A COMPLETELY GENETIC APPROACH TO THE DESIGN OF LAMINATES WITH MINIMUM NUMBER OF PLIES

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ABSTRACT: *The design of laminates having the minimum number of layers for obtaining given elastic properties is addressed in the paper. The topic of this study is the fact that the problem is treated and solved in a general case, since no simplifying hypotheses are made on the type of the stacking sequence. The problem is stated as a non-linear programming problem, where a unique objective function includes all the requirements to be satisfied by the solutions. The optimal solutions are found in the framework of the polar-genetic approach, since the objective function is written in terms of the laminate's polar parameters, while a non-classical genetic algorithm is used as optimization scheme. The optimization variables include the number of layers, layer orientations and layer thicknesses. In order to include the number of plies among the design variables, some modification of the genetic algorithm have been done, and new genetic operators have been developed. Some examples and numerical results, concerning the design of laminates with the minimum number of layers for obtaining some prescribed elastic symmetries, like orthotropy, bending-extension uncoupling, quasi-homogeneity and so on, are shown in the paper.*

1. INTRODUCTION

The design of elastic properties is very important in many applications, e.g. for aircraft and space structures. Unlike classical materials, composite laminates can be designed to obtain certain properties such as bending-extension uncoupling, in-plane and/or bending orthotropy, isotropy and so on. This can be mainly done by a correct design of the stacking sequence of the laminate's plies. The problem of tailoring a composite plate to realize a given elastic or hygral-thermal-elastic behaviour has attracted the attention of several researchers. A rather complete state of the art, at least for what concerns the design with respect to stiffness, can be found in the two recent papers by Ghiasi *et al* [1, 2]. The design of laminates considered as an optimisation problem is rather cumbersome and difficult to be solved due to the high non-linearity and non-convexity of the objective function; these circumstances are brought by the fact that the laminate's properties depend upon a combination of powers of the circular function of the layers orientations, these last being normally the natural design variables. As a consequence, designers generally limit the search of solutions to a restricted class of laminates, usually to symmetric stacking sequences to ensure bending-extension uncoupling, or balanced sequences to have in-plane orthotropy and so on.

The problem of designing laminates as an optimisation problem has received a general formulation, especially concerning the design of elastic symmetries, with the works of Vannucci, Vincenti and Verchery [3, 4, 5]. They have shown that it is possible to construct, through the so-called "polar method", a unique objective function which is able to take into account several design criteria, e.g. elastic properties, such as uncoupling, orthotropy and many others, and given thermal responses in extension and/or in bending. The general problem is therefore reduced to a classical non-linear programming problem and its solutions are the minima of a non-linear, non-convex function in the design space of the layer orientations. In these studies many optimal solutions have been found for several different problems. In all the researches made by these authors the number of plies was always fixed "a priori", the design process focusing only on the importance of the geometry of the stacking sequence, i.e. the only design variables were the layer orientation angles.

As natural continuation of [3, 4, 5, 6] the topic of this paper is a new formulation for the laminate's design with the minimum number of layers which satisfies assigned elastic symmetries. To this purpose the number and orientations of plies, as well as the thickness of each layer, are taken into account as design variables. More precisely, this paper tries to give an answer to a question which is usually left apart by designers, but which is a classical and fundamental question in any mathematical problem, i.e. the question about the existence of a solution. In the case of laminates design, this question should be: which is the minimal number of layers that guarantees the existence of at least one solution to a given problem of tailoring the elastic properties of a laminate?

To the knowledge of the authors, only in one case the minimal number of layers to obtain some prescribed properties is known exactly thanks to a theoretical result. This is the case of in-plane isotropy, solved by Werren and Norris [7]: at least three unidirectional plies are needed to obtain a laminate that will be isotropic in extension, although membrane-bending coupled. But as soon as one adds another or a different requirement, like for instance uncoupling or bending isotropy, the result is unknown. Finding the minimum number of layers for which a given optimum laminate's design problem can be solved is actually a very difficult task. In fact, the minimal number of layers varies with the type of elastic requirements to be obtained: the results are strictly problem-dependent and unfortunately in all the cases, the optimal solutions are unknown and there is no analytical model describing their evolution with the number of layers. Therefore, a numerical investigation seems to be an appropriate approach for investigation.

An important remark is that the optimal design of a laminate in terms of number and properties (orientation, material and thickness) of its layers is a combinatorial optimization problem, which is harder to solve for small numbers of layers. In fact, the fewer the number of plies, the smaller becomes the design space, and the number of available solutions decreases. However, solutions with minimum number of plies are important when the problem of minimum weight of laminates is addressed.

The way proposed in this paper is to formulate the problem in the form of a search for the minimum of a positive semi-definite form, including the number of the layers (n) among the variables. The function takes into account the (n) in the form of a penalization term, in order to strongly drive the search of the optimal solution towards laminates with the lowest number of layers.

The numerical technique adopted here is a genetic algorithm (GA), BIANCA, which stands for *Biologically Inspired Analysis of Composite Assemblages*, created by the authors and already used in previous research [3, 8]. This code has been specially conceived for the optimal design of laminates; here, a modified version of this code, able to include the number of layers among the design variables, is used. In this way, the code BIANCA becomes a GA able to cross not only *individuals*, but also *species*, see section 5. In order to obtain an effective formulation of the problem, the polar formalism has been used. It is based upon an algebraic formulation making use of tensor invariants for representing plane tensors, see [9, 10] and it has proven to be rather effective in the resolution of several design problems concerning laminates.

The paper is organized as follows: in the first section the general equations of the Classical Laminated Plate Theory (CLPT) are recalled and then the design problem is stated in the framework of the polar method and formulated as an optimisation problem. Further, an account of the numerical procedure, in particular a concise description of the new genetic algorithm scheme for solutions' search, is presented. Finally several numerical examples are given in order to show the effectiveness of the proposed approach and then some general considerations ends the paper.

2. GENERAL EQUATIONS

The general equations of the Classical Laminates Plate Theory (CLPT), see for instance [11], describing the behaviour of a composite laminate, in the case of zero hygro-thermal loads, are:

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\chi} \end{Bmatrix}, \tag{1}$$

where:

$$\mathbf{N} = \begin{Bmatrix} N_x \\ N_y \\ N_s \end{Bmatrix}, \quad \mathbf{M} = \begin{Bmatrix} M_x \\ M_y \\ M_s \end{Bmatrix}, \quad \boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_s \end{Bmatrix}, \quad \boldsymbol{\chi} = \begin{Bmatrix} \chi_x \\ \chi_y \\ \chi_s \end{Bmatrix} \tag{2}$$

In Eq. (1) the classical Voigt notation is assumed and all the quantities are expressed in the laminate's global frame, i.e. $R=\{O; x, y, z\}$ (see Fig.1). **N** and **M** are the in-plane forces and bending moment tensors, $\boldsymbol{\varepsilon}$ is the in-plane strains tensor, for the middle plane, and $\boldsymbol{\chi}$ is the curvatures tensor. The **A** and **D** tensors describe, respectively, the in- and out-of-plane plate's behaviour, while **B** is the coupling tensor. For example, if **B** is not equal to zero, when in-plane forces are applied the laminate results in both in-plane strains and curvatures of the middle plane and vice-versa. The **A**, **B** and **D** tensors depend on layer's mechanical and geometrical properties such as material, orientation and thickness. For a multilayer laminate with n plies the expression of **A**, **B** and **D** are:

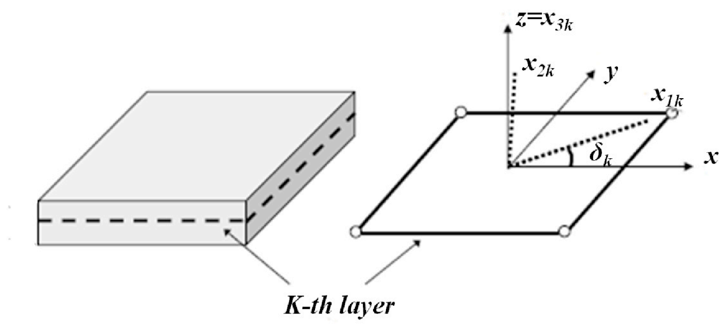


Fig. 1 Laminate reference systems

$$
\begin{aligned}
\mathbf{A} &= \sum_{k=-p}^{p} \mathbf{Q}_k(\delta_k)(z_k - z_{k-1}) \quad , \\
\mathbf{B} &= \frac{1}{2}\sum_{k=-p}^{p} \mathbf{Q}_k(\delta_k)(z_k^2 - z_{k-1}^2) \; , \\
\mathbf{D} &= \frac{1}{3}\sum_{k=-p}^{p} \mathbf{Q}_k(\delta_k)(z_k^3 - z_{k-1}^3) \; .
\end{aligned}
\tag{3}
$$

$\mathbf{Q}_k(\delta_k)$ is the k^{th} ply's reduced stiffness tensor oriented at the angle δ_k with respect to the global reference of the laminate, see Fig. 2; n is the number of plies and it is defined as:

$$
n = \begin{cases} 2p & \text{if even} \\ 2p+1 & \text{if odd} \end{cases}
\tag{4}
$$

z_k and z_{k-1} are the z coordinates of the top and bottom k^{th} layer's surfaces. Fig. 2 illustrates the definition of z_k used in this work. The homogenized stiffness tensors of the laminate, $\mathbf{A}^*$, $\mathbf{B}^*$, $\mathbf{D}^*$ and then, the homogeneity tensor **C,** are defined as:

$$
\begin{aligned}
\mathbf{A}^* &= \frac{1}{h_{tot}}\mathbf{A} \quad , \\
\mathbf{B}^* &= \frac{2}{h_{tot}^2}\mathbf{B} \quad , \\
\mathbf{D}^* &= \frac{12}{h_{tot}^3}\mathbf{D} \quad , \\
\mathbf{C} &= \mathbf{A}^* - \mathbf{D}^* \quad ,
\end{aligned}
\tag{5}
$$

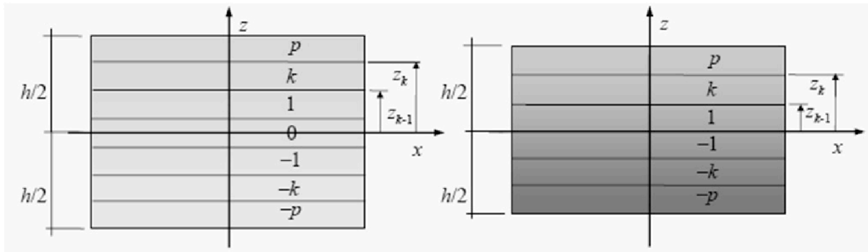
where h_{tot} is the laminate's total thickness. The inverse of Eq. (1) is:

$$
\begin{Bmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\chi} \end{Bmatrix} = \begin{bmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{b}^{\mathrm{T}} & \mathbf{d} \end{bmatrix} \begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix},
\tag{6}
$$

where:

$$
\begin{aligned}
\mathbf{a} &= \left(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{B}\right)^{1} \quad , \\
\mathbf{b} &= -\,\mathbf{a}\mathbf{B}\mathbf{D}^{-1} \quad , \\
\mathbf{d} &= \left(\mathbf{D} - \mathbf{B}\mathbf{A}^{-1}\mathbf{B}\right)^{1} \quad .
\end{aligned}
\tag{7}
$$

In Eq. (7) **a**, **b** and **d** are the in-plane, coupling and bending compliance tensors.



n odd *n* even

Fig. 2 Sketch of the laminate layers and interfaces.

3. POLAR REPRESENTATION METHOD

The polar representation method is a mathematical technique introduced in 1979 by Verchery [9]. By this method it is possible to represent a plane tensor of any order by its invariants. This technique has already been used to treat many optimal design problems of laminates. A complete overview of the polar method can be found in [12]. There are different ways to represent a tensor and the Cartesian representation is the most widely used. The main drawback of the Cartesian method is that the components are frame-dependent.

The idea of Verchery is to substitute these components by other parameters which are frame-independent, i.e. tensor invariants. There are several ways to choose these invariants. The quantities introduced by Verchery are directly linked to the elastic symmetries and to strain energy decomposition [12].

In the case of a fourth rank elasticity plane tensor **L**, like layer's stiffness tensor **Q** or layer's compliance tensor $\mathbf{S}=\mathbf{Q}^{-1}$, the polar method ensures that the Cartesian components can be expressed by 4 moduli, T_0, T_1, R_0, R_1 and 2 polar angles Φ_0 and Φ_1. The expression of the tensor's components **L** in its material frame $\{O;\, x_1, x_2, x_3\}$ are:

$$
\begin{aligned}
L_{1111} &= T_0 + 2T_1 + R_0\cos 4\Phi_0 + 4R_1\cos 2\Phi_1 ,\\
L_{1112} &= R_0\sin 4\Phi_0 + 2R_1\sin 2\Phi_1 ,\\
L_{1122} &= -T_0 + 2T_1 - R_0\cos 4\Phi_0 ,\\
L_{1212} &= T_0 - R_0\cos 4\Phi_0 ,\\
L_{1222} &= -R_0\sin 4\Phi_0 + 2R_1\sin 2\Phi_1 ,\\
L_{2222} &= T_0 + 2T_1 + R_0\cos 4\Phi_0 - 4R_1\cos 2\Phi_1 .
\end{aligned} \tag{8}
$$

A possible expression of an elasticity tensor's norm, as function of polar parameters, is the following one:

$$\|\mathbf{L}\| = \sqrt{T_0^2 + 2T_1^2 + R_0^2 + 4R_1^2}\ . \tag{9}$$

The converses of Eq. (8) are:

$$
\begin{aligned}
8T_0 &= L_{1111} - 2L_{1122} + 4L_{1212} + L_{2222} ,\\
8T_1 &= L_{1111} + 2L_{1122} + L_{2222} ,\\
8R_0\,e^{4\mathrm{i}\Phi_0} &= L_{1111} + 4\mathrm{i}\,L_{1112} - 2L_{1122} - 4L_{1212} - 4\mathrm{i}\,L_{1222} + L_{2222} ,\\
8R_1\,e^{2\mathrm{i}\Phi_1} &= L_{1111} + 2\mathrm{i}\,L_{1112} + 2\mathrm{i}\,L_{1222} - L_{2222} .
\end{aligned} \tag{10}
$$

The polar moduli T_0, T_1, R_0, R_1 and the angular difference Φ_0 - Φ_1 are the tensor's invariants. A frame's rotation through an angle δ causes the following change of the tensor's components:

$$
\begin{aligned}
L_{xxxx} &= T_0 + 2T_1 + R_0\cos 4(\Phi_0-\delta) + 4R_1\cos 2(\Phi_1-\delta),\\
L_{xxxy} &= R_0\sin 4(\Phi_0-\delta) + 2R_1\sin 2(\Phi_1-\delta),\\
L_{xxyy} &= -T_0 + 2T_1 - R_0\cos 4(\Phi_0-\delta),\\
L_{xyxy} &= T_0 - R_0\cos 4(\Phi_0-\delta),\\
L_{xyyy} &= -R_0\sin 4(\Phi_0-\delta) + 2R_1\sin 2(\Phi_1-\delta),\\
L_{yyyy} &= T_0 + 2T_1 + R_0\cos 4(\Phi_0-\delta) - 4R_1\cos 2(\Phi_1-\delta).
\end{aligned}
\tag{11}
$$

The above equations show that a rotation changes the polar angles Φ_0 and Φ_1 into Φ_0 - δ and Φ_1 - δ and that T_0, T_1 describe the tensor's isotropic part, while R_0, R_1 the anisotropic one. These are two reasons that make the polar method very effective when it is applied to represent layer's stiffness and compliance tensors in the framework of CLPT theory, especially for design problems. In particular, conditions for elastic symmetries are expressed in a simple way when using polar invariants and a summary of these conditions is given in Table 1. We recall here that square symmetry is the corresponding, in plane elasticity, of the cubic syngony, i.e. it is characterized by a periodicity of $\pi/2$ of the elastic moduli, while R_0-orthotropy is a special case of plane orthotropy, see [13] for more details.

In this paper T_0, T_1, R_0, R_1, Φ_0 and Φ_1 will denote the polar components of the reduced stiffness tensor of the basic layer, **Q**. It is possible to deduce the polar components of laminate's stiffness tensors **A**, **B** and **D** as function of polar components of the lamina's stiffness tensor **Q** according to Eq. (3) of the CLPT.

$$
\begin{aligned}
T_0^A, T_0^B, T_0^D &= \frac{1}{m}\sum_{k=-p}^{p} T_{0_k}\left(z_k^m - z_{k-1}^m\right),\\
T_1^A, T_1^B, T_1^D &= \frac{1}{m}\sum_{k=-p}^{p} T_{1_k}\left(z_k^m - z_{k-1}^m\right),\\
R_0^A\,e^{4i\Phi_0^A}, R_0^B\,e^{4i\Phi_0^B}, R_0^D\,e^{4i\Phi_0^D} &= \frac{1}{m}\sum_{k=-p}^{p} R_{0_k}\,e^{4i(\Phi_{0k}+\delta_k)}\left(z_k^m - z_{k-1}^m\right),\\
R_1^A\,e^{2i\Phi_1^A}, R_1^B\,e^{2i\Phi_1^B}, R_1^D\,e^{2i\Phi_1^D} &= \frac{1}{m}\sum_{k=-p}^{p} R_{1_k}\,e^{2i(\Phi_{1k}+\delta_k)}\left(z_k^m - z_{k-1}^m\right).
\end{aligned}
\tag{12}
$$

where T_0^A, T_1^A, R_0^A, R_1^A, Φ_0^A and Φ_1^A stand for polar components of tensor **A**, T_0^B, T_1^B, R_0^B, R_1^B, Φ_0^B and Φ_1^B stand for polar components of tensor **B** and T_0^D, T_1^D, R_0^D, R_1^D, Φ_0^D and Φ_1^D stand for polar components of tensor **D**. In Eq. (12), m=1, 2, 3 for the extension, coupling and bending stiffness tensor respectively.

From Eq. (12), it can be noticed that the symmetries of the laminate in terms of extension, coupling or stiffness behaviour depend on the stacking sequence, i.e. on layer properties, orientation, thickness and of course on the number of plies. When concerned with laminate's design, a designer has to satisfy several conditions at the same time, including not only common objectives, like buckling load or strength, but also general properties of the elastic response of the laminate, such as uncoupling, extension orthotropy, bending orthotropy and so on. In fact, it is not easy to take into account all these aspects, and normally designers use some shortcuts (rules of thumb), to get automatically some properties like uncoupling or extension orthotropy. Vannucci and Vincenti have shown in previous studies (see [4, 5, 6]) that it is possible, in the framework of the polar method, to formulate all the problems of optimal design of laminates including the requirements on elastic symmetries; therefore, a general approach to the design of laminates is possible. The reader is addressed to the works previously cited for a deeper insight in the matter. The next section presents an important modification to this approach that also includes the number of the layers among the design variables.

4. FORMULATION OF THE OPTIMISATION PROBLEM

In order to formulate the design of laminate's elastic properties as an optimisation problem, the key point is the construction of the objective function. For a laminate with n plies the design variables can be: the number of layers n, the vector of layer's orientation $\boldsymbol{\delta}$, the vector of layer's thickness $\mathbf{h}$. In order to formulate a laminate design problem in a general way, the objective function $f=f\,(n,\,\boldsymbol{\delta},\,\mathbf{h})$ should include all the design requirements, and in particular elastic symmetries of the laminate.

In the framework of the polar method, Vannucci and Vincenti [3, 4, 5, 6] showed that the laminate's design for given elastic properties can be reduced to the search of the minima of a positive semi-definite function of the laminate's polar elastic parameters. In those works, the number of layers, layer thickness and material properties were fixed, whilst the orientations of layers were assumed as the only optimisation variables. The optimisation problem was defined as:

$$\min_{\boldsymbol{\delta}}\left[f(\boldsymbol{\delta})\right]. \tag{13}$$

Since the objective function $f\,(\boldsymbol{\delta})$ is positive semi-definite, as mentioned above, its minima are also zeros of the function. For more details on the definition of this objective function for different combinations of elastic symmetries, see [3, 4, 5, 6].

As specified previously, the objective of this paper is the design of a laminate having assigned symmetries with the minimum number of layers. In such a case, the number of the plies and, eventually, the thickness of each layer must belong to the set of design variables: a modification of the objective function is necessary. The new unconstrained optimisation problem is:

$$\begin{gathered}\min_{n,\boldsymbol{\delta},\mathbf{h}}\left[F(n,\boldsymbol{\delta},\mathbf{h})\right]\\ \text{with :}\\ F(n,\boldsymbol{\delta},\mathbf{h})=f(n,\boldsymbol{\delta},\mathbf{h})n^{s}\end{gathered} \tag{14}$$

It can be noticed that the new objective function $F\,(n,\,\boldsymbol{\delta},\,\mathbf{h})$ is defined in such a way that it is a positive semi-definite function, whose zeros are solutions of our problem and also zeros of the function. The influence of the number of layers n is introduced as a penalization term, where s is a power whose value can be chosen in a certain range. The large number of numerical tests that the authors have been conducted, show that the best results are obtained when s is in the interval [1;4].

Eq. (14) formalizes a classical non-linear programming problem without constraints for which several numerical solving techniques are available. It can be noticed that, being $f\,(n,\,\boldsymbol{\delta},\,\mathbf{h})$ a non-convex function with several non-global minima, a suitable and robust solving algorithm must be chosen.

As a concluding remark of this section, it can be noticed that the proposed approach is general, i.e. no simplifying assumptions are introduced such as, for example, the limitation of the search for the laminates only to the set of symmetric, balanced, cross-ply or angle-ply stacking sequences.

5. SOLUTION'S SEARCH BY THE GENETIC ALGORITHM BIANCA

Some general considerations have determined the choice of a GA as numerical technique for the search of solutions to the optimisation problem expressed by Eq. (14). The key point is the objective function's nature which is a highly non-linear and non-convex function. Previous works, see for instance [14,15], showed that, due to the non-linearity of the problem, in many cases there are several sets containing an infinite number of solutions, that describe geometrical sets as continuous hyper-surfaces in the design space. Another very important aspect is the non-convexity of the problem. In fact, it is the rule that in problems concerning the optimal design of laminates there are many local and global minima: a GA is able to look for the global minima exploring the whole research space in an effective way.

Another important point is that, in the case of laminates, problems often depend on continuous, discrete or abstract variables. For example, layer orientations can be considered as continuous or discrete variables; for instance, angles can take all the values in the range [-90°, 90°] or they can be restricted to belong to a limited and discrete set of fixed orientations, such as 0°/ ±45°/ 90°. The same considerations can be done for the layer thickness. Finally an abstract variable can represent a group of different variables, such as in the case of the constitutive material of the layer, when the material is chosen within a database: once a particular material is associated to a ply, the whole set of the properties of the layer are determined, i.e. thickness, technical moduli and so on. Therefore, a suitable numerical technique should handle with the same effectiveness the different types of design variables; GAs can deal with all these aspects because they basically are zero-th order numerical strategies, which need only computations of the objective function.

A GA is a meta-heuristic method for solving optimisation problems, which is based upon the two biologic metaphors of natural evolution, according to the Darwinian principle of the survival of the fittest, and of genetic coding of the phenotype. A wide and detailed presentation of genetic algorithm can be found in the books of Goldberg [16] and Michalewicz [17].

Vannucci and Vincenti have developed a GA, BIANCA [8, 18], specially conceived to solve many optimisation problems concerning laminates. BIANCA is characterised by several original features, mainly concerning the representation of information, i.e. the coding of the genotype of the individual. This program applies all the classical genetic operators such as adaptation and selection, cross-over, mutation and elitism operators. In addition, specific strategies were developed within BIANCA in order to take into account constrained optimisation problems. We give here only some accounts of the code BIANCA, and more details can be found in [8, 18].

Unlike most of the GAs dedicated to laminates' problems, where generally a unique string of real variables composes a sort of surrogate of the genetic code of the laminate, in BIANCA the information concerning the laminate has an articulated structure: a laminate is considered as an individual *phenotype* whose genome *genotype* is stocked in a binary coded matrix. Each row of the matrix is a chromosome, coding all the data concerning a layer. In BIANCA a chromosome corresponds to each layer. In this way, the number of layers determines, in the biological metaphor, a *species* (in the sense that laminates with different number of layers belong to different *biological species*). Each chromosome is divided into *genes*; a gene codes a particular quantity concerning the chromosome, i.e. the layer, such as, e.g., orientation angle, material, thickness, etc. Corresponding genes for the various chromosomes form the columns of the genome matrix. It is possible to code real and discrete variables in the same genome at the same time. As programmed in BIANCA, the classical genetic operators, cross-over and mutation, mimic very closely Nature: they act on each gene separately and independently.

5.1 The modified genetic algorithm BIANCA

In order to include in the objective function the design with respect to the layer's number, we introduced some modifications in the code. As said previously, in BIANCA the number of the layers n determines the number of chromosomes, i.e. the biological species of the layer. So, if n has to be included among the variables, and has to evolve during generations, a cross-over of the species has to be performed: the new version of BIANCA is a *GA based upon the evolution of species and of individuals at the same time*. The modifications of the code BIANCA are substantially new genetic operators. To this purpose, we have been partially inspired by a previous work of Park *et al* [19]. In our case, their strategy was modified in order to be adapted to a scheme of genetic coding and genetic operations much more complicate and rich than those proposed in [19].

In particular, the classical reproduction phase, see Fig. 3, has been changed introducing new genetic operators called "chromosome shift operator", "chromosome reorder", "chromosome number mutation" and "chromosome addition-deletion". A brief description of these new operators and their use in the reproduction phase is given in the next paragraphs.

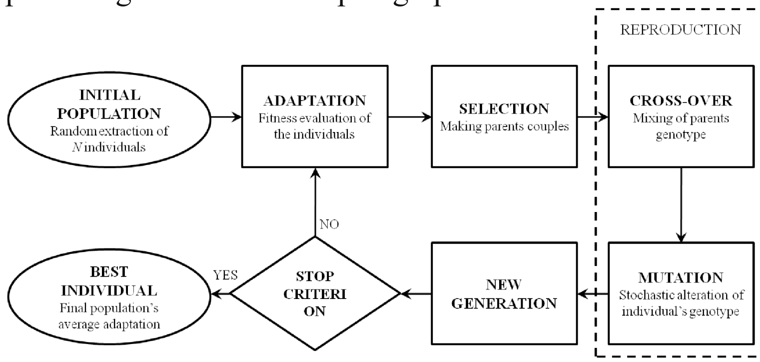


Fig. 3Standard GA scheme.

5.1.1 *The structure of the individual in BIANCA*

The first modification concerns the structure of the generic individual's genotype. As briefly said previously, in BIANCA the genotype of each individual is represented by a binary array shown in Fig. 4. In this figure the quantity g_{ij}^{k} represents the j^{th} gene of the i^{th} chromosome of the k^{th} individual. Letter *e* stands for empty location, i.e. there is no gene in this location, while *n* is the k^{th} individual's chromosomes number.

In this paper the number of layers, orientation angles and thickness are assumed as optimisation variables. The new structure of the individual-laminate is shown in Fig. 5. In this case the k^{th} laminate's number of layers is *n* while the orientation and the thickness of the i^{th} ply are δ_i and h_i, respectively.

Classical reproduction's operators, like cross-over and mutation, act on the parents couple's genotype. However, to take into account the variable number of layers, i.e. number of chromosomes, a new approach is required in executing the reproduction's phase. This new procedure is detailed below.

As mentioned previously, this new version of BIANCA is able to evolve at the same time species and individuals: the best value of *n* is the outcome of the biological selection and the most adapted species automatically issues as natural result of the Darwinian selection.

$(g_{11})^k$	$(g_{12})^k$	…	$(g_{1m})^k$	n
$(g_{21})^k$	$(g_{22})^k$	…	$(g_{2m})^k$	
…	…	…	…	
…	…	…	…	
$(g_{n1})^k$	$(g_{n2})^k$	…	$(g_{nm})^k$	
e	e	e	e	

Fig. 4 Structure of the individual genotype in BIANCA.

$(\delta_1)^k$	$(h_1)^k$	n
$(\delta_2)^k$	$(h_2)^k$	
…	…	
…	…	
$(\delta_n)^k$	$(h_n)^k$	
e	e	

Fig. 5 Individual-laminate in BIANCA.

5.1.2 *The modified cross-over phase and the role of chromosome shift and reorder operators*

To explain the way whereby the reproduction phase takes place one can consider the following case. There are two parents, *P1* and *P2*, with 3 and 5 layers respectively, see Fig. 6. In this example, for sake of synthesis, the maximum number of plies is assumed equal to 6. The minimum number can be chosen arbitrary between 1 and 6. Before realizing the cross-over among these two individuals, it can be noticed that there are different ways to pass the information restrained in the parents' genotype to the next generation, i.e. to their "children". At the successive generation two new individuals will be produced from this couple, one with 3 layers and another one with 5 layers. To improve the efficiency of the genetic algorithm in terms of exploration and exploitation of the information on the design space, the concept of "shift factor" is introduced. The shift factor is randomly sorted, with a given probability p_{shift}, in the range [0, $|n^{P1}-n^{P2}|$], where $|n^{P1}-n^{P2}|$ is the absolute value of the difference of the parents layers number. With the shift factor various combination of cross-over are possible. In the example mentioned before the minimum shift factor is 0 and the maximum is 2. For example, in the case that the shift factor is 1, all the genes of *P1*, which has smaller number of chromosomes, are shifted by a quantity equal to 1 up-to-down as shown in Fig. 7.

$(\delta_1)^{P1}$	$(h_1)^{P1}$	3
$(\delta_2)^{P1}$	$(h_2)^{P1}$	
$(\delta_3)^{P1}$	$(h_3)^{P1}$	
e	e	
e	e	
e	e	

$(\delta_1)^{P2}$	$(h_1)^{P2}$	5
$(\delta_2)^{P2}$	$(h_2)^{P2}$	
$(\delta_3)^{P2}$	$(h_3)^{P2}$	
$(\delta_4)^{P2}$	$(h_4)^{P2}$	
$(\delta_5)^{P2}$	$(h_5)^{P2}$	
e	e	

Fig. 6 Structure of the parents' couple.

e	e	3
$(\delta_1)^{P1}$	$(h_1)^{P1}$	
$(\delta_2)^{P1}$	$(h_2)^{P1}$	
$(\delta_3)^{P1}$	$(h_3)^{P1}$	
e	e	
e	e	

$(\delta_1)^{P2}$	$(h_1)^{P2}$	5
$(\delta_2)^{P2}$	$(h_2)^{P2}$	
$(\delta_3)^{P2}$	$(h_3)^{P2}$	
$(\delta_4)^{P2}$	$(h_4)^{P2}$	
$(\delta_5)^{P2}$	$(h_5)^{P2}$	
e	e	

Fig. 7 Effect of the shift operator on the parents couple.

After the shift operation the real cross-over phase takes place. The cross-over operator acts on every single gene. The position of cross-over is randomly chosen for each gene of both individuals. Naturally this operator involves all the chromosomes of the parent with the smaller number of plies, i.e. in the case shown in Fig. 8 all the genes of *P1*, while only the homologous of *P2* undergo the action of cross-over operator. At this point two new individuals can be created, *C1* and *C2* that have 3 and 5 chromosomes-layers respectively, see Fig. 9. It can be noticed that the 1st and 5th chromosome of *P2* haven't undergone the cross-over phase, so according to the notation of Figs. 8 and 9 it is possible to write the following equalities, $(\delta_1)^{P2} = (\delta_1)^{C2}$, $(\delta_5)^{P2} = (\delta_5)^{C2}$ and $(h_1)^{P2} = (h_1)^{C2}$, $(h_5)^{P2} = (h_5)^{C2}$. At this point, before the mutation phase, a readjustment of chromosomes position is required. The "chromosome reorder" operator achieves this phase by a translation of all chromosomes down-to-up in the structure of the individual with the smaller number of layers, see Fig. 10.

e	e	3
$(\delta_1)^{P1}$	$(h_1)^{P1}$	
$(\delta_2)^{P1}$	$(h_2)^{P1}$	
$(\delta_3)^{P1}$	$(h_3)^{P1}$	
e	e	
e	e	

$(\delta_1)^{P2}$	$(h_1)^{P2}$	5
$(\delta_2)^{P2}$	$(h_2)^{P2}$	
$(\delta_3)^{P2}$	$(h_3)^{P2}$	
$(\delta_4)^{P2}$	$(h_4)^{P2}$	
$(\delta_5)^{P2}$	$(h_5)^{P2}$	
e	e	

Fig. 8 Position of cross-over for every gene.

e	e	3
$(\delta_1)^{C1}$	$(h_1)^{C1}$	
$(\delta_2)^{C1}$	$(h_2)^{C1}$	
$(\delta_3)^{C1}$	$(h_3)^{C1}$	
e	e	
e	e	

$(\delta_1)^{C2}$	$(h_1)^{C2}$	5
$(\delta_2)^{C2}$	$(h_2)^{C2}$	
$(\delta_3)^{C2}$	$(h_3)^{C2}$	
$(\delta_4)^{C2}$	$(h_4)^{C2}$	
$(\delta_5)^{C2}$	$(h_5)^{C2}$	
e	e	

Fig. 9 Structure of new individuals after cross-over.

5.1.3 ***The modified mutation phase and the role of chromosomes' number mutation and addition-deletion operators***

Mutation is a random process applied to the genotype to better explore the feasibility domain. Mutation is articulated in two phases: it acts first on the number of chromosomes-layers, and then on the genes value.

In the first phase, the layers number is arbitrarily changed by one at time for each individual, with a given probability $(p_{mut})_{chrom}$, then the chromosome addition-deletion operator acts on the genotype of both individuals, by adding or deleting a chromosome-layer. The location of layer addition-deletion is also randomly selected. Naturally, if the number of layers is equal to the maximum chromosome number, only deletion occurs. Similarly if the plies' number is equal to the minimum number only addition is applied. In the case shown in Fig. 11 the number of layers of *C1* is increased by one and a new chromosome, $\left\{(\delta_a)^{C1},(h_a)^{C1}\right\}$, is randomly added in position 3, while the number of layers of *C2* is decreased by one and the chromosome deletion is randomly done in position 2.

In the second phase the mutation of the genes value, i.e. orientations and thickness, takes place, with a probability p_{mut}, after a rearrangement of chromosomes position. In the example of Fig. 12 the mutation is undergone by the genes $(\delta_2)^{C1}$ and $(h_a)^{C1}$ of the individual *C1* and by the gene $(\delta_1)^{C2}$ of the individual *C2*.

$(\delta_1)^{C1}$	$(h_1)^{C1}$	3
$(\delta_2)^{C1}$	$(h_2)^{C1}$	
$(\delta_3)^{C1}$	$(h_3)^{C1}$	
e	e	
e	e	
e	e	

$(\delta_1)^{C2}$	$(h_1)^{C2}$	5
$(\delta_2)^{C2}$	$(h_2)^{C2}$	
$(\delta_3)^{C2}$	$(h_3)^{C2}$	
$(\delta_4)^{C2}$	$(h_4)^{C2}$	
$(\delta_5)^{C2}$	$(h_5)^{C2}$	
e	e	

Fig. 10 Effect of the chromosome reorder operator on the new individuals.

$(\delta_1)^{C1}$	$(h_1)^{C1}$	4
$(\delta_2)^{C1}$	$(h_2)^{C1}$	
$(\delta_a)^{C1}$	$(h_a)^{C1}$	
$(\delta_3)^{C1}$	$(h_3)^{C1}$	
e	e	
e	e	

$(\delta_1)^{C2}$	$(h_1)^{C2}$	4
e	e	
$(\delta_3)^{C2}$	$(h_3)^{C2}$	
$(\delta_4)^{C2}$	$(h_4)^{C2}$	
$(\delta_5)^{C2}$	$(h_5)^{C2}$	
e	e	

Fig. 11 Effect of the chromosome mutation and addition-deletion operators on the new individuals.

$(\delta_1)^{C1}$	$(h_1)^{C1}$	4
$(\delta_{2m})^{C1}$	$(h_2)^{C1}$	
$(\delta_a)^{C1}$	$(h_{am})^{C1}$	
$(\delta_3)^{C1}$	$(h_3)^{C1}$	
e	e	
e	e	

$(\delta_{1m})^{C2}$	$(h_1)^{C2}$	4
$(\delta_3)^{C2}$	$(h_3)^{C2}$	
$(\delta_4)^{C2}$	$(h_4)^{C2}$	
$(\delta_5)^{C2}$	$(h_5)^{C2}$	
e	e	
e	e	

Fig. 12 Effect of the mutation operators on the new individuals.

6. SAMPLE PROBLEMS AND RESULTS

In order to demonstrate the effectiveness of the polar formulation and of the new genetic algorithm BIANCA in obtaining composite laminates with variable number of plies and with certain elastic properties, several calculations have been carried out and number of solutions that satisfy different combinations of design objectives are found. Among all the possible design cases, the following ones are discussed in this paper:

1. uncoupling, total orthotropy with K=0 and axis coincidence, i.e. in-plane and bending orthotropy with the same axes;
2. uncoupling, total orthotropy with K=1 and axis coincidence, i.e. in-plane and bending orthotropy, with the same axes;
3. uncoupling and total isotropy , i.e. in-plane and bending isotropy;
4. uncoupling and quasi-homogeneity, i.e. identical behaviour for the homogenized in-plane and bending stiffness tensors.

The corresponding polar conditions for elastic symmetries are resumed in Table 1. Here, uncoupling is intended to be the bending-extension uncoupling determined by the fact that tensor **B** is null.

6.1 Sample problems

Before formulating the expression of the objective function, for each case, it is worth to specify the general properties of F (n, $\boldsymbol{\delta}$, $\mathbf{h}$). In all the problems that are considered in the present section (cases 1 to 4), this function is a dimensionless, homogenized, convex function of the polar parameters of the tensors **A**, **B** and **D**, Eq. (12), while it is a highly non-linear, non-convex function of the design variables, i.e. number of layers, orientations and thickness. In all the cases the value of the power s in Eq. (14) is assumed equal to 2.

The objective function for each problem is defined in such a way that solutions, i.e. the minima, are also zeros of the function. Since each case corresponds to a given combination of elastic symmetries, the global objective function is constructed as a sum of partial objective functions, and values of each partial objective function are normalized between zero and unity.

Cases n. 1 and n. 2

In order to obtain elastic uncoupling, i.e. $\mathbf{B}=\mathbf{0}$, the norm of the coupling tensor **B** must be zero. In order to obtain orthotropy, the difference between the two polar angles Φ_0 and Φ_1 must be a multiple of $\pi/4$ (see the corresponding polar condition in Table 1), both for membrane and bending stiffness tensors. The last required elastic property, considered here, is the coincidence of the orthotropy axes, i.e. angle Φ_1 must be the same for **A** and **D**. The expression of the global objective function including all these conditions is:

$$F(n,\boldsymbol{\delta},\mathbf{h})=\left[\left(\frac{\left\|\mathbf{B}^*\right\|}{\left\|\mathbf{Q}\right\|}\right)^2+\left(\frac{\Phi_0^{A^*}-\Phi_1^{A^*}-K^{A^*}\frac{\pi}{4}}{\frac{\pi}{4}}\right)^2+\left(\frac{\Phi_0^{D^*}-\Phi_1^{D^*}-K^{D^*}\frac{\pi}{4}}{\frac{\pi}{4}}\right)^2+\left(\frac{\Phi_1^{A^*}-\Phi_1^{D^*}}{\frac{\pi}{4}}\right)^2\right]n^2 \quad (15)$$

In Eq. (15), $\left\|\mathbf{B}^*\right\|$ is the norm of the homogenized coupling tensor, while the normalization factor $\left\|\mathbf{Q}\right\|$ is the one of the layer's stiffness tensor. Both norms are calculated according to Eq. (9). All the other polar parameters are referred to their respective homogenized tensors, i.e. $\mathbf{A}^*$ and $\mathbf{D}^*$. The constants K^{A^*} and K^{D^*} can assume the values 0 or 1, depending upon the different kind of orthotropy (case 1: $K^{A^*}=K^{D^*}=0$; and case 2: $K^{A^*}=K^{D^*}=1$). The normalisation factor of the orthotropy and of the coincidence of the orthotropy axes requirements is assumed equal to $\pi/4$.

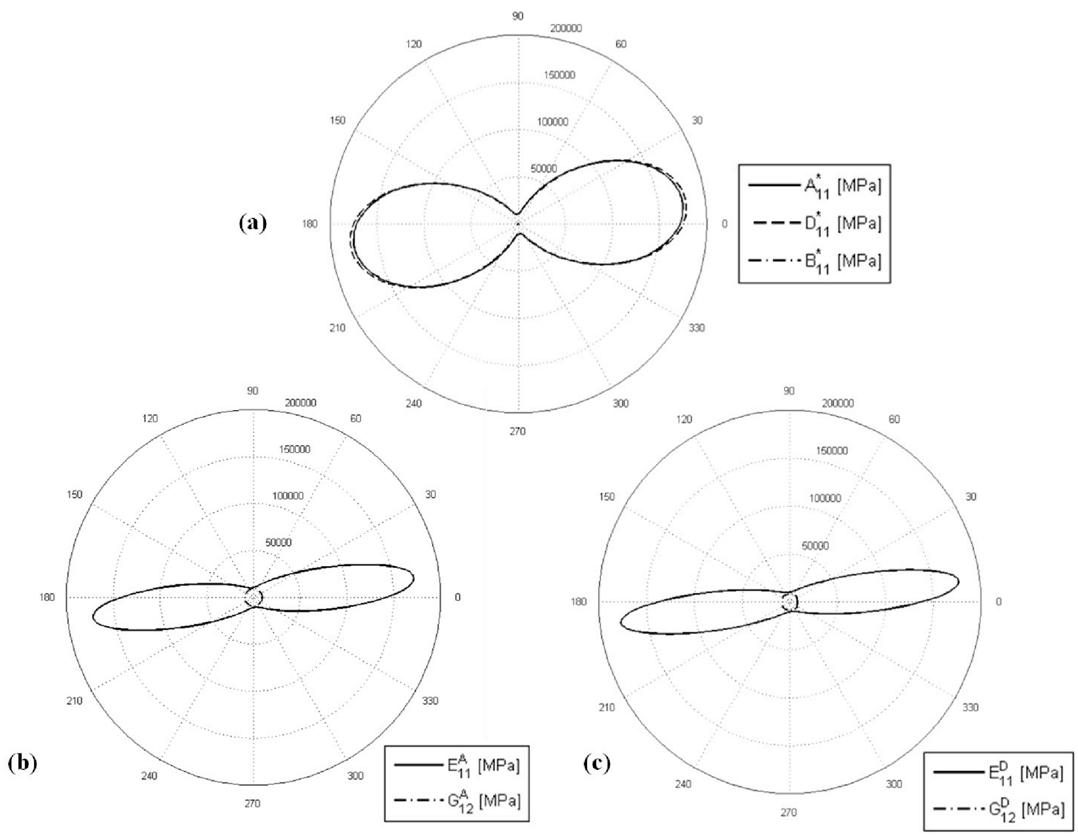


Fig. 13 Polar variations for laminate n.1, identical ply thickness. (a) Stiffness components, (b) membrane Young's modulus E_{11} and shear modulus G_{12} and(c) bending Young's modulus E_{11} and shear modulus G_{12}.

Case n. 3

In this case, along with the elastic uncoupling the total isotropy requirement has been formalized. The partial objective function for uncoupling is expressed as in cases 1 and 2, see Eq.

(15). In order to satisfy isotropy for in-plane and bending stiffness, the anisotropic part of tensors $\mathbf{A}^*$ and $\mathbf{D}^*$ must be zero. Therefore, the global objective function has the following form:

$$F(n,\boldsymbol{\delta},\mathbf{h})=\left[\left(\frac{\|\mathbf{B}^*\|}{\|\mathbf{Q}\|}\right)^2+\left(\frac{R_0^{A^*2}+4R_1^{A^*2}}{R_0^{Q2}+4R_1^{Q2}}\right)+\left(\frac{R_0^{D^*2}+4R_1^{D^*2}}{R_0^{Q2}+4R_1^{Q2}}\right)\right]n^2. \tag{16}$$

In Eq. (16), $R_0^{A^*}$, $R_1^{A^*}$ and $R_0^{D^*}$, $R_1^{D^*}$ are referred to the laminate's homogenized in-plane and bending stiffness tensor respectively, R_0^Q and R_1^Q are referred to the layer's stiffness tensor and they are used for normalization sake.

Case n. 4

In this last case, the requirements are uncoupling and quasi-homogeneity, i.e. the laminate has the same behaviour in extension and bending. To realize the objective of quasi-homogeneity, the polar parameters T_0, T_1, R_0, R_1, Φ_0 and Φ_1 must assume the same value for both the tensors $\mathbf{A}^*$ and $\mathbf{D}^*$, so the homogeneity tensor is equal to zero, $\mathbf{C}$=0. The objective function is:

$$F(n,\boldsymbol{\delta},\mathbf{h})=\left[\left(\frac{\|\mathbf{B}^*\|}{\|\mathbf{Q}\|}\right)^2+\left(\frac{\|\mathbf{C}\|}{\|\mathbf{Q}\|}\right)^2+\left(\frac{\Phi_0^C}{\frac{\pi}{4}}\right)^2+\left(\frac{\Phi_1^C}{\frac{\pi}{4}}\right)^2\right]n^2. \tag{17}$$

In Eq. (17) $\|\mathbf{C}\|$, Φ_0^C and Φ_1^C are, respectively, the norm and the polar angles of the homogeneity tensor.

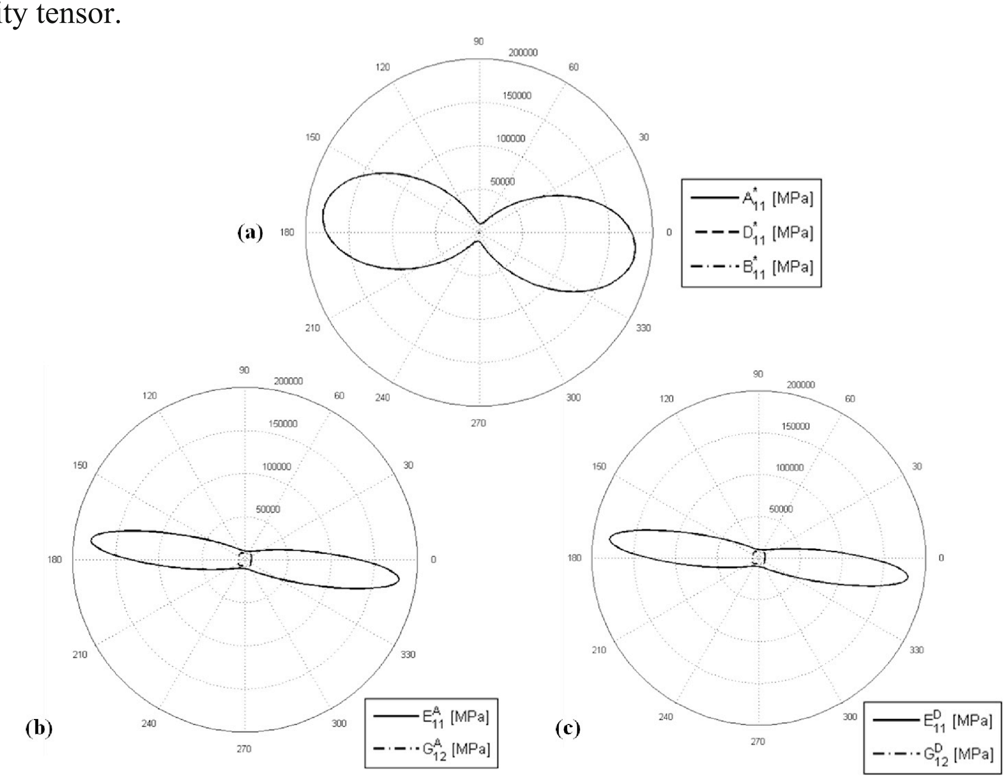


Fig. 14 Polar variations for laminate n.1, non-identical ply thickness. (a) Stiffness components, (b) membrane Young's modulus E_{11} and shear modulus G_{12} and(c) bending Young's modulus E_{11} and shear modulus G_{12}.

6.2 Numerical results

Since the laminate's elastic behaviour depends upon the elastic properties of the elementary ply, the results must refer to a given material; in this paper a highly anisotropic unidirectional carbon/epoxy ply (T300/5208) [20] has been chosen. Its properties are shown in Table 2.

For each case from 1 to 4 cited beforehand, two kinds of simulations have been performed. In the first one, the thickness of the elementary ply is assumed equal to 0.125 mm, so the design variables are only the number and orientations of the layers. In the second one, also the thickness is a variable of the optimisation.

We have considered also this possibility, of laminates composed by layers with variable thickness, with the aim to assess the influence of such a variable on the determination of the minimum number of layers needed to obtain some specified elastic properties. In practice, this corresponds to increase the number of design variables for the same kind of problem, and should result in a better quality of the results, with respect to the corresponding cases of fixed thickness, and perhaps in a lower final minimum number of layers. Actually, it is so and this can be observed in the results shown below. Of course, in doing this we do not consider the practical (e.g. manufacturing) aspects of such a choice, because here we are merely concerned with the theoretical solution of the mathematical problem of finding the laminate having the minimum number of layers to satisfy some elastic requirements; practical aspects have been discarded in this paper, at least for the cases of laminates with layers of different thickness. We consider only that all the layers have the same elastic properties but different thickness, which implies the assumption that the volume fraction and arrangement of the fibers are constant.

For each simulation the number of plies n varies in the range [4,16], while the orientation of each layer δ_k $(k=1,..,n)$ can assume any integer value in the domain [-90°,90°] discretized by a step of 1°. For the simulations in which also the layer's thickness is a design variable, the thickness h_k $(k=1,..,n)$ varies in a continuous way in the range [0.1,0.2] mm. Table 3 shows the common genetic parameters for all the cases. A remark regarding the values of mutation probability, p_{mut}, and chromosomes number mutation probability, $(p_{mut})_{chrom}$: such values have been chosen according to the De Jong's study [21]. In this study, it was suggested that mutation probability should be inversely proportional to the population size. Hence, for all simulations, the mutation probability has been chosen according to this rule.

Tables 4 and 5 show examples of stacking sequences for laminates responding to design criteria from cases n. 1 to 4, in the case of fixed and variable layer's thickness respectively. The residual in the last column is the value of the global objective function F (n, $\boldsymbol{\delta}$, $\mathbf{h}$) for the solution indicated aside (recall that exact solutions correspond to zeros of the objective function). As normal in a numerical technique, the real solution is found to within a small numerical tolerance; this tolerance is the *residual*. A discussion on the importance of the numerical residual in this type of problems can be found in [3, 18].

Tables from 6 to 13 show the polar parameters value for all the stacking sequences found in both cases of constant and variable thickness of the plies. It is possible to see that all the laminates are extension-bending uncoupled although the stacking sequences are not symmetric. Actually, some of these sequences are anti-symmetric (a condition that guarantees bending orthotropy, but not always bending-extension uncoupling, see [15]). For instance, the sequence of case n. 1 with plies of constant thickness, can be reduced to the sequence [2/-7/11/-11/7/-2], which is anti-symmetric, simply by a rotation of -7°. Actually, such an angle, see Table 6, corresponds to the value of the polar angle Φ_1, and in this case, having imposed K=0, also of Φ_0. In fact, for a given elastic tensor $\mathbf{L}$, in the case of orthotropy with K=0, the direction determined by the angle Φ_1 corresponds to the strong axis of orthotropy, i.e. to the direction of the highest value of the component L_{xxxx}, how can be easily seen from Eq. (11), see also [10]. A similar result is valid also for the case n. 2, always when the plies have a constant thickness, but not for all the other cases, which are completely unsymmetrical; in particular, all the cases with plies of variable thickness, have completely unsymmetrical stacking sequences, which is normal, because the condition of antisymmetrical stacks to obtain bending orthotropy is valid only for identical layers. In our calculations, we preferred not to fix the orthotropy direction, because the properties that we are looking for are intrinsic, i.e. frame independent. The use of the polar formalism allows in fact not only for fixing the frame, for instance imposing a given value of the polar angle Φ_1, but also, like in these cases, to do completely abstraction from the frame, whenever intrinsic properties are sought for independently from any frame. Of course, a post-processing operation of frame rotation, like the one described above, can always been done, if one wishes to have the final result in a particular frame.

Figs. From 13 to 20 show the directional plots of some of the elastic properties for the laminates solution for cases from n. 1 to 4; for the sake of clarity and of shortness, not all the elastic properties have been plotted, but those presented here are sufficient to show that the prescribed elastic properties have really been obtained.

For the sake of brevity, a detailed discussion of the results is presented below only for the laminates which are solution of cases 2 and 3, but similar considerations can be repeated *verbatim* also for the other cases.

One can consider first the requirement expressed by the case n.2. In Tables 8 and 9, it is possible to notice that the laminate solution, in both cases of constant and variable ply's thickness, respects the design criteria (such verifications are particularly simple to be done with the polar formalism, independently on the frame, see Table 1):

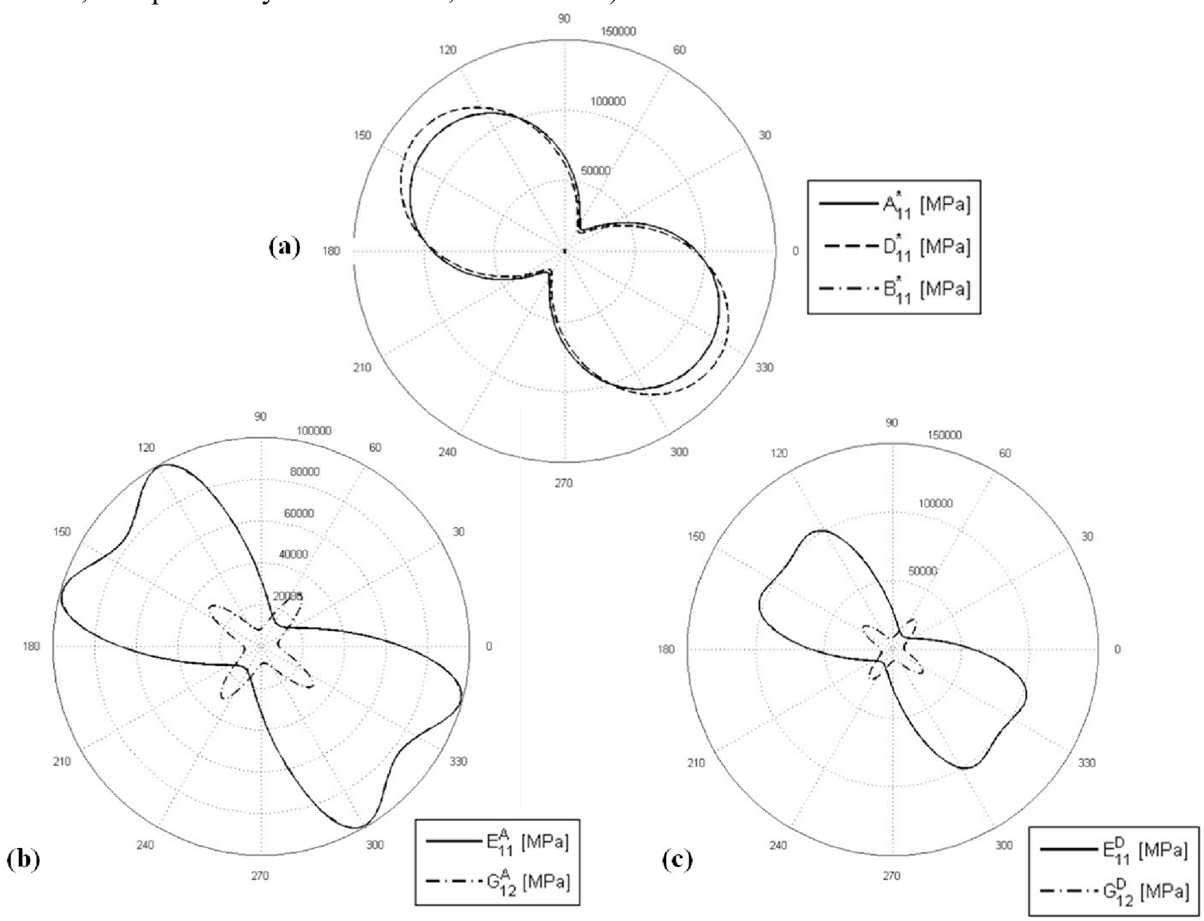


Fig. 15 Polar variations for laminate n.2, identical ply thickness. (a) Stiffness components, (b) membrane Young's modulus E_{11} and shear modulus G_{12} and(c) bending Young's modulus E_{11} and shear modulus G_{12}.

1. in-plane orthotropy with $K^{A^*}=1$:
    a. plies with identical thickness, $\Phi_0^{A^*}-\Phi_1^{A^*}=7.5°-(-37.5°)=45°$;
    b. plies with non-identical thickness, $\Phi_0^{A^*}-\Phi_1^{A^*}=-0.5°-(-45.5°)=45°$;
2. bending orthotropy with $K^{D^*}=1$:
    a. plies with identical thickness, $\Phi_0^{D^*}-\Phi_1^{D^*}=7.5°-(-37.5°)=45°$;
    b. plies with non-identical thickness, $\Phi_0^{D^*}-\Phi_1^{D^*}=-0.5°-(-45.5°)=45°$;
3. elastic uncoupling expressed by polar condition $\|\mathbf{B}^*\|=0$. The norm of tensor $\mathbf{B}^*$ is negligible compared to the one of tensor $\mathbf{A}^*$ or $\mathbf{D}^*$**:**
    a. plies with identical thickness, $\dfrac{\|\mathbf{B}^*\|}{\|\mathbf{A}^*\|}=0.0270$;
    b. plies with non-identical thickness, $\dfrac{\|\mathbf{B}^*\|}{\|\mathbf{A}^*\|}=0.0260$;
4. coincidence of orthotropy axes, polar condition $\Phi_1^{A^*}=\Phi_1^{D^*}$:
    a. plies with identical thickness, $\Phi_1^{A^*}=\Phi_1^{D^*}=-37.5°$;
    b. plies with identical thickness, $\Phi_1^{A^*}=\Phi_1^{D^*}=-45.5°$.

For the case n. 2 it is important to notice that, concerning the laminate with identical layer thickness, the solution which satisfies all the requirements with the minimum number of plies is only made by 6 layers, see Table 4. When the ply's thickness becomes a design variable the solution found has many improvements as shown in Table 5. First, the minimum layers number to obtain a solution is decreased from 6 to 4, then, this solution has a lower value of the residual, i.e. the laminate satisfies the requirements in a more satisfactory way. Figs. 15 and 16 show the polar diagrams of the elastic properties of the laminates solution of this case.

In Table 10 and 11 one can notice that the laminate solution of the problem n.3, in both cases of constant and variable ply's thickness, respects the design criteria:

1. in-plane isotropy, expressed by the polar condition that the anisotropic part of the tensor $\mathbf{A}^*$ must be zero, i.e. $R_0^{A^{*}2}+4R_1^{A^{*}2}=0$. The ratio between the tensor's anisotropic and isotropic part is very close to zero, i.e. the anisotropic components are negligible compared to the isotropic ones:
    a. plies with identical thickness, $\sqrt{\dfrac{R_0^{A^{*}2}+4R_1^{A^{*}2}}{T_0^{A^{*}2}+2T_1^{A^{*}2}}}=0.0023$;
    b. plies with non-identical thickness $\sqrt{\dfrac{R_0^{A^{*}2}+4R_1^{A^{*}2}}{T_0^{A^{*}2}+2T_1^{A^{*}2}}}=0.0017$;
2. bending isotropy, expressed by the polar condition that the anisotropic part of the tensor $\mathbf{D}^*$ must be zero, i.e $R_0^{D^{*}2}+4R_1^{D^{*}2}=0$. The ratio between the tensor's anisotropic and isotropic part is very close to zero, i.e. the anisotropic components are negligible compared to the isotropic ones:

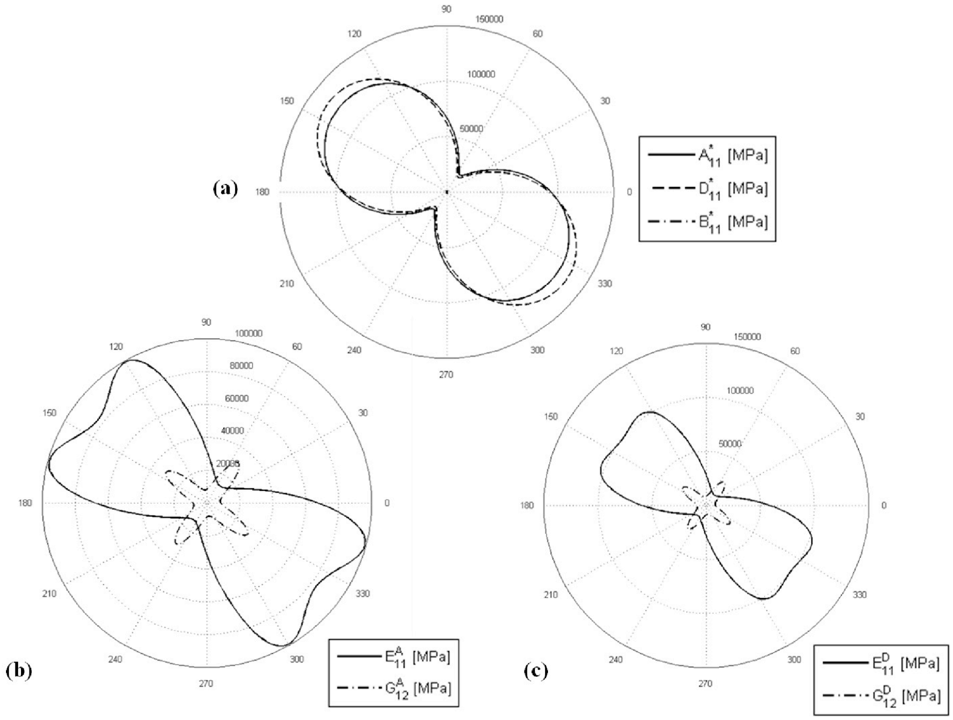


Fig. 16 Polar variations for laminate n.2, non-identical ply thickness. (a) Stiffness components, (b) membrane Young's modulus E_{11} and shear modulus G_{12} and(c) bending Young's modulus E_{11} and shear modulus G_{12}.

a. plies with identical thickness, $\sqrt{\dfrac{R_0^{D^{*}2} + 4R_1^{D^{*}2}}{T_0^{D^{*}2} + 2T_1^{D^{*}2}}} = 0.0040$;

b. plies with non-identical thickness, $\sqrt{\dfrac{R_0^{D^{*}2} + 4R_1^{D^{*}2}}{T_0^{D^{*}2} + 2T_1^{D^{*}2}}} = 0.0007$;

3. elastic uncoupling expressed by polar condition $\left\|\mathbf{B}^*\right\| = 0$. The norm of tensor $\mathbf{B}^*$ is negligible compared to the one of tensor $\mathbf{A}^*$ or $\mathbf{D}^*$**:**

   a. plies with identical thickness, $\dfrac{\left\|\mathbf{B}^*\right\|}{\left\|\mathbf{A}^*\right\|} = 0.0130$;

   b. plies with non-identical thickness, $\dfrac{\left\|\mathbf{B}^*\right\|}{\left\|\mathbf{A}^*\right\|} = 0.0060$.

In this case, concerning the laminate with identical layer thickness, the solution that satisfies all the requirements with the minimum number of plies is made by 13 layers, see Table 4. When the ply's thickness becomes a design variable, the best solution is really improved, as shown in Table 5. Like the previous case, the minimum layers number to obtain a solution is decreased from 13 to 10 and the value of the residual is lower, i.e. the laminate satisfies the requirements more accurately. Figs. 17 and 18 show the polar diagrams of the elastic properties of the laminates solution of this case.

7. CONCLUSIONS

The problem of determining which is the lowest number of layers, ensuring some given elastic properties of a laminate, has been addressed. The approach proposed in the paper to solve this problem is to reduce it to a classical unconstrained optimisation problem by searching the minima of a semi-definite positive function. This method is totally general, i.e. no simplifying assumptions are required, and it is based upon the so-called polar-genetic approach. The problem's formulation is based on the polar representation to express plane tensors while the numerical strategy for the search of the solutions is a genetic algorithm, BIANCA that has been modified to include the number of layers among the variables. The numerical results presented in this paper, which are completely new and non-classical examples show the effectiveness of the proposed method.

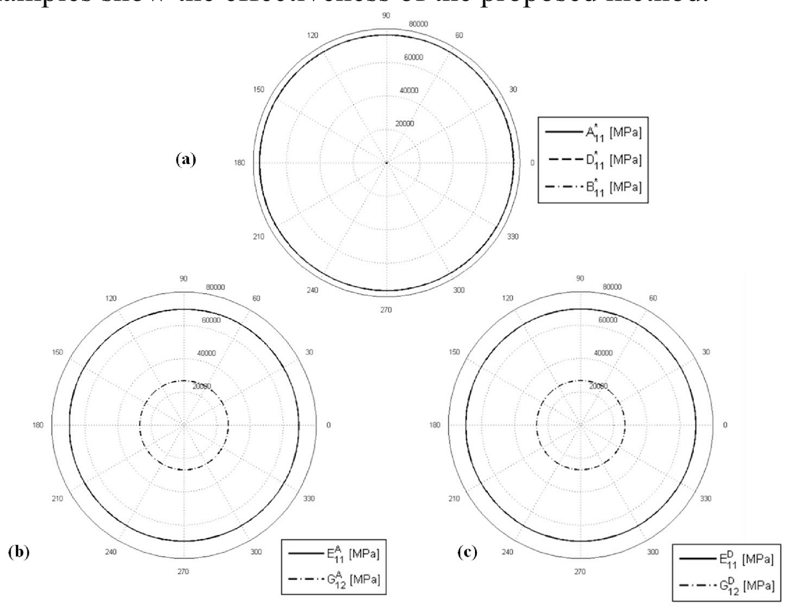


Fig. 17 Polar variations for laminate n.3, identical ply thickness. (a) Stiffness components, (b) membrane Young's modulus E_{11} and shear modulus G_{12} and(c) bending Young's modulus E_{11} and shear modulus G_{12}.

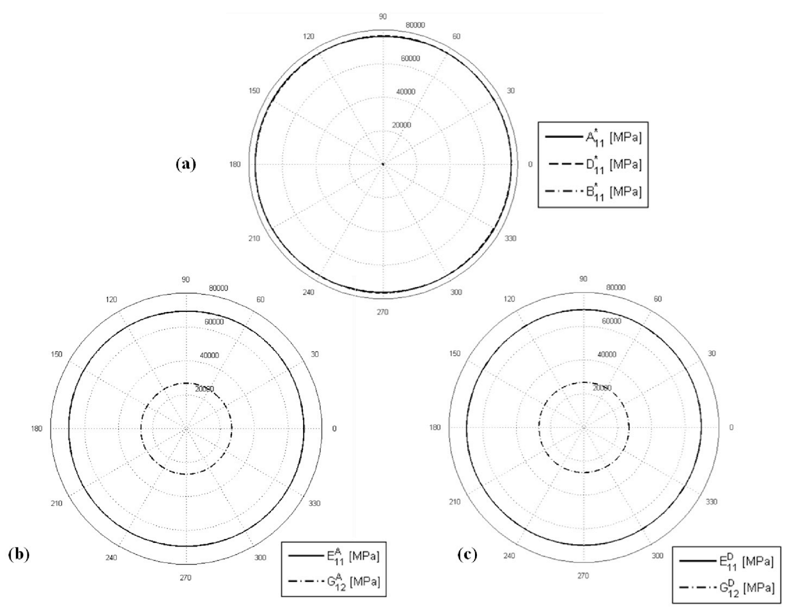


Fig. 18 Polar variations for laminate n.3, non-identical ply thickness. (a) Stiffness components, (b) membrane Young's modulus E_{11} and shear modulus G_{12} and(c) bending Young's modulus E_{11} and shear modulus G_{12}.

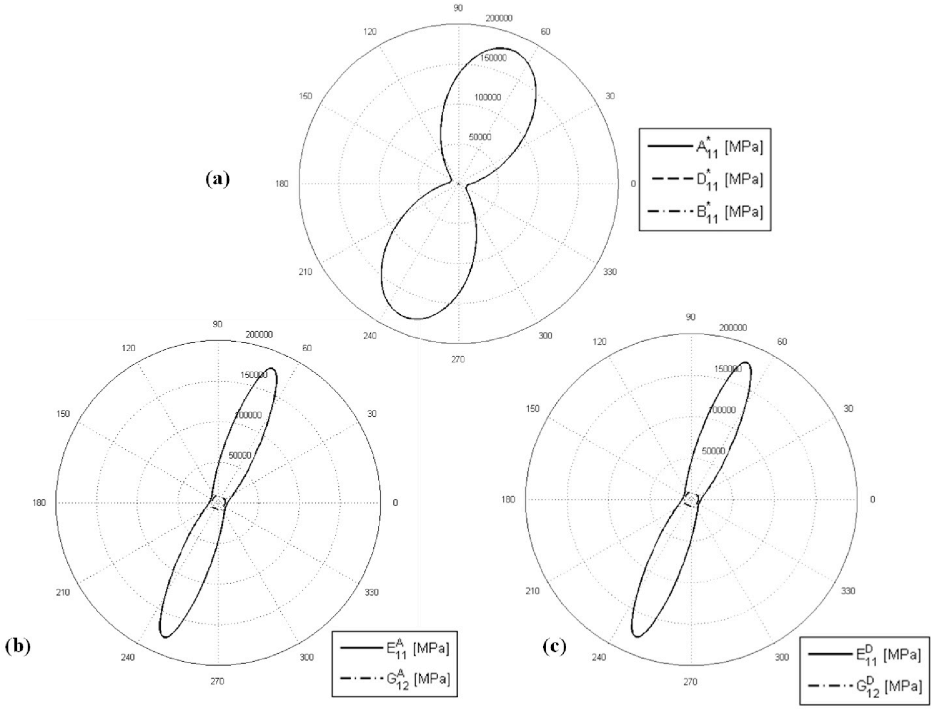


Fig. 19 Polar variations for laminate n.4, identical ply thickness. (a) Stiffness components, (b) membrane Young's modulus E_{11} and shear modulus G_{12} and(c) bending Young's modulus E_{11} and shear modulus G_{12}.

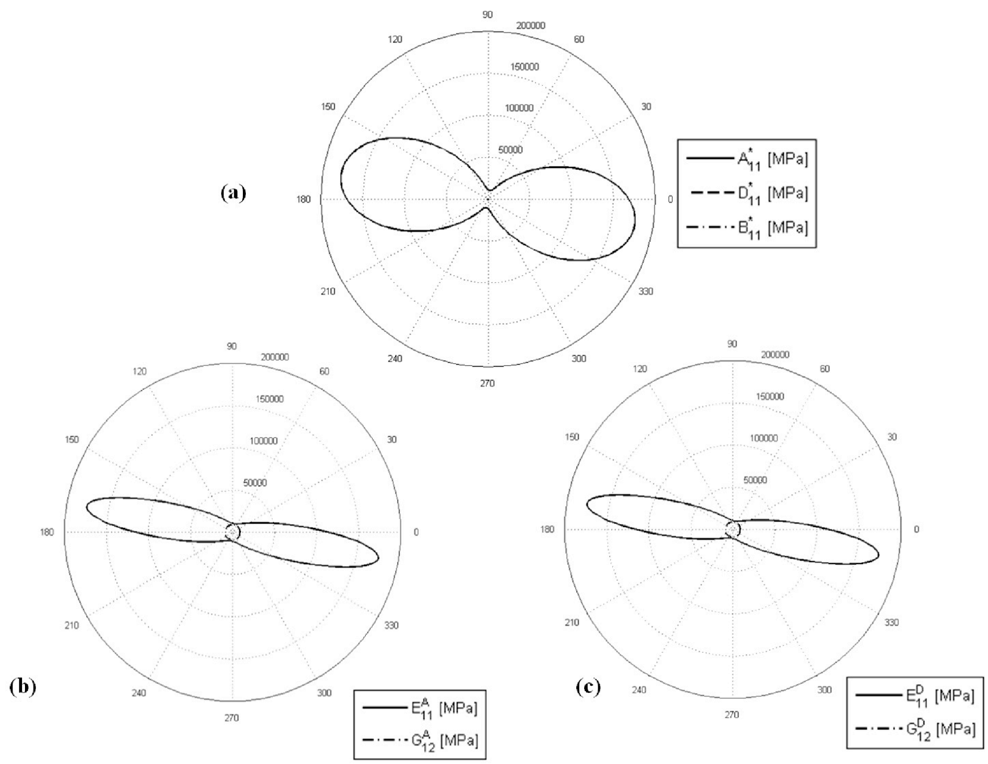


Fig. 20 Polar variations for laminate n.4, non-identical ply thickness. (a) Stiffness components, (b) membrane Young's modulus E_{11} and shear modulus G_{12} and(c) bending Young's modulus E_{11} and shear modulus G_{12}.

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TABLES

Table 1. Conditions for elastic symmetries in terms of polar invariants

Elastic symmetry	Polar condition
Orthtotropy	$\Phi_0 - \Phi_1 = K\frac{\pi}{4}$
R_0- orthtotropy	$R_0 = 0$
Square symmetry	$R_1 = 0$
Isotropy	$R_0 = R_1 = 0$

Table 2. Material properties for unidirectional carbone/epoxy ply T300/5208

E_1 [MPa]	E_2 [MPa]	G_{12} [MPa]	ν_{12}
181000	10300	7170	0.28

Table 3. Genetic parameters for all simulations

Genetic parameters	
N_{ind}	500
N_{gen}	500
p_{cross}	0.85
p_{mut}	$\frac{1}{N_{ind}}$
p_{shift}	0.5
$(p_{mut})_{chrom}$	$\frac{(n_{chrom})_{\max} - (n_{chrom})_{\min}}{N_{ind}}$

Table 4. Best stacking sequences for the design problems 1 to 4, fixed layer thickness

Objective	Stacking sequence (angles [°])	N. of plies	Residual
Case n. 1	[9/0/18/-4/14/5]	6	3.5873×10^{-6}
Case n. 2	[-16/-65/-67/-8/-10/-59]	6	1.7547×10^{-2}
Case n. 3	[0/-50/61/42/-87/-90/-49/-10/-12/36/26/-47/83]	13	1.6117×10^{-2}
Case n. 4	[-16/-65/-67/-8/-10/-59]	7	1.7547×10^{-2}

Table 5. Best stacking sequences for the design problems 1 to 4, variable layer thickness

Objective	Stacking sequence (angles [°]and thickness [mm])	N. of plies	Residual
Case n. 1	[-9/-6/-4/-11/-9/-6] [0.118/0.126/0.140/0.126/0.103/0.152]	6	3.2976×10^{-7}
Case n. 2	[-24/-73/-18/-67] [0.100/0.200/0.200/0.100]	4	7.3315×10^{-3}
Case n. 3	[-19/-74/45/20/75/-17/-53/-83/2/51] [0.113/0.168/0.200/0.137/0.149/0.190/0.199/0.113/0.106/0.116]	10	2.0476×10^{-3}
Case n. 4	[-15/-2/-12/-10/-21/-6] [0.156/0.188/ 0.158/0.151/0.111/0.182]	6	2.9945×10^{-5}

Table 6. Polar parameters for the laminate case n.1, constant ply thickness

Elastic properties	Tensor $\mathbf{A}^*$	Tensor $\mathbf{B}^*$	Tensor $\mathbf{D}^*$
T_0 [MPa]	26880.4311	0	26880.4311
T_1 [MPa]	24743.8933	0	24743.8933
R_0 [MPa]	17033.4619	19.5909	18772.4089
R_1 [MPa]	20683.2749	2.83	21173.5205
Φ_0 [°]	7	-	7
Φ_1 [°]	7	-	7

Table 7. Polar parameters for the laminate case n.1, variable ply thickness

Elastic properties	Tensor $\mathbf{A}^*$	Tensor $\mathbf{B}^*$	Tensor $\mathbf{D}^*$
T_0 [MPa]	26880.4311	0	26880.4311
T_1 [MPa]	24743.8933	0	24743.8933
R_0 [MPa]	19437.9543	2.5019	19583.8459
R_1 [MPa]	21358.8283	0.90806	21398.6655
Φ_0 [°]	-7.32	-	-7.32
Φ_1 [°]	-7.32	-	-7.32

Table 8. Polar parameters for the laminate case n.2, constant ply thickness

Elastic properties	Tensor $\mathbf{A}^*$	Tensor $\mathbf{B}^*$	Tensor $\mathbf{D}^*$
T_0 [MPa]	26880.4311	0	26880.4311
T_1 [MPa]	24743.8933	0	24743.8933
R_0 [MPa]	4873.3071	1407.9761	1122.9394
R_1 [MPa]	13002.5339	113.5579	14626.7514
Φ_0 [°]	7.5	-	7.5
Φ_1 [°]	-37.5	-	-37.5

Table 9. Polar parameters for the laminate case n.2, variable ply thickness

Elastic properties	Tensor **A***	Tensor **B***	Tensor **D***
T_0 [MPa]	26880.4311	0	26880.4311
T_1 [MPa]	24743.8933	0	24743.8933
R_0 [MPa]	4035.9329	1345.8389	1029.8979
R_1 [MPa]	13420.7539	158.8238	14673.2203$
Φ_0 [°]	-0.5	-	-0.5
Φ_1 [°]	-45.5	-	-45.5

Table 10. Polar parameters for the laminate case n.3, constant ply thickness

Elastic properties	Tensor **A***	Tensor **B***	Tensor **D***
T_0 [MPa]	26880.4311	0	26880.4311
T_1 [MPa]	24743.8933	0	24743.8933
R_0 [MPa]	101.0911	536.0837	120.9914
R_1 [MPa]	9.3146	90.7468	63.6677
Φ_0 [°]	-	-	-
Φ_1 [°]	-	-	-

Table 11. Polar parameters for the laminate case n.3, variable ply thickness

Elastic properties	Tensor **A***	Tensor **B***	Tensor **D***
T_0 [MPa]	26880.4311	0	26880.4311
T_1 [MPa]	24743.8933	0	24743.8933
R_0 [MPa]	45.4948	253.7272	30.4769
R_1 [MPa]	29.3042	46.5314	6.8212
Φ_0 [°]	-	-	-
Φ_1 [°]	-	-	-

Table 12. Polar parameters for the laminate case n.4, constant ply thickness

Elastic properties	Tensor **A***	Tensor **B***	Tensor **D***
T_0 [MPa]	26880.4311	0	26880.4311
T_1 [MPa]	24743.8933	0	24743.8933
R_0 [MPa]	18929.7208	6.4786	18921.6397
R_1 [MPa]	21219.3737	0.8899	21217.4539
Φ_0 [°]	-22.15	-	-22.14
Φ_1 [°]	67.85	-	67.85

Table 13. Polar parameters for the laminate case n.4, variable ply thickness

Elastic properties	Tensor **A***	Tensor **B***	Tensor **D***
T_0 [MPa]	26880.4311	0	26880.4311
T_1 [MPa]	24743.8933	0	24743.8933
R_0 [MPa]	18090.7546	44.438	18088.1731
R_1 [MPa]	20982.3915	19.2386	20983.6104
Φ_0 [°]	-10.05	-	-10.05
Φ_1 [°]	-10.08	-	-10.08