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Abstract. The interacting boson model (IBM) of Arima and Iachello is applied to calculate nuclear charge radii and electric monopole transitions of even-even nuclei in the rare-earth region. Consistent operators are used for the two observables. A relation between summed M1 strength and $\rho(\mathrm{E0})$ values is pointed out.

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1. INTRODUCTION

According to the geometric model of Bohr and Mottelson [1], a nucleus with an elipsoidal equilibrium shape may undergo oscillations of two different types, β and γ. While γ vibrations are an acknowledged feature of deformed nuclei, such is not the case for the β kind. A careful analysis of the observed properties of excited 0^+ states seems to indicate that very few indeed satisfy all criteria proper to a β-vibrational state [2]. In particular, very few decay to the ground state by way of an E0 transition of sizable strength [3], as should be for a β vibration [4]. It is therefore not surprising that alternative interpretations of excited 0^+ states in deformed nuclei, either as pairing isomers (see, *e.g.*, Refs. [5, 6]) or through shape coexistence and configuration mixing (see, *e.g.*, Ref. [7]) have gained advocates over recent years.

In this contribution it is shown that a collective interpretation of the ground and excited 0^+ states with the interacting boson model (IBM) of Arima and Iachello [8] can account for the data on charge radii and E0 transitions, as observed in the rare-earth region. Full details of this work can be found in Refs. [9, 10]. In addition, a correlation with summed M1 strength is pointed out.

2. CHARGE RADII AND ELECTRIC MONOPOLE TRANSITIONS

The probability for an E0 transition to occur between an initial state $|i\rangle$ and a final state $|f\rangle$ can be written as the product of an electronic factor Ω and the square of a nuclear factor $\rho(\mathrm{E0})$, the latter being equal to

$$\rho(\mathrm{E0};i\rightarrow f)=\frac{|\langle f|\hat{T}(\mathrm{E0})|i\rangle|}{eR^2}, \tag{1}$$

with $R = r_0 A^{1/3}$. In first approximation the E0 matrix element equals

$$\langle f|\hat{T}_{\rm p}({\rm E0})|i\rangle = \langle f|e\sum_{k\in{\rm p}} r_k^2|i\rangle. \tag{2}$$

where the sum is over all protons in the nucleus [11]. In the nuclei considered here not all protons can be treated explicitly and recourse should be taken to effective charges $e_{\rm n}$ and $e_{\rm p}$ for the neutrons (n) and protons (p), leading to the generalized expression [12]

$$\langle f|\hat{T}({\rm E0})|i\rangle = \langle f|\sum_{k=1}^{A} e_k r_k^2|i\rangle = \langle f|\left(e_{\rm n}\sum_{k\in{\rm n}} r_k^2 + e_{\rm p}\sum_{k\in{\rm p}} r_k^2\right)|i\rangle. \tag{3}$$

On the other hand, the mean-square charge radius of a state $|s\rangle$ is given by

$$\langle s|\hat{T}_{\rm p}(r^2)|s\rangle = \frac{1}{Z}\langle s|\sum_{k\in{\rm p}} r_k^2|s\rangle. \tag{4}$$

This is an appropriate expression insofar that a realistic A-body wave function is used for the state $|s\rangle$. This is often impossible and an effective charge radius operator $\hat{T}(r^2)$ should then be taken. The generalization of the expression (4), similar to the one carried out for the E0 operator, can therefore be written as

$$\langle r^2\rangle_s \equiv \langle s|\hat{T}(r^2)|s\rangle = \frac{1}{e_{\rm n}N + e_{\rm p}Z}\langle s|\sum_{k=1}^{A} e_k r_k^2|s\rangle. \tag{5}$$

The basic hypothesis of the present study is to assume that *the effective nucleon charges in the charge radius and E0 transition operators are the same*. If this is so, comparison of Eqs. (3) and (5) leads to the operator relation

$$\hat{T}({\rm E0}) = (e_{\rm n}N + e_{\rm p}Z)\hat{T}(r^2). \tag{6}$$

This is a general relation between the effective operators to be used for the calculation of charge radii and E0 transitions. Equation (6) can, in principle, be tested in the framework of any model and here the implied correlation is tested in the framework of the IBM [8]. This requires that all states involved [*i.e.*, $|i\rangle$ and $|f\rangle$ in Eq. (1), and $|s\rangle$ in Eq. (5)] are collective in character and can be described by the IBM.

In the IBM-1 the charge radius operator is taken as the most general scalar expression, linear in the generators of U(6) [13],

$$\hat{T}(r^2) = \langle r^2\rangle_{\rm c} + \alpha N_{\rm b} + \eta\frac{\hat{n}_d}{N_{\rm b}}, \tag{7}$$

where $N_{\rm b}$ is the total boson number, $\hat{n}_d$ is the d-boson number operator, and α and η are parameters with units of length^2. The first term in Eq. (7), $\langle r^2\rangle_{\rm c}$, is the square of the charge radius of the core nucleus. The second term accounts for the (locally linear) increase in the charge radius due to the addition of two nucleons (*i.e.*, neutrons since

isotope shifts are considered in this study). The third term in Eq. (7) stands for the contribution to the charge radius due to deformation. It is identical to the one given in Ref. [13] but for the factor $1/N_\mathrm{b}$. This factor is included here because it is the *fraction* $\langle \hat{n}_d \rangle / N_\mathrm{b}$ which is a measure of the quadrupole deformation (β_2^2 in the geometric collective model) rather than the matrix element $\langle \hat{n}_d \rangle$ itself.

Two quantities can be derived from charge radii, namely isotope and isomer shifts. The former measure the difference in charge radius of neighboring isotopes. For the difference between even-even isotopes one finds from Eq. (7)

$$\Delta\langle r^2 \rangle \equiv \langle r^2 \rangle_{0_1^+}^{(A+2)} - \langle r^2 \rangle_{0_1^+}^{(A)} = |\alpha| + \eta \left(\langle \frac{\hat{n}_d}{N_\mathrm{b}} \rangle_{0_1^+}^{(A+2)} - \langle \frac{\hat{n}_d}{N_\mathrm{b}} \rangle_{0_1^+}^{(A)} \right). \tag{8}$$

Isomer shifts are a measure of the difference in charge radius between an excited (here the 2_1^+) state and the ground state, and are given by

$$\delta\langle r^2 \rangle \equiv \langle r^2 \rangle_{2_1^+}^{(A)} - \langle r^2 \rangle_{0_1^+}^{(A)} = \frac{\eta}{N_\mathrm{b}} \left(\langle \hat{n}_d \rangle_{2_1^+}^{(A)} - \langle \hat{n}_d \rangle_{0_1^+}^{(A)} \right). \tag{9}$$

Once the form of the charge radius operator is determined, the E0 transition operator follows from Eq. (6). In the IBM-1 the E0 transition operator is therefore

$$\hat{T}(\mathrm{E0}) = (e_\mathrm{n} N + e_\mathrm{p} Z) \eta \frac{\hat{n}_d}{N_\mathrm{b}}. \tag{10}$$

Since for E0 transitions the initial and final states are different, neither the constant $\langle r^2 \rangle_\mathrm{c}$ nor αN_b in Eq. (7) contribute to the transition, and its $\rho(\mathrm{E0})$ value equals

$$\rho(\mathrm{E0}; i \to f) = \frac{e_\mathrm{n} N + e_\mathrm{p} Z}{eR^2} \frac{\eta}{N_\mathrm{b}} |\langle f | \hat{n}_d | i \rangle|, \tag{11}$$

3. APPLICATION TO THE RARE-EARTH REGION

To test the relation between charge radii and E0 transitions, proposed in the previous section, a systematic study of all even-even isotopic chains from Ce ($Z = 58$) to W ($Z = 74$) is carried out. This analysis requires the knowledge of structural information concerning the ground and excited states which is obtained by adjusting an IBM-1 Hamiltonian to observed spectra in the rare-earth region. The details of the energy calculation can be found in Ref. [10]. The procedure is closely related to the one followed by García-Ramos *et al.* [14] and yields a root-mean square deviation for an entire isotopic chain which is typically of the order of 100 keV.

Isotope shifts $\Delta\langle r^2 \rangle$, according to Eq. (8), depend on the parameters $|\alpha|$ and η in the IBM-1 operator (7). The parameter $|\alpha|$ is adjusted for each isotope series separately, while η is kept constant for all isotopes, $\eta = 0.50$ fm^2. The resulting isotope shifts are shown in Fig. 1. The peaks in the isotope shifts are well reproduced in all isotopic chains with the exception of Yb. The largest peaks occur for $^{152-150}$Sm, $^{154-152}$Gd, and $^{156-154}$Dy, that is, for the difference in radii between $N = 90$ and $N = 88$ isotopes. The

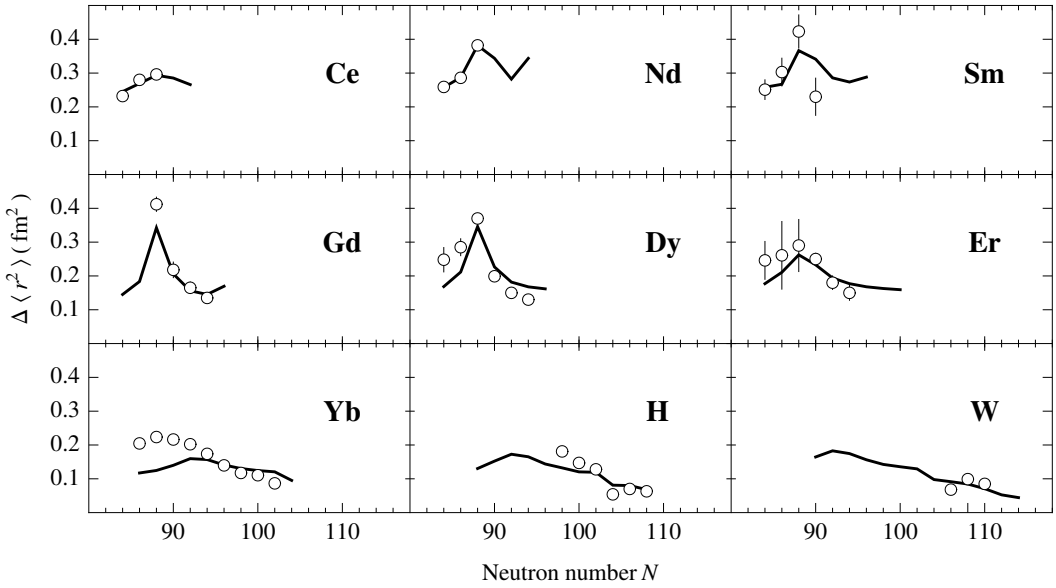

FIGURE 1. Isotope shifts $\Delta\langle r^2\rangle$ in the rare-earth region. The points are the experimental values and the lines are calculated with Eq. (8). References to experiments are given in Ref. [10].

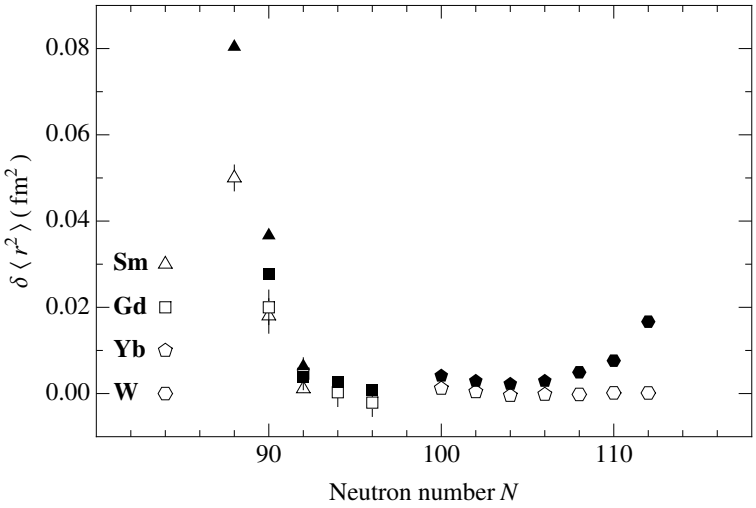

FIGURE 2. Isomer shifts $\delta\langle r^2\rangle$ in the rare-earth region. The open symbols are the experimental values and the full symbols are calculated with Eq. (9). References to experiments are given in Ref. [10].

peak is smaller below $Z = 62$ for Ce and Nd, and fades away above $Z = 66$ for Er, Yb, Hf, and W. The calculated isotope shifts broadly agree with these observed features but there are differences though. Notably, the calculated peak in the Sm isotopes is much broader than the observed one, indicating that the spherical-to-deformed transition occurs faster in reality than it does in the IBM-1 calculation.

A further test of the calculated charge radii is obtained from isomer shifts $\delta\langle r^2\rangle$,

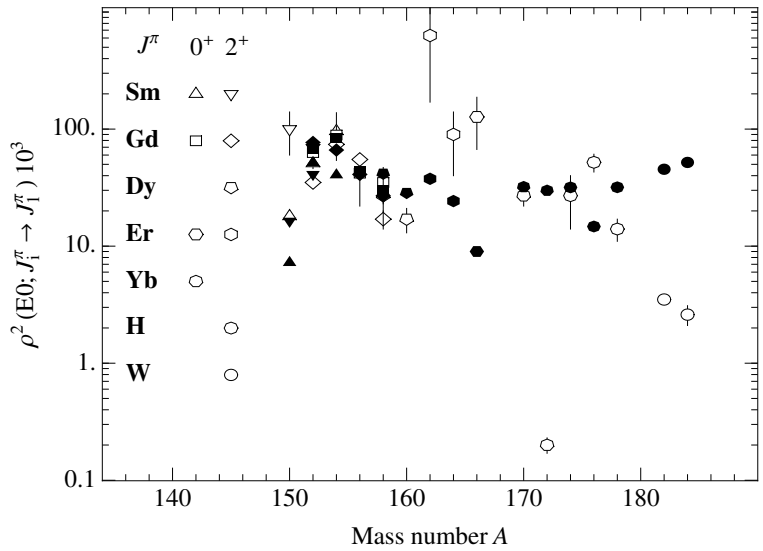

FIGURE 3. Electric monopole $\rho^2(\mathrm{E0}; J_i^\pi \rightarrow J_1^\pi)$ values (times 10^3) in the rare-earth region for $J^\pi = 0^+$ and 2^+. The open symbols are the experimental values and the full symbols are calculated with Eq. (11). References to experiments are given in Ref. [10].

depending only on η [see Eq. (9)]. The isomer shifts known experimentally are shown in Fig. 2. The data are more than 30 years old and often discrepant, in particular in the Yb and W isotopes. Nevertheless, a clear conclusion can be drawn from the isomer shifts measured in the Sm and Gd isotopes: they are an order of magnitude smaller in the deformed than they are in the spherical region.

The question is now whether the value $\eta = 0.50$ fm^2, obtained from isotope and isomer shifts, reproduces the E0 transitions observed in the rare-earth nuclei. In Fig. 3 the available E0 data for $0_i^+ \rightarrow 0_1^+$ and $2_i^+ \rightarrow 2_1^+$ transitions in the rare-earth region are compared with the results of the IBM-1 calculation. It is not the intention to give a detailed comparison for all transitions (which can be found in Ref. [10]) but the overall impression is that the present approach succeeds in reproducing the correct order of magnitude for $\rho^2(\mathrm{E0})$. However, some discrepancies can be observed in heavier nuclei and especially concern ^{172}Yb and $^{182-184}$W. A possible explanation is that the $\rho^2(\mathrm{E0})$ measured for these nuclei is not associated with collective states. This seems to be the case in ^{172}Yb where several $\rho^2(\mathrm{E0})$ have been measured, none of which is large. Only in the W isotopes does it seem that the observed E0 strength is an order of magnitude smaller than the calculated value. It is known that these nuclei are in a region of hexadecapole deformation [15] and this may offer a qualitative explanation of the suppression of the E0 strength [10].

4. RELATION TO SUMMED M1 STRENGTH

After the presentation of this work during the conference, J.N. Ginocchio asked a question concerning a possible correlation with summed M1 strength to the scissors

mode. The argument goes as follows. It is known from the work of Ginocchio [16] that the summed M1 strength from the ground state to the scissors mode (for a review on the latter, see Ref. [17]) is related to the ground-state matrix element of the d-boson number operator $\hat{n}_d$,

$$\sum_f B(\mathrm{M1}; 0_1^+ \to 1_f^+) = \frac{9}{8\pi}(g_\nu - g_\pi)^2 \frac{n\,z}{N_\mathrm{b}(N_\mathrm{b}-1)} \langle 0_1^+ | \hat{n}_d | 0_1^+ \rangle, \tag{12}$$

where n (z) is the number of valence neutron (proton) particles or holes, whichever is smaller, and g_ν and g_π are the g-factors of the neutron and proton bosons. Note that $n+z = 2N_\mathrm{b}$. The relation (12) was used in Ref. [18] to establish a connection between the summed M1 strength and the charge radii of the Nd, Sm, and Dy nuclei. It is therefore natural to ask whether in turn a connection exists between the summed M1 strength and $\rho(\mathrm{E0})$ values.

Such a connection can indeed be established. The E0 operator is directly proportional to $\hat{n}_d$, unlike the charge-radius operator which involves additional terms [see Eq. (7)] which complicate the relation between the summed M1 strength and charge radii. On the other hand, the matrix element of $\hat{n}_d$ appearing in the sum rule (12) is diagonal while a $\rho(\mathrm{E0})$ value involves a non-diagonal matrix element of $\hat{n}_d$. A relation between the two matrix elements can nevertheless be obtained in the SU(3) limit where they are [19]

$$\langle 0_1^+ | \hat{n}_d | 0_1^+ \rangle = \frac{4N_\mathrm{b}(N_\mathrm{b}-1)}{3(2N_\mathrm{b}-1)}, \qquad |\langle 0_\beta^+ | \hat{n}_d | 0_1^+ \rangle| = \left[\frac{8(N_\mathrm{b}-1)^2 N_\mathrm{b}(2N_\mathrm{b}+1)}{9(2N_\mathrm{b}-3)(2N_\mathrm{b}-1)^2} \right]^{1/2}. \tag{13}$$

From these expressions the ratio of matrix elements can be derived, resulting in the following relation, valid in the large-N_b limit:

$$B(\mathrm{M1}; 0_1^+ \to 1_1^+) \approx \frac{9}{4\pi}(g_\nu - g_\pi)^2 \frac{r_0^2}{\eta} g(N,Z,n,z) \rho(\mathrm{E0}; 0_\beta^+ \to 0_1^+), \tag{14}$$

where $g(N,Z,n,z)$ is the function

$$g(N,Z,n,z) = \frac{e(N+Z)^{2/3}}{e_\mathrm{n} N + e_\mathrm{p} Z} \frac{n\,z}{\sqrt{n+z}}. \tag{15}$$

Note that in the SU(3) limit only one 1^+ state is excited and only the β-vibrational state decays by E0 to the ground state, as is indicated in Eq. (14).

It should be emphasized that the relation (14) is valid only in the SU(3) limit which might jeopardize its use in transitional nuclei. One may nevertheless attempt to apply it to the entire rare-earth region. The ratio $r_0^2/\eta = 3.08$ and the effective charges $e_\mathrm{n} = 0.5e$ and $e_\mathrm{p} = e$ have been determined from a fit to radii [10]. The only remaining constant in Eq. (14), $g_\nu - g_\pi$, can be obtained by adjusting the expression (12) to the observed summed M1 strength in rare-earth nuclei [20], as shown in Fig. 4. This, incidentally, constitutes an additional test of the matrix elements $\langle 0_1^+ | \hat{n}_d | 0_1^+ \rangle$ as they are obtained in the IBM-1 fits to the different isotope series. An overall satisfactory agreement is obtained with the value $|g_\nu - g_\pi| = 0.83\ \mu_\mathrm{N}$ but it is seen that the calculated M1 strength

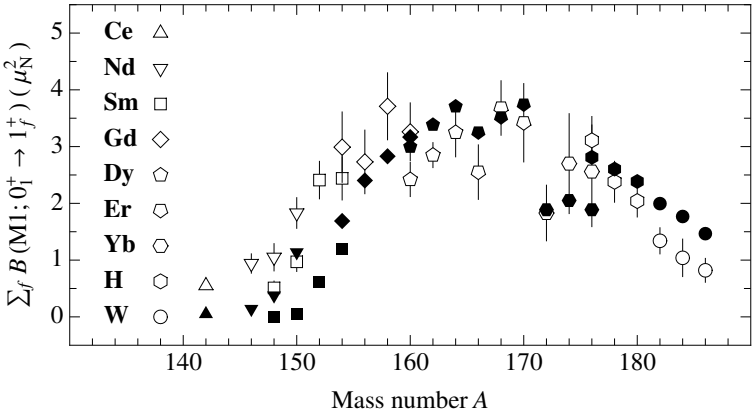

FIGURE 4. Summed M1 strength $\sum_f B(\mathrm{M1};0_1^+ \to 1_f^+)$ in the rare-earth region. The open symbols are the experimental values [20] and the full symbols are calculated with Eq. (12).

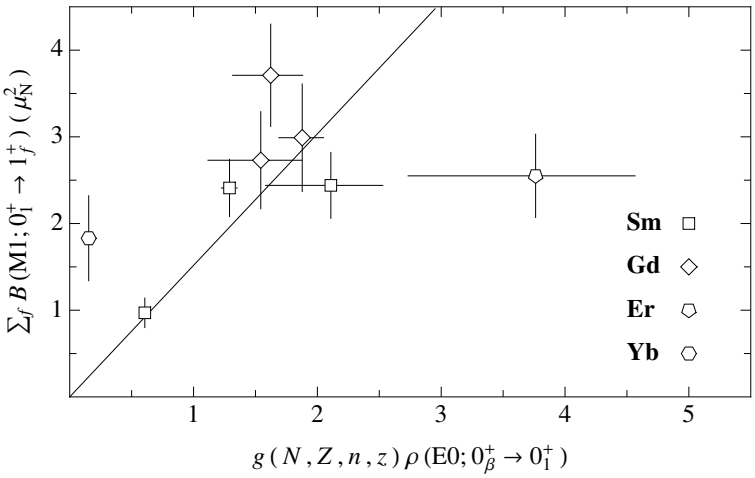

FIGURE 5. Correlation between the $\rho(\mathrm{E0};0_\beta^+ \to 0_1^+)$ value and the summed M1 strength $\sum_f B(\mathrm{M1};0_1^+ \to 1_f^+)$ for the eight nuclei in the rare-earth region where both properties are known. The function $g(N,Z,n,z)$ is defined in Eq. (15). The slope of the line is $9(g_\nu - g_\pi)^2 r_0^2/(4\pi\eta)\,\mu_N^2$.

in the Sm isotopes is well below its observed value, again indicating that the calculated transition is too slow for this isotope series.

The correlation (14) can now be tested and is shown in Fig. 5 for the eight nuclei in the rare-earth region where both E0 and M1 properties are known (^{150,152,154}Sm, ^{154,156,158}Gd, ^{166}Er, and ^{172}Yb). The ^{172}Yb point is conspicuously off the line, reinforcing the earlier remark about E0 strength in this nucleus. The ^{166}Er point follows from a recent experiment [21] where the *fourth* $J^\pi = 0^+$ level at 1934 keV has been identified as the band head of the β-vibrational band with a sizable E0 matrix element to the ground state.

5. SUMMARY

A consistent description of nuclear charge radii and E0 transitions was proposed and the validity of this approach was tested with the interacting boson model applied to even-even nuclei in the rare-earth region ($58 \leq Z \leq 74$). An additional correlation between $\rho(E0)$ values and the summed M1 strength to the scissors mode, valid in the SU(3) limit, was pointed out and tested in the rare-earth region. It would be of interest to derive a more general relation between these two properties, applicable to transitional and deformed nuclei.

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