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Bezalel Peleg and Hans Peters: Strategic Social Choice. Stable Representations of Constitutions

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Many sincere compliments should be addressed to Bezalel Peleg and Hans Peters, the authors of the book *Strategic Social Choice. Stable Representations of Constitutions* published last year by Springer. Our experience with the previous books of these authors on game theory (Peleg 1984; Peleg and Sudhölter 2003; Peters 1992, 2008) induces very high expectations with respect to this jointly written monograph. And we have not been disappointed with their new project.

The book studies stable representations of constitutions. The authors follow Gärdenfors (1981) who modeled constitutions as effectivity functions, a term introduced independently by Moulin and Peleg (1982) in the general context of game theory and its relation to social choice. An effectivity function assigns to each group in a society a collection of sets of social states. The manuscript is based on work that has appeared over the last thirty years, but it also contains many new results and new or improved proofs of existing results. Among various sources of the results collected in the book, the authors used, e.g., Abdou (1988), Abdou and Keiding (1991), Holzman (1986), Keiding and Peleg (2006), Peleg (1998, 2004), Peleg and Peters (2006, 2009), Peleg et al. (2002). The emphasis in the book by Peleg and Peters is on strategic stability of game forms that represent effectivity functions. As the authors mention themselves, their aim is not to contribute substantially to the purely philosophical literature on constitutions, and equity or fairness, but they focus on representation and game-theoretic stability, and rather contribute to the economic literature on mechanism design taken in a wide sense.

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The volume of about 160 pages is divided into two parts, and contains in total 11 chapters, preceded by a clear summary of the whole contents. Each chapter begins with a short motivation and summary section, followed by its main sections, and then it finishes by a short section with notes and comments. Apart from clearly presented definitions and numerous results, the authors deliver many illustrative examples. The first part, built by the first seven chapters, focuses on properties of game forms representing effectivity functions: strategic stability properties, Pareto optimality of equilibrium outcomes, and continuity properties of the game forms. In the introductory Chapter 1, after recalling Arrow's model of a social welfare function, Peleg and Peters present a definition of a 'rights-system' by Gärdenfors. The latter definition of a constitution and a more detailed formalization of the concept of rights by Peleg (1998) are adopted in the book. In Chapter 2 the authors formally describe a constitution and the effectivity function that it induces. They present a new proof of the representation theorem. Chapter 3 deals with a Nash consistent game form, i.e., a game form that has a Nash equilibrium for every profile of preferences. Necessary and sufficient conditions on an effectivity function to have a Nash consistent representation are delivered. The authors consider the case where the set of alternatives is finite, and then generalize the results to compact Hausdorff topological spaces of alternatives and topological effectivity functions. They also discuss the relation between their results and the Gibbard Paradox (Gibbard, 1974), and the 'liberal paradox' (Sen, 1970). Chapter 4 is devoted to acceptable game forms, i.e., Nash consistent game forms such that all Nash equilibrium outcomes for all preference profiles are Pareto optimal. A complete characterization of the effectivity functions which can be represented by an acceptable game form is delivered. Chapter 5 considers strongly consistent representations. Strongly consistent game forms admit a strong Nash equilibrium for any profile of preferences, where a strong Nash equilibrium is a strategy profile which is resistant to deviations of all coalitions (including one-person coalitions). First the authors study necessary and sufficient conditions for the existence of strongly consistent representations in case the set of alternatives is finite, and then they extend their results to topological effectivity functions and deliver a new result. Chapter 6 concerns representations by Nash-consistent game forms when even-chance lotteries on subsets of alternatives are allowed. The authors show that in such a case, constitutions have Nash consistent representations without the restrictive intersection assumption. In Chapter 7 Peleg and Peters investigate the continuity of the outcome function of a representation when the outcome space and the strategy sets are compact metric spaces. In particular, they present a new result on Nash consistency of upper semicontinuous representations.

The remaining four chapters of the book constitute its second part and consider a special kind of game forms, social choice functions. Since a social choice function assigns an alternative to any profile of individual preferences, it is a game form with the strategies of the players being their individual preferences. Chapter 8 is an introduction to the second part of the book. In particular, the authors recall the Gibbard-Satterthwaite theorem for social choice functions

and discuss some approaches to the implications of this theorem. Peleg and Peters investigate exactly and strongly consistent social choice functions, i.e., social choice functions that have for each profile of (true) preferences a strong Nash equilibrium which yields the sincere outcome. They briefly discuss voting on restricted domains, and the concept of equilibrium with threats. The class of exactly and strongly consistent social choice functions is analyzed in the rest of the book, where several existence and characterization results are presented. In Chapter 9 these social choice functions are investigated by using feasible elimination procedures. The authors present a new proof of the characterization result. In Chapter 10 the analysis of exactly and strongly consistent representations is extended to general effectivity functions. It contains characterizations of elimination stable effectivity functions, i.e., effectivity functions such that the set of alternatives resulting from applying feasible elimination procedures is non-empty. Chapter 11 concerns consistent voting systems with a continuum of voters. In particular, Peleg and Peters extend the Gibbard-Satterthwaite theorem and the concept of feasible elimination procedures to this framework, and extend most of their results of Chapters 9 and 10 to voting games with a continuum of voters. An almost complete characterization of anonymous exactly and strongly consistent social choice functions in this model is presented.

The book has several merits. First of all, I think that the choice of the subject in question for the book was quite fortunate. It was a long time ago when a specialized book on effectivity functions in social choice appeared. I have in mind, of course, the book by Joseph Abdou and Hans Keiding (1991) in which both formal properties of effectivity functions and their applications in Social Choice Theory (in particular, in the implementation problem) have been presented. While some parts of the book by Peleg and Peters owe to several publications by Abdou and Keiding, this new book has taken a different road. In particular, as the authors mention themselves, the theory in their monograph should be distinguished from what is typically called implementation theory. Moreover, during the last twenty years significant contributions on the subjects in question have appeared in the literature, and it is important that we can find them all recalled in the book by Peleg and Peters.

Another merit of the book, desirable although not always evidently found in a scientific production, is the very high expertise and professionalism. When we take the book in our hands for the first time and combine the information on the contents of the monograph and on its authors, we feel immediately that what we are going to read is an excellent book. The authors do know perfectly what they are writing about, something that is also confirmed by the list of cited works: almost 20% out of 73 references are the own contributions of the two authors.

Moreover, as I have already emphasized when describing the contents of the manuscript, the book is very well written and perfectly organized, something we also recognize in the former publications of the two authors. Despite that the topics are far from being easy, the material is presented in a clear way. Very helpful are motivating discussions and numerous nice examples. Finally,

we can fully enjoy the beauty of the results and their rigorous proofs presented in the book.

Summarizing, I can highly recommend the book by Bezalel Peleg and Hans Peters: a real pleasure for scholars working on effectivity functions and social choice functions, and a great learning opportunity for those who are not that familiar with the topics in question. It is an excellent contribution, bringing together a lot of material in a single manuscript in a coherent way. However, one warning should be repeated as well. If you are searching for an easy and straightforward literature, you should rather look for something else. Also, if you like to investigate a new field which could easily contain many open questions, this book is not your best choice. The authors present material that is dense and contains many deep results, and consequently they do not provide ‘tips’ for further research. And if one could still find an open question in the book, like the one in Chapter 11 on characterization of anonymous exactly and strongly consistent social choice functions in the model with a continuum of voters, it will be indeed a real challenge.

Let me finally express one more thought, in line with what I have just written above. Reading the monograph by Bezalel Peleg and Hans Peters brought to my mind a comparison with climbing in some beautiful and challenging mountains. It is very difficult to climb there and in principle not everybody can do it. First of all, you need a formal rigorous training, otherwise you are lost after one minute of the escapade, in the first chapter. And there might be moments that you say ‘Come on! Why am I doing that? Would it not be better just to stay quietly at home and not to lose so much energy with something that I can never achieve? ...’ But! If you are determined and patient enough, step by step you go up and up, and finally there is this desirable moment when you are on the top of the hill. And it recompenses all suffering that you have just experienced. You are so happy and proud of yourself, and you enjoy so much the beauty of the landscape, the beauty of mathematics ... And then you notice another hill, even higher and more tempting ... and you move to a subsequent chapter, to a more complicated demonstration of another result ...

Since no particular criticism has struck me during reading the book, I did not force myself to search artificially some drawbacks. I like to wish to all readers of this excellent book many wonderful moments of climbing in the beautiful challenging mountains called *Strategic Social Choice. Stable Representations of Constitutions*.

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