can be modeled by r_p. The value of r_p depends on the balancing circuit, r_p is infinite if there is no balancing circuit or for a voltage-switched resistive circuit, finite for a resistive circuit or bound to u_{sc} in the case of an active circuit. The capacitance $C_{sc}=f(u_c)$ reflects the real stored energy. In this paper, we show the influence of the choice to introduce the voltage-capacitance dependency.

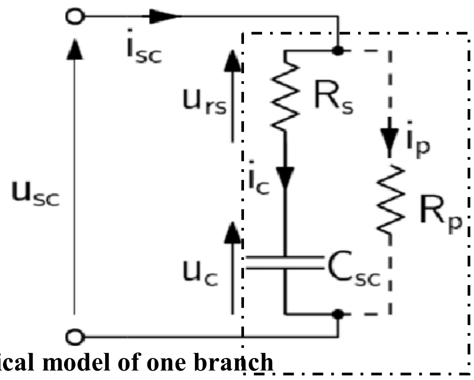

Figure 1 - Supercapacitor electrical model of one branch

Thanks to the analytical model of the figure 1, we write the electrical and energy equations (5-7). These electrical equations are useful to know the expression of the energy inside the supercapacitor during a charging mode. These equations also show the simple relation between the parameter voltage u_{sc} and the capacitance voltage u_c. The voltage u_c is also the idle voltage state parameter. The last controllable parameter i_{sc} is divided in two parts, i_c to charge the real capacitance and i_p to balance the voltage u_{sc}.

$$P_{sc} = u_{sc} \cdot i_{sc} = r_s \cdot i_c^2 + u_c \cdot i_c + r_p \cdot i_p^2 \tag{5}$$

$$u_{sc} = r_s \cdot i_c + u_c = r_p \cdot i_p \tag{6}$$

$$\underbrace{\int u_{sc} \cdot i_{sc} \cdot dt}_{\text{Input Energy}} = \underbrace{\int u_c \cdot i_c \cdot dt}_{\text{Stored Energy}} + \underbrace{\int r_s \cdot i_c^2 \cdot dt}_{\text{Transfer Energy}} + \underbrace{\int r_p \cdot i_p^2 \cdot dt}_{\text{Balancing Energy}} \tag{7}$$

With a self-discharge test, we know the value of r_p and with high constant-current step we know the r_s value. The capacitance is identified with the MuSSiCC method that we describe later. In this method, we consider a high value of r_p and a low value of r_s. In order to limit the transfer losses into the internal resistance r_s, the capacitance identification tests are made at low current range. We consider the influence of the current on the capacitance as negligible for our current range.

In the identification MuSSiCC method, we set multiple constant-current steps with constant idle time between them in order to measure the elevation voltage between each idle state. During an idle state, the supercapacitor voltage u_{sc} and the capacitance voltage u_c are equal. To understate the losses in r_s and the losses in r_p, we choose a constant current signal with active state and an idle state period of 5 seconds. The figure 2 illustrates the current and the voltage during the MuSSiCC method. The upper part shows the voltage u_{sc} during the multiple constant-current steps while the lower part shows the constant-current i_{sc} evolution. In order to minimize the voltage measurement dispersion, we calculate the u_{sc} mean voltage during the idle state ([P10; P11] and [P20; P21]) with:

$$u_{P1} = \left\langle u_{[P11-P10]} \right\rangle = \frac{1}{\Delta t_{P1}} \cdot \int_{P10}^{P11} u_{sc} \cdot dt = \frac{1}{t_{P11} - t_{P10}} \cdot \int_{t_{P10}}^{t_{P11}} u_{sc} \cdot dt \tag{8}$$

We also calculate the average current value of i_{sc} between each idle state. The time associated to each idle state are $t_{P1} = \frac{1}{2} \cdot \left(t_{P11} + t_{P10} \right)$ and $t_{P2} = \frac{1}{2} \cdot \left(t_{P21} + t_{P20} \right)$, they correspond to the middle of each idle segments. Thus, the capacitance is determined by equation (9) and the capacitance attached voltage $u_{P2P1} = \left\langle u_{[P2;P1]} \right\rangle = \frac{1}{2} \cdot \left(u_{P2} + u_{P1} \right)$.

$$C\left(\left\langle u_{[P2;P1]} \right\rangle \right) = \frac{i_{sc} \cdot \Delta t}{\Delta u} = \frac{\int_{t_{P1}}^{t_{P2}} i_{sc} \cdot dt}{u_{P2} - u_{P1}} \tag{9}$$

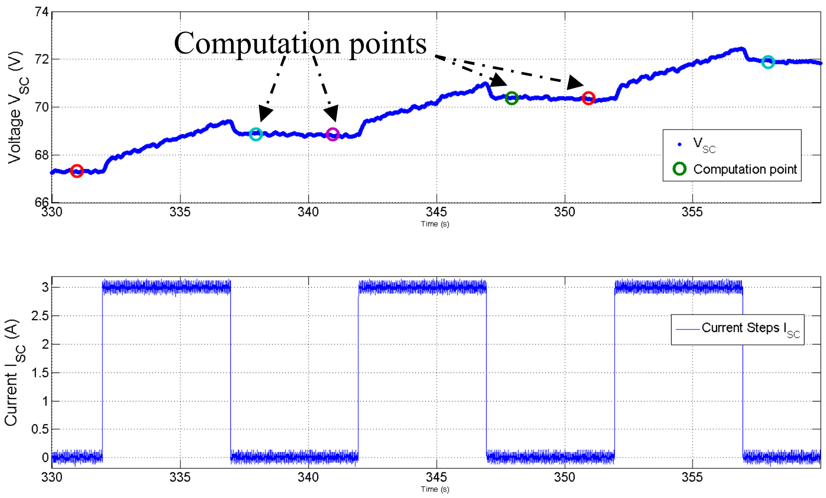

Figure 2 - Voltage and current evolution during MuSSiCC method

Bench used for the accurate sizing of a supercapacitor bank

The method described in this paper is based on the energy model developed on the figure 3 and in the paper [8]. For vehicular application, the electrical dynamic is slow enough to consider a quasi-static mode. We are aware that in the design of this kind of supercapacitors applications, the balancing circuit should have the highest resistive value while avoiding the voltage cell overflow.

The bench used to identify the capacitance is composed of a power converter with its smoothing inductance and Maxwell BOOSTCAP units (BMOD Series) on the figure 3. The supercapacitors use non-water soluble electrolyte (acetonitrile) technology. The energy storage bank is composed of a branch with each 7 supercapacitors under 105 V with a resistive balancing. According to the manufacturer information, the branch of supercapacitors composes an 8.3 F – 105 V supercapacitor-based energy storage unit.

Figure 3- Supercapacitor bench used for the accurate sizing method

After the identification of the resistance r_s, r_p and the capacitance $C(u)$, in order to approximate the variations of the capacitance according to the voltage [5][7-8], we compared a first order approximation, a 2nd order approximation, a 3rd polynomial approximation, the average value of the capacitance measured on the thorough voltage range and the manufacturer capacitance value. We use the MuSSiCC method to identify the parameters a_1 and c_1 of the first order function:

$$C(u_c) = a_1 \cdot u_c + c_1 \quad (10)$$

The figure 4 shows the linear polynomial approximation and its average value obtained by the MuSSiCC identification. We add the manufacturer capacitance value which is the minimum value of the capacitance and its supposed maximum capacitance. We know that the manufacturer gives the average capacitance for the thorough range [12] and this value cannot outrun the maximum value (8.3F+20%). Indeed the manufacturer identification method [12] is the MuSSiCC used with two computation points ($U_{sc\,min}$, $U_{sc\,max}$).

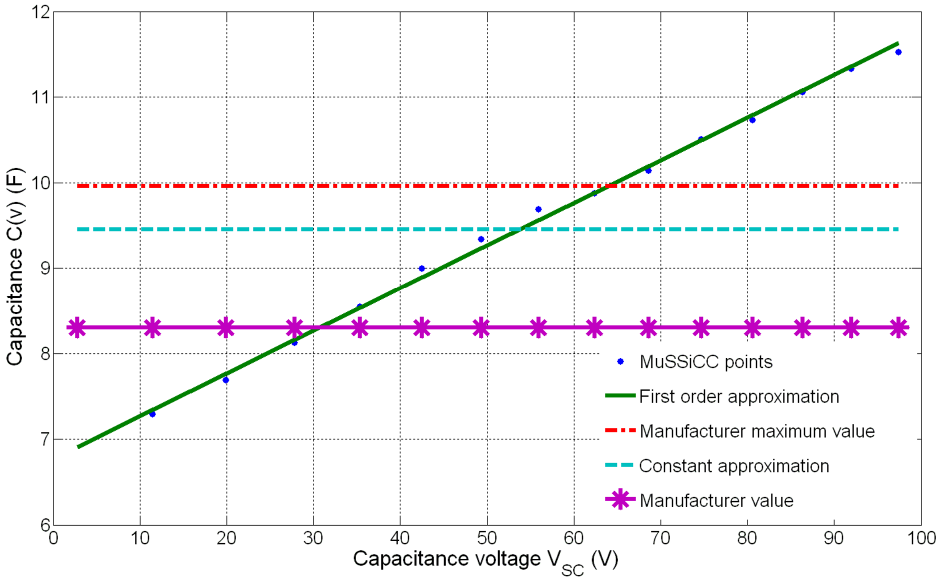

Figure 4 - Bench capacitance approximation versus voltage (branch n°3)

Of course the approximation is important for the energy stored accuracy but this approximation has to be as simple as possible. We compute the residual norm of each approximation and the table 1 confirms that the 3rd order polynomial approximation have a lower residual norm than the 1st order polynomial.

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Table 1 - residual norm of the approximation orders

	Constant	1st	2nd	3rd
Residual norm	2.12 F	0.12 F	0.11 F	0.10 F
Residual norm on average value (%)	24	1.4	1.3	1.1

We agreed to use the 1st order polynomial approximation because it is the best compromise. Thanks to the identification on 6 racks, it is possible to compute an average first order approximation with maximum relative error of 10% on the parameter a_1 and a maximum relative error of 1.3% for the parameter c_1. The identification data are summarized in the table 2. To compute the energy error and the total losses, we used the average value of the parameter a_1 and c_1.

Table 2 - First order coefficient of the 6 supercapacitor branches (105 V - 8.3 F)

	n°1	**n°2**	**n°3**	**n°4**	**n°5**	**n°6**	**Average**
a_1 **(10^{-3} F/V)**	45.8	49.9	40.8	45.8	42.2	49.8	45.7
Error (%)	0.2	9	- 10.7	0.2	- 7.7	8.9	
c_1 **(F)**	6.69	6.77	6.67	6.69	6.83	6.80	6.74
Error (%)	- 0.7	0.4	- 1.1	- 0.74	1.30	0.89	

Accurate sizing and energy errors

With this supercapacitor branch model, it is possible to give the equation (11) of the energy related to the linear approximation capacitance according the voltage depth of discharge $\gamma(\%)=100*(u_{initial}/u_{final})$ when we focus only on the stored energy without the balancing and transfer losses.

$$E_{cs1} = \frac{1}{2}\cdot c_1 \cdot u_{final}^2 \cdot \left(1-\gamma^2\right) + \frac{2}{3}\cdot a_1 \cdot u_{final}^3 \cdot \left(1-\gamma^3\right) \qquad (11)$$

The usual equation for a constant capacitance is defined by the equation (12). In the variable capacitance case, there is an additional part due to the first order capacitance approximation. This is why we have to accurate the sizing method because for some starting and ending voltage it may cause important errors.

$$E_{sc0} = \frac{1}{2}\cdot c_0 \cdot u_{final}^2 \cdot \left(1-\gamma^2\right) \qquad (12)$$

The classical sizing method is enough for a small power application when it does not require many supercapacitors. In our case, we need to sharply design the energy storage bank because the number of supercapacitors is very important for the whole-life cost of the application. In addition, we added the importance of the depth of discharge and the extreme voltage value for the energy transfer. In certain cases, the relative energy stored in the capacitance could reach an error value up to 20% in the worst case (figure 5) or the energy stored could be underestimated if a failing voltage range is used.

The figure 5 shows the energy stored error versus the initial voltage. This energy stored error is computed between the average capacitance C_0 or the manufacturer value C_{83} = 8.3 F and the first order capacitance approximation *C(u)*. These errors are computed for one rack of the figure 3. The energy stored error $E_r = E_{csc1} - E_{csc0}$ (right part of figure 5) between the linear capacitance approximation and its average value confirms the existence of an energy error. This error is greater if we compare the least capacitance value and *C(u)*. This figure 5 confirms the interest for applications with many supercapacitors to take into account the linear variation because the electrical designer could save supercapacitor modules and improve the purchasing cost of its application. This relative energy error $E_{rr} = (E_{csc1} - E_{csc0})/E_{csc0}$ represents at least 7% but if we compare to the manufacturer value it could reach 20%. Obviously, we focus on the energy errors between the first order capacitance approximation and its average value because it is only related to take or not into account the first order variation.

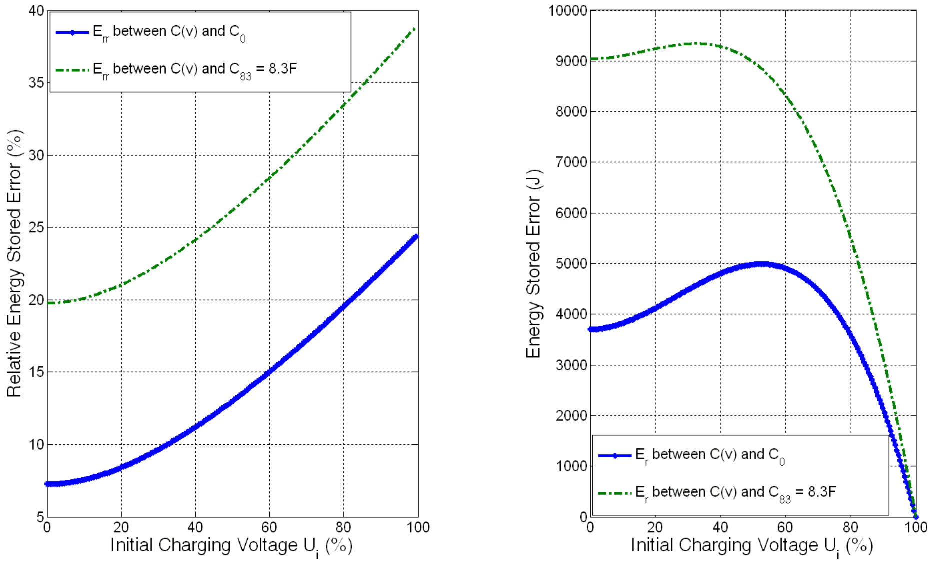

Figure 5 - Relative energy stored error (left) and the energy stored error (right) versus initial voltage

The figure 6 contains the relative error surface showing the energy differences between the capacitance C_0 and the capacitance *C(u)* for the stored energy according to the starting and the ending voltage value. This figure 6 is a representation of multiple computations of the figure 5 for different final voltages. It highlights the error in the case of supercapacitor bank over sizing. In this case, the energy can be underestimated.

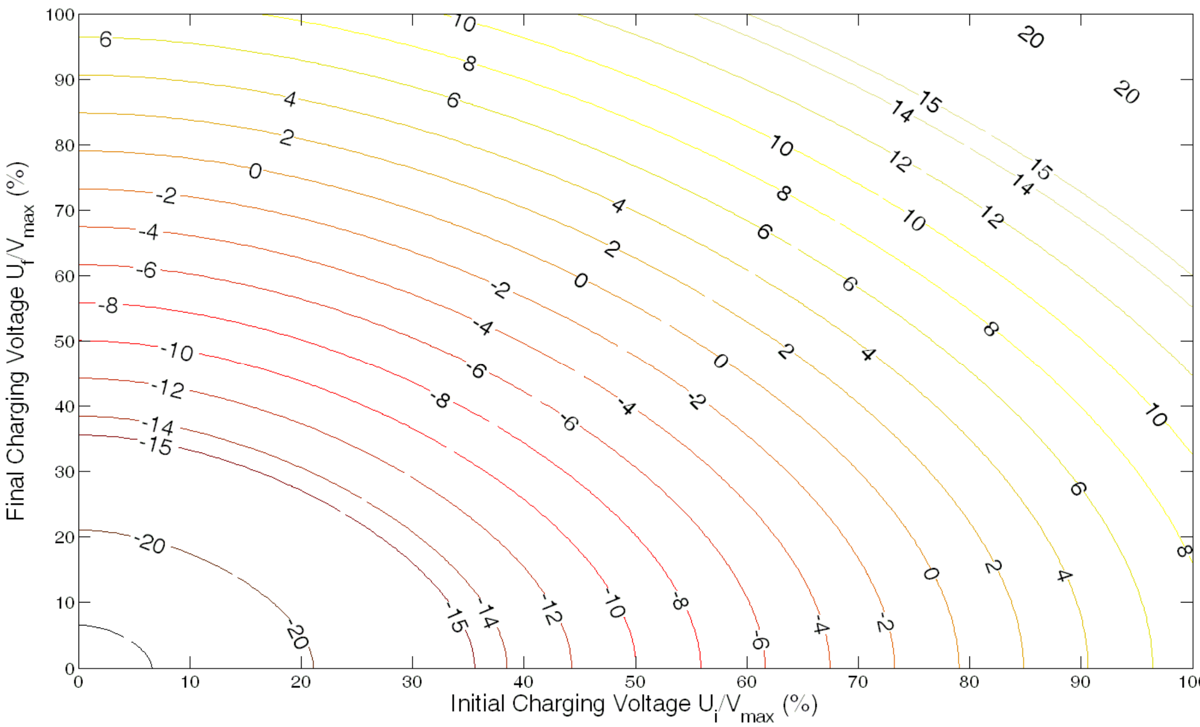

Figure 6 - Relative energy stored error surface versus initial and final voltage

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Accurate sizing and charging parameter choice

We focus on the common constant-current charging mode for the supercapacitors controlled by the parameter i_{sc} of the figure 1. From a starting voltage to a final voltage, the mode is associated with an efficiency of the mode which is directly link to the balancing and the transfer energy losses. This paper proposed to understand the best choice of the charging parameter i_{sc} to reach the best efficiency. This parameter choice can change according to the capacitance approximation choice. We based this energy stored estimation on the equations (10-12). We sum up the total losses as the sum of the transfer energy and the balancing energy. These losses are calculated by the Matlab® solver ode15i on the (13) differential equations. Here after is the differential equation (13) which determines the evolution of $u_c(t)$ according to the I_{sc0} parameter value.

$$r_s \cdot (a_1 \cdot u_c + c_1) \cdot \frac{du_c}{dt} + \frac{r_s}{r_s + r_p} \cdot u_c = \frac{r_s r_p}{r_s + r_p} \cdot I_{sc0} \tag{13}$$

The computation method is a trapezoidal method. We choose two typical depths of discharge related with the maximum charging voltage:

- a depth of discharge $\gamma = 50\%$ ($u_{c\ initial} = 52.5$ V)
- a depth of discharge $\gamma = 70\%$ ($u_{c\ initial} = 31.5$ V)

On the figure 7, the current-constant result for 100% of depth of discharge shows the decrease of the efficiency out of 5A to 10A. The transfer energy is link to the parameter I_{sc0} whereas the balancing energy is related to the voltage u_{sc} during the charging mode. This figure is also applicable if there is no balancing circuit. In this case, the total losses would be equal only to the transfer losses.

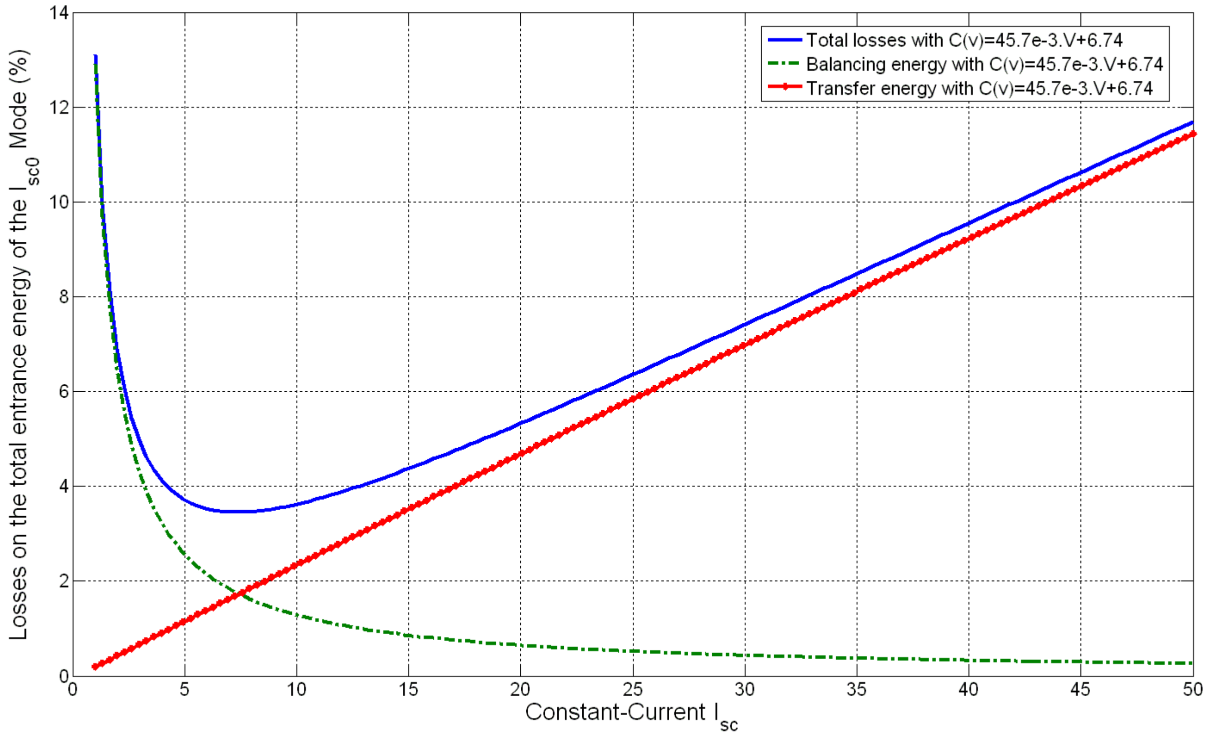

Figure 7 - Total losses with first order approximation for γ = 70%

For a constant-current charging mode, the optimal energy zone is only related to the supercapacitor parameters (r_s,r_p) and not to the approximation order as the figure 8 shows. The depth of discharge does not have a great influence on the energy losses minimum point. So the accurate sizing of the supercapacitor bank could set a number parallel branches to have the best charging efficiency and then the electrical designer choose the number of modules in serial association. This supercapacitor modules association will be optimize after when we try to optimize the design of the power converter and its attached supercapacitor bank.

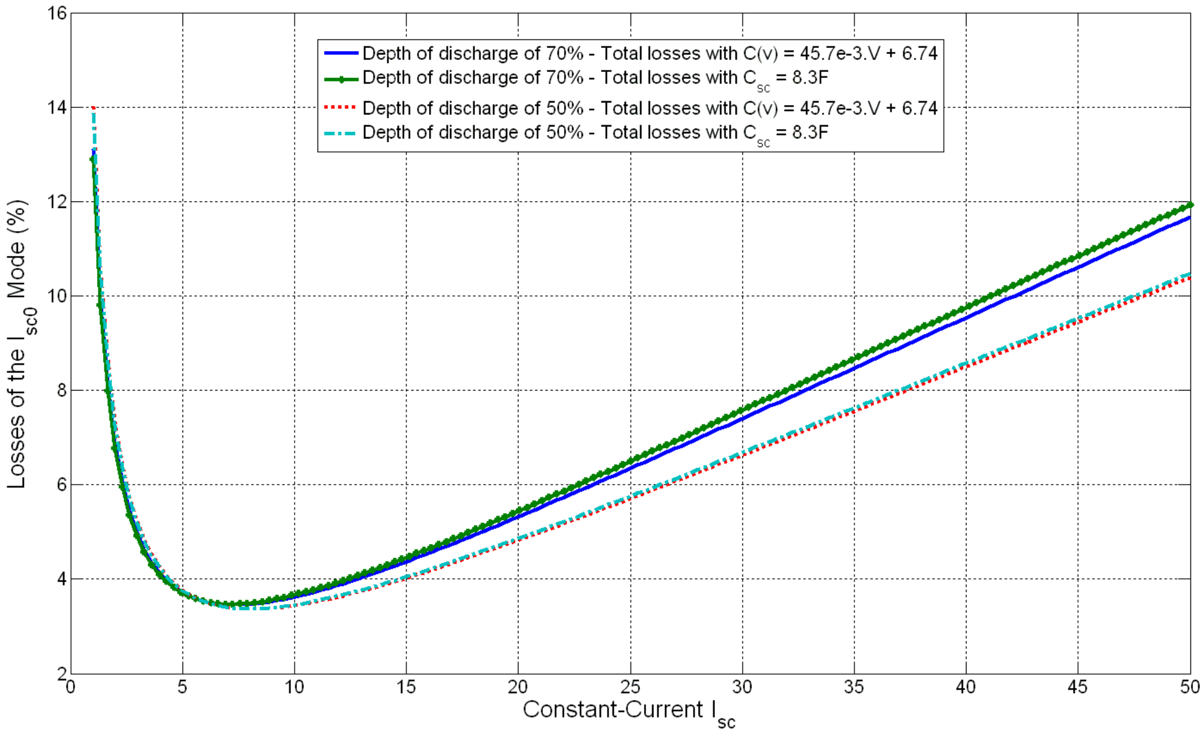

Figure 8 - Total losses versus constant-current charging according to the depth of discharge (50%, 70%)

The measurable voltage is u_{sc} so the final voltage depends on the u_{sc} value. Of course, for charging phase the u_c final voltage at 50 A is lower than the u_c voltage at 5 A. Otherwise, the choice of the balancing technology is very important because it depends on the application cycle. Of course, the balancing technology avoids overtaking the maximum admissible voltage. It has also to care about the energy lost during a charging cycle.

Application

In order to illustrate the energy errors between the manufacturer capacitance and the zero order approach versus the first order approximation onto the supercapacitor bank design, we sized a bank of BMOD Maxwell supercapacitor for a bus. The energy delivered is 8.6 kWh to the electrical motor. All bus characteristics are available in the paper [9]. The sum up the sizing results the relative errors are the same than the table 3 results presented. Thanks to first order approximation, it is possible to save between 9% and 13% of supercapacitors modules according to the depth of discharge.

Table 3 - Number of supercapacitor modules needed for 8.6 kWh and a final voltage of 105V

Number of needed branches	Manufacturer capacitance	Constant approximation	First order approximation	Relative Error Manufacturer/First Order (%)	Relative Error Constant/ First Order (%)
Depth of discharge @ 50 %	906	811	718	26.2	12.9
Depth of discharge @ 70%	742	665	607	22.2	9.6

We know the number of supercapacitors modules and we can optimize the serial and parallel connection of the modules. This optimization also depends on the application and the supercapacitor choice. For constant current charge, we want to charge around its minimum losses point while stay unchanged the charging time (12 to 15 minutes). This connection optimization has to take into account the power electronics.

Conclusion

Thanks to the first order approximation, it is possible to save supercapacitors modules. This is very important for the application purchasing cost. This accurate method also defines the best accurate constant-current to charge the supercapacitor bank and thanks to the paper we are aware of the best serial and parallel connection to reach the optimal efficiency zone. This optimal current charging value depends mainly on the internal losses parameters of the supercapacitors modules.

In this particular case, the charging-discharging cycle reaches up to 15 minutes. That is why, the electrical designer choice has an impact only on the balancing losses parameter and this paper highlights how the choice of the balancing technology influences the optimal losses zone.

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