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# Combined Compressive Sampling and Distribution Discontinuities Detection Approach to Wideband Spectrum Sensing for Cognitive Radios

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**Abstract**—This paper <sup>1</sup> presents a new sensing technique for cognitive radio systems which combines algebraic tools and compressive sampling techniques. The proposed approach consists of the detection of spectrum holes using spectrum distribution discontinuities detector fed by a compressed measurements. The compressed sensing algorithm is designed to take advantage from the primary signals sparsity and to keep the linearity and properties of the original signal in order to be able to apply algebraic detector on the compressed measurements. The complexity of the proposed detector is also discussed and compared with the energy detector as reference algorithm. The comparison shows that the proposed technique outperforms energy detector in addition to its low complexity.

**Index Terms**—Compressed sensing, compressive sampling, spectrum sensing, cognitive radio, distribution discontinuities, algebraic detection, sensing algorithm, wideband, change point detection.

## I. INTRODUCTION

Recently, compressed sensing/compressive sampling (CS) has been considered as a promising technique to improve and implement cognitive radio (CR) systems. The increasing demand for spectrum from various wireless devices and networks emerges the technical society to use the radio spectrum more efficiently. Cognitive radio is a smart wireless communication system that is able to promote the efficiency of the spectrum usage by exploiting the free frequency bands in the spectrum, namely spectrum holes [1], [2]. Detection of spectrum holes is of the first steps of implementing a cognitive radio system. In wideband radio one may not be able to acquire a signal at the Nyquist sampling rate due to the current limitations in Analog-to-Digital Converter (ADC) technology [3]. Compressive sensing makes it possible to reconstruct a *sparse* signal by taking less samples than Nyquist sampling, and thus wideband spectrum sensing is doable by CS. A sparse signal or a compressible signal is a signal that is essentially dependent on a number of degrees of freedom which is smaller than the dimension of the signal sampled at Nyquist rate. In general,

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signals of practical interest may be only nearly sparse [3]. And typically the wireless signal in open networks are sparse in the frequency domain since depending on location and at some times the percentage of spectrum occupancy is low due to the idle radios [1], [4].

In CS a signal with a sparse representation in some basis can be recovered from a small set of nonadaptive linear measurements [5]. A sensing matrix takes few measurements of the signal, and the original signal can be reconstructed from the incomplete and contaminated observations accurately and sometimes exactly by solving a simple convex optimization problem [3], [6]. In [7] and [8] conditions on this sensing matrix are introduced which are sufficient in order to recover the original signal stably. And remarkably, a random matrix fulfills the conditions with high probability and performs an effective sensing [5], [9].

Apart from reconstructing the original signal, detection is more required and interesting in the context of cognitive radio. Generally, for detection purposes it is not necessary to reconstruct the original signal, but only an estimate of the relevant sufficient statistics for the problem at hand is enough. This leads to less required measurements and lower computational complexity [10]. We are interested to skip the estimation of the original signal and directly use the measurements for detection purpose, and so reduce the complexity of the system as much as possible.

In [4] a wavelet-based detection approach using CS to identify the spectrum holes is introduced. To find the frequency band boundaries they derive a convex optimization formulation that the solution gives the band boundaries of the spectrum without requiring to reconstruct the original signal.

In this paper we develop a combined compressive sampling and distribution discontinuities detection technique based on algebraic method for the sensing task of identifying the spectrum holes. The proposed algebraic detector is a linear detector and we would like to feed the algorithm directly with the compressed measurements. For this purpose we find a proper sensing matrix that gives the possibility of feeding the algebraic detector with the measurements directly.

The rest of the paper is organized as follows. Section II states

the motivation and problem formulation. Section III details the proposed sensing technique based on combined compressive sampling and distribution discontinuities detection. Section IV is dedicated to performance results, and section V concludes this paper.

## II. MOTIVATION AND PROBLEM FORMULATION IN THE CONTEXT OF SENSING IN CR

### A. Motivation

The increasing demand for spectrum from various wireless devices and networks emerges the technical society to use the radio spectrum more efficiently. Measurements lead by the FCC (Federal Communication Commission) in the USA have shown that in some regions and/or at some day intervals up to 70 percent of the statically allocated spectrum is left idle [11]. Facing this inefficient usage of spectrum, the FCC recommends deploying unlicensed users in the wireless networks. These unlicensed users, also called secondary user (SU), are allowed to use those idle wireless resources only when the licensed users, also called primary user (PU), is not using them so they do not interfere with their transmissions. In order to make such a concept of spectrum sharing feasible, SUs are cognitive radios (CRs) deployed in the primary networks. CR as introduced by Joseph Mitola [12] is a self aware and intelligent device that can adapt itself to the wireless environment changes by first detecting them, and then adapting its radio parameters to the new opportunities. Cognitive radio technique is an smart wireless communication system that is able to promote the efficiency of the spectrum usage by exploiting the idle frequency bands in the spectrum, namely spectrum holes [1], [2]. Detection of spectrum holes is of the first steps of implementing a cognitive radio system. Different statistical approaches already exist. The easiest to implement and the reference one in terms of complexity is the energy detector (ED) [13]. Nevertheless, the ED is highly sensitive to noise and does not perform well in low signal to noise ratio (SNR). Other advanced techniques based on signals modulations and exploiting some of the transmitted signals inner features were also developed [14]. For instance, the cyclostationary features detector (CFD) exploits the built-in cyclic properties of the PU received signal. The CFD has a great robustness to noise compared to ED but its high complexity is still a consequent drawback. Other techniques that were developed by researchers at Eurécom Institute are based on model selection tools and entropy investigation [15]–[17].

An important issue of complexity in spectrum sensing by cognitive radio is the sampling rate used to sample the received signal at a CR. Specially that usually the spectrum to be sensed is wide which makes the sampling more challenging. In wideband radio one may not be able to acquire a signal at the Nyquist sampling rate due to the current limitations in Analog-to-Digital Converter (ADC) technology [3].

Furthermore, cooperative sensing is generally required to detect hidden nodes to a CR, that is several CR nodes sense the spectrum cooperatively to detect PUs and available holes.

In such scenarios, the amount of data processing at each CR node, in both centralized and decentralized schemes, and the amount of data exchange between CR nodes and the fusion center in the centralized scheme are important factors in complexity and power consumption of the system. Also, high number of samples, sampled at Nyquist rate, increase the power consumption and complexity at CR nodes.

In order to actualize sensing in wide spectrum and to reduce the complexity and power consumption at CR nodes, sampling at a smaller rate than Nyquist rate, while reconstruction or detection of signal is accurately possible, is a prominent key. Hence, compressive sampling or compressed sensing (CS) becomes a promising solution in realization of cognitive radio and reducing the complexity and power consumption. Compressive sampling enables us to do the sampling at a smaller rate than Nyquist rate, sometimes much smaller, and accurately reconstruct the *sparse* signal, or perform detection or estimation.

The first step of cognitive radio is to sense the spectrum and identify the spectrum holes, or in other words, detect the occupied frequency bands. Typically the wireless signal in open access networks is sparse in the frequency domain since depending on location and at some times the percentage of spectrum occupancy is low due to the idle radios [1], [4]. For example, we can model the spectrally sparse wideband signals as

$$s(t) = \sum_{j=0}^{N-1} \beta_j e^{i2\pi jt/N}, t = 0, \dots, N-1 \quad (1)$$

where  $N$  is very large but the number of nonzero coefficients  $\beta_j$  is much less than  $N$ . In this sense we can say that the signal is spectrally sparse [9]. Therefore, we would like to implement spectrum sensing in the context of cognitive radio by performing compressed sensing combined with distribution discontinuities detection. To avoid signal reconstruction burden we find a sensing matrix that enables the algebraic detector properly works while accepting the compressed samples directly as input.

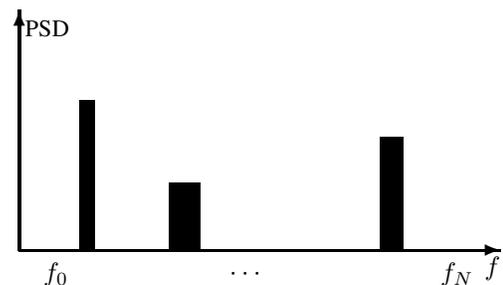


Fig. 1. An example of power spectral density vs. the frequency of a spectrally sparse wideband signal. PSD stands for power spectral density and  $f$  is frequency.

## B. Problem Formulation

Let us consider a discrete representation of the received signal given by:

$$x(n) = A_n s(n) + e(n) \quad (2)$$

where  $A_n$  is modeling the channel,  $s(n)$  represents the discrete signal, that is  $s(t)$  sampled at Nyquist rate, and  $e(n) \sim \mathcal{N}(0, \sigma^2 I_n)$  is i.i.d. Gaussian noise where  $I_n$  is an identity matrix of size  $n$ .

We would like to distinguish between two classified hypothesis  $\mathcal{H}_0$  and  $\mathcal{H}_1$ :

$$\mathcal{H}_0 : x(n) = e(n) \quad (3)$$

$$\mathcal{H}_1 : x(n) = A_n s(n) + e(n) \quad (4)$$

where  $\mathcal{H}_0$  means that the sensed frequency band is white containing only noise and  $\mathcal{H}_1$  means that the sensed frequency band is occupied with a signal corrupted by noise. The key parameter of all spectrum sensing algorithms are the false alarm probability  $P_F$  and the detection probability  $P_D$ .  $P_F$  is the probability to determine a frequency band as occupied while it is free, thus  $P_F$  should be kept as small as possible.

$$P_F = P(\mathcal{H}_1 | \mathcal{H}_0) = P(\text{a signal (user) is present} | \mathcal{H}_0) \quad (5)$$

$P_D$  is the probability to determine a frequency band as occupied when there exists a signal, thus  $P_D$  should be kept as large as possible.

$$P_D = 1 - P_M = \quad (6)$$

$$1 - P(\mathcal{H}_0 | \mathcal{H}_1) = 1 - P(\text{no signal (user) is present} | \mathcal{H}_1)$$

where  $P_M$  denote the probability of missed detection. To design the optimal detector on Neyman-Pearson criterion, we try to maximize the overall  $P_D$  under a given overall  $P_F$ .

In order to infer on the nature of the received signal, we calculate a threshold for each of the detectors. The decision threshold is determined using the required probability of false alarm  $P_F$  given by (5). The threshold  $Th$  for a given  $P_F$  is determined by solving the equation:

$$P_F = P(\text{a signal is present} | \mathcal{H}_0) = 1 - F_{\mathcal{H}_0}(Th) \quad (7)$$

where  $F_{\mathcal{H}_0}$  denote the cumulative distribution function (CDF) under  $\mathcal{H}_0$ .

The algebraic approach is able to detect the signal distribution discontinuities and find their positions in the spectrum, having the complete signal (Nyquist rate samples) as input to the detector. The problem is that sampling a wideband signal with Nyquist rate is constrained due to the reasons highlighted in section I. In order to make the detection possible with less number of samples or smaller sampling rate, relatively to Nyquist rate, we would like to implement compressed sensing technique. In this sense by considering the sparseness of the signal we observe the received signal compressively with a smaller rate than Nyquist rate such as:

$$y = \Phi x + e \quad (8)$$

where  $y \in \mathbb{R}^M$  is the compressed measurements,  $\Phi$  is the sensing matrix,  $x \in \mathbb{R}^N$  is the received signal like  $A_n s(n)$  as above,  $e$  is the additive noise, and  $M \ll N$ . It is shown that with some conditions on  $\Phi$  it is possible to recover  $x$  accurately based on  $y$ .

We would like to use the compressed samples directly to detect the frequency holes without recovering the signal itself. Since the algebraic detection of distribution discontinuities is a linear approach we should find a proper sensing matrix that makes it possible to use the compressed samples as the input to the linear detector. In following section we discuss about compressed sensing and the selection of the sensing matrix.

## III. COMBINED COMPRESSIVE SAMPLING AND DISTRIBUTION DISCONTINUITIES DETECTION

### A. Compressed Sensing

Let  $x \in \mathbb{R}^N$  be a signal with expansion in an orthonormal basis  $\Psi$  as

$$x(t) = \sum_{j=0}^{N-1} \alpha_j \psi_j(t), \quad t = 0, \dots, N-1 \quad (9)$$

where  $\Psi$  is the  $N \times N$  matrix with the waveforms  $\psi_j$  as rows. To use convenient matrix notations we can write the decomposition as  $x = \Psi \alpha$  or equivalently,  $\alpha = \Psi^* x$  where  $\Psi^*$  denotes conjugate transpose of  $\Psi$ . A signal  $x$  is sparse in the  $\Psi$  basis if the coefficient sequence  $\alpha$  is supported on a small set. We say that a vector  $\alpha$  is  $S$ -sparse if its support  $\{j : \alpha_j \neq 0\}$  is of cardinality less or equal to  $S$  [3]. Consider that we would like to recover all the  $N$  coefficients of  $x$ , vector  $\alpha$ , from measurements  $y$  about  $x$  of the form

$$y_m = \langle x, \phi_m \rangle = \sum_{n=0}^{N-1} \phi_{mn} x[n], \quad m = 0, \dots, M-1 \quad (10)$$

or

$$y = \Phi x = \Phi \Psi \alpha = \Theta \alpha \quad (11)$$

where we are interested in the case that  $M \ll N$ , and the rows of the  $M \times N$  sensing matrix  $\Phi$  are incoherent with the columns of  $\Psi$ . Then it is shown that signal  $x$  can accurately and sometimes exactly be recovered, considering that the recovered signal  $x^*$  is given by  $x^* = \Psi \alpha^*$ , and  $\alpha^*$  is the solution to the convex optimization program

$$\min_{\tilde{\alpha} \in \mathbb{R}^N} \|\tilde{\alpha}\|_{l_1} \quad \text{subject to} \quad \Phi \Psi \tilde{\alpha} = \Theta \tilde{\alpha} = y \quad (12)$$

where  $\|\tilde{\alpha}\|_{l_1} := \sum_{j=1}^N |\tilde{\alpha}_j|$ . The compressed sensing (CS) theory states that there exists a measuring factor  $c > 1$  such that only  $M := cS$  incoherent measurements  $y$  are needed to recover  $x$  with high probability. We also have to mention that except  $l_1$ -minimization solution other methods such as greedy algorithms in [18] exist for recovering the sparse signal [3], [6], [9], [10], [19], [20].

In case of noisy measurements, i.e.,  $y = \Phi x + e$ , where  $e$  is noise with  $\|e\|_{l_2} \leq \epsilon$ , [6] shows that solution to

$$\min_{\tilde{\alpha} \in \mathbb{R}^N} \|\tilde{\alpha}\|_{l_1} \quad \text{subject to} \quad \|\Theta \tilde{\alpha} - y\|_{l_2} \leq \epsilon \quad (13)$$

recovers the sparse signal with an error at most proportional to the noise level. Also, [6] discuss the conditions for stable recovery from noisy measurements.

We are interested in doing the spectrum holes detection using algebraic approach *directly* from the compressed measurements without reconstructing the original signal itself. For this reason we must find out the appropriate sensing matrix according to the detection technique. The proposed detection technique is a linear algebraic algorithm. This technique uses the Fourier transform of the observed signal to detect the occupied frequency bands in the observed spectrum. Therefore the compressed measurements of the observed signal must keep the linearity and properties of the original signal in order to apply the detection algorithm successfully on the compressed measurements. To find the sensing matrix we start by looking at the Fourier transform of the signal  $x \in \mathbb{R}^N$ .

$$X_l = \sum_{n=0}^{N-1} x[n] \exp(-\omega l n), l = 0, \dots, N-1 \quad (14)$$

where  $\omega = \frac{2\pi i}{N}$  and  $i$  is the imaginary unit. The Fourier transform of the measured signal is

$$Y_k = \sum_{m=0}^{M-1} y[m] \exp(-\omega k m), k = 0, \dots, M-1. \quad (15)$$

From (11) we replace  $y[m]$  and we have

$$Y_k = \sum_{m=0}^{M-1} \left( \sum_{n=0}^{N-1} \phi_{mn} x[n] \right) \exp(-\omega k m), k = 0, \dots, M-1 \quad (16)$$

where  $\phi_{mn}$  denotes the element of  $\Phi$  at the cross of row  $m$  and column  $n$ . Then by linearity properties we have

$$Y_k = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \phi_n[m] \exp(-\omega k m) x[n], k = 0, \dots, M-1 \quad (17)$$

where  $\phi_n[m]$  denotes the  $m^{\text{th}}$  element of the  $n^{\text{th}}$  column vector of  $\Phi$ ,  $\phi_n$ , and we see that

$$\sum_{m=0}^{M-1} \phi_n[m] \exp(-\omega k m) = \hat{\Phi}_{nk}, k = 0, \dots, M-1 \quad (18)$$

that is the Fourier transform of the  $n^{\text{th}}$  column vector of  $\Phi$ ,  $\hat{\Phi}_n$ . Then from (17) and (18)

$$Y_k = \sum_{n=0}^{N-1} \hat{\Phi}_{nk} x[n], k = 0, \dots, M-1. \quad (19)$$

And, as we said, in order to feed the detection algorithm directly by the compressed measurements we seek that

$$Y_k(\omega) = a X_l(\omega), k \in \{0, \dots, M-1\}, l \in \{0, \dots, N-1\} \quad (20)$$

where  $a > 0$  is a constant. From (19) and to satisfy (20) we find that

$$\hat{\Phi}_{nk} = a \exp(-\omega z n), z \in \{1, \dots, N\}, k = 0, \dots, M-1 \quad (21)$$

and therefore from inverse Fourier transform we have

$$\phi_n = a \delta(n - z), z \in \{1, \dots, N\} \quad (22)$$

which means that any row vector of the sensing matrix is a Dirac function, that is, only one column of each row is nonzero.

Now that the general format of the sensing matrix is clear, we should find a way to generate it. The  $\Phi^T$  matrix can be generated by randomly selecting  $M$  columns of an identity matrix  $I_N$ .  $\Phi$  is given by transpose of  $\Phi^T$ , and we define  $a = 1$  to make sure that the columns of the sensing matrix are unit-normed. So the sensing matrix  $\Phi$  that we achieved has a form like this

$$\Phi \sim \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 \end{bmatrix}_{M \times N}. \quad (23)$$

This form of sensing matrix gives us the opportunity to use the compressed measurements directly as input to the algebraic detection algorithm and thus avoiding the computation complexity of reconstructing the original signal. Following, the algebraic detection technique with compressed measurements as the input to the algorithm is explained.

### B. Algebraic Detection Based on Compressive Sampling

The algebraic detection (AD) is a new approach based on advanced differential algebra and operational calculus. In this method, the primary user's presence is rather casted as a change point detection in its transmission spectrum [21]. In this approach, the mathematical representation of the spectrum of the compressed measurements, i.e., the observed signal  $Y_n$  in frequency domain, is assumed to be a piecewise  $P^{\text{th}}$  polynomial signal expressed as following:

$$Y_n = \sum_{k=1}^K \mathcal{Y}_k[n_{k-1}, n_k](f) p_k(n - n_{k-1}) + E_n \quad (24)$$

where  $\mathcal{Y}_k[n_{k-1}, n_k]$  is the characteristic function,  $p_k$  is a polynomial series of order  $P$ ,  $E_n$  is the additive corrupting noise,  $K$  is the number of subbands defined in the frequency range of observation interest, and  $n = \frac{f}{f_s}$  is the normalized frequency, where  $f_s$  is the sampling frequency and  $f$  is the signal frequency.

Let us define the clean version of the received signal  $S_n$  as:

$$S_n = \sum_{k=1}^K \mathcal{Y}_k[n_{k-1}, n_k](f) p_k(n - n_{k-1}) \quad (25)$$

And let  $b$ , the frequency band, is such that one and only one change point occurs in the interval  $I_b = [n_{k-1}, n_k] = [\nu, \nu + b]$ ,  $\nu \geq 0$ . Denoting  $S_\nu(n) = S(n + \nu)$ ,  $n \in [0, b]$  as the restriction of the signal in the interval  $I_b$  and redefine the change point  $n_\nu$  relatively to  $I_b$  such as:

$$\begin{cases} n_\nu = 0 & \text{if } S_\nu \text{ is continuous} \\ 0 < n_\nu \leq b & \text{otherwise} \end{cases} \quad (26)$$

Then, the primary user presence on a sensed sub-band is equivalent to find  $0 < n_\nu \leq b$  on that band. The AD gives the opportunity to build a whole family of spectrum sensing detectors, depending on a given model order  $P$ . Depending on this model order, we can show that performance of the AD is increasing as the order  $P$  increases.

The proposed algorithm is implemented as a filter bank which composed of  $P$  filters mounted in a parallel way. The impulse response of each filter is:

$$h_{k+1,n} = \begin{cases} \frac{(n^l(b-n)^{P+k})^{(k)}}{(l-1)!}, 0 < n < b \\ 0, otherwise \end{cases} \quad (27)$$

where  $k \in [0 \cdots P-1]$  and  $l$  is chosen such that  $l > 2 \times P$ . The proposed expression of  $h_{k+1,n}$ ,  $k \in [0 \cdots P-1]$  is determined by modeling the spectrum with a piecewise regular signal in frequency domain and casting the problem of spectrum sensing as a change point detection in the primary user transmission [21]. Finally, in each detected interval  $[n_{\nu_i}, n_{\nu_{i+1}}]$ , we compute the following equation:

$$\lambda_{k+1} = \sum_{m=n_{\nu_i}}^{n_{\nu_{i+1}}} W_m h_{k+1,m} X_m \quad (28)$$

where  $M$  is the number of samples of the observed signal,  $W_m$  is the weight for numeric integration defined by:

$$\begin{cases} W_0 = W_M = 0.5 \\ W_m = 1 \quad otherwise \end{cases} \quad (29)$$

In order to infer whether the primary user is present in its interval, a decision function is computed as following:

$$Df = \left\| \prod_{k=0}^P \lambda_{k+1}(n_\nu) \right\| \quad (30)$$

The decision is made by comparing the threshold  $Th$  to the mean value of the decision function over the detected intervals.

#### IV. SIMULATIONS

In this section we investigate the performance of the proposed algorithm in comparison with the energy detector (ED). First we consider a frequency band in the range of [50, 250]MHz, in order to compare the compressive sensing using the algebraic method and the wavelet approach introduced in [22]. The signal is fully described in [22]. During the observed burst of transmissions in the network, there 6 bands, with frequency boundaries at  $n_{\nu_{n=0}}^6 = [50, 120, 170, 200, 220, 224, 250]$ MHz.

Comparing with the wavelet approach, in the algebraic detection technique change points are detected only in one shot, while in the wavelets approach, many detections have to be conducted and fused to make a final decision.

Figure 2 shows the algebraic detection performance on this signal. Now, comparing the proposed compressed sensing algorithm to the reference algorithm, let us give some key notes on the ED. ED is the most common method for spectrum sensing because of its non-coherency and low complexity. The

energy detector measures the received energy during a finite time interval and compares it to a predetermined threshold. That is, the test statistic of the energy detector is:

$$\sum_{m=1}^M \|y_m\|^2 \quad (31)$$

where  $M$  is the number of samples of the received signal  $y$ . Traditional ED can be simply implemented as a spectrum analyzer. A threshold used for primary user detection is highly susceptible to unknown or changing noise levels. Even if the threshold would be set adaptively, presence of any in-band interference would confuse the energy detector.

Since the complexity of sensing algorithms is a major concern in implementation and ED is well known for its simplicity, we choose ED as the comparison reference. Denoting  $N$  the number of Nyquist samples of the observed signal  $y$  and  $P$  the model order of AD, we show that the complexity of AD is  $PN$  and the complexity of ED is  $N$ . From these results, we clearly see that the exploited sensing algorithm has a comparable complexity to the energy detector. For the proposed AD based compressed sensing algorithm, the complexity is equal to:  $P \frac{M}{N} N = PM$ , where  $M$  is the number of compressed measurements of the received signal and  $M \ll N$ .

Table I summarizes the complexity of each detector.

Sensing technique	Complexity
Energy detector	N
Algebraic detector	PN
Combined compressive sampling and distribution discontinuities detection	PM

TABLE I  
COMPLEXITY COMPARISON OF THE THREE SENSING TECHNIQUES;  $M \ll N$ .

In order to achieve realistic and well founded simulations, DVB-T signals based on DVB-T 2K recommendations are used as the signals to be sensed. This choice can be justified by the fact that almost all licensed primary networks are DVB-T and secondary users are CR deployed in these networks. The signal parameters are given in Table II. Figure 3 shows the performance of the following simulated detectors: energy

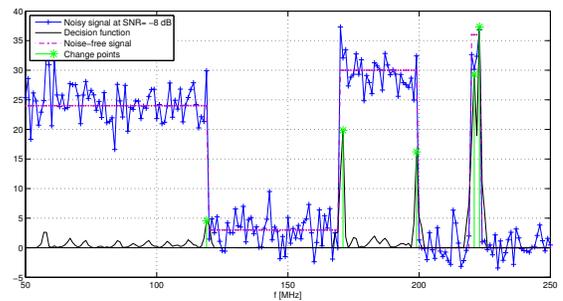


Fig. 2. Edge detection using the algebraic technique. The signal in red is the original signal, the one in blue is the noisy observation with SNR=-8dB. The black signal is the computed decision function and the green stars are the detected change points.

Bandwidth	8MHz
Mode	2K
Guard interval	1/4
Channel models	AWGN
Frequency-flat	Single path
Sensing time	1.25ms

TABLE II  
THE TRANSMITTED DVB-T PRIMARY USER SIGNAL PARAMETERS

detector (ED), first order algebraic detector ( $AD_1$ ),  $AD_1$  with compression rate of  $M/N = 20\%$ ,  $30\%$ ,  $40\%$  and  $50\%$  and second order algebraic detector ( $AD_2$ ) with  $M/N = 50\%$ . We note that ED,  $AD_1$  and  $AD_2^{50\%}$  all have the same complexity and the figure 3 shows that  $AD_2^{50\%}$  have a much better performance than ED and at low SNRs it is outperforming  $AD_1$ .

Another key metric in the sensing problems is the receive operating characteristics (ROC) curve which helps giving an idea about the reliability of the proposed technique. For instance we plot the ROC curve at  $SNR = -25dB$  for ED,  $AD_1$  and  $AD_2^{50\%}$ .

Figure 4 shows how reliable the compressed sensing technique is, as the detector operates at high probability of detection under a low false alarm rate.

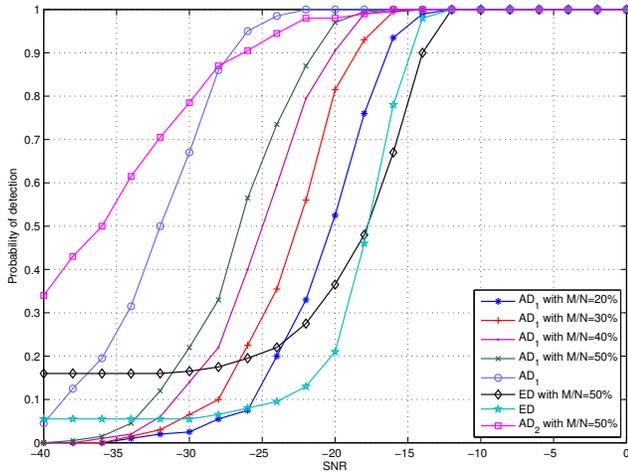


Fig. 3.  $P_D$  vs. SNR at  $P_{Fa}=0.05$ ;  $AD_P$ : Algebraic detection of order  $P$ ; ED: Energy detector;  $\frac{M}{N}$ : Compression ratio.

## V. CONCLUSION

We present in this work a new sensing technique which combines compressive sampling and algebraic method to detect spectrum holes. In a first step, we designed a compressed sensing matrix which keeps the linear properties of the sampled primary signal. Then, we applied the compressed measurements to algebraic detector to localize spectrum distribution discontinuities and identify spectrum holes. The analysis of the complexity of the proposed technique shows that it can be dramatically reduced when the model order of the algebraic detector increases. The performance comparison at different sampling rates shows that the new designed scheme achieves

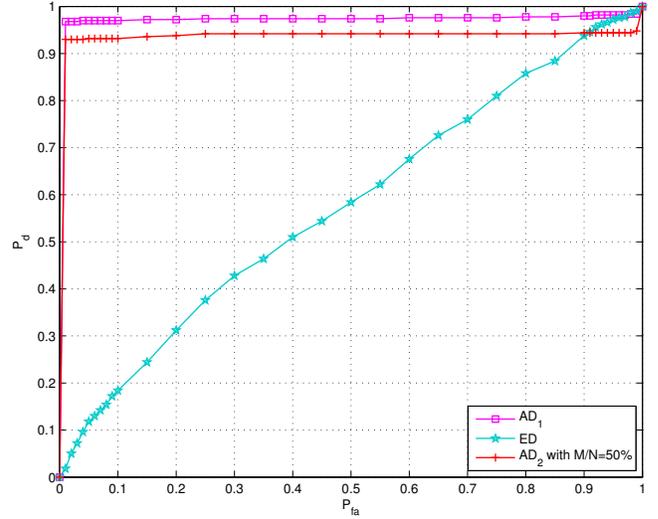


Fig. 4. ROC curve at  $SNR=-25dB$ ;  $AD_P$ : Algebraic detection of order  $P$ ; ED: Energy detector;  $\frac{M}{N}$ : Compression ratio.

better performance than energy detector while preserving a low computational complexity.

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