Heuristics and lower bound for energy management in hybrid-electric vehicles

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HEURISTICS AND LOWER BOUND FOR ENERGY MANAGEMENT IN HYBRID-ELECTRIC VEHICLES

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ABSTRACT: *In an hybrid electrical vehicle, the electrical powertrain system uses multiple energy sources among which a compromise of usage must be found to ensure that the vehicle can satisfy the power demand whilst minimizing the total energy consumption. This paper focusses on the minimization of the hydrogen consumption over a given set of constraints. Based on global optimization approaches, the heuristics proposed find solutions that best split the power required between the multi-electrical sources available. A lower bounding procedure is introduced for a better evaluation of the solutions quality. Computational results show a significant improvement over previous results from the literature in both the computing time and the solution quality.*

KEYWORDS: *Heuristics, Lower Bound, Lagrangian Optimization, Hybrid-Electric Vehicles, Energy Management*

1 INTRODUCTION

The growing interest in environment friendly hybrid vehicles during the design of new cars, has opened several research fields such as energy management for power distribution. A hybrid vehicle uses at least two energy sources for its function:(1) an internal combustion engine, a fuel cell with an hydrogen tank or a solar panel, these sources are not reversible and can only furnish a limited power quantity at a time, combined with (2) a reversible source such as a battery or super capacitor which can store braking energy and give it back later. The operation of such vehicles is restricted by several constraints depending on the energy sources chosen: fuel cells have a slow responsiveness due to the chemical reaction inside their stack and due to the air compressor; whereas the cost, durability and energy capacity of reversible sources limit their proliferation in such vehicles. Today, the storage element plays the role of a secondary energy source that supports the primary source, but it represents an intermediate step towards full electrical vehicles.

Several operational research approaches have been applied to respond to the increasing concern with power control and energy management, specifically from combinatorial or global optimization. For hybrid systems, the goal is to minimize the energy consumption. Often, various methods are implemented and compared with each other. Two different situations can arise depending on whether the profile of power demand is considered entirely known in advance or not. In the first case, the best 'offline strategy" is searched for, whereas in the second case, an "online strategy" has to be designed to allow real time optimization. This paper focusses on the first case, although the best offline algorithms, if fast enough, may be applicable in an online context. The paper is organized as follows: Section 2 specifies the problem studied, Section 3 presents an overview of classical resolution methods from the literature, Section 4 highlights the main weaknesses identified in the best known method from the literature, Section 5 describes the heuristics proposed, Section 6 introduces the lower bounding procedure and section 7 analyses the results obtained, before the conclusion.

2 PROBLEM DESCRIPTION

The hybrid electric system considered in this paper has two energy sources, as illustrated on Figure 1: a fuel cell stack and a storage element (supercapacitor). It derives from a hybrid full electric vehicle classified as *hybridization series*. The fuel cell is the main energy source used to produce electricity from hydrogen (fuel). Usually it provides the electricity needed for the traction whereas the storage element can recover the energy generated during braking, or from the fuel cell, for a later reuse.

The goal is to minimize the overall cost of hydrogen consumption for a given vehicle that follows a given profile of power demand, by optimizing the distribution of power on the two sources, in the presence of several constraints of availability, performance and state of charge of the super-capacitator.

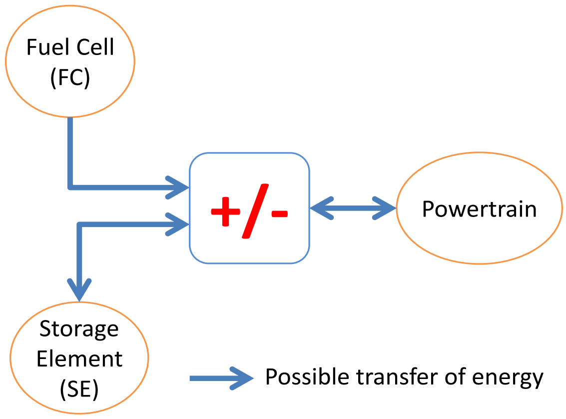


Figure 1: Series-hybrid architecture of the vehicle.

2.1 Fuel Cell System (FC)

A fuel cell (FC) is composed of several elementary cells and has several uses. In this system, it produces electricity from hydrogen and oxygen. The optimization criterion of our problem has to take into account the efficiency coefficient of the FC, which varies with the amount of power delivered. Such characteristic is given in the form of a yield curve (or efficiency curve).

2.2 Storage Element System (SE)

A storage element (SE) is needed for high accelerations of a few seconds that cannot be provided by the main source (which otherwise would be over-dimensioned). To fulfill this requirement, a super-capacitor is more suitable than a battery, because it presents a higher dynamic to quickly deliver the power during a short period of time. It also has an almost constant efficiency coefficient. Let $X(t)$ be the amount of energy contained in the storage element at instant-time t. $X_{\min}$ and $X_{\max}$ are respectively the minimum and the maximum amount of energy SE that can contain. Generally the storage element can only be used between 25% and 100% of its energy capacity. The state of charge (SOC) is defined by the following equation:

$$SOC(t) = \frac{X(t)}{X_{max}}. \tag{1}$$

If the SE supplies power to the powertrain system (discharge) then $P_{SE} > 0$ and $\dot{X}(t) < 0$. Otherwise, if SE absorbs the power recovered by the braking (recharge) then $P_{SE} < 0$ and $\dot{X}(t) > 0$. Therefore, following equation is always valid.

$$\dot{X}(t) = -P_{ES}(t). \tag{2}$$

2.3 Powertrain System

The powertrain is responsible of the traction in the hybrid vehicle. This is the system that consumes the electrical energy provided by both sources. If the power of this moto-propulsion-group is positive then the system is in traction, if it is negative then the vehicle is in braking. At the end, if the power demand is zero then the vehicle is stopped.

2.4 Power demand profile

An instance for our problem corresponds to a profile of power required (or demand) over time by a chosen powertrain to perform its mission. The profile provided by INRETS (Institut National de REcherche sur les Transports et leur Sécurité) (Figure 2) and used in this paper represents the path of electric vehicles in urban environments. The validity domain of the energy variation of the storage element is deduced from this power profile and illustrated on Figure 3 (The validity domain is all possible states of energy in the Storage Element).

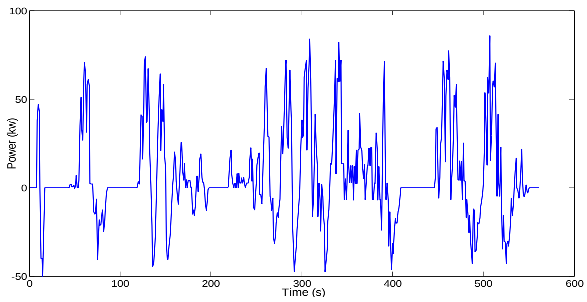


Figure 2: Power profile for urban electric vehicles.

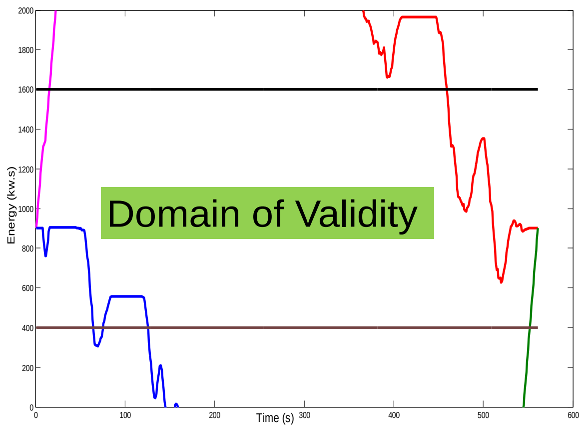


Figure 3: Validity domain of the storage element.

2.5 System characteristics

The design of the power sources is outside the scope of this paper. The optimization problem considered here is done on a pre-defined system whose characteristics are summarized in Table 1.

Value	Description
$P_{SEmin} = -60kW$	min power extractible from SE
$P_{SEmax} = 60kW$	max power extractible from SE
$E_{smin} = 400kW.s$	min energy storable in SE
$E_{smax} = 1600kW.s$	max energy storable in SE
$E_{sinit} = 900kW.s$	initial energy in SE
$P_{FCmax} = 70kW.s$	max power deliverable by FC

Table 1: Characteristics of the storage element.

3 Literature review

The general formulation of the problem of energy management can be represented in the form of dynamic equation with equality and inequality constraints. In this case, the objective function considered is non-linear with nonlinear constraints.

Dynamic equation:

$$\dot{x}(t) = f(x(t), u(t), t), \qquad x(t_0) = x_0. \quad (3)$$

Cost criterion to be minimized:

$$\int_{t0}^{tf} \gamma(x(t), u(t), t)dt. \quad (4)$$

Constraints:

$$\begin{cases} \chi(x(t), u(t), t) = 0, \\ \varphi(x(t), u(t), t) \leq 0, \end{cases} \quad (5)$$

where $x(t)$, $u(t)$ and γ are respectively the state, control variables and the representative of the global efficiency (which is not equal to the sum of the source characteristics).

Some solving approaches focus on defining the evolution of the state of charge variation (SOC) that leads to minimize the cost of hydrogen consumption, and then use these SOC values to deduce the fuel cell powers. It is the case of dynamic programming, classically applied once the problem has been discretized as explained in Subsection 3.1. Other approaches try to directly compute powers delivered by the fuel cell and then use the values obtained to deduce the SOC. The remainder of this section presents an overview of the latests solving methods from the literature that have been applied on energy management problems for electric power systems.

3.1 Dynamic programming

Dynamic programming (DP) is a well known method, widely used to solve a variety of optimization problems. For a detailed description see for example the book of Bertsekas (2011). In papers from the energy management literature, DP provided the best results, sometimes after an additional discretization of the data (Yuan-Bin Yu *et al.*, 2009). For our problem, any given profile of the power demand is already discretized in function of time ("horizontally"). Hankache (2008) proposed a DP algorithm, summarized in Algorithm 1. It requires an additional discretization of the power levels ("vertically"), where: $Cost(j, i)$ is the optimal cost to go from the state of energy j at time i to the final state at time N whereas E_i is the vector of all feasible (in the validity domain) states of charge at instant i. $Sequence(j, i)$ is the set of commands $u(t)$ related to the cost $Cost(j, i)$. In this case, if $Cost'(i, j, p)$ is the cost of moving from state j at time i to state p at time $i + 1$, then recurrence relation is given as follows:

$$\text{Cost(j,i)= Cost(p,i+1) + Cost'(i,j,p).} \quad (6)$$

Algorithm 1 DP Principle

```
for j ∈ E_{N−1} do
    Compute Cost(j, N−1) ; Sequence(j, N−1)
end for
for i = N − 1 to 1 do
    for j ∈ E_i do
        for k ∈ E_{i+1} do
            Compute Cost(j, i) ; Sequence(j, i)
        end for
    end for
end for
return 2 Matrix: Cost and Sequence
```

3.2 Equivalent Consumption Minimization Strategy (ECMS)

This is a control strategy based on the minimization of the equivalent consumption. Briefly, a cost of solicitation of the storage element (SE) is defined by equating SE to a secondary fuel tank. In this case, any discharge of SE becomes equivalent to some energy consumption (positive fuel flow), and any recharging becomes equivalent to some energy refill (negative fuel flow). Therefore the resulting strategy is to minimize the total equivalent consumption (Paganelli, 2002). The resulting objective is given by:

$$\min(Flow_{fuel/hydrogen} + Flow_{equivfromSE}). \quad (7)$$

ECMS uses the rules of the optimal control to solve the resulting problem: the Hamiltonian function is applied to find a minimum.

3.3 Optimal Control

This theory is in general used in conjunction with the Bellman principle and/or the Pontryagin maximum principle, classified as efficient tools for solving global optimization problems. Consider O an open interval of $\mathbb{R}^n$. To solve the optimal control problem we have to introduce the Hamiltonian of equations (3)-(5). The Hamiltonian equation is expressed with equation (8).

$$H(x(t), u(t), \lambda(t)) = \gamma(x(t), u(t)) + \lambda^T(t) f(x(t), u(t)) \tag{8}$$

The principle is to search the trajectory of the optimal control $u^*(t)$ associated to the optimal solution $x^*(t)$. The variation of the trajectory of the command u remains in the validity domain of the state of charge, see domains (9). The control u defines the energy distribution of the SE usually included in the "validity domain". But $\lambda(t)$ is not easy to obtain when constraints are added.

$$\begin{array}{ll} f : I \times V \times U \longrightarrow \mathbb{R}^n, & \text{such as} \\ I \in \mathbb{R}, V \in \mathbb{R}^n, U \in \mathbb{R}^n, (t_0, x_0) \in I \times V & \end{array} \tag{9}$$

3.4 Fuzzy logic

Among the tools from the artificial intelligence field, the most used for energy optimization is the fuzzy logic. Zadeh (1965) 's method is essentially based on the mathematical theory of fuzzy sets. This approach was explained and used by Caux et al (2010). It appeared especially in the translation of linguistic rules to states other than boolean. Thus, it allows a combination of multiple inputs. Linguistics rules are defined using the membership functions that will take values between 0 and 1. Therefore we define classes of membership; more we get closer to a class given more the membership is strong (see Figure 4).

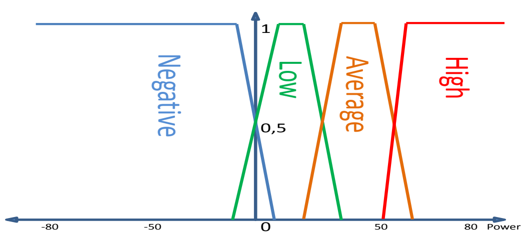


Figure 4: Membership function of the required power.

4 Main weaknesses identified in DP

4.1 Recovery of energy braking

It is obvious that recovering energy from braking for a later reuse can help minimizing the cost of a mission. However, contrary to the common assumption, imposing that all braking energy has to be recovered may lead to worse solutions, especially if applied in dynamic programming. Figure 5 illustrates an example, on a 15 seconds profile, where P_r is power requested (or power demand), P_{SE} and P_{FC} are the power provided by respectively SE and FC.

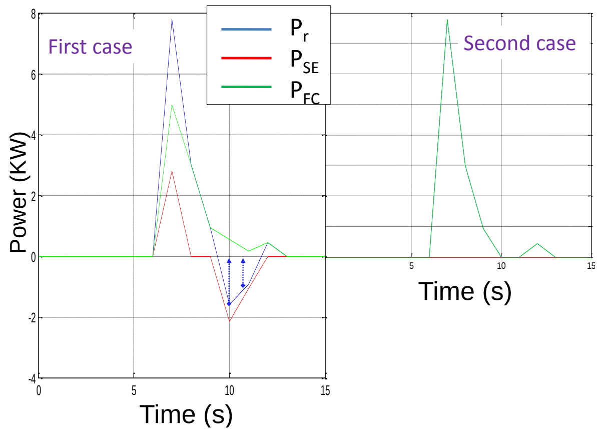


Figure 5: Contrary example where all braking energy should not be recovered.

In the first case, when retrieving braking energy, the cost of the mission is equal to 56.45 kWs. In the second case, no braking energy is recovered and the cost is reduced to 47 kWs. Unrecovered braking energy can be simply dissipated as heat. Another counter-example can be generated by considering a vehicle descending a downhill path. In this case, the amount of braking energy is greater than the amount of traction, therefore there is no feasible solution that allows to recover all of the braking energy. As a consequence, the equality constraint $P = P_{FC} + P_{SE}$. should be replaced with the inequality $P \leq P_{FC} + P_{SE}$. This means that during braking, the recovery of all or part of the energy generated is authorized but not mandatory.

4.2 Final state of charge of the SE

To facilitate comparisons between different algorithms Hankache (2008) proposed to impose an equality between the initial (at instant-time t_0) and the final (at instant-time t_f) states of charge of the SE. However, for some instances, this means that the vehicle had to "burn" excess energy towards the end of the profile to return the energy level to its initial state. As a solution, we propose to forbid the driver to return the SE at the end with less energy than the quantity he had at the beginning of the profile, but to allow the final energy level to be higher than at start (but with no special reward for the additional energy). In a DP algorithm, this corresponds to open the validity domain and to allow the final state of the storage element to vary in an interval, instead of being reduced to a single point (see for example Figure 6).

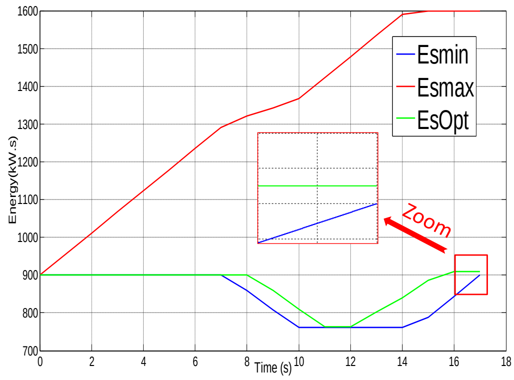


Figure 6: Excess energy at the end (final charge state greater than the starting state of charge).

4.3 Penalties due to SE discretization

Figure 7 illustrates an example where, although the braking energy is greater than the traction energy which should lead to a cost of zero, DP produces a solution which uses the FC (and therefore has a cost strictly positive). Basically, since the power levels have been discretized, the system tends to respond too strongly to demands that are not exact multiples of the discretization step. This flaw has been handled when designing our new heuristics.

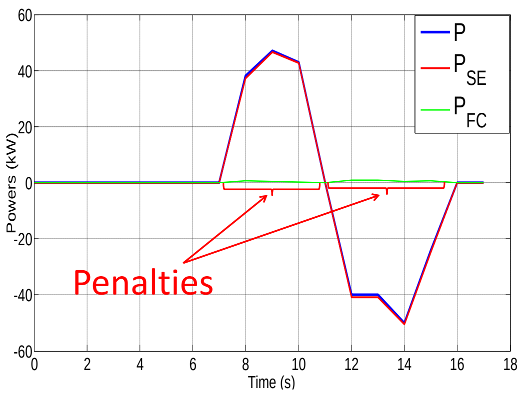


Figure 7: Penalties during the optimization

To summarize, although dynamic programming produces good results in the literature, even better than the fuzzy logic and optimal control, there is an increasing need for a new and more efficient approach to find better solutions in terms of total cost as well as computing time.

5 New Heuristic Methods

5.1 Heuristic 1

This heuristic has a simple principle, inspired from electrical filters: filtering is applied on the required power to determine when power should be delivered by the fuel cell. It requires two parameters B_l and B_u called respectively lower and upper bands:

- B_l, called the lower band, is chosen between 0 and the maximum power FC can provide.
- B_u, called the upper band, must be strictly higher than B_l and less than the maximum power the FC can provide.

The principle of this heuristic is illustrated on Figure 8, and summarized with Equations (10) and (11). Notice that SE is never charged by FC, and the validity domain must be open.

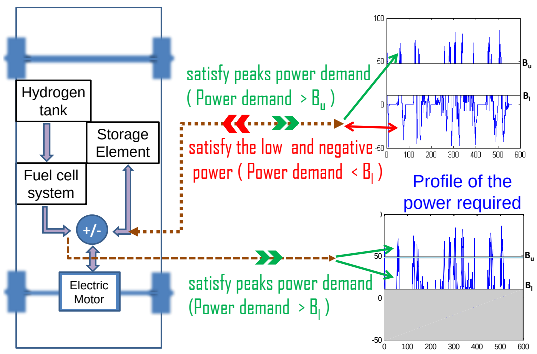


Figure 8: Principle of Heuristic 1.

$$P_{rmin} < P_r < P_{rmax}. \tag{10}$$

$$\begin{cases} \text{if} \quad P_r \leq B_l & \Longrightarrow \quad P_r = P_{SE}. \\ \text{if} \quad B_l \leq P_r \leq B_u & \Longrightarrow \quad P_r = P_{FC}. \\ \text{if} \quad P_r \geq B_u & \Longrightarrow \quad P_r = P_{SE} + P_{FC}. \end{cases} \tag{11}$$

Figure 9 shows the selection of bands and the proportionality between the power of the fuel cell and the power of the powertrain.

This approach has several advantages:

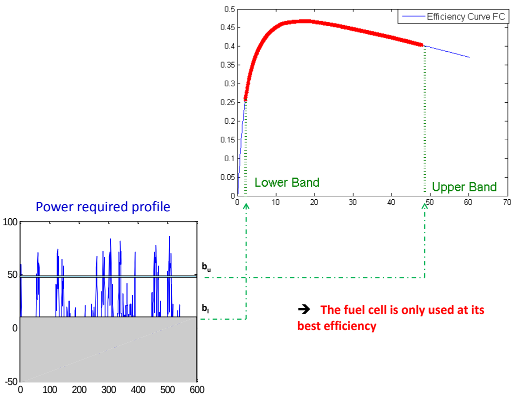


Figure 9: How to choose the upper and lower bands depending on the efficiency curve of the Fuel Cell.

- The computation time is very short (less than 10 seconds for a sample of 6000 time steps), which makes it possible to run several tests and fine-tune the parameter settings.

- A simple modification of the interval $[B_l, B_u]$ ensures that the desire final state of energy is reached.

- The best solution reachable with such a strategy is attained after a few iterations with visual verification that the variation of the state of charge is still in the domain of validity.

This heuristic uses the same principle as the method called 'Thermostat' where the power required is filtered by looking at the energy storage elements and not at the performance of the fuel cell as in our heuristic.

Recall that, the SE turns on when the power demand is outside the chosen bands and turns off otherwise, whereas the FC turns on when the power demand is between the chosen bands (which usually corresponds to high FC efficiency coefficients).

5.2 Heuristic 2

In most scientific papers, as in Hankache (2008), the application of dynamic programming is done after sampling the time (horizontally) and the state of charge of the SE (vertically). It has several drawbacks, as described in Section 4. Heuristic 2, instead of sampling the variation area of the energy storage element, applies dynamic programming on the variation of the power of the fuel cell. It computes the sum of local minimum of power to avoid losses due to the powers correction (see penalties illustrated on Figure 7) .

5.3 Heuristic 3

During the DP from Hankache (2008), the storage element is required to collect all the braking energy generated. As explained in Section 4, this may lead to additional consumption and thus increased cost of the mission. The goal of Heuristic 3 is to get only the amount of energy that would not deteriorate the quality of the final solution. As a consequence, the following constraint is relaxed for all instant times $i \in I$:

$$\begin{array}{lll} \text{Basic equation} & P_r(i) & = P_{FC}(i) + P_{SE}(i), \\ & & \Downarrow \\ \text{Improved equation} & P_r(i) & = P_{FC}(i) + P_{SE}(i) + \varepsilon(i). \end{array} \tag{12}$$

The idea is to get less energy from braking and provide more than the required energy if it leads to a better point of efficiency curve (thus a better solution). The excess electrical energy can be dissipated as heat (in a resistance inserted in the braking system).

5.4 Heuristic 4: Using Quasi-Newton method

We formulated our problem as a nonlinear problem with nonlinear constraint as follows:

- Decision variables

  $\circledast\, x_i$ = power (instantaneous) generated by the fuel cell (FC) at time i, $x_i \geq 0$,
  $\circledast\, y_i$ = power (instantaneous) generated ($y_i \geq 0$) or stored ($y_i \leq 0$) by the (SE) at time i,
  $\circledast\, SOC_i$ = State Of Charge at time i.

- Bounds

$$\begin{array}{l} x_{imin} \leq x_i \leq x_{imax}, \\ y_{imin} \leq y_i \leq y_{imax}. \end{array} \tag{13}$$

- Objective-function: Minimize the total energy generated by the fuel cell

$$\min F(X) = \min \textstyle\sum_{i=1}^{n} f(x_i), \tag{14}$$

  $\circledast$ *definition of the function* $f(x_i)$

$$f(x_i) = \frac{x_i}{\rho(x_i)}, \tag{15}$$

  where ρ is the efficiency function of the fuel cell. Since the data are obtained experimentally, there are two ways to represent this function: as a linear per piece function or as a polynomial function which is generated by interpolation using the experimental curve (we use here *polyfit* of Matlab).

- Constraints

  ⊛ *Reach the power demand at each instant time*

$$\forall i \in [1, ..., n], \;\; x_i + y_i \geq P_r(i), \tag{16}$$

(Recall that using $x_i + y_i = P_r(i)$ would force all energy generated during braking to be recovered by the SE, which can be sub-optimal.)

⊛ *Emptying the storage element at the end:*

$$\sum_{i=1}^{n}[y_i + \tilde{\rho}(y_i)] = 0. \tag{17}$$

where $\tilde{\rho}(y_i)$ is the efficiency function of the SE. With this condition, the storage element ends with a state of charge equal to the one it began with. In this case, it rejects the amount of energy recovered from braking. If the energy level is not imposed to the SE at the end of the profile, then the equation becomes:

$$\sum_{i=1}^{n}[y_i + \tilde{\rho}(y_i)] \geq 0. \tag{18}$$

⊛ *Bounds of variation of the state of charge:*

The state of charge can vary between 25% and 100% of the maximum energy of the SE:

$$\begin{array}{c} 25\%\, SOC_{max} \leq SOC_{initial} - \sum_{k=1}^{i}[y_k + \tilde{\rho}(y_k)] \\ SOC_{initial} - \sum_{k=1}^{i}[y_k + \tilde{\rho}(y_k)] \leq 100\%\, SOC_{max}. \end{array} \tag{19}$$

Feasible solutions for our problem can be found by applying a non linear solver on this formulation. Heuristic 4 combines the nonlinear solver 'fmincon' (it performs subgradient optimization taking into account nonlinear constraints) with random multi-start or from a specific starting point (given in the form of a vector). The difficulty is to find a good starting point that may lead to an optimal solution in a short computing time. Because of the use of Quasi-Newton optimization, the starting point is not required to be a feasible solution.

6 Lower bounding procedure

An upper bound of the consumption can be obtained by using only the fuel cell to satisfy all positive power requested. All solutions cost can be compared to this upper bound, but to have more certainties on the quality of solutions, it is better to evaluate their gap with the optimal solution. Since the optimal solution is yet unknown, a good lower bound is required.

For our problem, such lower bound is computed by assuming ideal conditions: that the fuel cell efficiency remains at its maximum value. Assuming that $\rho(x_i) = \alpha$ (constant) in Equation (15), the objective function becomes the linear following function:

$$f(x_i) = \frac{x_i}{\alpha}. \tag{20}$$

The new function is convex and the global optimum of the resulting problem can be found easily (by a local search algorithm), which is a lower bound of the original problem. The proof of convexity of the objective function is the following:

Let be $(\lambda^1, \lambda^2, ..., \lambda^n,) \in [0;1]^n$ *where* $\sum_{i=1}^{n} \lambda^i = 1$ and f is defined by Equation (20).

$$\begin{aligned} f(\textstyle\sum_{i=1}^{n} \lambda^i x_i) &= \frac{\sum_{i=1}^{n} \lambda^i x_i}{\alpha} \\ &= \sum_{i=1}^{n} \frac{\lambda^i x_i}{\alpha} \\ &= \sum_{i=1}^{n} \lambda^i \left(\frac{x_i}{\alpha}\right) \\ &= \sum_{i=1}^{n} \lambda^i f(x_i). \end{aligned} \tag{21}$$

$\Longrightarrow$ According to Jensen (1906)'s inequality, applied to Equation (21), our objective-function is convex.

7 Computational analysis

7.1 Instance, Parameters Settings and Table headings

The instance solved is the profile provided by INRETS (see figure 2) which represents the path of electric vehicles in urban environments, obtained after experimentation. All heuristics have been programmed with MATLAB 7.9(R2009b) on a desktop computer Intel Pentium 4 with 3 GB of RAM. Heuristic 4 and the lower bound use the non-linear solver 'fmincon' from the Optimization Toolbox. In both cases, the efficiency curve can be represented in the form of a linear interpolation or a polynomial representation.

Table 2 summarizes the results of the DP of Hankache (2008) and our heuristics with the following headings:

- DP: Dynamic Programming from the literature.
- H1, H2, H3, H4: refer to the heuristics proposed.
- Cost (kWs): solution cost = total hydrogen consumption in (kWs).
- ET : Execution Time = CPU time.
- SH: amount of hydrogen stored in the storage element in the form of electricity (kWs).
- GLB: Gap to lower-bound $= 100 * \frac{\text{Solution Cost} - \text{Lower Bound}}{\text{Lower Bound}}$.

- HL = Hybridization Level (HL). It is a classification criterion for hybrid vehicles. It corresponds to the percentage of participation of the storage element in the traction power train (positive power required) [see among others (Bolvashenkov,2006),(Buecherl,2009), (Lukic,2004)].

$$HL = \frac{\text{Total traction power of Electric element}}{\text{Sum of all traction power sources}} = \frac{\text{Total traction power of Storage element}}{\text{Total traction power of (SE ,FC)}} = \frac{\text{Total traction power of Storage element}}{\text{Total traction of power required}}.$$

	DP	H 1	H 2	H 3	H4
Cost	10131	8869	8797	9000	8750
ET	22 h	5 s	5 min	20 h	23 min
SH	1115	5	34	335	163
GLB	14.65 %	2.5 %	1.71 %	3.92 %	1.18 %
HL	51.67 %	36.6 %	37 %	39.94 %	38.75 %

Table 2: Result of our heuristics vs literature (DP).

7.2 Results summary

The maximum efficiency of the fuel cell is 0.4668 (as shown on the efficiency curve on Figure 9) and the lower bound of the total consumption cost, on the INRETS profile was computed as explained in Section 6. The value obtained is 8647 KWs.

Table 2 shows that our heuristics outperform the previous DP from the literature, both in computing time and solution cost. It also shows that our best known solution is now proven to be less than 2% far from the optimal solution.

One advantage of using various methods is to acquire a good insight on the management of power in the powertrain and have an order of magnitude when comparing the different strategies. Figure 10 shows the comparison between the different methods used to solve the problem (Time is represented in a logarithmic scale).

Figure 11 shows the state of charge variation in the storage element for the solution given by each method. DP has the longest path thus more energy loss, but other methods have a less energy exchanged.

7.3 Focus on Heuristic 4: Quasi-Newton method from starting point

- Version 1: multi-start technique

A starting point is in fact a vector of dimension n where n is the number of considered instant-times. In this version of Heuristic 4, 200 random starting points were generated to try to scan various parts of the solution space. The subgradient optimization is

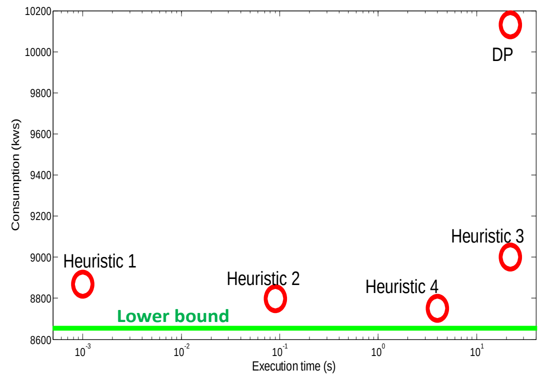


Figure 10: Comparison of different methods

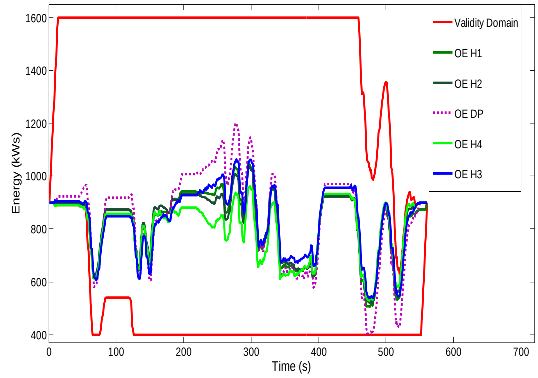


Figure 11: Energy variation in the storage element

launched from each of these starting points. Results obtained showed that most solution costs converged to a value around 9600kWs ± 50kWs. However, with this strategy, it is possible after several attempts to find an interesting local minimum, in our case the best solution was equal to 8800kWs. Nonetheless, using this strategy, the state of charge of the storage element is under great fluctuations, which, as illustrated on Figure 11, tends to be detrimental to the solution quality.

To avoid that, we modified the random starting point generation as follows: generate each initial vector by picking random values between -70kW and 70kW, but before the introduction of this vector in the solver 'fmincon', cancel (reset to 0) all values from it that are less than a predefined "decision value". The results of this strategy are presented on Figure 12.

- Version 2: specific starting point

The idea is to start the optimization from the vector for which each value corresponds exactly to the power required (demand profile). The result obtained is given in Figure 13. After 48 iterations in 23 minutes, the cost of consumption obtained is 8750 kWs. Figure 13 shows that the algorithm takes the starting point equal to the power required, but modifies

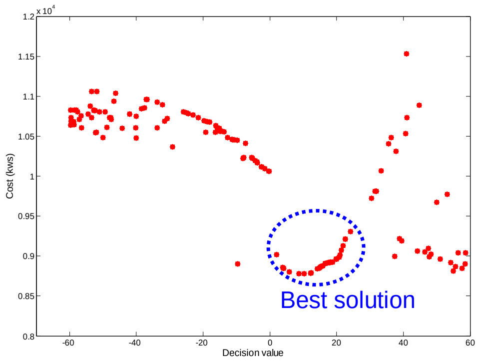


Figure 12: Impact on the solution cost of the cancellation of values from the initial solution

the power a little in order to find another interesting minimum. A small variation from the starting point can lead to a large change in the energy profile but not in cost of consumption. To confirm that, 35 starting points with random variations from the demand profile have then been tested.

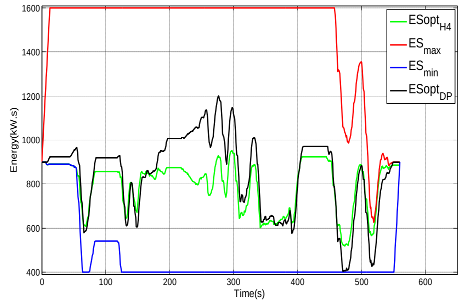
Figure 14(a) shows the surface displacement of the state of charge for these tests. This figure hints that it may be possible to do a Taboo search from any starting point to follow the path towards the end of the journey. Figure 14(b) shows the variation on the consumption cost for each variation of the 35 starting points.

8 Conclusion

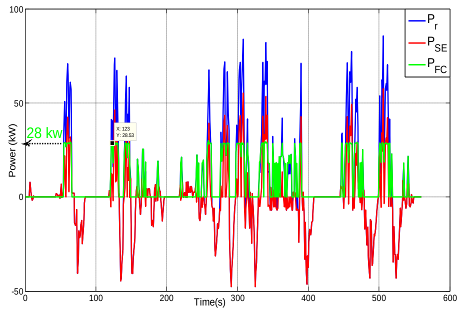
Hybrid vehicles offer the possibility to recover the energy generated during braking and use it to minimize fuel consumption and subsequently to minimize the emission of "greenhouse" gas emissions. In this paper, we developed several global optimization heuristics for the distribution of powers between the fuel cell and the storage element of an hybrid electric vehicle. The objective is to minimize the cost of fuel consumption whilst satisfying the power demand.

We conclude that the application of the Quasi-Newton method through the 'fmincon' function associated with a specific starting point seems to give us the best results, less than 2% far from our newly computed lower bound. However, if time is an issue our other heuristics can produce very good solutions (less than 3 % far from the lower bound and thus from the optimal solution) in a few seconds.

To summarize, the efficiency of the new approaches and resulting heuristics is proven, in comparison to traditional methods such as dynamic programming. Also, the novelty in this work compared to previous research is that the results are compared with lower bounds of consumption and not only to any other

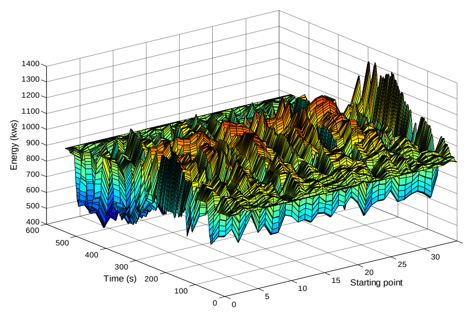


(a) Variation of the optimal energy in the validity domain

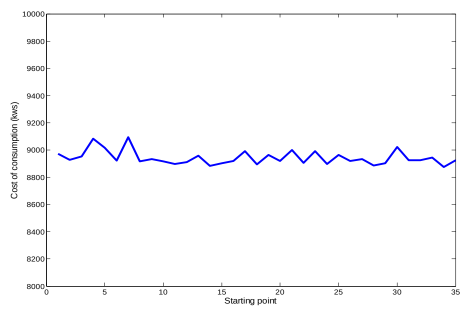


(b) Variation of the splitted powers

Figure 13: Quasi-Newton method results



(a) Energy surface



(b) Variation of the cost

Figure 14: Study of a small variation of a starting point.

upper bounds as it usually done in electric energy management research. For this reason, we can guarantee that the cost of our best solution obtained on the data set provided by INRETS profile is less than 2 % far from the optimal global solution.

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