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Abstract

In this study, we investigate the Belief Propagation (BP) algorithm proposed in [1] as an inference method to estimate the joint probability distribution of a given Bayesian network. Many works have raised up the use of BP in numerous applications as in image processing, computer vision, neural network, statistical physics and channel coding.

Among examples in the channel coding realm, we focus on the Low-Density Parity-Check (LDPC) codes used to protect a sequence of bits sent through a noisy channel. Any LDPC code can be represented by a graph whose edges (resp. the vertices) stand for the parity-check equations (resp. the bits) which protect the information against the noise of the channel. This graph is equivalent to a Markov random field. It is used to run the algorithm by iteratively passing messages along the edges in such a way that a message is the probability distribution of the destination conditionned to the emitter. LDPC codes are sufficiently robust against the channel noise for the algorithm to converge after some iterations to the most likely state of the estimate of the joint probability distribution. LDPC codes provide a lower bit-error rate than a no coded transmission for any values of the Signal-to-Noise Ratio (SNR).

However, most LDPC codes exhibit loop-like topological structures that ruin the BP performance in terms of the number of corrected errors. The short loops and the combinations thereof are especially harmful. These defects drive the algorithm toward states that are not the most likely ones, or even toward complex attractors that need to be defined.

A few studies have been carried out that propose an analytic approach to the BP convergence [2, 3]. Other authors have focused on experimental results to extract properties about the behavior of the BP algorithm [4]. In our work, we make use of well-known estimators and new ones to describe the dynamics of the algorithm in terms of the SNR.

One is the bifurcation diagram which we use to extract the critical values of the SNR for which the algorithm blatantly changes its behavior. Experiments we conducted on many noise realizations that lead to complex attractors provide three critical values that divide the SNR values into four intervals. Each interval

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corresponds to particular dynamics.

To visualize these dynamics, we create a three dimensional reduced state space by the phase space reconstruction method. It brings out four different behaviors announced by the bifurcation diagram. Whatever the harmful noise realization we get back these four patterns as the SNR is increased: small sized attractor, limit cycle, chaos and fixed point.

To assert that the algorithm suffers from chaos and to evaluate its intensity we compute the largest Lyapunov exponent λ. The simulations provide the confirmation of these four behaviors and assert that the third interval matches with chaos of the BP.

We use the Jacobian matrix to underline the state parameters responsible for chaos. Though, the loop-like structure propagates the information along most edges, whatever its nature. Thereby we obtain from this estimator that all the messages suffer from a chaotic dynamic.

To complete the investigation, we propose a new estimator. It consists in computing the radius of the circumscribed hypersphere of the true trajectory in a given temporal window. By sliding this window, we get a sequence of radii. This sequence has the advantage to be one dimensional, whereas the trajectory is multi dimensional. Furthermore, it describes accurately the shape of the attractor and its diameter. This helps to understand the nature of the considered attractor. An extra use of these method is to compute the iterative evolution of the mutual distance between the hyperspheres of two initially close trajectories. Thus this estimator highlights the stability brought out by the largest Lyapunov exponent and it also provides the attractor nature.

To conclude, this study helps to classify four distinguished dynamical behaviors of the BP along the SNR values. It confirms that the BP is widely infuenced by the topology of the considered LDPC code. The estimators, either well-known or new, provide qualitative and quantitative descriptions of the algorithm, especially for the complex dynamics.

Keywords: Inference, message-passing, loops, stability, chaos.

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