

# Probabilistic Formulation of General Relativity and the Postulates of Relativity.

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Recent developments at CERN have given more credibility to the postulated faster than the speed of light particles[1]. Regardless of the accuracy of the findings, the compounding evidence of the inaccuracies observed in Relativistic predictions of astronomical phenomena is enough to compel one to consider an alternative approach to explaining physical phenomena. We will also highlight some difficulties exposed by the postulates of Relativity, subsequent to which we will propose a probabilistic approach to GR.

## 1. The Postulates.

The two postulates upon which Relativity is built can be stated as follows[P]:

POSTULATE 1. *It is impossible to measure, or detect, the un-accelerated translation of a system through free space or through any aether-like medium which might be assumed to pervade it.*

POSTULATE 2. *The speed of light in free space is the same for all inertial observers, independent of the relative velocity of the source and the observer.*

From here on we will refer to these as (P1) and (P2) respectively.

Before considering any thought experiments, let us make a few observations regarding physical phenomena of movement. Let  $B$  and  $B'$  be two frames of reference, where we place an observer at  $B$  and let  $B'$  be in strict horizontal transversal w.r.t  $B$ . Furthermore, let  $B'$  be an object in **non accelerated** motion being observed. Let us arbitrarily subdivide this object into spatial intervals of  $\Delta S$  along its length  $L$  as observed by  $B$ . For any arbitrary  $\Delta S_i$  along  $L$ , let  $(M_{(l)}, M_{(r)})$  be a measure (in frame  $B$ ) of the lengths respectively along either side of  $\Delta S_i$ . It is simple to see that it is necessary for all  $(M_{(l)}, M_{(r)})$  associated with the segments  $\Delta S$  to be constant throughout the motion of  $B'$  (as observed by  $B$ ), for if this were not the case, then not all parts of  $B'$  are in constant motion with regard to  $B$ . If further, we formed a mapping between spatial positions associated with any stationary (**horizontally**) objects within  $B'$  to  $\Delta S$ , one can again make the claim that such objects are also traversing horizontally with regard to  $B$  with the same velocity as that of  $B'$ , for if this were not the case, then again not all parts of  $B'$  are in constant motion. Consider now an observer within  $B'$  observing objects including herself. From our previous argument, it is easy to conclude that should observer  $O'$  observe objects in her frame to be stationary, then it is necessary that such objects are in horizontal motion with respect to  $B$ . A second consideration that is expressed in Relativistic Mechanics is that length contraction is experienced only in the

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<sup>1</sup>Dedicated with much love to my Mother U.Devaraj and my Father R.Pillay.

direction of observed motion[P](p29-p36).

Consider now a thought experiment involving an event  $(E, E')$  in frame  $B'$ , that begins when both frames are synchronous at some point on  $B$ . Let  $E'$  be the emission of a beam of light in frame  $B'$ . In order now for (P1) to hold, it is required that  $M_{(l)}, M_{(r)}$  associated with  $\Delta S_{E'}$ , the position on  $L$  of the emission of the beam, remain constant through out the beams' trajectory with respect to  $L$ . For if this were not the case, and  $M_{(l)}, M_{(r)}$  associated with the beginning and end at the height  $H$  of the object within which the observation is made, then the observer within  $B'$  would be able to tell of her movement. On the other hand, should we observe that no such deviation occurs, then the observer in  $B'$  would note no change in the trajectory of the emitted beam of light and as such, the  $M_{(l)}, M_{(r)}$  measurements remains constant, then it is necessarily also traversing horizontally with some velocity with respect to  $B$ . From our second consideration,  $B$  observes no vertical length contraction of  $B'$ , thus the vertical height of the object would be the same of both  $B, B'$ . From the second postulate, it is required for the beam to traverse  $H$  in  $c$ . This is not a problem except for one thing, for the beam as observed by  $B$ , must both hold its horizontal equilibrium along with its vertical velocity which must be  $c$  as the height of the object  $H$  is the same in both frames. In order then to achieve this, the observer in  $B$  must observe the beam of light to be traversing faster than  $c$ . This is because the beam holds its horizontal position in  $B'$  throughout its movement, in addition to travelling vertically at  $c$ .

There is some mechanistic fault at play here for it forces to choose between one of the two postulates. To further investigate this claim, consider the beam of light as observed by  $B$ . In keeping with Relativity, the beam remains equi-measured from either side of the object in motion being observed. A natural question to consider this behavior is that, aside from such an experiment, is there any natural phenomena that can concur with such behavior. Specifically, is there some observation that gives suggestion to the possibility that an emitted quantity of light remain on the same path as that of its emitter, given that the emitter's motion is perpendicular to that of the beam of light? For this is exactly what is mechanically required of the beam of light in order to be horizontally stationary in  $B'$ , for we must admit horizontal movement in frame  $B$  otherwise not all parts of  $B'$  is in constant motion with regard to  $B$ . Relativity would have us view the situation as the beam actually being horizontally stationary though it appears to be moving horizontally as viewed from  $B$ . The Relativistic view is that a traversing beam within  $B'$  is in fact traversing a different distance outside of  $B'$  i.e. when viewed from  $B$  and time dilation accounts for the differences in distance traversed by light. But if this is the case then as we saw above, then there should be no instances when lengths within  $B'$  are equivalent to that of  $B$  for if this is possible, and both frames coincided whilst two separate beams of light began traversing the stipulated lengths, and still it were required that the speed of light and the lengths remain constant, then no time dilation can exist during the motion of  $B'$ . In addition there is physical evidence that post emission, a beam of light continues on its path while the source is free to move

arbitrarily without interfering with the trajectory of the beam emitted. The most notable such observation is the aberration of light[B]. Here clearly, the position of a star being observed does not concur with its actual position. If this is true, then post emission, a beam of light continues on its path, and experiences no 'tug' from its emitter. Then the same should also be true for an observer in  $B'$  observing a beam of light. The beam would appear to linearly deviate from its intended trajectory. The postulate P(1) is in fact limited to experiments that involve bodies capable of receiving an impulse. It is for this precise reason that one can't ascertain absolute speed with regard to another such body for this body may itself have been subject to other impulses. If light behaved as such then it would be capable of travelling faster than  $c$ , for it would be capable of admitting a horizontal impulse whilst traversing vertically. However if this is not the case, then light does not admit an impulse and deviations in its path is observable from within the system emitting the beam.

## 2. The Mechanics of Acceleration.

An explanation for the curvature in trajectory experienced by a beam of light in acceleration fields can be viewed in the following manner. For every differential quantity  $\Delta W$  of photons emitted from a point like source, this quantity remains on its trajectory. Should the source be positioned in a manner that emits light perpendicular to its direction of acceleration, then for each quantity of light emitted and the next, the source would have travelled a distance  $\frac{\partial^2 V}{\partial t^2}$  from some point  $p_1$  on a vertical scale to the next such point of emission of a similar quantity. In this exact time  $t$  within the same frame of reference, the quantity travels a horizontal distance  $ct$ . Combining the two measures one obtains the following formulae for the trajectory of the beam:

$$ct\mathbf{i} + \frac{\partial^2 V}{\partial t^2}\mathbf{j} \quad (1)$$

Leaving aside the above for the moment, consider the nature of the metric tensor:  $ds^2 = g_{ik}dX^i dX^k$ . Under certain relativistic conditions, as expressed by Einstein's field equations, one can solve for the metric components  $g_{ik}$ . Once accomplished, it is possible to use these components to solve problems involving the motion of bodies in a gravitational field. In a completely mathematical sense, it is quite possible that small variations in the metric components themselves can either predict such motion more precisely or extend error in its predictions. This is important for two reasons, one is simply that Lorentz type conditions have little influence on  $g_{ik}$  formed, and the all other aspects are more influential, in which case, the promising results obtained via Relativity are primarily due to the nature of its calculus. The other being that small variations in the nature of the formation of  $g_{ik}$  will preserve this integrity.

We draw attention to the property that varying the conditions under which one obtains the components  $g_{ik}$  does not interfere with the designated function of the metric tensor or the nature of the mechanism by which it functions. The calculus basically makes use of the metric components as in conjunction with the calculus of variations to make predictions regarding movement in a gravita-

tional field.

We deviate our thoughts for a moment to the remarkable formulation of Quantum Mechanics via the use of path integrals. Here Feynman makes use of the calculus of variations to maximize the probability of events in order to extrapolate most probable paths travelled by particles[F-H]. One could formulate metrics in a similar fashion. Specifically, here the conditions under which the metric components are derived are structured in such a manner that these produce probability fields (produced by many particles) as functions that need to be minimized in order to obtain the paths traversed by objects in gravitational fields.

One could view Gravity as restricting probabilistically, the movement of objects in a gravitational field. By doing this we accomplish the task of formulating a consistent theory that is developed outside going in. Specifically, all aspects are expressed probabilistically, and should it be discovered that some particle is responsible for gravitational behavior, we need only explain how the particle behaves in order to accomplish the already expressed values.

### 3. A Probabilistic Approach.

Our approach involves interpreting gravity as an entity that alters the probability associated with where a particle appears (when within a gravitational field) at each subsequent time interval. That is, the probability associated with a path travelled by a particle in a gravitational field is affected by the field itself. Just as the presence of light alters the probably associated with the path travelled by an electron[F-H], one would expect that such probabilities vary with the intensity of the field, the number of particles in the field at the same time and (amongst others), the state of the particles (such as whether these are in the form of an atom) themselves. This view has many advantages. Firstly, one can account for the possibility that at the particle level, slight deviances from the conventional inverse square law can be accounted for furthermore, so can the changing values associated with possibly varying probabilities with the intensity of such fields.

To achieve this, we follow the path integral approach of Feynman, to the formulation of QM[F-H].

Every sub segment associated with every possible path possible between two points that a particle is capable of traversing, is associated with a probability. The summation of the integrals of all such paths forms a kernel, the square of which yields the probability of arriving at the latter of the points. The integrals expressing such kernels are of the form :

$$K(b, a) = \int \int \dots \int \Phi[x(t)] dx^1 dx^2 \dots dx^N \quad (2)$$

Our hypothesis is that the span of such paths available decreases with the intensity of the gravitational field. One can conceivably obtain the (probability something) associated with the graviton[F] via observations of such changes in

the span. The limit of the span we formulate to be the Euler Lagrange equations:

$$\frac{d^2 X^i}{dS^2} + \Gamma_{kl}^i \frac{dX^k}{dS} \frac{dX^l}{dS} = 0 \quad (3)$$

Loosely known as the world lines as given by GR. Here :

$$\Gamma_{kl}^i = g^{im} \frac{1}{2} \left( \frac{\partial g_{ik}}{\partial X^l} + \frac{\partial g_{il}}{\partial X^k} - \frac{\partial g_{kl}}{\partial X^i} \right) \quad (4)$$

To account for the probabilistic decrease in span, and to relate the metric components to certain probabilities, we form the relation :

$$\lim_{b \rightarrow a} K(b, a) = Pr([b, a] \in \Phi(X^l, X^k, X^i)) \quad (5)$$

Here  $\Phi(X^l, X^k, X^i)$  is the solution to (3). Given certain small proposed alterations to the formulation of the field equations governing GR, we can derive the metric components in a similar fashion[P]<sup>2</sup>. On small scales involving particles in a gravitational field, the functional nature of the probabilities associated with path integrals in such fields can then be used to determine metric components associated. By doing this we would have achieved a consistent probabilistic approach to gravitation, which is devoid of the cause for such fields which can be filled in later on without altering the nature of the formulations involved.

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<sup>2</sup>In a follow up article, we will specify the changes which will mainly involve certain alterations to the Lorentz transform.