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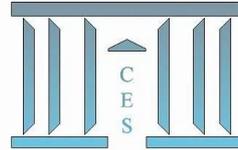
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**The user cost of natural resources and the optimal
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Antoine d'AUTUME

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The user cost of natural resources and the optimal exploitation
of two non-renewable polluting resources

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Abstract

We study the optimal extraction of two non-renewable resources when extraction costs depend on cumulative previous extraction. Defining a complete user cost of natural resources, including environmental damages, allows us to greatly simplify the resolution. It allows us to describe, in a first stage, potential optimal extraction paths of the two resources. It also reduces the problem to one with a unique stock variable, which can easily be solved through time elimination. We also characterize the evolution of the carbon-price and firm rents.

This framework is applied to a study of oil and coal optimal extraction. The extraction cost of oil is initially lower than the one of coal, but it increases more rapidly with extraction. In a business as usual scenario, without taking into account environmental costs, the optimal path is to use only oil in a first time phase, before using simultaneously the two resources in a second phase, until the backstop becomes profitable. As coal becomes cheaper to extract, it provides for the largest part of extraction in the second time phase.

When the carbon price is taken into account, through a tax or emission permits, the optimal path relies much less on the more polluting coal, prices are higher and the backstop is reached much earlier.

If the backstop price is very high, extraction lasts much longer and it is possible that it becomes optimal to revert to the less polluting oil only in a third time phase.

We consider a model of optimal extraction of two non-renewable polluting resources with a renewable non-polluting backstop¹. As in Heal[1976] we make the realistic assumption that unit extraction costs increase with the cumulative amount of the resource which has already been extracted, as extraction becomes more and more difficult. Extraction of the resource will stop not when it is exhausted but rather when extraction costs become too high. This set-up makes more sense than the original Hotelling set-up where the overall quantity of the resource is given and can be extracted at a zero or possibly constant unit cost. In the Hotelling setting the resource scarcity is absolute and the only problem is when to consume the existing quantity. In the setting we adopt, scarcity is economic, which appears much more satisfactory. Extraction will stop when it becomes too costly in comparison to other non-renewable resources as well as renewable backstops presently too costly to be used. Oil for instance will not cease to be used when resources will be exhausted but when the high level of extracting costs will choke extraction. As Sheikh Yamani famously said, "the Stone Age did not end for lack of stone". More concretely the level of exploitable oil resources has been constantly reevaluated upward as new discoveries were made but also, and more and more, as higher extraction costs became acceptable.

Heal's approach has been followed by Hanson[1980] and more recently, Hoel[2011], Bureau[2008], Van der Ploeg and Withagen[2011] and others

Our first aim, in this paper, is to put forward a new and synthetic interpretation of the intertemporal trade-offs faced by economic agents when using natural resources. To this end, we define a complete user cost of natural resources. This notion is standard in the case of physical capital. If K is capital, δ the physical rate of depreciation and $I = \dot{K} + \delta K$ the investment flow, the user cost of capital is $(r + \delta)K$, including the physical cost of depreciation and the interest charge of holding an amount K of capital. The user cost is the true cost of using capital and allows to define instantaneous profit in a meaningful way. The total cost of using capital over time may then be represented alternatively by the discounted value of investment flows or by the discounted value of user costs. The equivalence between these two measures follows from pure accounting and, more specifically, by an integration by parts of the discounted integral of profits.

We define in a similar way the complete user cost of natural resources. Its first component is the true measure of the extraction costs currently supported. It takes the form of an interest charge on cumulative extraction costs. A second component of the cost is environmental, as the use of fossil

¹I acknowledge the support of the French National Research Agency (ANR) under the CLEANER project (ANR_NT09_505778).

fuels generates greenhouse gas emissions which contribute to global warming. It takes the usual form of a damage function.

To our knowledge, the notion of an user cost of natural resources has not been stated explicitly in the literature. It appears implicitly in Van der Ploeg and Withagen[2011]² but the corresponding equation is derived from optimality conditions, rather than from an a priori reformulation of the problem in terms of complete user costs. On the other hand, the integration by parts which leads to this reformulation appears in a number of articles on renewable or non renewable resources: see Spence-Starett[1975], who developed a general analysis of models where the Hamiltonian is linear with respect to the control variable and studied most rapid paths converging to a singular solution; see also Leonard-Long[1992], who use it in various contexts. But again no general presentation of the user cost and its importance is offered. In d'Autume[2012], we also use the user cost of natural resources to study the optimal behavior of an extracting firm, that is a model where the Hamiltonian is linear with respect to the extraction rate.

The definition of complete user costs provides an intuitive and simple method to characterize the optimal extraction path of two non-renewable resources, say oil and coal. This issue has been examined in numerous articles and in particular in recent papers by Van der Ploeg and Withagen[2011] and Chakravorty, Moreaux and Tidball[2008]. Both articles stress a possible energy reversal, that is a return to a resource which had earlier been forsaken, a phenomenon which may also appear in our own setting. Our framework is close to the one of Van der Ploeg and Withagen[2011], but we treat the case of two non-renewable resources whereas they treat, for simplification, the second resource, coal, as a dirty renewable backstop. Our framework differs more markedly from the one used in Chakravorty, Moreaux and Tidball[2008]. On one hand, they do not introduce extraction costs increasing with the quantity already extracted. On the other hand, we do not take into account natural emission absorption, which is admittedly a limitation of our approach.

Our approach allows us to characterize, in a first stage, possible extraction paths, before studying optimal dynamics. We then reduce the dynamic system to one state variable, namely total resource extraction.

Two different typical configurations appear. In the first one, the economy begins to exploit the less costly resource and after some time switches to a second regime where both resource are simultaneously extracted. During this second time phase, the economy follows an iso-"user cost" curve, and does so until the moment where the two costs simultaneously reach the cost of the backstop, say solar energy. In the second configuration, it proves impossible

²See equation (6).

to follow the iso-cost after some point, as this would require that the extracted stock of one of the resource decrease, which is impossible by definition. We then face an irreversibility problem akin to the one considered by Arrow-Kurz[1970] in the case of physical investment and the solution method is similar to the one they proposed. We may then observe an energy reversal. The economy may revert to a regime where one resource, the less polluting one, is the sole to be extracted.

We use our framework to simulate the ordering of oil and coal extraction. The extraction cost of oil is initially lower than the one of coal, but it increases more rapidly with extraction. In a business as usual scenario, without taking into account environmental costs, the optimal path is to use only oil in a first time phase, before using simultaneously the two resources in a second phase, until the backstop becomes profitable. As coal becomes cheaper to extract, it provides for the largest part of extraction in the second time phase.

When the carbon price is taken into account, through a tax or emission permits, the optimal path relies much less on the more polluting coal, prices are higher and the backstop is reached much earlier. It may be the case that the economy has to revert to an oil only regime if the polluting power of oil is large.

1 The framework

The two resources and the backstop are perfectly substitutable in consumption. Let q_1 , q_2 and x be the consumption levels of the three goods. Current utility is an increasing and concave function of total consumption:

$$U(q_1 + q_2 + x).$$

The resources differ by their extraction costs as well as by their polluting character.

For $i = 1, 2$, we denote at time t by $q_{i,t}$ and $Z_{i,t}$ the flow of extraction of resource i and the cumulated sum of past extractions, with an arbitrary and unspecified initial point. Thus,

$$\dot{Z}_{i,t} = q_{i,t}. \tag{1}$$

As in Heal[1976], the unit extraction cost of resource i is an increasing, differentiable and convex function $G_i(Z_i)$. This unit cost does not depend on the flow q_i currently extracted.

Extraction also has a detrimental environmental impact. More specifically, the two resources are fossil fuels the consumption of which generates

GreenHouse Gases emissions which contribute to global warming. We do not take into account the natural absorption of GreenHouse Gases by the environment and simply assume that the stock of pollutants created by the use of resource i is proportional to the stock already extracted,

$$E_i = \phi_i Z_i$$

with a positive emission coefficient ϕ_i of resource i . This implies³ $\dot{E}_i = \phi_i q_i$.

Current environmental damages are then defined as

$$D(\phi_1 Z_1 + \phi_2 Z_2),$$

where D is an increasing, differentiable and convex function.

Lastly, we denote by p_b the constant unit production cost of the backstop.

Under these assumptions, the social welfare takes the following form, where r is rate of discount, identical in this framework to the consumers rate of time preference⁴:

$$\int_0^{\infty} e^{-rt} [U(q_1 + q_2 + x) - p_b x - G_1(Z_1) q_1 - G(Z_2) q_2 - D(\phi_1 Z_1 + \phi_2 Z_2)] dt. \quad (2)$$

2 The complete user cost of natural resources

We now introduce a user cost of natural resources.

For a moment, we consider a unique resource. Assuming that the unit extraction cost does not depend on the quantity currently extracted implies that the time required to extract a given quantity, or indeed the time profile of extraction, does not affect the overall cost of extraction. Starting at time T_1 from an extraction level Z_{T_1} and extracting between T_1 and T_2 a flow q_t , the integral of which is Z_{T_2} has the following cost

$$\int_{T_1}^{T_2} G(Z_t) q_t dt = \int_{Z_{T_1}}^{Z_{T_2}} G(z) dz = H(Z_{T_2}) - H(Z_{T_1}) \quad (3)$$

³More generally, we might assume that the pollution stock associated with a resource is an increasing and convex function $E_i = f_i(Z_i)$ of cumulative extraction. This would imply $\dot{E}_i = f'_i(Z_i) q_i$, so that the emission coefficient of new extraction of resource i would be an increasing of Z_i . The pollution associated with extraction would then increase over time as extraction becomes more and more difficult.

⁴All variables depend on time. To simplify notations we omit the time index.

It does not depend on the time profile of extraction

The economic total cost of course does depend on this profile, as it incorporates interest charges. For generality, let us consider a time varying discount rate r_t and define $R_t = \int_0^t r_s ds$. A simple integration by parts yields⁵

$$\int_{T_1}^{T_2} e^{-R_t} G(Z_t) q_t dt = e^{-R_{T_2}} H(Z_{T_2}) - e^{-R_{T_1}} H(Z_{T_1}) + \int_{T_1}^{T_2} e^{-R_t} r_t H(Z_t) dt \quad (4)$$

The total economic cost now includes the integral of $r_t H(Z_t)$. As mentioned in the introduction, this relation is similar to the one which applies in the case of physical investment. Let K_t be the stock of capital, δ its rate of depreciation and $I_t = \dot{K}_t + \delta K_t$ gross investment. A similar integration by parts yields:

$$\int_{T_1}^{T_2} e^{-R_t} I_t dt = e^{-R_{T_2}} K_{T_2} - e^{-R_{T_1}} K_{T_1} + \int_{T_1}^{T_2} e^{-R_t} (r_t + \delta) K_t dt$$

$r_t H(Z_t)$ thus appears as the user cost of the resource stock, as $(r_t + \delta) K_t$ is the user cost of physical capital. In the simple capital model we consider, one unit of capital is produced with one unit of generic good, so that K is the cumulative cost which has to be supported to reach a capital level equal to K . In the resource model, $H(Z)$ is the cumulative cost which has to be supported to reach a resource level Z ⁶.

In our setting with two polluting non-renewable resources, this leads us to the following definition and proposition⁷.

Definition 1 *The complete user cost of natural resources is*

$$\Gamma(Z_1, Z_2) = rH_1(Z_1) + rH_2(Z_2) + D(\phi_1 Z_1 + \phi_2 Z_2) \quad (5)$$

Marginal complete user costs are the derivatives

$$\Gamma_i(Z_1, Z_2) = rG_i(Z_i) + \phi_i D'(\phi_1 Z_1 + \phi_2 Z_2), \quad i = 1, 2 \quad (6)$$

⁵This detailed in appendix 1.

⁶The two models differ regarding the utility side. In the physical capital model, utility derives from the production made with the capital stock, so that K has a positive social value. In the resource model, utility derives from consuming the flow of extraction. Moreover Z has a negative social value, as a higher Z means higher marginal extraction costs.

⁷The interest rate is the consumers' discount rate and we treat it as constant.

Proposition 2 *Up to initial constants, social welfare is equal to*

$$\int_0^{\infty} e^{-rt} [U(q_1 + q_2 + x) - p_b x - \Gamma(Z_1, Z_2)] dt \quad (7)$$

The complete user cost of natural resources $\Gamma(Z_1, Z_2)$ is the sum of the user cost of extraction and the environmental damage caused by emissions.

Social welfare may be expressed as the discounted value of the difference between the utility derived from consuming natural resources and their complete user cost.

We also define the two marginal complete user costs. As environmental damages depend on the two stocks Z_1 and Z_2 , the two marginal user costs are linked and each one depends on the two stocks.

3 The ordering of the two resources extraction

Knowledge of the user costs is sufficient to characterize possible orderings of extraction of the resources.

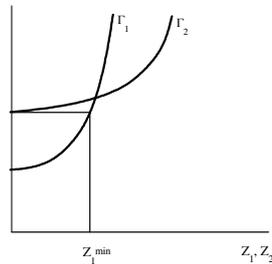


Figure 1: Extraction costs

3.1 The case without pollution

Let us first consider the case without pollution. marginal user costs are simply $rG_1(Z_1)$ and $rG_2(Z_2)$.

As the two resources as well as the backstop are perfectly substitutable in consumption, they can only be produced simultaneously if they have the same price. The price of a resource is not equal to its cost, as it includes a

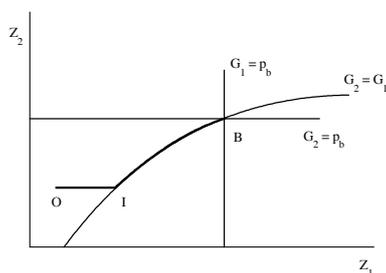


Figure 2: Extraction path 1

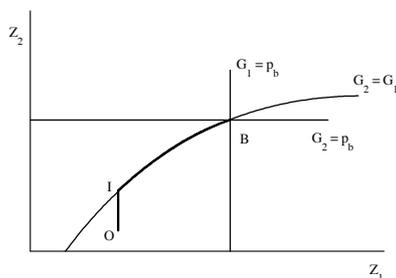


Figure 3: Extraction path 2

scarcity rent. As we shall see⁸, however, the equality of prices over a time interval is only possible if user costs are equal.

Without pollution, this reduces to the condition that current extraction costs be equal:

$$G_1(Z_1) = G_2(Z_2).$$

Let us consider the case of figure (1). Resource 1 is initially less costly to extract than resource 2, but its cost increases more steeply. The two resources may be extracted simultaneously only if cumulative extraction of resource 1 has reached the minimum level Z_1^{\min} such that $G_1(Z_1^{\min}) = G_2(0)$. Figure (2) plots the equal cost locus in the plane (Z_1, Z_2) . As each cost increases with its cumulative extraction level, the locus has a positive slope $dZ_2/dZ_1 = G'_1(Z_1)/G'_2(Z_2)$. At each point located above or to the left of the curve, the extraction cost of the first resource is lower than the one of resource 2 and marginal user costs rG_1 and rG_2 are ranked in the same way.

The economy switches to the backstop at point B on the figure, when both

⁸We develop in this section an intuitive analysis. All assertions will be proved later on.

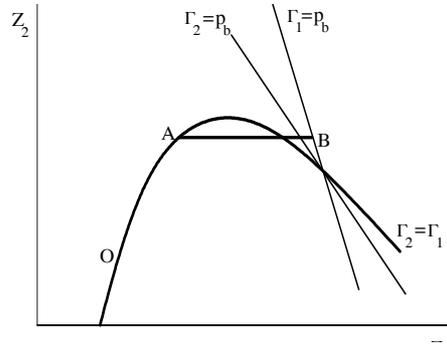


Figure 4: Forsaking resource 2

extraction costs reach the price p_b of the backstop⁹. The vertical and horizontal lines through B are respectively the loci $G_1(Z_1) = p_b$ and $G_2(Z_2) = p_b$.

The optimal path is intuitive. Remember that Z_1 and Z_2 cannot decrease. Assume for example that the initial amounts already extracted correspond to point O on figure (2). The initial user cost of resource 2 is higher than the one of resource 1. It is optimal to extract only resource 1. As it is exploited, the user cost of resource 1 increases. The economy moves on segment OI until the two resources costs are equalized. The economy then starts extracting simultaneously the two resources, at such rates that the two costs remain equal. The economy moves on the curved segment IB .

The shares of the two resources may be read on the diagram. We have $q_2/q_1 = \dot{Z}_2/\dot{Z}_1 = dZ_2/dZ_1 = G'_1(Z_1)/G'_2(Z_2)$ which, as we have seen, is the slope of the equal cost curve. This slope, i.e. the relative share of resource 2 extraction, decreases as exploitation goes on.

During this time period the price of energy increases until it reaches the price of the backstop. Extraction then has to stop. The economy switches to consumption of the backstop and the price of energy remains later on equal to p_b . Rents disappear so that the extraction costs are equal to the price of the backstop.

Figure (3) considers the opposite case of a relatively high initial level of Z_1 . The initial extraction cost of resource 1 is larger than the one of resource 2 so that the economy first uses resource 2 only and moves on a vertical straight line¹⁰ until it reaches the iso-cost curve.

⁹As Γ_i is a user cost, it cannot be compared to the price of the backstop. The relevant comparison is between the discounted value of future user costs, which reduces here to $\Gamma_i/r = G_i$ and p_b .

¹⁰Other cases would be the ones of initial very high levels of Z_1 or Z_2 . The economy would never extract simultaneously the two resources and would move on an horizontal or

We thus have been able to characterize intuitively the extraction path of the two resources. The formal dynamic analysis will only determine the speed at which this path will be followed and, of course, confirm our intuitive results.

3.2 The case with pollution

We now consider the general case with environmental damages.

We again define the equal cost curve $\Gamma_1(Z_1, Z_2) = \Gamma_2(Z_1, Z_2)$ curve in the (Z_1, Z_2) plane. If this curve always has a positive slope, not much is changed. The optimal ordering of exploitation of the two resources follows the same principles than in the previous case.¹¹

A second configuration arises when the iso-cost curve reaches a maximum, as in the case of figure (4). As Z_2 cannot decrease, the economy cannot go on following this curve. It ceases at some point to exploit resource 2 and engages on the horizontal segment AB until the moment when the cost of extracting resource 1 reaches the cost of the backstop.

For reasons which will be explained later, the economy does not reach the maximum of the curve and forsakes extraction of resource 2 before reaching it. The situation is one of irreversibility, as the Z_i s cannot decrease, and is reminiscent of the Arrow-Kurz[1970] analysis of irreversible physical investment.

The optimal switch point is endogenous. In this more complicated case, the optimal ordering of extraction cannot be determined ex ante, without solving the dynamic model. It depends on all the elements of the model and, in particular, on the utility function, that is on the demand side.

Let us now clarify the shape of the iso-cost curve. We may show that the case of forsaking one resource only occurs if this resource is more polluting than the other one, and sufficiently so.

The slope of the curve is now

$$\left. \frac{dZ_2}{dZ_1} \right|_{\Gamma_2=\Gamma_1} = \frac{\Gamma_{11} - \Gamma_{21}}{\Gamma_{22} - \Gamma_{12}} \quad (8)$$

From (6),

$$\begin{aligned} \Gamma_{11} &= G'_1 + (\phi_1^2/r)D'' > 0 \\ \Gamma_{22} &= G'_2 + (\phi_2^2/r)D'' > 0 \\ \Gamma_{12} &= \Gamma_{21} = (\phi_1\phi_2/r)D'' > 0 \end{aligned}$$

vertical straight line until it switches to the backstop. If both initial Z_1 and Z_2 were very high, the economy would of course switch at once to the backstop.

¹¹One difference is that the $\Gamma_i(Z_1, Z_2)/r = p_b$ are no more vertical or horizontal.

$$\begin{aligned}\Gamma_{11} - \Gamma_{21} &= G'_1 + (\phi_1 - \phi_2) (\phi_1/r) D'' \\ \Gamma_{22} - \Gamma_{12} &= G'_2 + (\phi_2 - \phi_1) (\phi_2/r) D''\end{aligned}$$

Assume resource 2 to pollute more, so that $\phi_2 > \phi_1$. The denominator of slope (8) is always positive, while the numerator may become negative, in particular when D'' is high that is when total emissions become high. The iso-cost curve then reaches a maximum and the economy forsakes extraction of the polluting resource at some point before reaching the maximum¹².

The previous derivatives also allow to calculate the slopes of the curves describing the switch to the backstop :

$$\left. \frac{dZ_2}{dZ_1} \right|_{\Gamma_1=p_b} = -\frac{\Gamma_{11}}{\Gamma_{12}}, \quad \left. \frac{dZ_2}{dZ_1} \right|_{\Gamma_2=p_b} = -\frac{\Gamma_{21}}{\Gamma_{22}}.$$

Both are negative. In the more natural case, the cross terms are smaller which implies that the $\Gamma_1/r = p_b$ curve is steeper than the $\Gamma_2/r = p_b$ curve.

4 The dynamic analysis

4.1 Optimality conditions

The problem is to maximize social welfare (7) under the resource constraints.

$$\int_0^\infty e^{-rt} [U(q_1 + q_2 + x) - p_b x - \Gamma(Z_1, Z_2)] dt$$

$$\dot{Z}_1 = q_1 \geq 0, \quad \dot{Z}_2 = q_2 \geq 0$$

$$Z_{1,0} \text{ and } Z_{2,0} \text{ given.}$$

Let p_i , $i = 1, 2$ be the shadow prices of the two resource constraints.

Proposition 3 *First order Conditions are the following*

$$x \geq 0, \quad U'(q_1 + q_2 + x) - p_b \leq 0 \quad (9)$$

$$q_i \geq 0, \quad U'(q_1 + q_2 + x) - p_i \leq 0, \quad i = 1, 2 \quad (10)$$

$$\dot{p}_i = r p_i - \Gamma_i(Z_1, Z_2), \quad i = 1, 2 \quad (11)$$

with slackness conditions for the static conditions.

The proof is not reproduced.

¹²More complicated configurations, with several maxima, may occur.

4.2 Resources prices, the carbon price and rents

Equation (11) states that the shadow price of a resource is the discounted value of its future complete marginal user costs. This relation extends the Hotelling rule to the case of endogenous extraction costs and environmental costs. As the complete user cost is non decreasing, its discounted cumulative value, namely the price, is also non decreasing.

Condition may also be written as a no arbitrage condition

$$r = \frac{\Gamma_i(Z_1, Z_2)}{p_i} + \frac{\dot{p}_i}{p_i} = \frac{\phi_i D'}{p_i} + \frac{rG_i(Z_1, Z_2)}{p_i} + \frac{\dot{p}_i}{p_i}. \quad (12)$$

Distinguishing the two components of the marginal user cost leads to distinguish two components in the price of a resource.

The first one is the discounted value of the marginal environmental damages it generates. It is the product of the emission coefficient ϕ_i and the carbon price τ , defined as the discounted value of future marginal environmental damages:

$$\dot{\tau} = r\tau - D' \quad (13)$$

The second part, say p_i^2 , is the discounted value of marginal extraction user costs:

$$\dot{p}_i^2 = rp_i^2 - rG_i$$

which yields

$$p_{it}^2 = \int_t^\infty e^{-r(s-t)} rG_{is} ds.$$

An integration by parts yields¹³

$$p_{it}^2 = G_{it} + \int_t^\infty e^{-r(s-t)} \dot{G}_{is} ds \stackrel{def}{=} G_{it} + \psi_{it}$$

with

$$\dot{\psi}_i = r\psi_i - \dot{G}_i \quad (14)$$

ψ_i is the discounted value of future extraction cost increases. As the analysis of the competitive extraction firm behavior¹⁴ would show, it is the producer rent gained by the resource field owner. This rent reflects the scarcity of the resource in the sense that extraction will become more and more costly. It may increase with time, but will ultimately decrease and become zero when the exploitation of the resource ceases.

¹³Letting $u = G_{is}$, $dv = e^{-r(s-t)} r ds$, $du = \dot{G}_{is}$, $v = -e^{-r(s-t)}$ and taking into account the fact that G_{is} will remain constant after the backstop starts to be used.

¹⁴See d'Autume[2012]

Proposition 4 *The price of a resource*

$$p_i = G_i + \phi_i \tau + \psi_i \quad (15)$$

is the sum of current marginal extraction cost, the carbon value of emissions and the producer rent.

The carbon price is the discounted sum of future marginal environmental damages, while the producer rent is the discounted value of future extraction cost increases.

The price thus appears as the sum of the current extraction cost and the discounted sum of all marginal damages inflicted by current extraction to natural capital and the environment. Extraction reduces future extraction possibilities and thus the value of natural capital. Extraction also leads to emissions which affect the environment and reduce its value.

In a competitive market economy the first element takes the form of a rent gained by the owner of the resource field. The second element is the cost of carbon emissions, which has to be internalized through a carbon-tax or a market for emission permits.

4.3 The dynamics of aggregate resources and the two regimes

As resources are perfectly substitutable we may define the total amount of already extracted resources and the total extraction flow

$$Z = Z_1 + Z_2 \quad (16)$$

$$q = q_1 + q_2 = \dot{Z} \quad (17)$$

We also define the demand function $q^d(p)$ as the inverse function of marginal utility.

Two types of regimes are possible depending on which resources are currently extracted.

Proposition 5 *The regime with joint extraction of the two resources, $q_1 > 0$, $q_2 > 0$, is characterized as follows.*

i) The two resources have the same price and the backstop is unused :

$$U'(q) = p < p_b \quad (18)$$

ii) They have the same complete user cost:

$$\Gamma_1(Z_1, Z_2) = \Gamma_2(Z_1, Z_2) \quad (19)$$

This cost can be expressed as a function $\Gamma(Z)$ of total cumulative extraction.

iii) Cumulative extraction levels and extraction flows may be expressed as functions of total cumulative extraction and the total extraction flow:

$$Z_1 = \psi_1(Z), \quad Z_2 = \psi_2(Z)$$

$$q_1 = \psi'_1(Z)q, \quad Z_2 = \psi'_2(Z)q$$

iv) The dynamics in this regime is described by the following system:

$$\dot{Z} = q^d(p) \quad (20)$$

$$\dot{p} = rp - \Gamma(Z) \quad (21)$$

v) Eliminating time reduces the dynamics to the unique differential equation

$$p'(Z) = \frac{rp(Z) - \Gamma(Z)}{q^d(p(Z))}, \quad (22)$$

The two resources can obviously be simultaneously exploited if and only if they have the same price, which has to be smaller than the price of the backstop, in order to make production of the latter unprofitable.

From conditions (11) describing price evolutions, the two prices may be equal for a finite time period only if both complete costs are equal¹⁵. Note that prices are not equal to costs as implicit rents are present, but these rents have to be equal.

From (19) and (16) we derive the shares of cumulative extractions $Z_i = \psi_i(Z)$, as well as the shares of current extraction rates $q_i = \dot{Z}_i = \psi'_i(Z) \dot{Z} = \psi'_i(Z) q$.

We thus are led to a model with only one stock. Resolution is also simplified by the fact that the terminal point is not be a saddle-point and will be reached in finite time. This allows us to eliminate time and use a one dimensional differential equation in the (Z, p) plane.

We have

$$\dot{p}/\dot{Z} = (dp/dt) / (dZ/dt) = dp/dZ = p'(Z),$$

¹⁵As we stated earlier in our intuitive analysis.

and $\dot{Z} = q^d(p)$. Using (11), and the equality of prices and costs, we are led to differential equation (22).

Let us now consider the regime with extraction of the first resource only. Cumulative extraction of the second resource remains constant at level \bar{Z}_2 and we have

$$\begin{aligned} Z &= Z_1 + \bar{Z}_2 \\ q &= q_1 > 0, \quad q_2 = 0 \end{aligned}$$

Proposition 6 *The regime with extraction of the sole resource one is characterized as follows*

i) *Prices and extraction flows are such that*

$$U'(q_1) = p_1 < \min[p_2, p_b]$$

ii) *The dynamics is*

$$\begin{aligned} \dot{Z} &= q^d(p_1) \\ \dot{p}_1 &= rp_1 - \Gamma_1(Z - \bar{Z}_2, \bar{Z}_2). \end{aligned}$$

iii) *Eliminating time yields*

$$p_1'(Z) = \frac{rp_1(Z) - \Gamma_1(Z - \bar{Z}_2, \bar{Z}_2)}{q^d(p_1(Z))} \quad (23)$$

iv) *The price of resource 2 varies according to*

$$p_2'(Z) = \frac{rp_2(Z) - \Gamma_2(Z - \bar{Z}_2, \bar{Z}_2)}{q^d(p_1^*(Z))} \quad (24)$$

where $p_1^*(Z)$ is the solution of (23), with an appropriate boundary condition.

The price of the sole resource which is extracted is smaller than the prices of the two competing energy sources, which makes their use non-profitable. As we shall see, this does not require the complete user cost of the extracted resource to be lower than the one of the second resource, as the two rents are not equal.

These propositions validate our previous intuitive analysis of possible extraction paths. The cost relations we described in section 3, and in particular the equality of the two complete user costs of two resources when they are simultaneously extracted are simply necessary conditions of the minimization of the discounted total cost of producing a given flow of resource $q_{1t} + q_{2t} + x_t$, for $0 \leq t < \infty$. The maximization of social welfare clearly requires this minimization and it is easy to check that the description of the regimes follows from this second and simpler problem.

4.4 Global resolution in the case of an ever increasing iso-cost curve

Let us consider a case, with pollution, where the iso-cost curve in the plane (Z_1, Z_2) is always increasing up to the backstop point. The configuration is similar to the one in figure (2) but environmental damages are now part of the problem. The iso-cost curve is represented by a function $Z_2^*(Z_1)$. We assume the initial point $(Z_{1,0}, Z_{2,0})$ to be such that $\Gamma_2 > \Gamma_1$, as in figure (2).

The solution is of the following type:

- first, a regime with resource 1 only extracted;
- second, a regime with the two resources extracted.

The model can be solved by simple backward recursion.

The terminal point is the switch point to the backstop. Cumulative extraction is Z^B such that $\Gamma(Z^B)/r = p_b$ and the price level is $p(Z^B) = p_b$.

The dynamics in the regime with the two resources extracted is determined by differential equation (22) with terminal condition $p(Z^B) = p_b$.

The switch point (Z_1^I, Z_2^I) between this regime and the first one with only resource 1 extracted is determined by $Z_2^I = Z_{2,0} = Z_2^*(Z_1^I)$ and we have $Z^I = Z_1^I + Z_{2,0}$. The price $p(Z^I)$ is known from the dynamics of the second regime.

The dynamics of the first regime is determined by (23) with a known $p(Z^I)$ as terminal condition.

This completes the determination of the optimal solution $p^*(Z)$, for all $Z \in [Z_0, Z^B]$.

It remains to check that this solution satisfies $p_2 > p_1$ at all point on the first regime trajectory, where $Z_2 = Z_{2,0}$. Price $p_2^*(Z)$ is then solution to equation (24), with terminal condition $p_2(Z^I) = p^*(Z^I)$.

Let $\Delta p = p_2^*(Z) - p^*(Z)$ and $\Delta\Gamma = \Gamma_2(Z - Z_{2,0}, Z_{2,0}) - \Gamma_1(Z - Z_{2,0}, Z_{2,0})$. Δp satisfies equation

$$\Delta p'(Z) = \frac{r\Delta p(Z) - \Delta\Gamma(Z - Z_{2,0}, Z_{2,0})}{q^d(p^*(Z))} \quad (25)$$

In the time dependent equivalent equation, Δp is the discounted value of future $\Delta\Gamma$ up to Z^A at which point it is zero. As $\Delta\Gamma$ is positive for all Z between Z_0 and Z^A , Δp is positive on this interval, as it should to prevent extraction of resource 2.

The optimal trajectory $p^*(Z)$ is thus determined for all $Z \in [Z_0, Z^B]$. The evolution of p as a function of time then follows from the resolution of differential equation

$$\dot{Z}(t) = q^d(p^*(Z(t))), \quad Z(0) = Z_0$$

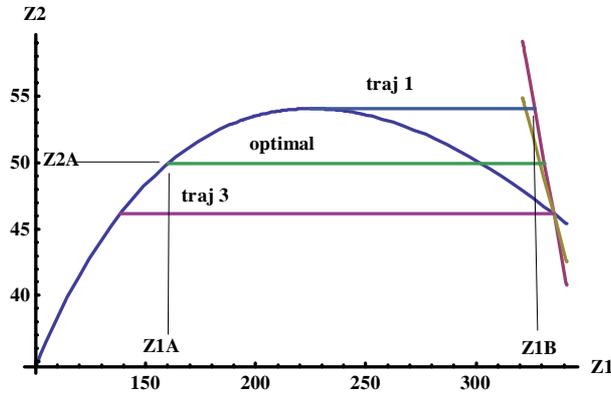


Figure 5: Possible solutions

The evolution of all other variables follows.

4.5 The resolution in the case of an iso-cost curve with a maximum

Let us now consider the case of figure (4). The point where the two user costs, divided by r , simultaneously reach the cost of the backstop is now over the maximum of the iso-cost curve, in its decreasing section.

We now face an irreversibility problem as cumulative extraction Z_2 cannot be decreasing. The optimal solution ends with a phase where only resource 2 is extracted, following the phase where the two resources are jointly extracted.

Consider figure (5). The problem is to determine the point (Z_1^A, Z_2^A) where the trajectory leaves the upward sloping iso-cost curve to engage in an horizontal trajectory. The figure identifies two possible solutions.

i) Point (Z_1^A, Z_2^A) is at the maximum of the iso-cost curve. The path followed in the final "resource one only" regime would lie completely in the $\Gamma_2 > \Gamma_1$ region, that is in a region where $\Delta\Gamma = \Gamma_2 - \Gamma_1 > 0$. On the other hand the price gap $\Delta p = p_2 - p_1$ has to be non negative at all points of the last regime¹⁶, in order to forbid the use of resource 2. Following equation (11) and its interpretation, the price gap is an average of future *positive* cost gaps. It would be strictly positive at the initial point $Z^A = Z_1^A + Z_2^A$ of the regime. This is impossible as the price gap has to be zero at any point on the iso-cost curve. This first trajectory is ruled out.

¹⁶It is strictly positive at the terminal point where $\Gamma_1/r = p_b < \Gamma_2/r$.

ii) The initial point (Z_1^A, Z_2^A) is such that the economy reaches, at the end of its horizontal trajectory, the point on the iso-cost curve where $\Gamma_1/r = \Gamma_2/r = p_b = p_1 = p_2$. A symmetric argument rules out this path, as it lies completely in the $\Delta\Gamma < 0$ region. Δp would be strictly negative at the initial point of the regime.

This suggests that the optimal path lies between the previous two paths we just considered. It is indeed possible to find a path, lying alternatively in the $\Delta\Gamma < 0$ and $\Delta\Gamma > 0$ regions, such that at the initial point we have $\Delta p = 0$. As we show in the appendix a simple one dimensional loop allows to determine the optimal path.

Note that during the first part of this optimal path, which lies below the iso-cost curve, the complete user cost of resource 2 is lower than the one of resource 1 and yet resource 2 is not extracted. The static comparison of complete user costs is insufficient to determine which resources should currently be exploited.

5 Simulations

We use the following functions

$$G_i(Z_i) = c_i + g_i Z_i, \quad D'(E) = \delta E^2/2$$

Then

$$\Gamma_i(Z_1, Z_2) = r(c_i + g_i Z_i) + \phi_i \delta (\phi_1 Z_1 + \phi_2 Z_2)^2 / 2$$

The slope of the curve is now

$$\begin{aligned} \left. \frac{dZ_2}{dZ_1} \right|_{\Gamma_2=\Gamma_1} &= \frac{\Gamma_{11} - \Gamma_{21}}{\Gamma_{22} - \Gamma_{12}} \\ &= \frac{g_1 - \phi_1(\phi_2 - \phi_1)(\delta/r)(\phi_1 Z_1 + \phi_2 Z_2)}{g_2 + \phi_2(\phi_2 - \phi_1)(\delta/r)(\phi_1 Z_1 + \phi_2 Z_2)} \end{aligned}$$

We assume a higher emission coefficient for resource 2, coal, than for resource 1, oil, ie $\phi_2 > \phi_1$.

The denominator is always positive. The numerator is positive if the stock $\phi_1 Z_1 + \phi_2 Z_2$ of emissions is low but becomes positive when it is larger than

$$\frac{r g_1}{\delta \phi_1 (\phi_2 - \phi_1)}$$

We thus have the configuration of figure (4), with a possible decreasing part of the iso-cost curve.

The marginal utility is $U'(q) = dq^{-1/\varepsilon}$, which yields the following total energy demand

$$q_1 + q_2 + x = q^d(p) = (d/p)^\varepsilon$$

Table 1 describes the 2009 oil and coal supplies and the CO2 emissions they generate. We use these data as a benchmark for our rough calibration. To make results more transparent, we express the price of oil in terms of dollars per barrel, in a range of 50/100, while extraction annual flows are expressed in terms of billions of ton of oil equivalent, in a range of 1/5.

The calibration is the following.

$$\begin{aligned} c_1 = 10, c_2 = 25, g_1 = 1/3, g_2 = 1/6, \phi_1 = 1, \phi_2 = 1.5 \\ r = .02, \varepsilon = .1, d = 4 \cdot 10^7, \delta = 2.2 \cdot 10^{-5} \\ Z_{10} = 0, \quad Z_{20} = 30. \end{aligned}$$

Coal induces 50% more CO2 emissions than oil. Coefficient δ describes the weight of the environmental cost in the complete cost of the resources. It is chosen so that the environmental cost of oil is equal to 6 when the extraction cost is 30, that is when $Z_1 = 60$ $Z_2 = 30$, $\Gamma_1 = 36$.

The cost of the backstop is $p_b = 80$.

Tables 2, 3 and 4 and Figures 6, 7 and 8 describe the results of the simulation.

Source : EIA Key World Energy Statistics

Table 1

| | Date 0 | oil | 1 st switch | oil/coal | 2 ^d switch | backstop |
|---------|--------|-----|------------------------|----------|-----------------------|----------|
| Z_1 | 0 | | 60 | | 210 | |
| Z_2 | 30 | | 30 | | 330 | |
| q_1 | 4.0 | | 3.9/1.3 | | 1.2 | |
| q_2 | 0 | | 0/2.6 | | 2.5 | |
| p_1 | 41.4 | | 49.2 | | 80 | |
| p_2 | 44.2 | | 49.2 | | 80 | |
| G_1 | 10 | | 30 | | 80 | |
| G_2 | 30 | | 30 | | 80 | |
| μ_1 | 31.4 | | 19.2 | | 0 | |
| μ_2 | 14.2 | | 19.2 | | 0 | |
| time | 0 | | 15.3 | | 134.3 | |

Table 2 Business as usual

| | Date 0 | oil | 1 st switch | oil/coal | 2 ^d switch | bs |
|--------------|--------|-----|------------------------|----------|-----------------------|----|
| Z_1 | 0 | | 71.0 | | 123.6 | |
| Z_2 | 30 | | 30 | | 70.9 | |
| q_1 | 3.8 | | 3.8/1.9 | | 2.3 | |
| q_2 | 0 | | 0/1.9 | | 1.4 | |
| p_1 | 56.7 | | 71.1 | | 80 | |
| p_2 | 60.0 | | 71.1 | | 80 | |
| G_1 | 10 | | 33.7 | | 51.2 | |
| G_2 | 30 | | 30 | | 36.8 | |
| μ_1 | 29.2 | | 13.7 | | 0 | |
| μ_2 | 3.7 | | 5.4 | | 0 | |
| $\phi_1\tau$ | 17.5 | | 23.8 | | 28.8 | |
| $\phi_2\tau$ | 26.2 | | 35.7 | | 43.2 | |
| time | 0 | | 18.7 | | 43.8 | |

Table 3 With environmental damages

| | Date 0 | oil | 1 st switch | oil/coal | 2 ^d switch | oil | 3 ^d switch | bs |
|--------------|--------|-----|------------------------|----------|-----------------------|-----|-----------------------|----|
| Z_1 | 0 | | 89.6 | | 160.4 | | 331.3 | |
| Z_2 | 30 | | 30 | | 50.0 | | 50.0 | |
| q_1 | 3.6 | | 3.5/3.5 | | 2.9 | | 3.3 | |
| q_2 | 0 | | 0/1.1 | | 0.4 | | 0 | |
| p_1 | 115.6 | | 170.8 | | 230.2 | | 300 | |
| p_2 | 120.0 | | 170.8 | | 230.2 | | 302.7 | |
| G_1 | 10 | | 39.9 | | 63.5 | | 120.4 | |
| G_2 | 30 | | 30 | | 33.3 | | 33.3 | |
| μ_1 | 46.7 | | 38.9 | | 35.5 | | 0 | |
| μ_2 | 1.6 | | 2.7 | | 0 | | 0 | |
| $\phi_1\tau$ | 58.8 | | 92.1 | | 131.2 | | 179.6 | |
| $\phi_2\tau$ | 88.2 | | 138.2 | | 196.8 | | 269.4 | |
| time | 0 | | 25.5 | | 52.3 | | 104.4 | |

Table 4 A return to oil

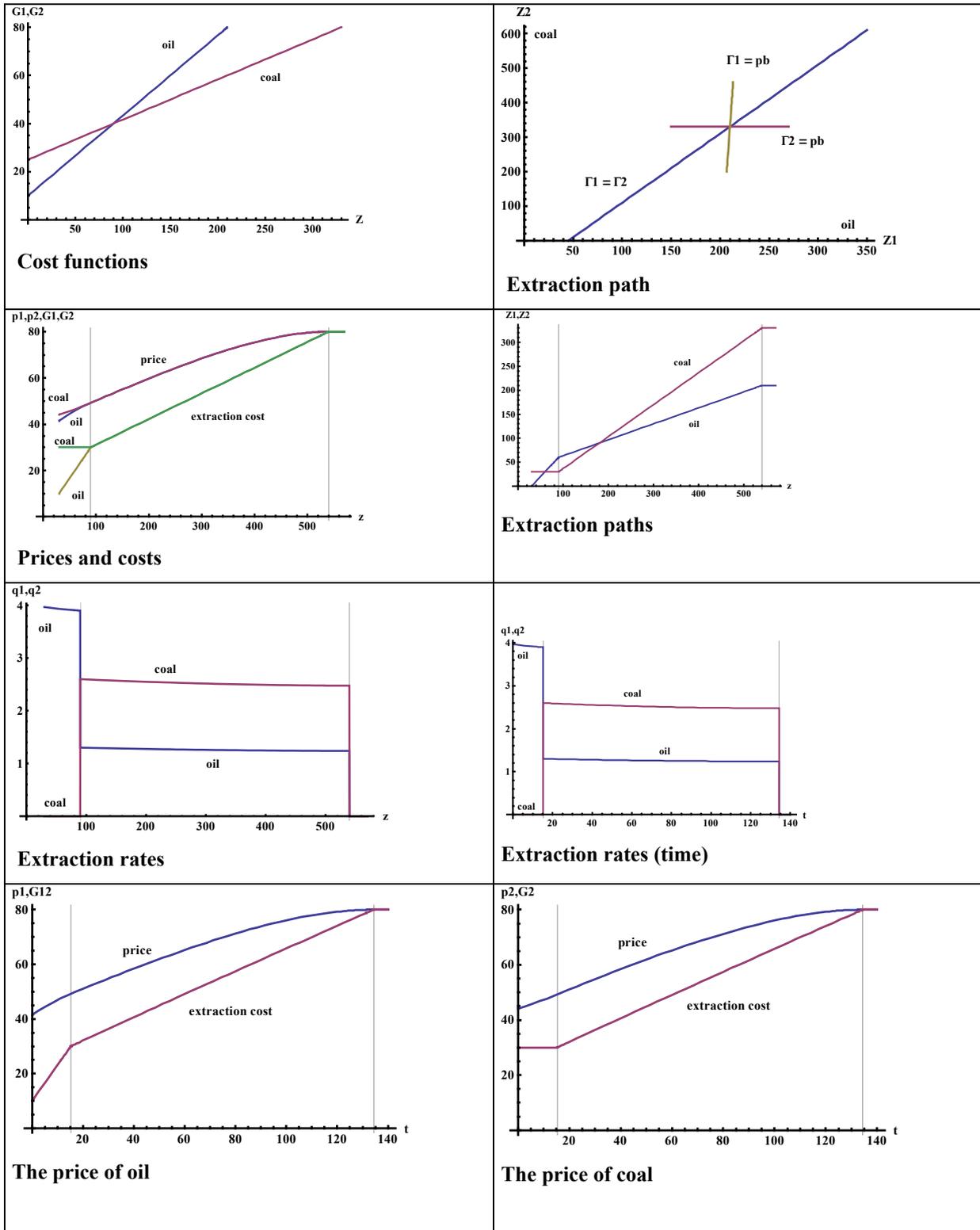


Figure 6 Business as usual

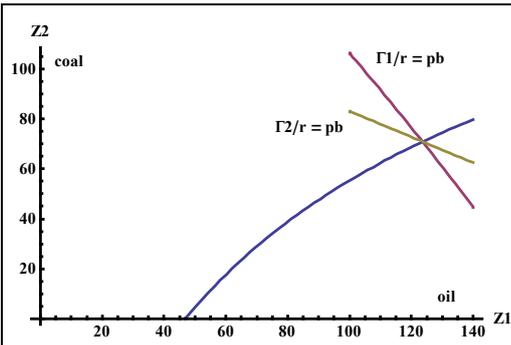


Figure 1 The extraction path

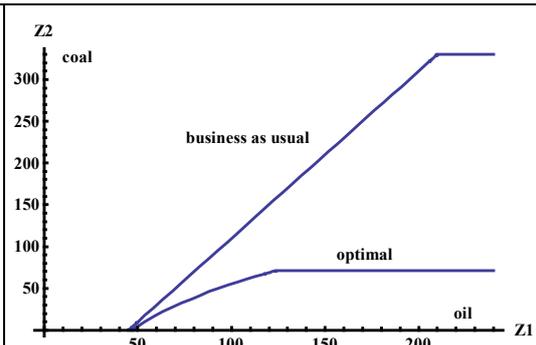


Figure 2 Business as usual and optimal extraction paths

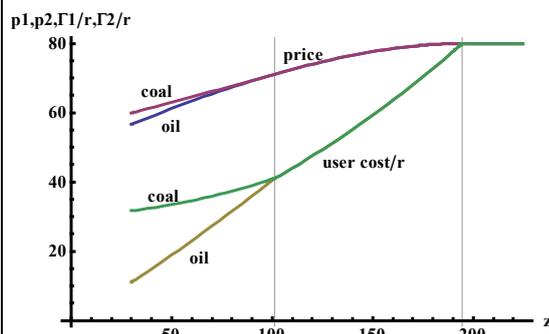


Figure 3 Prices and complete user costs

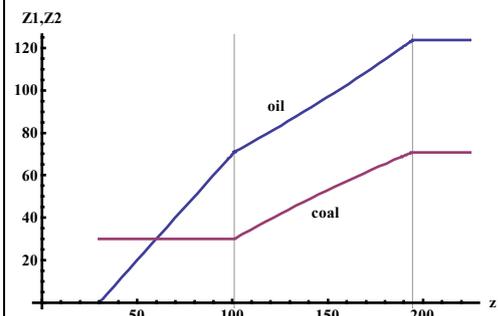


Figure 4 Extraction paths

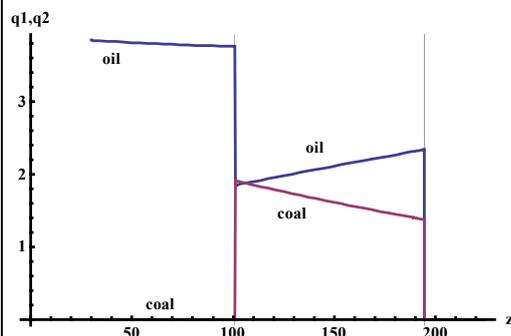


Figure 5 Extraction rates

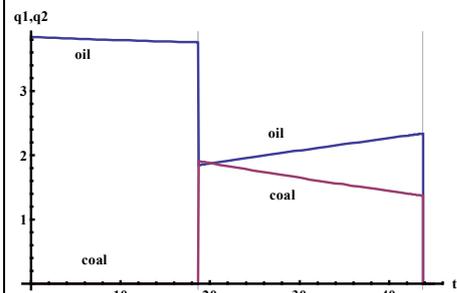


Figure 6 Extraction rates (time)

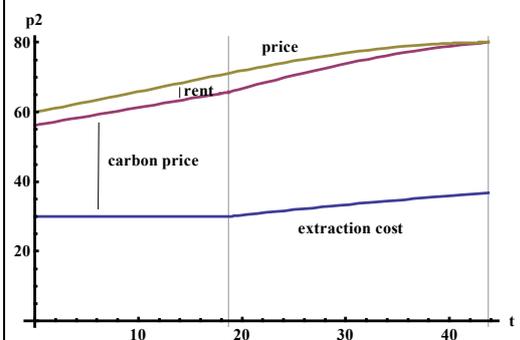
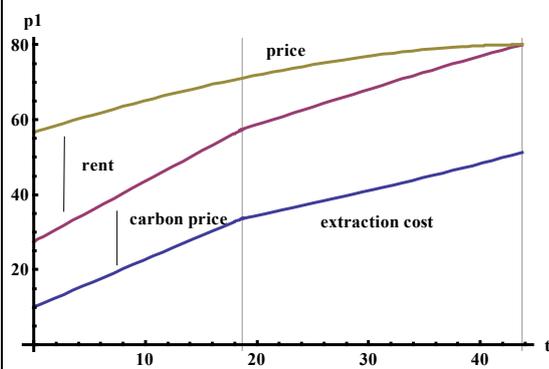


Figure 7 Taking into account environmental dammages

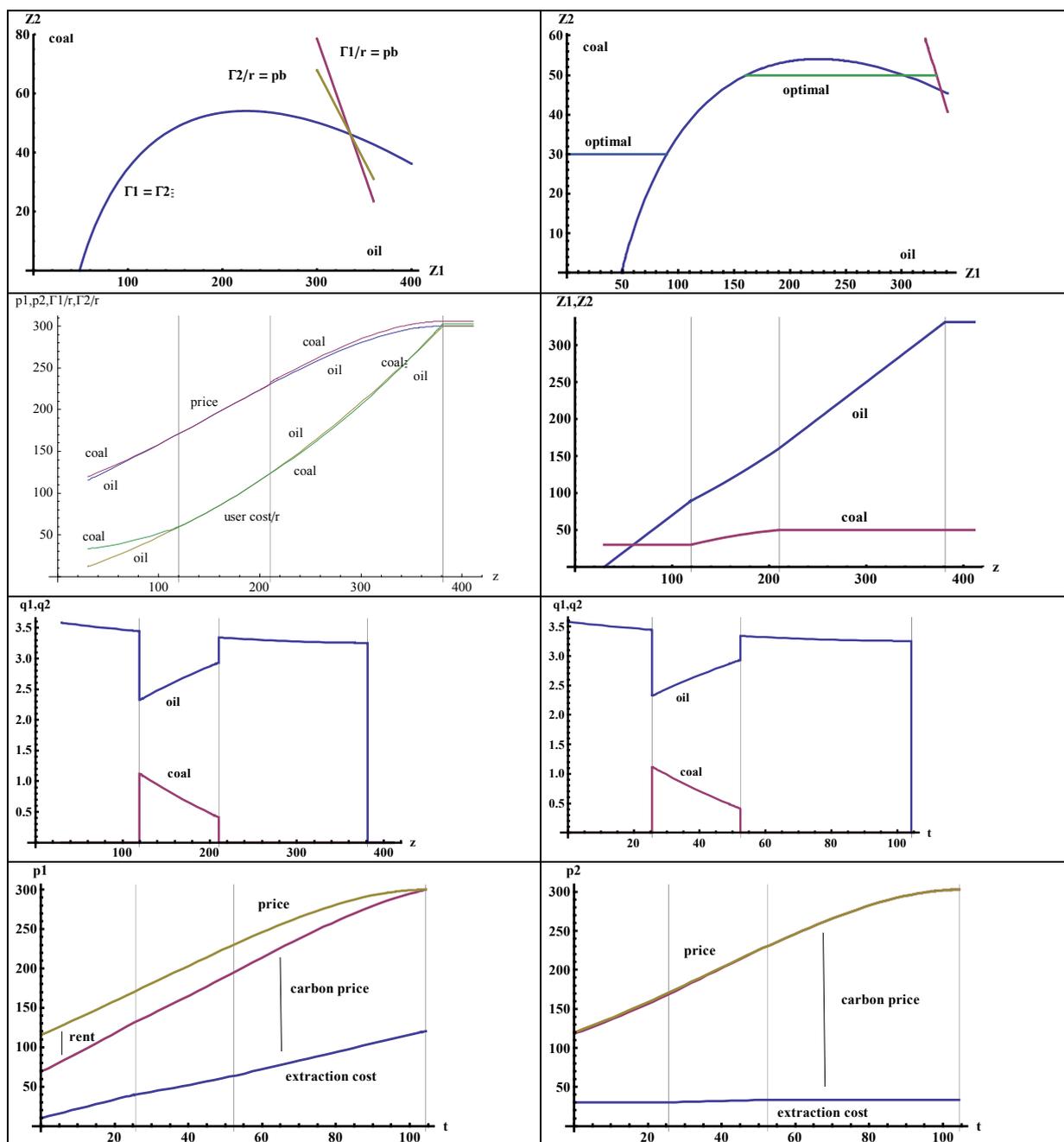


Figure 8 A case with a return to oil

i) Business as usual

We first assume that agents do not take into account environmental costs. The extraction cost of oil is initially lower than the one of coal, but increases more rapidly. After a first regime where oil only is exploited a joint oil-coal regime is sustained until the backstop become profitable when cumulative extractions reach respectively $Z_1 = 210$ and $Z_2 = 330$. In the first phase only oil is extracted at a level q_1 close to 4. In the second phase, oil extraction q_1 falls to a level equal to 1.3 while coal extraction begins at $q_2 = 2.6$. The last three figures describe evolution as functions of time. As the carbon price is not taken into account, fossil energy prices remain low for a long time and their exploitation lasts for 134 years ! The last two figures describe the two components of the two prices, namely extraction costs and rents. Producer rents reach zero when extraction ceases. Coal rents are much lower than oil rents.

ii) Taking into account carbon prices

We now assume that optimal carbon prices are incorporated into energy prices. The economy thus follows an optimal path. Energy prices are higher and extraction now stops much earlier, when $Z_1 = 123.6$ and $Z_2 = 70.9$, after 43.8 years. Contrary to the business as usual case, the economy relies much less on polluting coal. The last two pictures describe the decomposition of the two energy prices, which now include carbon prices, which leads to lower rents, in particular in the case of oil.

iii) The case of a return to oil

With our calibration, the extraction path in the plane (Z_1, Z_2) reaches a maximum when $Z_1 = 416$ and $Z_2 = 130$. The common complete cost of the two energy is then equal to 353. Thus, if the price of the backstop was higher than this level, the economy would eventually forsake coal extraction and revert to a final phase with oil only extraction.

To make stronger this phenomenon, we modify our calibration and now assume higher environmental damages. The weight δ of damages in the utility function is doubled to $\delta = 4.4 \cdot 10^{-5}$. The backstop prices as assumed to be $p_b = 300$, which corresponds to a point located well after the maximum of the iso-cost curve.

Coal exploitation is now temporary and takes place between years 25.5 and 52.3. Carbon prices are much higher, which reduces producers rents. The rent of coal producers almost disappears.

6 Appendix 1 Reformulation of Social Welfare

Consider

$$A = \int_{T_1}^{T_2} e^{-Rt} G(Z_t) q_t dt$$

Integrating by parts with $u = e^{-Rt}$, $dv = G(Z_t) q_t$, $du = -r_t e^{-Rt} dt$, $v = H(Z_t)$ yields

$$A = [e^{-Rt} H(Z_t)]_{T_1}^{T_2} + \int_{T_1}^{T_2} e^{-Rt} r_t H(Z_t) dt$$

which is relation (4) in the text.

The same reasoning is applied to social welfare (2), where the integral is taken between 0 and infinity. The existence of the backstop prevents Z_{it} to tend to infinity and ensures that $\lim_{t \rightarrow \infty} e^{-rt} H_i(Z_{it}) = 0$. Relation (7) follows.

6.1 Appendix 2 Determination of the optimal solution in the case of an iso-cost curve with a maximum

Let us choose an arbitrary Z_1^A in the increasing zone of the iso-cost curve and define (as functions of Z_1^A) Z_2^A , Z^A , Z_1^B and Z^B such that

$$\Gamma_1(Z_1^A, Z_2^A) = \Gamma_2(Z_1^A, Z_2^A)$$

$$\Gamma_1(Z_1^B, Z_2^A) / r = p_b$$

$$Z^A = Z_1^A + Z_2^A$$

$$Z^B = Z_1^B + Z_2^A$$

Let $p^{sol}(Z; Z_1^A)$ be the solution of differential equation (23) with terminal condition $p(Z^B) = p_b$, on interval (Z^A, Z^B) .

Let $p_2^{sol}(Z; Z_1^A)$ be the solution of differential equation

$$p_2'(Z) = \frac{rp_2(Z) - \Gamma_2(Z - Z_2^A, Z_2^A)}{q^d(p^{sol}(Z; Z_1^A))}$$

with terminal condition $p_2(Z^B) = \Gamma_2(Z_1^B, Z_2^A) / r$, as p_2 will remain constant later on.

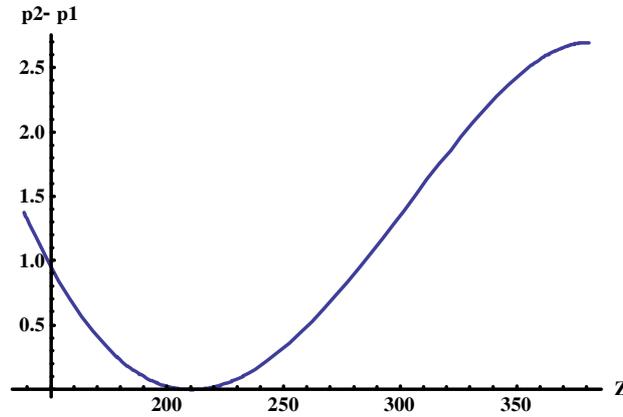


Figure 9: The solution

We now must find the optimal Z_1^{Asol} . The corresponding point on the iso-cost curve must be such that both $p_1 = p_2$ and $\dot{p}_1 = \dot{p}_2$ that is, after time elimination, such that

$$p_1^{sol}(Z^{Asol}, Z_1^{Asol}) = p_2^{sol}(Z^{Asol}, Z_1^{Asol})$$

and

$$p_1^{tsol}(Z^{Asol}, Z_1^{Asol}) = p_2^{tsol}(Z^{Asol}, Z_1^{Asol})$$

Figure (??) describes the optimal path of $p_2 - p_1$, associated with the optimal value of Z_1^A , in the exemple we use later in our simulations. The optimal point Z^A is the one for which the curve reaches a local minimum where $p_2 - p_1 = 0$. For a larger value of Z_1^A , the value $p_2 - p_1$ is always positive. For a lower value, $p_2 - p_1$ takes negative values but there is no point where both $p_2 - p_1$ and its derivative are simultaneously zero.

A simple one dimensional loop thus allows to determine the solution.

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