Long-range gravitational-like interaction in a neutral atomic cold gas

M. Chalony,[1] J. Barré,[2] B. Marcos,[2] A. Olivetti,[2] and D. Wilkowski[1,3,4]

[1] *Institut Non Linéaire de Nice, Université de Nice Sophia-Antipolis, CNRS, 06560 Valbonne, France.*
[2] *Laboratoire J.-A. Dieudonné, Université de Nice Sophia-Antipolis, CNRS, 06109 Nice, France.*
[3] *Centre for Quantum Technologies, National University of Singapore , 117543 Singapore, Singapore.*
[4] *PAP, School of Physical and Mathematical Sciences, Nanyang Technological University, 637371 Singapore, Singapore.*

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A quasi resonant laser induces a long-range attractive force within a cloud of cold atoms. We take advantage of this force to build in the lab quasi 1D systems of particles with a gravitational-like interaction, at a fluid level of modeling. We give experimental evidences of such an interaction in a Strontium cold gas, studying the density profile of the cloud, its size as a function of the number of atoms and its breathing oscillations.

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When interactions between the microscopic components of a system act on a length scale comparable to the size of the system, one may call them "long range". For instance, the inverse-square law of the gravitational force between two point masses is one of the most celebrated and oldest laws in Physics. In the many particles world, it is responsible for dramatic collective effects such as the collapse of a gravothermal catastrophe [1] or the gravitational clustering which is the main mechanism leading to the formation of the structure of galaxies in the present universe. Beyond gravitation, such long range interactions are present in various physical fields, either as fundamental or as effective interactions: plasma physics, two dimensional fluid dynamics, wave-particles systems. . . They deeply influence the dynamical and thermodynamical properties of such systems. At the thermodynamic equilibrium, long range interaction is at the origin of very peculiar properties, especially for attractive systems: the specific heat may be negative; canonical (fixed temperature) and microcanonical (fixed energy) ensembles are not equivalent. These special features have been known for a long time in the astrophysics community, in the context of self gravitating systems.

After the seminal works of Lynden-Bell and Wood [2] and Thirring [3], many contributions followed on this subject (see for instance [4] for a recent review), so that the equilibrium characteristics of attractive long range interacting systems are theoretically well established. This situation is in striking contrast with the experimental side of the problem: there is currently no controllable experimental system exhibiting the predicted peculiarities. There have been some proposals to remedy to this situation: O'Dell et al. [5] have suggested to create an effective $1/r$ potential between atoms in a Bose-Einstein condensate using off-resonant laser beams; recently, Dominguez et al. have proposed to take advantage of the capillary interactions between colloids to mimic two-dimensional gravity [6], and Golestanian has suggested experiments using thermally driven colloids [7]. However, these proposals have not been implemented yet, and so far the dream of a tabletop galaxy remains elusive. The key results of this paper are to show some experimental evidences of a gravitational-like interaction in a quasi one-dimensional (hereafter 1D) test system consisting in a cold gas of Strontium atoms in interaction with two counter-propagating quasi-resonant lasers.

First let us show that the quasi 1D {cold atomic gas + 1D quasi-resonant laser beams} system can indeed exhibit an 1D gravitational-like force. For that purpose, we consider an atomic gas with a linear density $n(z)$ in interaction with two counter-propagating laser beams. The two beams intensities $I_+(z)$ and $I_-(z)$, where $I_+(-\infty) = I_-(+\infty) \equiv I_0$, respectively propagating in the positive and negative direction, are much smaller than the atomic line saturation intensity I_s. Thus the atomic dipolar response is linear. The radiation pressure force of the lasers on a single atom having a longitudinal velocity v_z is given by [8]:

$$F_\pm(z, v_z) = \pm\hbar k \frac{\Gamma}{2} \frac{\Gamma^2}{4(\delta \mp k v_z)^2 + \Gamma^2} \frac{I_\pm(z)}{I_s}, \qquad (1)$$

where $\hbar$ is the reduced Planck constant, Γ the bare linewidth of the atomic transition, k the wavenumber and δ the frequency detuning between an atom at rest and the lasers. The attenuation of the laser intensity is given by:

$$\mathrm{d}I_\pm = \mp \frac{\sigma}{2\pi L_\perp^2} n(z)\mathrm{d}z, \qquad (2)$$

where

$$\sigma = \frac{6\pi}{k^2}\Gamma^2 \int \frac{\mathfrak{f}(v_z)}{4(\delta + k v_z)^2 + \Gamma^2} \mathrm{d}v_z \qquad (3)$$

is the absorption cross-section for a cloud of atoms having a normalized longitudinal velocity distribution $\mathfrak{f}(v_z)$, and $2\pi L_\perp^2$ is the transverse section of the cloud. The optical depth is defined as

$$b = \sigma \int_{-\infty}^{+\infty} n(z)\mathrm{d}z\,. \qquad (4)$$

Atoms also experience a velocity diffusion due to the random photon absorptions and spontaneous emissions. In experiments, $\delta < 0$ such that the force, given in Eq. (1), is a cooling force counteracting the velocity diffusion. In this context, we can derive a fluid equation starting from the appropriate 1D Fokker-Planck equation for the one-point probability density function $f(z, v_z, t)$ describing particles in a trap with a longitudinal trap frequency ω_z and interacting with the pair force (1). We assume that f may be written as

$$f(z, v_z, t) = mn(z,t)\frac{1}{\Delta(t)}\mathfrak{f}\left(\frac{v_z - u(z,t)}{\Delta(t)}\right), \quad (5)$$

where $\mathfrak{f}(v_z)$ is even and homogeneous along the sample; u is a (small) macroscopic velocity and the velocity dispersion is characterized by a time modulation $\Delta(t)$. Integrating the Fokker-Planck equation over $v_z\ dv_z$ and looking for a stationary solution we obtain:

$$k_BT\frac{\partial n}{\partial z} + m\omega_z^2 zn - C\int_{-\infty}^{+\infty}\mathrm{sgn}(s-z)n(s)\mathrm{d}s = 0\,. \quad (6)$$

m the atom mass, k_B the Boltzmann constant and $\bar{v}_z$ the rms velocity. The temperature, defined as $k_BT = m\bar{v}_z = m\int_{-\infty}^{+\infty} v_z^2\mathfrak{f}(v_z)\mathrm{d}v_z$, results from the equilibrium of cooling forces and the velocity diffusion. It appears in the first term of Eq. (6) which governs the spreading of the density distribution. One can note that, in contrast with the 2D and 3D cases, this term always prevents the collapse of the cloud. The second term is due to the external holding trap, supposed here to be harmonic. The long range attractive interaction appears in the third term. Eq. (6) is derived in the weak absorption limit, i.e. assuming $b \ll 1$. One defines a characteristic interaction length as

$$L_i = \frac{k_BT}{Cm}\frac{1}{N} \quad (7)$$

with N is the number of atoms, $C = \frac{3\hbar\Gamma}{2kL_\perp^2}\frac{I_0}{I_s}\eta^2$, and $\eta = \int_{-\infty}^{+\infty}\frac{\Gamma^2}{\Gamma^2+4(\delta-kv_z)^2}\mathfrak{f}(v_z)dv_z$. The self-gravitating regime is then reached if $L_i \ll L_{ni}$ where

$$L_{ni} = \sqrt{\frac{T}{T_{\mathrm{trap}}}}z_R \quad (8)$$

is the characteristic length of the non-interacting gas in its external harmonic holding potential. The quantities z_R and $k_BT_{\mathrm{trap}}/2$ are respectively the Rayleigh length and the depth of the trapping potential. If the inequality $L_i \ll L_{ni}$ is fulfilled, Eq. (6) yields the profile of a 1D self-gravitating gas in thermal equilibrium [9]:

$$n(z) = \frac{N}{4L_i}\mathrm{sech}^2\left(\frac{z}{2L_i}\right). \quad (9)$$

The attractive force coming from the beams absorption, as described above, is known since the early days of laser cooling and trapping [10]. Unfortunately, in a 3D setting this attractive force is usually dominated by the repulsive force due to photons reabsorption [11], which in the small b limit may be seen as an effective repulsive Coulomb force. By contrast, in a 1D configuration with an elongated cloud along the cooling laser beams, the probability of photons reabsorption is reduced by a factor of the order of $L_\perp/L_z$, in comparison with the isotropic cloud having the same longitudinal optical depth. $L_\perp$ and L_z are respectively the rms transverse and longitudinal size of the cold cloud. In our experiment, the reduction factor is about 2×10^{-2}, so that the repulsive force can be safely ignored.

The sample preparation is done in the same way as depicted in [12]; more details about laser cooling of Strontium in a magneto-optical trap (MOT) can be found in [13]. After laser cooling, around 10^5 atoms at $T \simeq 3\,\mu$K are loaded into a far detuned dipole trap made of a 120 mW single focused laser beam at 780 nm. We directly measure the longitudinal trap frequency $\omega_z = 6.7(0.5)$ Hz from relaxation oscillations of the cold cloud (see example of temporal evolutions in Fig. 1). The radial trap frequency $\omega_\perp = 470(80)$Hz is deduced from cloud size measurements. The beam waist is estimated at $23(2)\,\mu$m leading to a potential depth of $T_{\mathrm{trap}} \simeq 20\,\mu$K.

50 ms after loading the dipole trap (corresponding to $t = 0$ in Fig. 1), a counter-propagating pair of laser beams, red-detuned with respect to the ${}^1S_0 \rightarrow {}^3P_1$ intercombination line at 689 nm (radiative lifetime: $21\,\mu$s), are turned on for 400 ms. These beams, aligned with respect to the longitudinal axis of the cloud, generate the effective 1D attractive interaction. When the 1D lasers are on, we apply a $B = 0.3$ G magnetic bias field, for two important reasons: First, the Zeeman degeneracy of the excited state is lifted such that the lasers interact only with a two-level system made out of the $m = 0 \rightarrow m = 0$ transition which is unsensitive to the residual magnetic field fluctuation. Second, the orientation of magnetic field bias, with respect to the linear polarization of the dipole trap beam, is tuned to cancel the clock (or transition) shift induced by the dipole trap on the transition of interest [12].

The temperature along the 1D laser beams, in our experimental runs, is found to be in the range of $1-3\,\mu$K. Even at such low temperatures, and in sharp contrast with standard broad transitions, the frequency Doppler broadening $k\bar{v}_z$ remains larger than Γ. As a direct consequence, the optical depth b depends on the exact longitudinal velocity distribution $\mathfrak{f}(v_z)$ (see Eqs. (3) and (4)) which are not necessarily gaussian [12]. Since we have not access to $\mathfrak{f}(v_z)$ but only to $\bar{v}_z$ or T, one has enough control to assert the $b \ll 1$ limit, thus the occurrence of the self-gravity regime, but we can perform only qualitative tests of our theory described above.

At $t = -50$ms, the MOT cooling laser beams are turned off, leaving the trapped atomic cloud in an out-

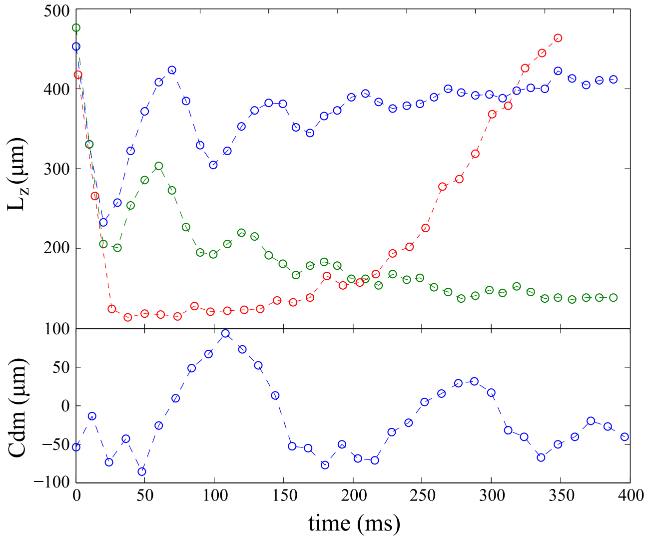

FIG. 1: Upper part: Typical temporal evolutions of L_z the rms longitudinal size of the atomic cloud for three different 1D beam intensities, $I = 0$ (blue circles), $I = 0.015 I_s$ (green circles) and $I = 0.1 I_s$ (red circles). The laser detuning is $\delta = -5\Gamma$ for all curves. Lower part: The center of mass position of the atomic gas without the 1D lasers ($I = 0$). The y axis origin is arbitrary.

of-equilibrium macroscopic state. Without the 1D lasers, we observed a weakly damped oscillation of the breathing mode and of the center of mass position (blue circles in Fig. 1). One notes that damping is caused by anharmonicity of the dipole trap and not by thermalization of the gas which is negligible on the experimental timescale. In presence of the 1D laser beams, overdamped or underdamped oscillations of the cloud are observed. Both regimes will be now analyzed in detail.

Let us first consider the stationary state in the overdamped situation (red circles in Fig. 1). After the transient phase ($t < 30$ ms), the rms longitudinal size of the atomic gas reaches a plateau at a minimal value of $L_z \simeq 120\,\mu$m with $T \simeq 2\,\mu$K. The slow increase of the cloud's size after the plateau ($t > 150$ ms) goes with an increase of the temperature up to $4\,\mu$K at the end of the time sequence. The origins of the long time scale evolution are not clearly identified, but it is most likely due to coupling of the longitudinal axis with the uncooled transverse dimensions because of imperfect alignment of the 1D laser beams with the longitudinal axis of the trap and nonlinearities of the trapping forces. At the plateau where temperature is around $2\,\mu$K, we stress that, according to Eq. (8), the non-interacting gas is expected to have a rms longitudinal size of $L_z = L_{ni} \simeq 370\,\mu$m. Hence, here we clearly observe a compression of the gas, by a factor of three, due to the attractive interaction induced by the absorption of the 1D laser beams. Moreover, the estimated optical depth is $b \simeq 0.4$. We then

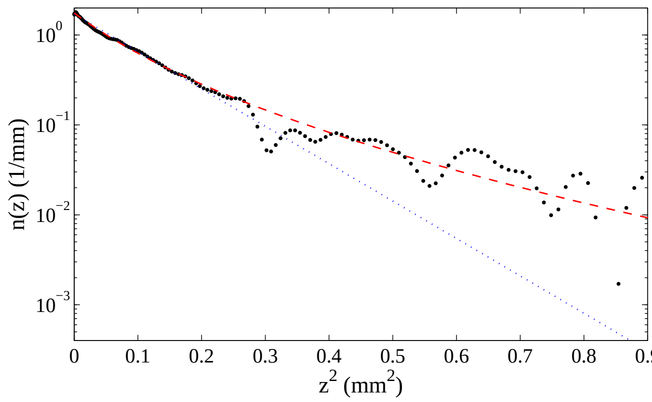

FIG. 2: Density linear distribution in self-gravitating limit. The black dots are the experimental data. The profiles were symmetrized to improve the signal to noise ratio. The blue dotted and red dashed curves are fits of the experimental data with respectively, a gaussian and a sech^2 profile. The fits are performed taking the distribution width and height as the two free fitting parameters.

almost fulfill the two previously mentioned conditions — $b \ll 1$ and $L_z \ll L_{ni}$ — for being in the 1D self-gravitating regime. We note that the corresponding experimental linear density distribution (black dots in Fig. 2) has longer tails than the standard Gaussian profile (blue doted curve in Fig. 2). Moreover, we find an excellent agreement with the expected sech^2 profile (red dashed curve in Fig. 2). We have also checked that in absence of the 1D laser beams, the experimental linear density distribution corresponds to the expected profile of a non interacting gas in our dipole trap having a $z_R = 1.2(1)$ mm Rayleigh length.

In the self-gravitating regime a $1/N$ dependency of L_z is expected at fixed temperature (see Eqs. (7) and (9)). Fig. 3 shows that the cloud's size L_z is in agreement with this prediction for two temperature ranges: $1.5(2)\,\mu$K (blue circle) and $2.1(2)\,\mu$K (red star). Linear fits correspond to the blue dashed line for $1.5(2)\,\mu$K and the red dashed line for $2.1(2)\,\mu$K. The fitting expression is $N^2 = a_1/L_z^2 + a_2$, where a_1 and a_2 are free parameters. The presence of the holding trap is revealed when N goes to zero by the finite value of L_z. The fit gives $L_z = \sqrt{-a_1/a_2} \simeq 400\,\mu$m for $N = 0$, slightly larger but still in reasonable agreement with the expected value of L_{ni} at these temperatures. The $1/N$ dependency of L_z in the self-gravitational regime is a clear signature of the long range nature of the interaction.

Let us now consider the evolution of the trapped cold cloud in the underdamped situation (as an example see green circles in Fig. 1). Without the 1D lasers, the ratio of the eigenfrequencies of the breathing mode $\omega_{\rm br}$ and the center of mass ω_z is found to be close to two, as expected for a non-interacting gas in a harmonic trap.

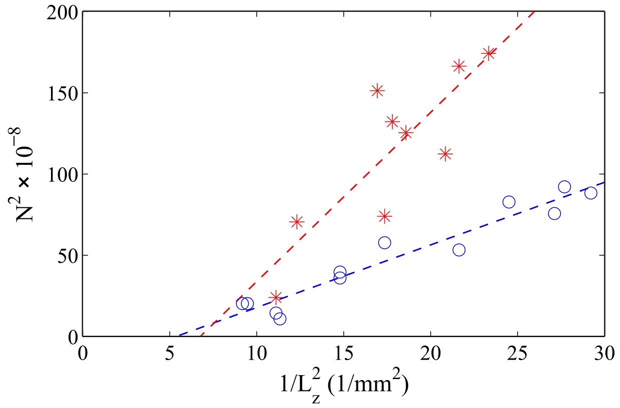

FIG. 3: Dependency of the longitudinal size of the cloud with the number of atoms for $\delta = 5.7(5)\Gamma$ and $I = 0.3 I_s$. The blue circle (red star) data points correspond to temperature $1.5(2)\,\mu$K ($2.1(2)\,\mu$K). The optical depth is in the range of 0.6-0.2 according to atoms number variations. The blue and the red dashed lines are linear fits.

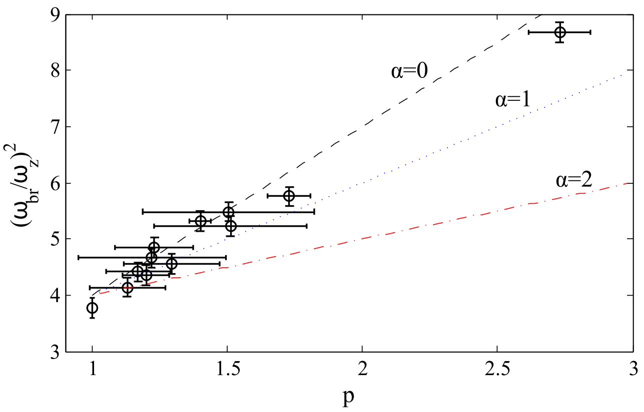

FIG. 4: Comparison for $\alpha = 0, 1$ and 2 of the experimental ratio $(\omega_{\rm br}/\omega_z)^2$ and the predictions deduced from the relation (10). The values of p are measured on the experiment.

As an example the blue curve, shown in Fig. 1, gives $\omega_{\rm br}/\omega_z = 1.9(1)$. If now the attractive long-range interaction is turned on, some modifications of $\omega_{\rm br}$ are expected whereas ω_z should remain unchanged. More precisely, assuming a power law two-body interaction force in the gas $1/r^\alpha$, it is possible to find a simple relation for $\omega_{\rm br}$ in the weak damping limit [14]:

$$\omega_{\rm br} = \omega_z \left((3-\alpha)(p-1)+4\right)^{\frac{1}{2}} . \qquad (10)$$

This formula relates $\omega_{\rm br}$ to α, the index of the interaction potential, and $p = L_{ni}^2/L_z^2$. Eq. (10) is derived in [14] assuming a velocity independent interaction term, which would be obtained by linearizing the radiation pressure force (1) in velocity. Since $0.3 < k\bar{v}_z/|\delta| < 1.2$ for our experimental data points shown in Fig. 4, the force cannot reasonably be linearized. However in the limit of small optical depth, we check, using the Fokker-Planck equation with the ansatz (5), that Eq. (10) with $\alpha = 0$ still provide a reasonable approximation for the breathing frequency. Fig. 4 summarizes the comparisons between the measured ratio $(\omega_{\rm br}/\omega_z)^2$ and the predictions deduced from the relation (10). p is computed from the experimental data in the stationary state. We expect $\alpha = 0$, however to judge the nature of the long-range attractive interaction, three plots respectively for $\alpha = 0$, 1 and 2 are shown. If the $\alpha = 2$ case can be excluded, the experimental uncertainties does not allow to clearly discriminate between $\alpha = 0$ and $\alpha = 1$.

To conclude, we give some experimental evidences of an 1D gravitational-like interaction in a Strontium cold gas, induced by quasi resonant contra-propagating laser beams. First, we show that in the self-gravitating limit, the density distribution follows the expected sech^2 profile. Moreover, the scaling of the cloud size with the number of atoms follows the predicted $1/N$ law. Finally, the modification of breathing frequency of the cloud, due to the long range interaction, is correctly described by a self-gravitating model. This suggests interesting consequences: by contrast with the 1D case, a 2D self-gravitating fluid undergoes a collapse at low enough temperature, or strong enough interaction. Hence, it is conceivable that an experiment similar to the one presented in this paper, in a pancake geometry, would show such a collapse.

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