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CONDITION-BASED MAINTENANCE FOR AVAILABILITY OPTIMIZATION OF PRODUCTION SYSTEM UNDER ENVIRONMENT CONSTRAINTS

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ABSTRACT: *This paper deals with a condition-based maintenance policy for continuously degrading system to reduce the excessive environment degradation. This maintenance policy consists on maintaining the system with respect to a given threshold level of degradation. Such a level is measured by sequential inspections. According to statistical knowledge of degradation level, a preventive maintenance action is performed on the system, while a corrective maintenance action is executed at system failures. Two kinds of system availability are considered and for each of which a mathematical formulation is given. The objective of the inspection and maintenance policy is to find an optimal vector of inspection times in order to maximize the system availability. On the basis of Nelder-Mead method, an algorithm is implemented to calculate optimum solutions. Numerical experiments are conducted to illustrate the present work.*

KEYWORDS: *System degradation, Condition-based maintenance policy, Stationary availability, Nelder-Mead algorithm.*

1 INTRODUCTION

This paper deals with a condition-based maintenance policy for continuously degrading system to reduce the excessive environment degradation. In the existing literature, several research papers deal with system inspection and maintenance problems. Early works are those initiated by Barlow and Hunter. The authors in (Barlow and Hunter, 1960) proposed two preventive maintenance policies for a system subject to random failures and whose state is assumed to be known at any moment, i.e. the system is assumed to be continuously monitored. In (Barlow and Hunter, 1960), an optimization model is developed where the objective is either to minimize the total cost generated by maintenance activities, or to maximize availability. Barlow *et al.* (Barlow *et al.*, 1963) introduced an inspection policy where the objective is to minimize the average total cost of inspection activities. An algorithm based on a recurrence relation is proposed to calculate the optimal dates of inspection. Several extensions of the work by (Barlow *et al.*, 1963) have been proposed in the literature. In (Munford and Shahani, 1972), a nearly optimal inspection policy has been suggested and an approximate solution to that of (Barlow *et al.*,

1963) is proposed. The policy developed in (Munford and Shahani, 1972) has been exploited in the work by (Munford and Shahani, 1973) to solve the same problem while considering the particular case where the system lifetime is Weibull distributed. In (Munford and Shahani, 1973), a numerical and empirical methods are used to solve the problem initially investigated in (Munford and Shahani, 1972). On the basis of the work by (Munford and Shahani, 1972) and (Munford and Shahani, 1973), Tadikamalla (Tadikamalla, 1979) proposed methods to derive the optimal dates of inspection. The lifetime of the system investigated in (Tadikamalla, 1979) follows a gamma distribution. Turco and Parolini (Turco and Parolini, 1984) proposed a condition-based maintenance policy for a system subject to random failures and whose state is assumed to be known at any moment (i.e. revealed failure). In (Turco and Parolini, 1984), two inspection policies, sequential and periodic, are studied and each of which optimal dates of inspection are evaluated. In (Turco and Parolini, 1984), the optimality criterion is the total cost induced by inspection and maintenance actions. Several extensions of the work by (Turco and Parolini, 1984) have also been proposed in the literature. For example, (Pellegrin, 1992) studied the

case where inspection and maintenance durations are not negligible. Pellegrin (Pellegrin, 1992) suggested three criteria and proposed a graphical method to calculate the optimal inspection period. The policy developed in (Turco and Parolini, 1984) has been exploited in the work by (Chelbi and Aït-Kadi, 1999) to solve the same problem while dealing with the case of unrevealed failure, i.e. the state of the system is known only through inspection. In (Chelbi and Aït-Kadi, 1999), the optimality criterion considered is the total cost induced by inspection and maintenance actions as well as by the penalty cost due to the inactivity of the system between occurrence and detection of the failure. Grall *et al.* (Grall *et al.*, 2002) developed a mathematical maintenance cost model for a system which is subjected to a condition-based maintenance policy. In (Grall *et al.*, 2002), the optimal inspection schedule as well as the optimal replacement threshold are derived for the studied system. The authors in (Grall *et al.*, 2002) included in their modeling the cost engendered by the inactivity of the system. In (Grall *et al.*, 2002), (Chelbi and Aït-Kadi, 1998), and (Chelbi and Aït-Kadi, 1999), the cost induced by the inactivity of the system is also taken into account as a penalty cost. Badía *et al.* (Badía *et al.*, 2002) considered a system whose state is assumed to be known with some uncertainty, and a maintenance policy which depends on the nature of the information gathered from inspection is proposed. The objective in (Badía *et al.*, 2002) is to minimize the average total cost induced by inspection and maintenance actions.

To deal with system availability optimization, (Chelbi *et al.*, 2008) extended the work of (Badía *et al.*, 2002). In (Chelbi *et al.*, 2008), numerical solutions have been presented for Normal and Weibull failure distributions. The authors in (Sarkar and Sarkar, 2000) studied the availability of a periodically inspected system subjected to a perfect repair whose the time is assumed to be non negligible. In order to determine the inspection period, (Sarkar and Sarkar, 2000) expressed the system availability function as well as the limiting average availability. The inspection period have been evaluated in (Sarkar and Sarkar, 2000) in the case where the system lifetime distribution is either gamma or exponential. On the basis of the work by (Sarkar and Sarkar, 2000), (Cui and Xie, 2005) investigated the availability of periodically inspected system with random repair or replacement times. In (Cui and Xie, 2005), the proposed models are the same as those of (Sarkar and Sarkar, 2000), where the instantaneous availability and the limiting average availability, together with the steady-state availability are derived and studied. Numerical results are then given in (Cui and Xie, 2005) and compared to those obtained in (Sarkar and Sarkar, 2000). More recently, Liao

and his co-authors (Liao *et al.*, 2006) proposed a condition-based availability limit policy for continuously degrading and monitoring system. The maintenance policy investigated in (Liao *et al.*, 2006) aims to achieve the maximum availability value of a system subject to imperfect maintenance actions and short-run availability constraint. Using a search algorithm, the optimum preventive maintenance threshold is determined in (Liao *et al.*, 2006) for a degrading system modeled by a Gamma process. Because of the difficulty to monitor the degrading system continuously, Aït-Kadi and Chelbi (Aït-Kadi and Chelbi, 2010) studied an inspection policy for systems subject to shocks with unrevealed failures. In (Aït-Kadi and Chelbi, 2010), the inspection strategy suggested aims to reduce the frequency of the random failures and to increase the system availability. In the work by (Aït-Kadi and Chelbi, 2010), a computational procedure based on cubic spline interpolation has been implemented to determine the inspection sequence to insure a required availability value. For detailed studies on various inspection and maintenance policies, readers are referred to (Chelbi and Aït-Kadi, 2009) where authors give a survey of different inspection strategies.

In the present work, a condition-based maintenance policy is used to take into account the impact of the system degradation on the environment. This maintenance policy aims to reduce the excessive environment degradation. The system considered is subject to random degradations, and system failures are assumed to be determined and known at the point of occurrence (i.e. case of revealed failures). The value of the measured quantity of environment impact is however known only through inspection. The objective consists on the determination of optimal inspection times to maximize the system average availability. Two mathematical models are then proposed and studied. The first model considers the system stationary availability while the second focuses on the system availability by considering duration of excessive degradation as unavailability duration (i.e. in this case, the system is assumed to be unavailable whenever the threshold level of degradation is exceeded). Such availabilities are derived from inspection and maintenance durations as well as from system operating cycle. To solve the problem, an algorithm based on the Nelder-Mead method is proposed as a solution technique. The Nelder-Mead algorithm is a direct search method exploited in the literature to solve nonlinear optimization problems. Indeed, especially in maintenance problems, (Li and Pham, 2005) and (Roux *et al.*, 2010) used the Nelder-Mead algorithm to optimize maintenance policies.

The rest of this paper is organized as follows.

Notations and assumptions are given in the next section. The mathematical formulation of the inspection and maintenance problems are given in Section 3, where proofs of propositions are sketched in Appendix. In Section 4, numerical experiments are provided to illustrate the proposed work, and results obtained are then analyzed and discussed. Conclusions and perspectives are drawn in Section 5.

2 NOTATIONS AND ASSUMPTIONS

In this work, the system considered is subject to random failures and assumed to be in one of two possible functioning states, an operational state or a failed state. Because of its degradation, the system functioning may have an impact on the environment, i.e. the environment degraded by the degradation of the system. This impact becomes more significant whenever the degradation level of the system reaches a given threshold value. Preventive maintenance actions are then scheduled whenever the degradation level of the environment exceeds a specified value. Such value can be measured via environment inspection. Figure 1 gives an example of the environment degradation versus time. This figure gives also the distribution of inspections on the axis of operating time. At system failure, corrective maintenance is performed. In this work, it is assumed that the system becomes as good as new after either a corrective or a preventive maintenance actions. A cycle is assumed to be the interval between two consecutive replacements, either corrective or preventive maintenance. During this interval, the system is operating and subjected to inspections. In the beginning of the cycle, the system is assumed to be new. The following notations are used throughout this paper.

T_c	duration of corrective maintenance.
T_i	inspection duration.
T_p	duration of preventive maintenance.
T	continuous non-negative random time to exceed the threshold level of environment degradation.
τ	realization of T on the axis of operating time, from the beginning of the cycle (see Figure 1).
f	pdf of T .
X	continuous non-negative random time elapsed from instant τ to failure occurrence.
x	realization of X , from instant τ (see Figure 1).
g, G	pdf and cdf of X , respectively.
N	random number of inspections during a cycle.
P_c	probability that the system undergoes a corrective maintenance.

P_p	probability that the system undergoes a preventive maintenance.
H	time to perform preventive maintenance, from inspection time in which the exceed of alarm threshold is detected.
Θ	vector of inspection times θ_j , ($j = 1, 2, 3, \dots, n$), on the axis of operating time.
A_{1s}	system stationary availability.
A_{2s}	stationary availability of the system without exceeding the alarm threshold.
D_c	random down-time of the system in a cycle.
T_d	continuous non-negative random time of excessive environment degradation.
U_c	random up-time of the system in a cycle.

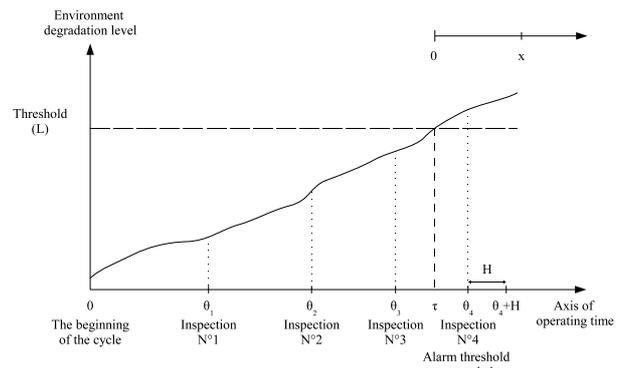


Figure 1: Scheduling of preventive maintenance

The basic hypotheses are:

- After each inspection, only two kind of actions are possible: do nothing, or replace preventively.
- The preventive maintenance is planned after the predetermined time H from the instant of the inspection in which the degradation level is greater than the threshold level (see Figure 1). Any possible inspection within this interval is canceled, e.g. within the interval $[\theta_4, \theta_4 + H]$, inspections are canceled (Figure 1).
- Durations of inspection and maintenance actions are non negligible.
- Inspection and maintenance actions do not degrade the system.
- Inspection and maintenance actions are perfect.
- Durations T_c, T_p and T_i , together with duration interval H as well as pdfs (f and g) are assumed to be known.

3 MATHEMATICAL MODELS

The maintenance policy studied in the present paper aims to reduce the excessive environment degradation. The cycle of the system is considered to be

the interval between two consecutive replacements either a corrective or a preventive one. The preventive replacement is performed whenever the level of environment degradation measured at inspection exceeds a specified threshold. In this section, two models of system availability are developed.

3.1 First model: system stationary availability A_{1s}

The first model considers the system stationary availability A_{1s} expressed as the ratio of the average up-time of the system $E(U_c)$ and the average cycle time $E(U_c) + E(D_c)$:

$$A_{1s} = \frac{E(U_c)}{E(U_c) + E(D_c)}. \quad (1)$$

In order to determine the optimal vector of inspection times, the system stationary availability A_{1s} is considered as the objective function:

$$\max_{\Theta} A_{1s} \quad (2)$$

The following proposition gives the formula of the average up-time of the system in a cycle. Such a time is measured from the beginning to the end of an operating cycle (see Figure 2 and Figure 3).

Proposition 1 *The average up-time $E(U_c)$ of the system is given by:*

$$E(U_c) = \sum_{j=1}^{\infty} \int_{\theta_{j-1}}^{\theta_j} \left(\int_0^{\theta_j + H - \tau} (\tau + x)g(x)dx \right) f(\tau)d\tau + \sum_{j=1}^{\infty} (\theta_j + H) \int_{\theta_{j-1}}^{\theta_j} (1 - G(\theta_j + H - \tau)) f(\tau)d\tau. \quad (3)$$

Proof. See Appendix. ■

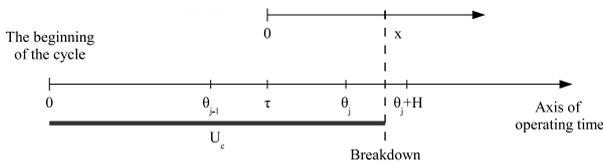


Figure 2: The case where an operating cycle ends with a corrective maintenance

The first term of Equation (3) corresponds to the case where the system undergoes a corrective maintenance (see Figure 2), while the second term corresponds to the case where a preventive maintenance is performed on the system (see Figure 3).

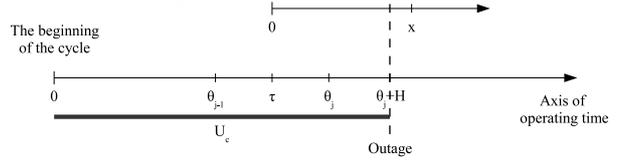


Figure 3: The case where an operating cycle ends with a preventive maintenance

At the end of a system operating cycle, either a preventive or a corrective maintenance is performed. The average time of a corrective maintenance is $T_c P_c$, while that of a preventive maintenance is $T_p P_p$. During an operating cycle, the system is subjected to a random number N of inspections whose the average time induced is given as $T_i E(N)$. The following formula gives the average down-time of the system in a cycle:

$$E(D_c) = T_c P_c + T_p P_p + T_i E(N), \quad (4)$$

where $E(N)$ is the average number of inspections in a cycle.

To derive the explicit formula of the average down-time $E(D_c)$, some propositions are given. The first one allows to determine the probability P_c that a corrective maintenance is performed on the system at the end of an operating cycle.

Proposition 2 *The probability P_c that the cycle ends with a corrective maintenance is given by:*

$$P_c = \sum_{j=1}^{\infty} \int_{\theta_{j-1}}^{\theta_j} G(\theta_j + H - \tau) f(\tau) d\tau. \quad (5)$$

Proof. See Appendix. ■

From the result of the above proposition, it follows that the probability P_p that the system undergoes a preventive maintenance is such that $P_p = 1 - P_c$. The following proposition gives the average number of inspections in a cycle.

Proposition 3 *The average number $E(N)$ of inspections in a cycle is:*

$$E(N) = \sum_{j=1}^{\infty} j \left(\int_0^{\theta_{j+1}} G(\theta_{j+1} - \tau) f(\tau) d\tau - \int_0^{\theta_j} G(\theta_j - \tau) f(\tau) d\tau \right). \quad (6)$$

Proof. See Appendix. ■

The denominator of Equation (1) is then deduced from Equations (3)-(6).

3.2 Second model: stationary availability A_{2s} of the system without exceeding the alarm threshold

The second model focuses on the system availability by considering the system as unavailable within duration T_d of excessive degradation, i.e. the system is unavailable whenever the threshold level of degradation is exceeded. In this case, the system stationary availability is denoted by A_{2s} and defined to be:

$$A_{2s} = \frac{E(U_c) - E(T_d)}{E(U_c) + E(D_c)}, \quad (7)$$

where $E(T_d)$ is the average time of excessive environment degradation in a cycle.

The numerator of Equation (7) represents the functioning time of the system under an acceptable level of environment degradation. The following proposition gives the formula of the average time $E(T_d)$ of excessive environment degradation in a cycle. Let us recall that such a time is measured from the instant τ until the end of an operating cycle (see Figure 4 and Figure 5).

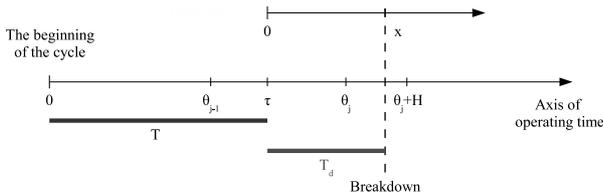


Figure 4: The case where the system undergoes a corrective maintenance

Proposition 4 *The average time $E(T_d)$ of excessive environment degradation in a cycle is given by:*

$$E(T_d) = \sum_{j=1}^{\infty} \int_{\theta_{j-1}}^{\theta_j} \left(\int_0^{\theta_j + H - \tau} x g(x) dx \right) f(\tau) d\tau + \sum_{j=1}^{\infty} \int_{\theta_{j-1}}^{\theta_j} (\theta_j + H - \tau) (1 - G(\theta_j + H - \tau)) f(\tau) d\tau. \quad (8)$$

Proof. See Appendix. ■

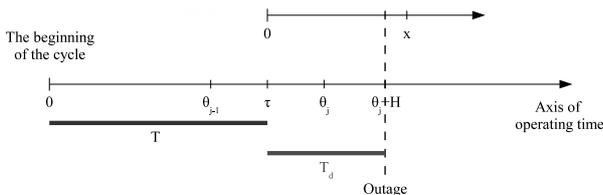


Figure 5: The case where the system undergoes a preventive maintenance

In Equation (8), the first term corresponds to the case where a cycle ends with a corrective maintenance, while the second one corresponds to the case where a cycle ends with a preventive maintenance.

In order to determine the optimal vector of inspection times with a minimum level of environment degradation, A_{2s} is considered as the objective function:

$$\max_{\Theta} A_{2s}. \quad (9)$$

4 NUMERICAL EXAMPLE

In this section, a numerical example is investigated to illustrate proposed models. Closed forms of solutions of the nonlinear optimization problems given by (2) and (9) are difficult to obtain. Therefore, in order to determine inspection dates, an algorithm based on the Nelder-Mead method is proposed as a solution technique. The Nelder-Mead simplex algorithm, first published in 1965, is a popular direct search method widely exploited in the literature to solve nonlinear optimization problems. Several authors have suggested the Nelder-Mead simplex algorithm as a solution technique to optimize maintenance policies. (Li and Pham, 2005), for example, proposed an algorithm on the basis of Nelder-Mead method to solve a nonlinear optimization problem while dealing with inspection and maintenance of systems subjected to multiple sources of degradation. Inspection sequences and preventive maintenance thresholds for degradation processes are derived in (Li and Pham, 2005) to calculate the optimum policy minimizing the average long-run maintenance cost rate. More recently, the Nelder-Mead simplex method has been used in (Roux *et al.*, 2010) to optimize the maintenance-production problems while minimizing the unavailability of the production system. The authors in (Roux *et al.*, 2010) developed a generic modeling tool for the simulation model to determine an efficient preventive maintenance period.

The Nelder-Mead method consists firstly on initializing a simplex of $n + 1$ vertices, where n is the dimension of the solution vector. Then, the objective function is evaluated at each vertex and the $n + 1$ vertices are ordered. The centroid of the best n vertices is computed in order to generate a new vertex for the simplex through operations of reflection, expansion, or contraction. If the new vertex is better than the worst one, it replaces the worst vertex to form a new simplex. Readers are referred to (Lagarias *et al.*, 1998) for more details about Nelder-Mead algorithm.

Figure 6 gives the principle of the algorithm implemented in the present work. The input data of

the numerical procedure includes probability density functions f and g , durations of maintenance and inspection actions (T_c , T_p , and T_i) and duration H . The time is assumed to be given in time unit. Probability density functions f and g , as defined in Section 2, are both Weibull distributed such that:

- The pdf f of the random time to exceed the threshold level of environment degradation is given by:

$$f(\tau) = \left(\frac{\beta_f}{\alpha_f}\right) \left(\frac{\tau}{\alpha_f}\right)^{\beta_f-1} e^{-\left(\frac{\tau}{\alpha_f}\right)^{\beta_f}}, \quad (10)$$

where $\alpha_f = 1164.1$ and $\beta_f = 8.7$.

- The pdf g of the random time elapsed from instant τ to failure occurrence is given by:

$$g(x) = \left(\frac{\beta_g}{\alpha_g}\right) \left(\frac{x}{\alpha_g}\right)^{\beta_g-1} e^{-\left(\frac{x}{\alpha_g}\right)^{\beta_g}}, \quad (11)$$

where $\alpha_g = 144.2$ and $\beta_g = 3.6$.

Algorithm 4.1: SORT (Θ^*)

comment: On the basis of the Nelder-Mead simplex method, this algorithm sort the optimal vector Θ^* of inspection times.

input Durations (T_c , T_p , and T_i), Pdfs (f and g), H , and n .

output (Θ^* : the best solution found.)

while the maximum of objective function does not reached

do $\left\{ \begin{array}{l} \text{Increment the dimension } n \text{ of solution vector,} \\ \text{Compute the solution vector } \Theta^* = (\theta_1^*, \dots, \theta_n^*) \text{ by using Nelder-Mead algorithm.} \end{array} \right.$

return (Θ^*)

Figure 6: Algorithm of the numerical procedure

The duration H as well as durations of maintenance and inspection actions (T_c , T_p , and T_i) will be

considered as sensitivity parameters. These four parameters are then made variable within the following experiments. In the rest of this paper, we denote by δ the ratio $\delta = \frac{E(T_d)}{E(U_c)}$, i.e. the proportion of the average time of excessive environment degradation per the average up-time of the system.

In order to analyze the sensitivity of the proposed performance criteria (i.e. A_{1s} and A_{2s}) with respect to the four sensitivity parameters, only one of those parameters is made variable while the others are considered constant. Therefore, four experiments are conducted as follows.

4.1 Experiment 1: impact of T_c on A_{1s} and A_{2s}

This experiment investigates the case where only the corrective maintenance duration T_c is made variable. The durations of preventive maintenance and inspection actions are respectively set to: $T_p = 12$ and $T_i = 2$. The duration H is set to 0, i.e. preventive maintenance actions are not delayed. For different values of T_c , Tables 1 and 2 give the optimal vector Θ^* of inspection times for each model (i.e. Table 1 shows the results obtained by the maximization of A_{1s} , while Table 2 depicts the results obtained by the maximization of A_{2s}). For each vector Θ^* , Table 1 gives also the dimension of the vector Θ^* noted n , the optimal system stationary availability A_{1s}^* , and δ . Table 2 gives also n , the optimal stationary availability of the system without exceeding the alarm threshold A_{2s}^* , and δ .

T_c	Θ^*	n	A_{1s}^*	δ (%)
24	1135.9, 1272.7, 1410.2	3	0.9844	6.92
48	1017.0, 1171.3, 1259.7, 1356.3, 1412.7, 1706.3	6	0.9803	5.51
72	1017.0, 1171.3, 1259.7, 1356.3, 1412.7, 1706.3	6	0.9767	5.51

Table 1: Impact of T_c on decision variable to maximize A_{1s}

T_c	Θ^*	n	A_{2s}^*	δ (%)
24	1062.6, 1158.2, 1259.3	3	0.9271	5.57
48	1017.0, 1171.3, 1259.7, 1356.3, 1412.7, 1706.3	6	0.9262	5.52
72	1017.0, 1171.3, 1259.7, 1356.3, 1412.7, 1706.3	6	0.9228	5.52

Table 2: Impact of T_c on decision variable to maximize A_{2s}

These tables show that the optimal values of stationary availabilities (A_{1s}^* and A_{2s}^*) are decreasing by the increase of corrective replacement duration, while the dimension n of optimal solution vector increases

with the increasing of corrective replacement duration. The most important result is that the increase of corrective replacement duration induced the decrease of the ratio δ , i.e. the excessive environment degradation is reduced. For the case where $T_c = 24$, the vector (1135.9, 1272.7, 1410.2) in Table 1 represents the optimal vector to maximize A_{1s} , while the vector (1062.6, 1158.2, 1259.3) in Table 2 represents the optimal vector to maximize A_{2s} . It is also shown, as one may expect, that the value of δ has been reduced from 6.92% to 5.57% according to the change of criterion from maximization of A_{1s} to the maximization of A_{2s} .

4.2 Experiment 2: impact of T_p on A_{1s} and A_{2s}

This experiment considers the case where the preventive maintenance duration T_p is made variable. For this case study, we set the corrective replacement duration T_c to 24 and the inspection duration T_i to 2. Preventive maintenance actions are not delayed, i.e. $H = 0$. The evolution of optimal decision variable and of the optimal objective function as well as the optimal ratio δ are illustrated in Tables 3 and 4. For different values of T_p , Tables 3 and 4 give the optimal vector Θ^* of inspection times for each model. For each vector Θ^* , Table 3 gives also n , A_{1s}^* , and δ . Table 4 gives also n , A_{2s}^* , and δ .

T_p	Θ^*	n	A_{1s}^*	δ (%)
6	1073.8, 1199.5, 1288.2, 1379.2, 1417.6, 1714.8	6	0.9882	5.66
12	1135.9, 1272.7, 1410.2	3	0.9844	6.92
18	1270.3, 1443.4	2	0.9815	8.77

Table 3: Impact of T_p on decision variable to maximize A_{1s}

T_p	Θ^*	n	A_{2s}^*	δ (%)
6	1017.0, 1171.3, 1259.7, 1356.3, 1412.7, 1706.3	6	0.9336	5.52
12	1062.6, 1158.2, 1259.3	3	0.9271	5.57
18	1062.6, 1158.2, 1259.3	3	0.9224	5.57
24	1062.6, 1158.2, 1259.3	3	0.9179	5.57

Table 4: Impact of T_p on decision variable to maximize A_{2s}

Obviously, the optimal value of ratio δ (resp. A_{1s}^* , A_{2s}^* , and n) increased (resp. decreased) when the preventive replacement duration increased. This means that the preventive replacement duration has a negative impact on the percentage of excessive degradation (i.e. the ratio δ increases by the increasing of T_p). However, the maximization of A_{2s} tends to reduce the ratio δ ; e.g. δ increases from 5.66 % to 8.77 % and from 5.52% to 5.57% according to the increase of T_p

respectively on Tables 3 and 4. We can also note that either the increase of T_p or the decrease of T_c seeks to avoid more frequent inspections performed on the system.

4.3 Experiment 3: impact of T_i on A_{1s} and A_{2s}

The case where the inspection duration T_i is made variable is studied here. We set the corrective replacement duration T_c to 24. Preventive maintenance action whose duration T_p is set to 12 is not delayed, i.e. $H = 0$. For different values of T_i , Tables 5 and 6 give the optimal vector Θ^* of inspection times for each model. For each vector Θ^* , Table 5 gives also n , A_{1s}^* , and δ . Table 6 gives also n , A_{2s}^* and δ .

T_i	Θ^*	n	A_{1s}^*	δ (%)
2	1135.9, 1272.7, 1410.2	3	0.9844	6.92
4	1169.3, 1286.9, 1550.9	3	0.9822	7.37
6	1339.5, 1449.6, 1837.2	3	0.9815	9.44
12	1328.7, 1348.4, 1661.9	3	0.9781	9.35

Table 5: Impact of T_i on decision variable to maximize A_{1s}

T_i	Θ^*	n	A_{2s}^*	δ (%)
2	1062.6, 1158.2, 1259.3	3	0.9271	5.57
4	1062.6, 1158.2, 1259.3	3	0.9233	5.57
6	1062.6, 1158.2, 1259.3	3	0.9196	5.57
12	1062.6, 1158.2, 1259.3	3	0.9086	5.57

Table 6: Impact of T_i on decision variable to maximize A_{2s}

By varying the value of inspection duration T_i , Tables 5 and 6 report the results of this analysis. We can remark that the increase of inspection duration T_i induces the decrease of the optimal availabilities A_{1s}^* and A_{2s}^* against the increase of the ratio δ . This can be explained by the fact that the increase of maintenance actions durations induce generally the decrease of system availability. For different values of inspection duration T_i , the dimension n of optimal solution vector does not vary. In this configuration, the most sensitive parameter is the ratio δ . According to the increase of inspection duration T_i from 2 to 12, we can see in Table 5 that the ratio δ increases from 6.92% to 9.35%. This means that the inspection duration T_i has an important impact on the ratio δ against the other durations of maintenance actions.

4.4 Experiment 4: impact of H on A_{1s} and A_{2s}

This subsection concentrates on analyzing the impact of duration H on decision variable. The durations of maintenance and inspection actions are set

to: $T_c = 24$, $T_p = 12$, and $T_i = 2$. The results for this case are shown in Tables 7 and 8. For different values of H , Tables 7 and 8 give the optimal vector Θ^* of inspection times for each model. For each vector Θ^* , Table 7 gives also n , A_{1s}^* , and δ . Table 8 gives also n , A_{2s}^* and δ . The increase of duration H induces the increase of the ratio δ against the decrease of optimal stationary availabilities A_{1s}^* and A_{2s}^* . This last analysis shows that important values of maintenance delay durations, in a more general way, can reduce the system availability. Instead of the evolution of duration H , the dimension n of optimal solution vector is invariant as shown in the above experiment. The important increase of δ (from 6.92% to 7.49% in Table 7 and from 5.57% to 6.35% in Table 8) can be explained by the fact that the delay of preventive maintenance actions have a negative influence on the system degradation.

H	Θ^*	n	A_{1s}^*	δ (%)
0	1135.9, 1272.7, 1410.2	3	0.9844	6.92
6	1138.8, 1272.3, 1407.5	3	0.9843	7.21
12	1142.0, 1272.2, 1405.3	3	0.9840	7.49

Table 7: Impact of H on decision variable to maximize A_{1s} .

H	Θ^*	n	A_{2s}^*	δ (%)
0	1062.6, 1158.2, 1259.3	3	0.9271	5.57
6	1062.6, 1158.2, 1259.3	3	0.9231	5.97
12	1062.6, 1158.2, 1259.3	3	0.9192	6.35

Table 8: Impact of H on decision variable to maximize A_{2s} .

The numerical results of these experiments show the importance of the evolution of maintenance durations. These sensitivity parameters have a significant impact on the system degradation which companies need to reduce especially when the excessive degradation induces a penalty cost.

5 CONCLUSION

This paper proposed a condition-based maintenance policy under environment constraint for a production system which has two functioning states. This maintenance strategy consists on maintaining the system with respect to a minimum level of environment degradation. Inspections are performed to measure the environment degradation level. The preventive maintenance action is then planned whenever the degradation level of the environment exceeds a given threshold value. In this paper, a vector of inspection times has been determined in order to maximize either the system stationary availability or the stationary availability of the system without exceeding the alarm threshold. On the basis of Nelder-Mead sim-

plex method, a numerical procedure has been developed to solve nonlinear optimization problems. It has been shown that the second proposed model which integrates the time of excessive degradation is important to reduce the impact of the system degradation on the environment. The extensions of this work can include the imperfect information gathered from inspection as well as the imperfect maintenance actions.

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A APPENDIX

Proof of proposition 1. The average up-time $E(U_c)$ can be expressed as:

$$E(U_c) = \sum_{j=1}^{\infty} \int_{\theta_{j-1}}^{\theta_j} E(U_c |_{\tau < T \leq \tau + d\tau}) Pr \{ \tau < T \leq \tau + d\tau \}$$

$$= \sum_{j=1}^{\infty} \int_{\theta_{j-1}}^{\theta_j} E(Z) f(\tau) d\tau,$$

where $E(Z) = E(U_c |_{\tau < T \leq \tau + d\tau})$,
or $E(Z) = E(E(Z|X))$
 $= E_1 + E_2$.

E_1 corresponds to the case where the cycle ends with a corrective maintenance ($X \leq \theta_j + H - \tau$), while E_2 corresponds to the case where the cycle ends with a preventive maintenance ($X > \theta_j + H - \tau$). These two terms are obtained according to:

$$E_1 = \int_0^{\theta_j + H - \tau} (\tau + x) g(x) dx,$$

$$\text{and } E_2 = (\theta_j + H) \int_{\theta_j + H - \tau}^{\infty} g(x) dx$$

$$= (\theta_j + H) (1 - G(\theta_j + H - \tau)).$$

$$\text{Hence } E(U_c) = \sum_{j=1}^{\infty} \int_{\theta_{j-1}}^{\theta_j} \left(\int_0^{\theta_j + H - \tau} (\tau + x) g(x) dx + \int_{\theta_j + H - \tau}^{\infty} (\theta_j + H) g(x) dx \right) f(\tau) d\tau. \blacksquare$$

Proof of proposition 2. Let us assume that $\theta_{j-1} < \tau \leq \theta_j$. The probability that the cycle ends with a corrective maintenance is:

$$P_c = \sum_{j=1}^{\infty} Pr \{ T + X \leq \theta_j + H \},$$

where $Pr \{ T + X \leq \theta_j + H \} =$

$$\int_{\theta_{j-1}}^{\theta_j} (Pr \{ X \leq \theta_j + H - \tau |_{\tau < T \leq \tau + d\tau} \}) Pr \{ \tau < T \leq \tau + d\tau \},$$

with $Pr \{ X \leq \theta_j + H - \tau |_{\tau < T \leq \tau + d\tau} \} =$

$$\int_0^{\theta_j + H - \tau} g(x) dx$$

$$= G(\theta_j + H - \tau). \blacksquare$$

Proof of proposition 3. The average number $E(N)$ of inspections in the cycle can be expressed like the expected value of a discrete random variable as:

$$E(N) = \sum_{j=1}^{\infty} j p(j), \text{ where the probability to have } j$$

inspections in a cycle is given by:

$p(j) = Pr \{ N = j \}$. We know that:

$$Pr \{ N < j + 1 \} = Pr \{ N < j \} + Pr \{ N = j \},$$

so $p(j) = Pr \{ N < j + 1 \} - Pr \{ N < j \}$

$$= Pr \{ T + X \leq \theta_{j+1} \} - Pr \{ T + X \leq \theta_j \}.$$

Or $Pr \{ T + X \leq \theta_j \} =$

$$\int_0^{\theta_j} (Pr \{ X \leq \theta_j - \tau |_{\tau < T \leq \tau + d\tau} \}) Pr \{ \tau < T \leq \tau + d\tau \},$$

$$\text{thus } p(j) = \int_0^{\theta_{j+1}} G(\theta_{j+1} - \tau) f(\tau) d\tau -$$

$$\int_0^{\theta_j} G(\theta_j - \tau) f(\tau) d\tau. \blacksquare$$

Proof of proposition 4. The average time $E(T_d)$ of excessive environment degradation in a cycle can be expressed as:

$$E(T_d) = \sum_{j=1}^{\infty} \int_{\theta_{j-1}}^{\theta_j} E(T_d |_{\tau < T \leq \tau + d\tau}) Pr \{ \tau < T \leq \tau + d\tau \}$$

$$= \sum_{j=1}^{\infty} \int_{\theta_{j-1}}^{\theta_j} E(W) f(\tau) d\tau,$$

where $E(W) = E(T_d |_{\tau < T \leq \tau + d\tau})$,

or $E(W) = E(E(W|X))$

$$= E_3 + E_4.$$

E_3 corresponds to the case where the system undergoes a corrective maintenance ($X \leq \theta_j + H - \tau$), while E_4 corresponds to the case where the cycle ends with a preventive maintenance ($X > \theta_j + H - \tau$). These two terms are obtained according to:

$$E_3 = \int_0^{\theta_j + H - \tau} x g(x) dx,$$

$$\text{and } E_4 = (\theta_j + H - \tau) \int_{\theta_j + H - \tau}^{\infty} g(x) dx$$

$$= (\theta_j + H - \tau) (1 - G(\theta_j + H - \tau)).$$

$$\text{Indeed } E(T_d) = \sum_{j=1}^{\infty} \int_{\theta_{j-1}}^{\theta_j} \left(\int_0^{\theta_j + H - \tau} x g(x) dx + \int_{\theta_j + H - \tau}^{\infty} (\theta_j + H - \tau) g(x) dx \right) f(\tau) d\tau. \blacksquare$$