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My prayer and my sacrifice and my life and my death
are surely for Allah, the Lord of the worlds
The Qur'an

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My feelings of gratitude go towards my mother's soul; I am asking God to cover her with his graciousness and keep her in his paradise. My feelings of gratitude also go towards my father for his love and infinite support. I will not forget to thank my brothers and friends for their moral support.

They say behind every great man there's a woman.
While I'm not a great man, there's a great woman behind me.
Meryll Frost, the "Port Arthur" News, February 1946

Never will I forget the support of my wife Mariem, and the energy that my daughter Zeineb gave me. Without them my PhD would have been hard and stressful.

When the people decide to live,
destiny will obey and the chains will be broken.
Aboul Kacem Chebbi, "the will to live", September 1933

This thesis is written in 2011, the year of the revolution in my country Tunisia; I have to say thanks to the martyrs and all revolutionists. Nothing can express my gratitude to you who gave me and all Tunisians the hope for a better life in Tunisia.

Malek Masmoudi
Toulouse, November 2011

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Introduction

A project is a set of partially sequenced activities that have to be executed by limited resources within a specific time horizon. Project management is a complex decision making process that consists of planning and scheduling while respecting resource and precedence constraints. Deterministic project management techniques are not longer applicable in situations in industry where uncertainties are highly present e.g. helicopter maintenance center. As alternative, proactive approaches are provided, and particularly robust planning based on uncertainty modelling techniques like probability and fuzzy sets.

Planning is a tactical process respecting temporal constraints and providing resources levels by defining activities time windows and decisions related to resource allocation: overtime, temporal hiring, hiring, firing and subcontracting. For tactical project planning under uncertainty, as far as we know, only few recent stochastic optimization papers and no fuzzy optimization papers exist in literature. Part of this thesis deals with tactical project planning problem under uncertainties and provides new models and algorithms based on both fuzzy and stochastic modelling techniques. We refer to these problems as the Fuzzy Rough Cut Capacity Problem (FRCCP) and the Stochastic Rough Cut Capacity Planning (SRCCP). Motivated by the fuzzy approach (see section 3.2.4), the FRCCP is studied more deeply.

Scheduling is the operational process that consists of determining in short term the time window of each task respecting the limit of capacity and the precedence constraints between tasks. For operational project planning under uncertainty, many stochastic and fuzzy optimization papers exist in literature. For fuzzy project scheduling particularly, resources had been considered within a deterministic way till we have provided recently a fuzzy modelling of resources usage based on the possibilistic approach. Part of this thesis explains in details the new modelling and provides algorithms to solve fuzzy time-driven and resource-driven scheduling problems. We refer to these problems as the Fuzzy Resource Leveling problem (FRLP) and Fuzzy Resource Constrained Project Scheduling Problem (FRCPSP).

This thesis is organized as follows. In Chapter 1, we introduce the framework of our study and explain our motivation and the originality of our contribution. In Chapter 2, we explain the project planning problem with the difference between tactical and operational level of planning while dealing with capacity issues. In Chapter 3, we recall basics of possibilistic and stochastic approaches, and show how they are used to deal with planning and scheduling problems under uncertainties. Chapter 4 contains the modelling of Rough Cut Capacity Problem (RCCP) under uncertainties and provides a generalization of several algorithms to handle fuzzy and stochastic parameters; a simulated annealing is provided and existing deterministic exact Branch- and -Price algorithm and linear-programming-based heuristics are adapted to uncertainties. In Chapter 5, a new fuzzy modelling approach is provided to deal with fuzzy scheduling problem. A fuzzy greedy algorithm (parallel SGS) and a fuzzy genetic algorithm are provided to deal with FRCPSP and FRLP problems, respectively. Conclusions and perspectives of the thesis are provided at the end.

Chapter 1

Context of study

Contents

1.1 Helimaintenance project

Helimaintenance is a project approved by the Aerospace Valley cluster in 2006. It consists of establishing a center of civil helicopter maintenance and aims at reaching the rank of European leader in maintenance, customisation and compete with civil helicopter obsolescence [Thenaisie, 2005].

Civil helicopter is the unique mean of transport to carry out several important missions such as medical evacuation and rescue. Moreover, helicopter maintenance is very costly (*e.g.* The PUMA helicopter Heavy maintenance Visit (HMV) costs around 2 *Million euros*, equivalent to one third the cost of possession). The development of civil helicopters maintenance is limited in Europe, because actors are not able to invest in R&D. In fact, expensive and sophisticated facilities are needed to make development, while, actors in this domain have very small structures (6 to 10 employees in average), except for a few centres in United Kingdom, Norway, and Spain.

The objective of the Helimaintenance project is to make improvements in the field and reduce the helicopter maintenance cost, particularly decreasing the helicopters immobilization. To reach these objectives, two strategical axes are considered: to establish a strong industrial organisation, and make a research and development program in the field.

The Industrial organisation is created by the association of 12 local partners of the Helimaintenance Industry; the project lead and first sponsor. The structure is located in the aerodrome of Montauban near Toulouse in France within the area of $4400m^2$ (the covered surface is equal to $1300m^2$) (see Figure 1.1).

Figure 1.1: Helimaintenance center geographic area.

The research and development program started in 2008 with a first project called Helimaintenance R&D 1. This project is supported by the FUI (Abbreviation of the french government funds called "Fonds Unique Interministériel" given by the French state to finance projects approved by French clusters). Two industrials (C3EM, SEMIA) and three Research centres (ONERA, EMAC, ISAE) are involved in this project. The objective is to develop a complete integrated system (see Figure 1.2):

- Smart sensors are to be installed on critical components. These sensors are to contain radio DATALOGGER to transmit data through wireless network to an embedded calculator,
- The embedded calculator is to be manufactured to transmit, through GPRS network, this data to a database on the ground to be exploited by the centre,
- Real time software is to be developed for sensors and calculator control,
- An automatic embedded data driver is to be developed,

– Software tools are to be developed to exploit data on the ground. They are to be connected to Helimaintenance Industry production pilot tools,

– A *Decision Support System* (DSS) is to be developed and integrated to Helimaintenance Industry production pilot tools to optimize the industrial process,

– A web ASP computing platform with a database are to be developed and connected to all computing tools.

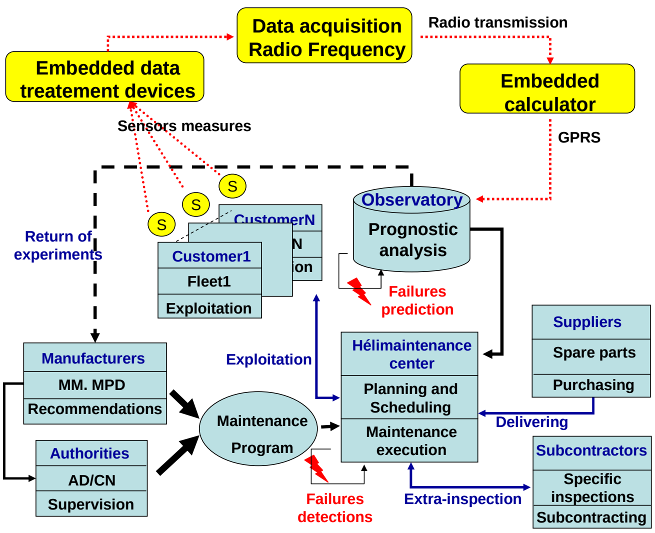


Figure 1.2: Complete integrated logistics support; Project Hélimaintenance R&D1.

The DMIA-ISAE laboratory is particularly involved in the industrial maintenance process optimisation. Among the complex tasks to study in this topic, we cite the mastering of obsolescence and the anticipation of component failure, in addition to the managerial aspect; activity planning, scheduling and execution. We were engaged in the Helimaintenance R&D 1 to work on the maintenance planning and scheduling optimization, which represents the downstream part of the project. Our contribution to the Helimaintenance R&D 1 consists of developing approaches and algorithms to be implemented in a *Decision Support System* (DSS) to optimize the industrial activity. The main objective is to reduce the maintenance cost by 30%, which is a key factor for a successful development of the civil helicopters maintenance.

1.2 Helicopter maintenance

Helicopters have some specificities compared to aircraft; particular flight conditions (vibrations, way of landing), frequent inspections of some equipments, and limited volume that constraints the realization of maintenance (number of operators concurrently working). Helicopter maintenance consists of carrying out all the actions necessary to guarantee the required level of reliability, safety and operational capacity of the aircraft. In Helicopter maintenance and for civil and military domains, we distinguish between three levels of maintenance according to the inspections complexity and localisation [Fabricius, 2003]:

– Line maintenance: contains simple checks that do not need sophisticated facilities, such as small checks before and after flights, daily and weekly inspections, some diagnosis tasks, small reparations, some replacements, cleaning and conditioning,

– Light maintenance: contains visual inspections and several checks that need specific facilities such as detailed check and diagnosis of components and systems, high-level check and inspection and modifications,

– Heavy maintenance: contains great inspections such as *Heavy Maintenance Visits* (HMV) that needs sophisticated facilities for parts removal, disassembly, and structural checks.

The off-line inspections (Light and Heavy maintenance) are usually subcontracted and carried out at *Maintenance, Repair and Overhaul shops* (MROs). The most extensive and demanding check is the *Heavy Maintenance Visit* (HMV). This visit is particularly detailed in this thesis, because the Helimaintenance project stakeholders decided to start the activity with HMVs on one particular type, namely the PUMA helicopter. The result of our study must be generic because it is expected that the activity will be extended to cover all off-line checks for a series of helicopters; PUMA, Superpuma, Ecureuil and others. The HMV contains several maintenance tasks that affect all aspects (structure, avionics, mechanics) and can last up to several months. On PUMA helicopters for example, HMV is generally carried out every twelve years. Moreover, HMV contains planned maintenance tasks and also corrective maintenance tasks since several failures are only discovered during the execution. Precedence constraints exist between the tasks, due to technical or accessibility considerations. Consequently, HMVs are managed in MROs like projects that share several resources. This will be explained in details in section 1.3 and real data will be exposed.

Helicopter maintenance is a highly regulated domain, due to the potential criticality of failures. Actors in this field must be continuously in relation with the manufacturers and the local authorities *i.e.* a French MRO making inspections on PUMA helicopters regularly receives documents from Eurocopter manufacturer and *Direction Générale de l'Aviation Civile* (DGAC) authority.

From a product point of view, various documents are delivered by the manufacturer to explain how to exploit the helicopter and keep it safe. Among them, the *Maintenance Planning Document* (MPD), established by the manufacturer on the basis of reliability studies, gives the periodicity of inspection of the equipments (calendar limits and/or number of flight hours and/or number of take-off-landing flight cycles). The MPD is periodically updated so the maintenance tasks may change all along the aircraft life cycle. From a process point of view, the *Aircraft Maintenance Manual* (AMM) that is delivered by the manufacturer describes how to perform the maintenance actions. Regulations from authorities also constrain the maintenance activity: ratio of permanent operators, number of hours per week, and operator skills. On this basis, the aircraft owner establishes the *maintenance program*, that must be approved by the authorities. The list of tasks to be performed during a maintenance visit depends on the aircraft exploitation and equipment history, while considering the limits set in the MPD. It also depends on decisions of anticipating some tasks in order to balance the number of visits and their durations (*i.e.*, to balance aircraft exploitation and maintenance cost). Finally, unexpected failures may force to anticipate a visit.

1.2.1 MROs and HMVs management

Maintenance, repair and Overhaul centres operate in a very regulated area. The organisation has to be approved as PART145 by the regulatory authority (EASA in Europe) [EASA, 2010]. Moreover, Each MRO department is organised differently with varying activities and structures. The global MROs management is a very complex subject that is out of scope of this thesis; we refer readers to [Fabricius, 2003, Kinnison, 2004]. Nowadays, MROs are managed using computers. There are more than 200 commercial *Computerized Maintenance Management System* (CMMS) tools dealing with aeronautical maintenance planning and control [Fabricius, 2003]. In this thesis we will focus on HMVs management. A HMV may be seen as a project involving various resources; technicians, equipments, documents, and spare parts. Below, we provide an overview about these resources and how they are managed within HMVs.

Depending on the type of helicopter to be inspected, the assigned work team to a HMV is composed of at least one avionic, one mechanic, and one structure specialist. Technicians and engineers need to hold the certified licence Part66 for each specific subsystem to inspect. Each licence is checked every 5 years. Moreover, any licence is lost when it is not exploited for at least 6 months each 2 years. In France, the number of weekly working hours is 35 per person. Overtime is acceptable and equal at most to 25 hours per person, but not for two successive weeks. On the other hand, it is required by the regulatory authority that the external human resources must not exceed the regular capacity.

Several tools, facilities and specific infrastructure are needed to make HMVs (*e.g.* test bench, Non-destructive testing equipment). The set of necessary equipments are listed in the AMM. To be operational, An MRO structure and facilities must be approved Part145 or Part21 from EASA. These equipments are expensive, consequently they are limited and thus they are to be shared between projects (HMVs) in the operational level of planning.

More than 50 documents are used to exploit an helicopter and maintain its airworthiness. The main documents used for HMVs are MPD, AMM and the Minimum Equipment list (MEL). Based on these technical documents, job-cards (the document that provides a technician with all the information needed to execute an inspection) are prepared. The MEL contains information about the criticity of components which is useful to take the decision of carrying out or delay several checks. The MPD contains all maintenance tasks coming from:

- the Maintenance Review Board Report (MRBR), and the Airworthiness Limitations Section(ALS) documents (Certification Maintenance requirements(CMR) and Airworthiness Limitation Items(ALI)): documents provided by the manufacturers and approved by the authority. They are delivered with the helicopter,

- the Service Bulletin(SB) and the Service Information Letter(SIL): updates of the MRBR sent by the manufacturers to the Helicopter's users and MROs,

- the Airworthiness Directives (AD/CN): updates sent by the authorities containing mandatory modifications on helicopters.

Spare parts must be approved as PART21. They can be expendable (consumable), repairable and rotable. The MEL document contains the list of equipments classified according to their criticity into three categories: Go (non critical whose failure does not have impact on the helicopter airworthiness), Go if (critical, implicates a restriction of the airworthiness) and No Go (very critical lead to the immobilisation of the helicopter). The type of a component, its criticity and cost are the main information that are used to define the corresponding quantity to be stocked to reach a specific degree of security [Masmoudi and Hait, 2011]. MacLeod and Petersen [1996] present different benchmark policies for spare parts management. Spare parts management is out of scope of this thesis.

Dealing with uncertainties is the main issue in helicopter maintenance planning and scheduling, the next subsection describes this problem.

1.2.2 Uncertainties in helicopter maintenance

At the tactical level of planning, we can identify three main sources of uncertainty:

– Uncertainty in the release date: a customer enters into a contract for a HMV with MRO several months in advance. According to the exploitation of the helicopter, the real start date may vary in order to reach the limits specified in the MPD. The release date is fixed only 6 to 8 weeks in advance,

– Uncertainty in activity work content: e.g. the corrective maintenance part that is significant in these projects is only known after the first inspection tasks of the project,

– Uncertainty in procurement delays: though spare parts for planned maintenance can be purchased on time, corrective maintenance induces additional orders. Depending on whether these parts are available in the inventory or should be purchased, or even must be manufactured, the procurement delays may change radically.

At the operational level, we can identify three main sources of uncertainty:

– the regularly maintenance program updates: manufacturers and authorities regularly send new documents (Service Bulletin (SB), Airworthiness Directives (AD), etc...) to add, eliminate or modify some tasks from the maintenance program document,

- the variability of tasks durations: differs according to skills level of the assigned operator. It differs also from one helicopter to another according to the compactness, state, and mission use. Tasks starting dates are consequently uncertain,
- the absence of operators: the unexpected lack of resources causes the delay of several tasks and hence some tasks durations are increased.

To deal with uncertainties in our case, we propose a combined approach. First, considering the non repetitive aspect of the problem (each helicopter has its own history, the customers are numerous and the conditions of use are highly different), and the difficulty to predict the exploitation or establish statistics on corrective tasks or tasks durations, we propose a fuzzy set modelling for macro-tasks work contents to cope with uncertainty in tactical level of planning. Then uncertainty affecting the operational level is managed by a fuzzy set modelling for tasks dates and durations.

1.2.3 State of the art about helicopter maintenance planning

Looking on the literature, we remark that almost the totality of research in helicopter maintenance field are carried out in military domain. To the best of our knowledge, only little work has been published on civil helicopter maintenance [Glade, 2005, Djeridi, 2010], and none on planning and resource management of the heavy inspections. Addressing civil customers involves a great heterogeneity of helicopters. Indeed, the average number of helicopters by civil owner is between two and three, and the conditions of use can radically vary from one customer to another (sea, sand, mountain...). On the contrary, in military domain, there are important homogeneous fleets, and the missions for which the helicopters are assigned are quite similar. Moreover, the management process in Civil MROs is different from the process in military MROs. In fact, in military domain, the helicopters maintenance is managed respecting planned and expected missions [Sgaslink, 1994]. This is similar to the maintenance of machines in production industry that is managed respecting the orders due dates [Nakajima, 1989]. On the contrary, in civil domain, maintenance is carried out by an external maintenance center that is not concerned by the exploitation, but maintains a highly multi-customers relation, and considers each customer's helicopter as a unique project with its release and due date that should be respected. This is similar to *engineering-to-order*

(ETO) manufacturing [Hans, 2001] and heavy maintenance of (other) complex systems e.g. boats [de Boer, 1998].

The application of global optimization approaches, as can be found in the military domain for important homogeneous fleets and one single customer [Hahn and Newman, 2008], is not necessarily pertinent for civil helicopter maintenance. Moreover, according to our knowledge, the tactical planning problem under uncertainty has never been studied, even in the military domain. On the contrary, for scheduling problem under uncertainty, the Theory Of Constraints (TOC) technique has proven its effectiveness for military domain [Srinivasan et al., 2007, Mattioda, 2002] (see *www.realization.com*).

In this thesis, we consider a hierarchical approach instead of a global (monolithic) approach. A new modelling of uncertainties and several algorithms are provided to deal with non-deterministic planning and scheduling for civil MROs.

1.3 Case study: planning and scheduling in civil MROs

We are interested in PUMA Helicopter for which a HMV is composed of 18 macro-tasks, many of which do not need resources like purchasing and subcontracting (see Figure 1.3 and Table 1.1).

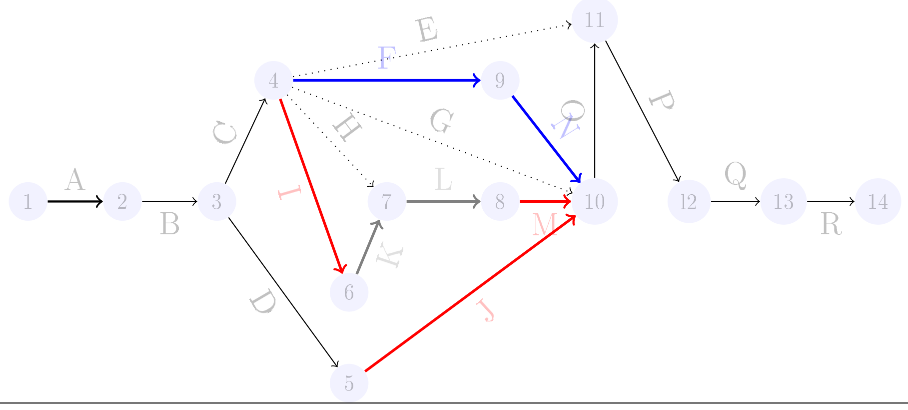


Figure 1.3: AOA network of a HMV project

The work contents to make macro-tasks are estimated on the basis of data from the MPD. However, HMV's planners are aware that the deterministic planning and scheduling they make are always incorrect. In fact, as explained earlier, many uncertainties and perturbations are expected and are hard to

Table 1.1: Example of a HMV project of PUMA helicopter.

Task Name	Task Id	Predecessors	Duration (weeks)	Processing time (hours)	Resource i=(1-2-3)
Waiting for the release date	A	-	8	0	0
First check	B	A	1	∼60	1/3-1/3-1/3
Removal structure and mechanics	C	B	3	∼160	1/2-0-1/2
Removal avionics	D	B	3	∼120	1/4-1/2 -1/4
Supplying procedure for finishing	E	C	14	0	0
Mechanical inspection I	F	C	5	∼360	2/3- 1/3-0
Supplying to assembling	G	C	7	0	0
Supplying to structural inspection	H	C	2	0	0
Subcontracted structure-cleaning	I	C	1	0	0
Subcontracted avionic repairs	J	D	3	0	0
Structural inspection I	K	I	3	∼160	1/4-0-3/4
Structural inspection II	L	H-K	1	∼120	1/4-0-3/4
Subcontracted painting	M	L	1	0	0
Mechanical inspection II	N	F	1	∼90	2/3-1/3-0
Assemble helicopter parts	O	G-J-M-N	1	∼120	1/2-1/4-1/4
Finishing before fly test	P	E-O	1	∼40	1/2-1/2-0
Test before delivering	Q	P	1	∼40	1/2-1/2-0
Possible additional work	R	Q	2	∼40	1/4-1/2-1/4

estimate and integrate into planning and scheduling. Table 1.1 contains data of a HMV project with uncertain macro-tasks processing time. The three main human resources categories needed to perform HMVs are: mechanic experts ($i = 1$), avionic experts ($i = 2$) and structure experts ($i = 3$).

At the strategic level of planning, capacity limits (regular capacity, overtime capacity, hired capacity and subcontracted capacity limits) are decided. In tactical level, the macro-tasks workloads are assigned to periods. If the quantity of workload exceeds the regular capacity, then we try to cover the excess with the overtime capacity. If we still have an excess we apply hired capacity and then subcontract what remains. Finally, in operational level, macro-tasks are split into several tasks to be scheduled within a small horizon. Below, we explain how to divide macro-tasks into small tasks within helicopter maintenance activity.

In HMV, tasks are grouped by subsystems respecting the Air Transport Association ATA100 classification. For example, the mechanical inspection (macro-tasks F and N in Table 1.1) is divided into macro-tasks to be executed on several mechanical parts inspections (Some are presented in Table 1.2):

– the Main Rotor: The work is carried out by 1 expert during 35 to 70 hours,

– the Tail Rotor: The work is carried out by 1 expert during 17 to 35 hours,

- the Main Gear Box: The work is carried out by 1 to 2 experts during 70 to 105 hours. It is often subcontracted to the manufacturer,
- the Propeller: The work is carried out by 1 expert during 70 to 105 hours,
- the Hydraulic System: The work is carried out by 1 to 2 experts during 18 to 35 hours,

Table 1.2: Mechanical tasks from a the HMV of PUMA helicopter.

Part name	Taks Id	Id	Pred.	Experts	Equipments	Duration (days)
Main Rotor	Put off Muff	1	-	1	-	∼0.8
	Put off bearings	2	1	1	-	∼1.3
	Put off flexible components	3	-	1	-	∼0.15
	Clean	4	2-3	1	Cleaning machine	∼1.3
	Non-destructive test	5	4	1	Testing equipment	∼0.4
	Assemble components	6	5	1	-	∼1.3
	Check water-tightness	7	6	1	-	∼0.35
	Touch up paint	8	7	1	-	∼0.15
	Tight screws	9	8	1	-	∼0.55
Propeller	Put off axial compressor	10	-	1	-	∼1.6
	Put off centrifugal compressor	11	10	1	-	∼1.7
	Purchase	12	10	0	-	∼1.5
	Put off turbine	13	-	1	-	∼0.75
	Clean	14	11-13	1	Cleaning machine	∼0.45
	Non-destructive test	15	14	1	Testing equipment	∼0.35
	Assemble components	16	12-15	1	-	∼2.6
	Touch up paint	17	16	1	-	∼0.15
	Tight screws	18	17	1	-	∼0.18
	Test	19	18	1	Test Bench	∼0.18
Hydraulic Sys.	Evacuate oil	20	-	2	-	∼0.15
	Put off servos	21	20	2	-	∼0.75
	Clean	22	21	1	Cleaning machine	∼0.35
	Non-destructive test	23	22	1	Testing equipment	∼0.4
	Assemble then remove joints	24	23	2	-	∼1.1
	Test	25	24	1	Test Bench	∼0.15
	Tight screws	26	25	2	-	∼0.15

Each part inspection can be considered as a small project containing several tasks subject to precedence constraints. The MRO capacity (technicians and equipments) is limited, thus resources are shared by all projects. Hence, the problem is to schedule small projects respecting precedence constraints and workshop resources constraints. We will have to transfer the work content of tasks into durations based on 35-hour working week and the assigned operators.

To show the scheduling problem in MRO within a simple example, we will consider one helicopter HMV and three parts to be checked which are the

Main Rotor, the Propeller and the Hydraulic System gear (see Table 1.2). We consider that our experts have the required qualifications to inspect the different parts. In addition, we consider that MRO has 3 available operators, 1 test bench, 1 Non-destructive testing equipment, and 1 cleaning machine.

Uncertain work contents in the tactical plan and uncertain task durations in the schedule are modelled with the fuzzy set and the possibilistic theory based on the knowledge of experts because no rich statistical information exists in the database. The resume of our contribution within this thesis is shown in the next section.

1.4 Thesis contribution

Maintenance planning aims at organizing the activity of a maintenance center. It deals with tasks to be performed on each aircraft, the workforce and equipment organization, and spare parts logistics (purchasing and inventory management). The challenge is to minimize aircraft down time, while maintaining good productivity and inventory costs.

As already noted, minimizing the overall visit duration gives a competitive advantage to the company. If the delivery date is not respected the company must pay to customer a penalty equivalent to 4 hours operational profitability per day *e.g.* 6 *thousand euros/day* for a PUMA Helicopter.

The MRO management is viewed as multi-project management, where every project duration should be minimized while respecting capacity constraints. Strategic, tactical and operational levels of planning are to be studied within a hierarchical approach to deal with MRO management. This will be the framework of our study, and especially tactical and operational levels are considered.

At the tactical level of planning, orders are studied and then prices and delivery times are negotiated with customers. After a project has been accepted, the macro-tasks are well specified and integrated into the global tactical plan. Moreover, resource capacities are fixed. At the operational level, macro-tasks are detailed into elementary tasks, and resources are assigned to different tasks according to their capabilities. At the tactical and operational planning levels, the capacity management problem is called *Rough-Cut Capacity Planning* (RCCP) and *Resource Constrained Project Scheduling Problem* (RCPSP), respectively. The difference between RCCP (tactical level) and RCPSP (operational level) is well clarified in [Gademann and Schutten, 2005]. To deal with capacity management problem is an important issue in project management and particularly in our application. In RCCP, the planning horizon is divided into periods, contrary to RCPSP where the time horizon is continuous. In RCCP the workload is defined in terms of macro-tasks work contents per required resources (*e.g.* a total of 150 hours of work content for avionicians) in contrast to RCPSP where the tasks duration and the number of operators assigned to the tasks are considered (*e.g.* the task must be performed by 2 avionics during 8 hours).

A significant amount (more than 30%) of unplanned work is discovered during inspections. Consequently, the initial preventive maintenance program usually does not fit with reality. Hence, additional corrective maintenance and purchasing spare parts are to be integrated into the initial tactical and operational planning.

Hans et al. [2007] classified multi-project organizations according to their projects' variability and dependency. Following this reference, our problem is considered with high variability (numerous uncertainties) and high dependency (shared resources and external influence on spare parts supply).

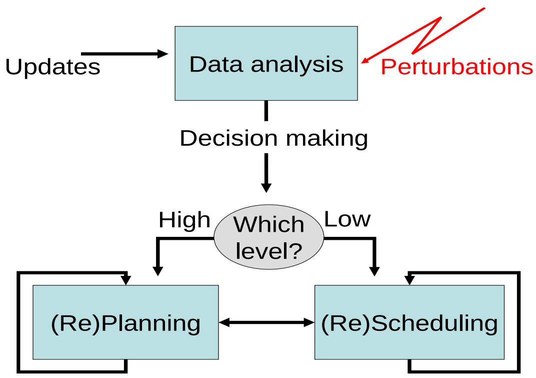


Figure 1.4: Decisional Scheme for proactive-reactive planning and scheduling.

Civil MROs management under uncertainties is the research topic of this thesis. Our contribution will be on planning and scheduling taking into account uncertainties. A proactive-reactive approach is envisaged within an optimization framework (see figure 1.4). Our contribution mainly focuses on the proactive part using fuzzy set modelling and possibilistic approach. In this regard, synthesis of a robust project planning at the tactical and operational levels, is provided with an original definition and modelling of the fuzzy workload plans. The modelling and algorithms provided in this thesis for civil MROs activity are generic and can be adapted to other domains.

Chapter 2

State of the art about project planning and scheduling

Contents

2.1 Project management

Project management is a management discipline that is receiving a continuously growing amount of attention from many organisations in production and service sectors. Project management is a complex task that deals with the selection and initiation of projects, as well as their operation and control. Complexity arises when considering several projects in parallel sharing the same resources. This is called Multi-project management and it is characterised by a high degree of complexity and dependency [Hans et al., 2007]. A survey of existing literature on approaches for multi-project management and planning is proposed in [Wullink, 2005]. Wullink, like other authors [Gademann and Schutten, 2005, Hans et al., 2007], adopted the hierarchical planning framework defined in [de Boer, 1998]. This hierarchical approach is deterministic, like almost all approaches in literature, even though its author mentions that, especially in project environments, uncertainties play an important role. Wullink [2005] proposed a partial generalisation of this hierarchical model to uncertainty consideration based on discrete stochastic scenarios. The de Boer's framework is explained in this section. The aim of our study is to extend several parts of this approach to uncertainty considerations based on continuous fuzzy set modelling with the perspective to provide a complete fuzzy hierarchical planning approach.

2.1.1 Hierarchical planning

Two approaches have been used in literature to address project management; the monolithic approach and the hierarchical approach. The monolithic approach solves the problems as a whole. On the other hand, the hierarchical approach partitions the global problem into series of sub-problems that are to be solved sequentially. In order to break down project and production management into more manageable parts, a hierarchical planning framework has been proposed in [de Boer, 1998] (Figure 2.1). This framework is suitable to our problem that gathers production and project features. Hans et al. [2007] adapt this hierarchical approach to discern the various planning functions with respect to material coordination and technological planning in addition to capacity planning. The framework is divided into the three levels of Anthony's classification [Anthony, 1965]: *strategic*, *tactical*, and *operational*. Each level has its own constraints, input data, planning horizon and review interval. Interactions between levels depend on the application environment [Hans et al., 2007].

Strategic planning involves long-range decisions such as make or buy decisions regarding to space, staffing levels, layouts, number of critical resources. At strategic resource planning level, senior managers define the strategic resource plan respecting a specific management vision within its overall goals with regard to strategic issues such as the hiring and release of staff, the acceptable level of under-utilisation, and the maximum amount of subcontracting. Other input data may be a market competitiveness strategy, agreements with external suppliers, and agreements with major customers, etc. The horizon of such a plan may vary from one to several years and the review interval should depend on the dynamics of the organisation's environment.

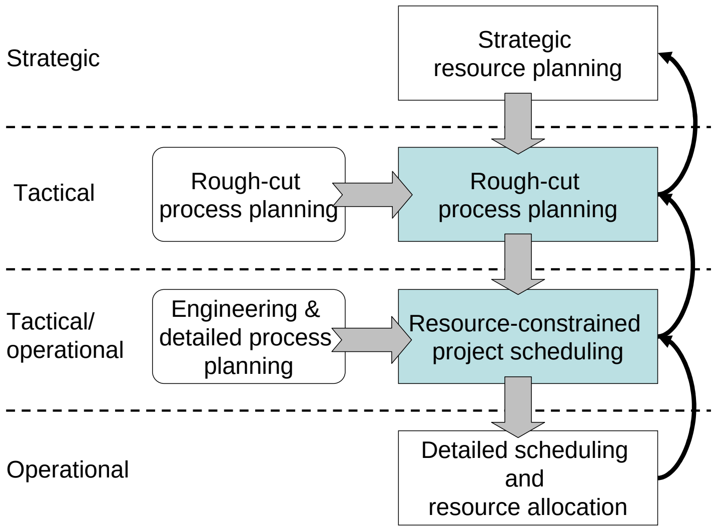


Figure 2.1: Hierarchical planning framework ([de Boer, 1998])

Tactical planning involves medium-range decisions. At this level, the rough-cut capacity planning (RCCP) method is applied to make adequate decisions about due dates and milestones of projects, overtime work levels and subcontracting. RCCP should be used during the negotiation and order-acceptance stage of a new project. Project networks already available and estimations of future resource availabilities are input for the RCCP. At the RCCP-level, it is assumed that the amount of regular capacity for each resource is given. Regular capacity is capacity that is normally available to the company and is to be distinguished from non-regular capacity, which is a result of working overtime, hiring extra personnel, subcontracting, etc. Employment of non-regular capacity will result in an extra cost and is decided at this level [Leus, 2003]. Two approaches to the RCCP-problem can be distinguished: resource-driven and time-driven planning [de Boer, 1998]. With resource-driven planning, the availability of each resource is constrained and the aim is to meet due dates as much as possible (the resource availability problem). In time-driven planning, on the other hand, time limits on the projects are given and the aim is to minimize the use of non-regular capacity such as overtime work (resource levelling problem) [Shankar, 1996]. In practice, a combination of the two methods is already used [Kim et al., 2005a], but for the operational level of planning which is, compared to the tactical level, characterized by a lower degree of capacity flexibility. The tactical planning horizon may vary from half a year to one or two years, depending on expected project durations.

Finally, operational planning involves short-range decisions. At this level, to deal with resource considerations, de Boer [1998] uses in his approach, as shown in Figure 2.1 the Resource constrained Project Scheduling Problem, although other models for project scheduling like the resource leveling and the resource allocation can also be plugged in. After a project is accepted, more detailed information about resource and material requirements becomes available from engineering, and process planning and a more detailed activity network can be drawn. The RCPSP assumes given resource levels. The work packages of the RCCP-level are broken down into smaller (possibly precedence-related) activities with specific duration and resource usage, based on engineering and detailed process planning information. These data are used as input for RCPSP. The operational planning horizon may vary from several weeks to several months.

Information is communicated to subsequent levels. Hence, constraints are imposed on lower levels and downward compatibility of the planning framework is ensured. Figure 2.1 shows feedback loops that ensure upward compatibility and reactivity of the planning.

2.1.2 Rough cut capacity planning survey

Deterministic planning approaches for the RCCP-problem have been proposed by [de Boer, 1998, Hans, 2001], and [Gademann and Schutten, 2005]. All these tactical planning approaches minimize the cost of using non-regular capacity. de Boer [1998] proposes two heuristics; a constructive heuristics called *Incremental Capacity Planning Algorithm* (ICPA) and a Linear Programming Based heuristic. Hans [2001] proposes an exact Branch and Price algorithm to solve the problem modelled as a MILP. Gademann and Schutten [2005] propose several heuristics and distinguish three categories of solution approaches: constructive heuristics, heuristics that start with infeasible solutions and convert these to feasible solutions, and heuristics that improve feasible solutions. Then they make an interesting comparison between their own heuristics, the heuristics of de Boer and the exact technique of Hans.

To deal with a non deterministic RCCP-problem, Elmaghraby [2002] affirms that the processing time of an activity is one of the most important sources of variability. Wullink [2005] proposes a proactive approach based on stochastic scenarios to model uncertain processing times and [Masmoudi et al., 2011] proposed a proactive approach based on a continuous representation of uncertain processing times using fuzzy sets and possibility approach. Taking into account uncertainties in the RCCP problem may require particular objectives such as expected cost of non-regular capacity and robustness of the plan. These criteria are studied in the aforementioned references. The approach provided in [Masmoudi et al., 2011] is extended and explained in details in chapter 4.

2.1.3 Project scheduling and resource leveling survey

Traditionally, scheduling theory has been concerned with allocation of resources, over time, to tasks or activities [Parker, 1995]. On the area of scheduling, rapid progress regarding models and methods has been made. Two techniques of resource management, namely resource-constrained project scheduling and resource leveling, are considered while dealing with renewable resources *e.g.* available workers per day [Herroelen, 2007]. Resource-constrained project scheduling explicitly takes into account constraints on resources and aims at scheduling the activities subject to the precedence constraints and the resource constraints in order to minimize the makespan (project duration). On the other hand, resource leveling takes into account the precedence constraints between the activities, and aims at completing the project within its due date with a resource usage which is as leveled as possible throughout the project duration.

Resource-constrained project scheduling problem is one of the most attractable classical problems in practice. It is clear that solving the RCPSP has become a flourishing research topic when observing the significant number of books that were published in this subject [Artigues et al., 2008]. Multiple exact techniques and heuristics and a number of meta-heuristics have been applied to solve the RCPSP problem [Herroelen, 2007]. This thesis does not aim to provide a survey of all models, algorithms, extensions and applications in this field because already many books deal with this issue in both deterministic and non-deterministic situations [Leus, 2003, Slowinski and Hapke, 2000]. We are particularly interested in studying the parallel *Schedule Generation Scheme* (SGS) technique. Chapter 5 explains in details how it was generalized to fuzzy parameters [Hapke and Slowinski, 1996, Masmoudi and Haït, 2011a].

The Resource Leveling problem has been originally studied for fixed project duration. Popular exact and heuristics methods were developed for deterministic situations. As an example of exact methods, Easa [1989] used integer programming techniques and Ahuja [1976] presented exhaustive enumeration procedures, and as example of heuristics, Harris [1990] developed the popular PACK model and Chan et al. [1996] proposed a model using genetic algorithms. In the last decades, authors have begun to study non-deterministic scheduling. On the contrary to RCPSP, few non-deterministic resource leveling models have been developed [Leu et al., 1999]. Chapter 5 explains how resource leveling techniques were generalized to fuzzy parameters based on a genetic algorithm [Leu et al., 1999, Masmoudi and Haït, 2011b].

2.2 Solution techniques for project planning and scheduling

Herroelen [2007] provides a remarkable academic book that surveys the existing techniques for project scheduling by means of illustrative examples. This section provides a small overview and not a survey about techniques for project planning and scheduling. However, references to interesting surveys and papers are included for readers who want to get more information in this field.

2.2.1 The PERT/CPM techniques

The *Program Evaluation Review Technique* (PERT) and the *Critical Path Method* (CPM) were developed in the 50's, within different contexts: the CPM was developed for planning and control of DuPont engineering projects and the PERT was developed for the management of the production cycle of the Polaris missile. They share the same objectives such as defining the project duration and the critical tasks [MacLeod and Petersen, 1996]. In this thesis, we use the expression PERT/CPM technique to express that both techniques are considered equivalent. The PERT/CPM technique is based on two successive steps; a forward propagation to determine the earliest start and finish dates (and consequently the project duration and the free floats), and a backward propagation for the latest start and finish dates (and the total floats). Originally, the activity times are fixed within the CPM technique and probabilistic within the PERT technique [Ika, 2004]. Over the last few decades, both CPM and PERT techniques have been generalized to fuzzy and stochastic areas [Lootsma, 1989, Chanas et al., 2002] to deal with uncertainty in project management. Particularly, Fuzzy PERT and CPM are to be considered to deal especially with fuzzy scheduling in chapter 5. On the contrary to PERT/CPM technique that omits any consideration of resources, other techniques that deal with resources constraints are recalled in next section.

2.2.2 Time and resource driven techniques

The trade-offs between lead time and due date on one hand, and resource capacity levels on the other hand is always present when dealing with project planning and scheduling within resource consideration. Hence, for both tactical and operational planning we distinguish two kins of problems: the resource driven and the time driven [de Boer, 1998]. In resource driven planning and scheduling, the resource availability levels are fixed, and the goal is to meet a project due date *e.g.* minimize lateness, minimize the number of projects or tasks that are late. In time driven planning and scheduling, due dates are considered as strict as deadlines, and the aim is to minimize the extra resources usage *e.g.* minimize the costs of using non regular capacity [Mohring, 1984]. In this thesis, the resource driven technique is applied to the RCPSP problem (see chapter 5), and the time-driven technique is applied to RCCP and RLP (see chapter 4 and chapter 5), respectively.

2.2.3 Algorithms: exact and heuristics

Most of the planning and scheduling problems are NP-hard. Hence, exact and approximate methods are considered (see Figure 2.2) depending on the complexity of the projects to manage. In fact, to solve small projects, exact methods can be applied, to obtain optimal solutions with guarantee of optimality. On the contrary, to solve projects with several hundreds of activities, only approximate procedures (heuristics) are computationally feasible and generate high quality solutions but without guaranty of optimality.

In the class of exact methods we find: branch and X family (B&Bound, B&Cut and B&Price), constraint programming, dynamic programming, and several algorithms developed in the artificial intelligence community like A^*. Particularly, the Branch & Price technique was applied to tactical capacity planning problems [Hans, 2001]. The Hans' B&P method is briefly explained in section 4.3.1 and generalized to fuzzy sets to deal with tactical capacity planning problems under uncertainties.

Approximate algorithms are classified into specific heuristics and meta-heuristics.

Specific heuristics are provided to solve specific problems and/or instances *i.e.* Parallel SGSs algorithms are provided to solve scheduling problems based on priority rules [Kolish and Hartmann, 1999] and Linear-programming-based

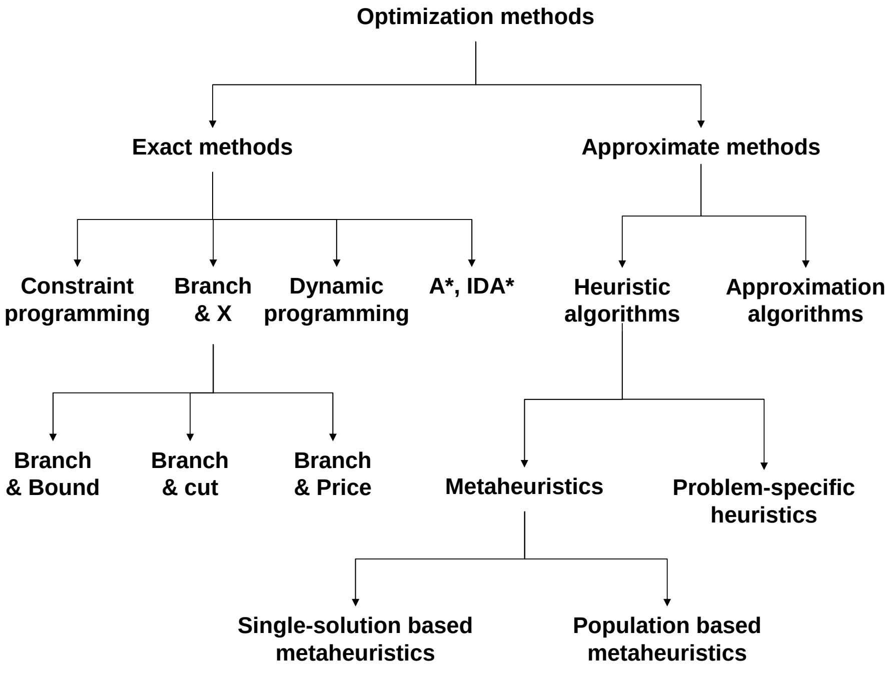


Figure 2.2: Classical optimization methods ([Talbi, 2009])

heuristics are provided to solve tactical capacity planning problems [Gademann and Schutten, 2005]. These two heuristics are particularly considered in this thesis and generalized to fuzzy sets to deal with scheduling and planning problems under uncertainties, respectively.

Meta-heuristics are generic and their application falls into a large number of areas. Talbi [2009] provides a genealogy of the 24 meta-heuristics (and the number is growing) that were developed from 1947 to 1996. Among them, there are the Genetic Algorithm that is developed in 1962 [Holland, 1962] and the Simulated Annealing that is developed in 1983 [Kirkpatrick et al., 1983]. In this thesis, these two meta-heuristics are explained and then adopted and generalized to fuzzy sets to deal with Resource leveling and tactical capacity planning problems under uncertainties, respectively.

2.2.4 Practical variants and extensions

The basic scheduling and planning models are too restrictive for many current practical applications. Consequently, different variants and extensions of project planning and scheduling problems have been studied in literature

[Shankar, 1996]. They can be divided into several branches; single project or multi-project, single resource or multi-resource constrained, uni-objective or multi-objective, uni-mode or multi-mode, time driven or resource driven, allowing task preemption or not, resources are renewable and/or not, etc.

The consideration of a uni-objective or a multi-objective function is among the most common extensions [Talbi, 2009]. The optimizing objective function could be to minimize project duration, minimize (weighted) project tardiness or lateness, minimize cost or maximize profit, smoothing resource usage, or maximize robustness or flexibility or stability in case of a non-deterministic problem. In addition, depending on the objective(s) we select, relevant structural constraints are to be included in the problem formulation *i.e.* while minimizing the weighted tardiness of projects, an upper limit of the tardiness is to be specified and set as a constraint in addition to the objective function.

Demeulemeester and Herroelen [2002] provide a survey of extensions of project scheduling problems and Hartmann and Briskorn [2010] provide a survey of variants and extensions of the RCPSP problem in particular. Deterministic and non-deterministic are among the common extensions, and project planning and scheduling under uncertainty is becoming one of the subjects that present a big interest in literature. Next chapter will deal with this topic of research.

Chapter 3

State of the art about planning and scheduling under uncertainties

Contents

3.1 Uncertainty and imprecision

Knowledge and perception are imperfect, due to the complexity of observed process, and the lack of clear observation limits *e.g.* the limit between young and old can not be expressed precisely. The imperfection can be seen as uncertainty, as imprecision or as both uncertainty and imprecision.

Dubois and Prade [1985] differentiate between imprecision and uncertainty: imprecision concerns the content of the information and uncertainty is relative to its truth. Imprecise information can not be expressed clearly *i.e.* Alain is 40 to 45 years old. Uncertain information contains a doubt on its validity *i.e.* Alain may be 45 years old. When we combine uncertainty with imprecision we get such information: Alain may be 40 to 45 years old.

Each information, whatever its quality, should not be neglected. Hence, different techniques are provided in literature to take into account uncertain and imprecise information. Next section contains a survey of these techniques and a motivation to use fuzzy/possibilistic approach for our application.

3.2 Uncertainty modelling techniques

We must specify the kind of uncertainty we are interested in, otherwise the models are numerous. For example, Dynamic and conditional Constrained-Satisfaction problems are used when the problem structure (variables or/and constraints) is uncertain, Bayesian Networks is used when variables are uncertain (represented by a probability distribution), dependent (associated with conditional probabilities) and non-time related, and Markov Chain Process modelling and dynamic programming are used when variables (such as states and decisions) are uncertain and time related [Bidot, 2005]. In this thesis, we consider uncertainty in work contents of activities; and durations and consider that activities are independent. Billaut et al. [2005] distinguish 4 modelling approaches to model such uncertainties: stochastic, fuzzy, by interval and by scenarios approaches. According to the experts' knowledge in helicopter maintenance domain, continuous and non uniform distributions fit well with the kind of uncertainty that we deal with. Consequently, only stochastic and fuzzy modelling approaches are considered and studied in details below.

3.2.1 Probability and stochastic modelling

Probability theory is the best developed mathematically, and the most established theory of uncertainty. In practice, there are two probability interpretations, whose communities possess different views about the fundamental nature of probability:

– Bayesians assign to any statement a probability that represents the degree of belief in a statement, or an objective degree of rational belief.

– Frequentists consider probabilities when dealing with experiments that are random and well-defined. The probability of a random event (outcome of the experiment) denotes the relative frequency of occurrence when repeating the experiment.

In probability theory, a distribution is a function that describes the probability of a random variable taking certain values. We consider a random variable X that can be instantiated to value v belonging to a discrete or continuous domain S. In the discrete case, one can easily assign a probability to each possible random variable (used by Bayesians). A probability distribution of a random variable X is discrete and completely known when X is discrete and $\sum_{v \in S} \Pr(X = v) = 1$. In contrast, in the continuous case, a random variable takes values from a continuum (continuous range of values) and probabilities are nonzero only if they refer to finite intervals (used by Frequentists). Formally, if X is a continuous random variable, then it has a probability density function $f(x)$, and therefore its probability to fall into a given interval, say $[A, B]$ is given by the integral $\Pr[A \leq X \leq B] = \int_A^B f(x)dx$. Hence, the probability distribution is completely characterized by its cumulative distribution function $F(x)$. This latter gives the probability that the random variable is not larger than a given value $F(x) = \Pr[X \leq x] \quad \forall x \in S$.

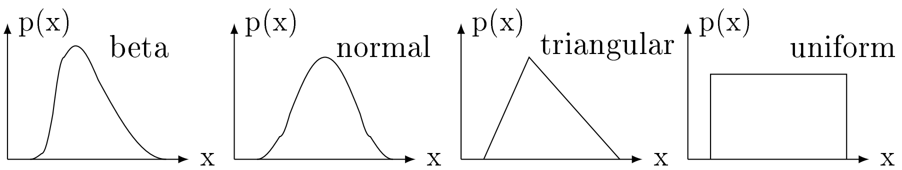


Figure 3.1: Some probability distributions.

There are many examples of continuous probability distributions: uniform, triangular, normal, beta, and others (see figure 3.1). Particularly, the beta distribution is well supported by several techniques in project management such as PERT and CPM techniques [Ika, 2004] because it is bounded, positive, continuous, uni-modal, and multi-shaped.

The sum of two independent random variables is the convolution of each of their density functions and the difference of two independent random variables is the cross-correlation of their density functions. Golenko-Ginzburg [1988] and MacCrimmon and Ryavec [1964] have defined mathematical operations that can be performed on uncertain numbers modelled with beta distributions.

Let A be an uncertain number represented with a beta distribution. The same profile of A can be numerically represented by three expressions:

$$A = [a_A, d_A, \alpha_A, \beta_A], \quad A = [a_A, d_A, m_A] \quad or \quad A = [a_A, d_A, E_A, V_A] \tag{3.1}$$

where:
a_A and d_A are the minimum and the maximum estimation values. They are the same for the different expressions,
m_A is the mode,
α_A and β_A are the parameters of the beta distribution that represents the number A,
and E_A and V_A are the expectation and variance of the number A, respectively.

These different parameters are formally related by the following expressions:

$$m_A = a_A + (d_A - a_A)(\alpha_A - 1)/(\alpha_A + \beta_A - 2) \tag{3.2}$$

$$E_A = a_A + (d_A - a_A)\alpha_A/(\alpha_A + \beta_A) \tag{3.3}$$

$$V_A = (d_A - a_A)^2\alpha_A\beta_A/[(\alpha_A + \beta_A)^2(\alpha_A + \beta_A + 1)] \tag{3.4}$$

Probabilities can be used to model imprecise macro-task processing times or task's durations, but they require statistical data that does not systematically exist. In addition, probabilities are easy to interpret, but can not represent full or partial ignorance [Bidot, 2005]. The stochastic approach explicitly represents uncertainty in the form of probability distributions. In project planning, a few works in literature have used probability distributions within discrete modelling [Wullink, 2005] and continuous modelling [Giebels, 2000]. In scheduling, on the contrary, probability distributions are widely used within discrete and continuous modelling. We refer readers to [Leus, 2003] where a state of the art about stochastic scheduling extensions such as stochastic RCPSP, stochastic activity interruptions, stochastic multi-mode, and stochastic discrete time/cost trade-off problems, are provided.

3.2.2 Fuzzy sets and possibilistic approach

Zadeh [1965] has defined a fuzzy set $\widetilde{A}$ as a subset of a referential set X, whose boundaries are gradual rather than abrupt. Thus, the *membership function* $\mu_{\widetilde{A}}$ of a fuzzy set assigns to each element $x \in X$ its degree of membership $\mu_{\widetilde{A}}(x)$ taking values in [0,1].

To generalize some operations from classical logic to fuzzy sets, Zadeh has given the possibility to represent a fuzzy profile by an infinite family of intervals called α-cuts. Hence, the fuzzy profile $\widetilde{A}$ can be defined as a set of intervals $A_\alpha = \{x \in X/\mu_{\widetilde{A}}(x) \geq \alpha\}$ with $\alpha \in [0, 1]$. It became consequently easy to utilize classical interval arithmetic and adapt it to fuzzy numbers.

Many profiles are used in the literature to model fuzzy quantities (Figure 3.2). Particularly, the trapezoidal profile is well-supported by the possibility approach [Dubois and Prade, 1988] that is presented below.

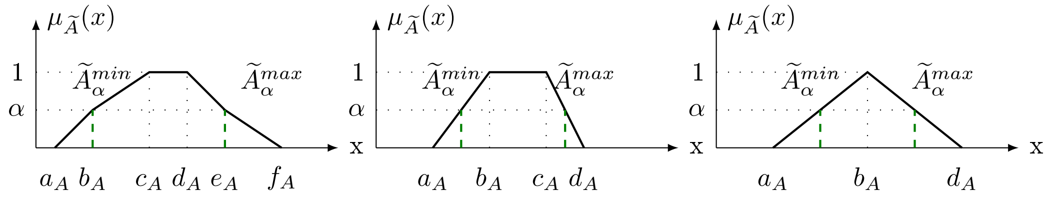


Figure 3.2: Some fuzzy profiles.

Dubois and Prade [1988], and Chen and Hwang [1992] have defined mathematical operations that can be performed on trapezoidal fuzzy sets. Let $\widetilde{A}(a_A, b_A, c_A, d_A)$ and $\widetilde{B}(a_B, b_B, c_B, d_B)$ be two independent trapezoidal fuzzy numbers, then:

$$\widetilde{A} \oplus \widetilde{B} = (a_A + a_B, b_A + b_B, c_A + c_B, d_A + d_B) \tag{3.5}$$
$$\widetilde{A} \ominus \widetilde{B} = (a_A - d_B, b_A - c_B, c_A - b_B, d_A - a_B) \tag{3.6}$$

$$\min(\widetilde{A}, \widetilde{B}) = (\min(a_A, a_B), \min(b_A, b_B), \min(c_A, c_B), \min(d_A, d_B)) \tag{3.7}$$
$$\max(\widetilde{A}, \widetilde{B}) = (\max(a_A, a_B), \max(b_A, b_B), \max(c_A, c_B), \max(d_A, d_B)) \tag{3.8}$$

$$\widetilde{A} \cup \widetilde{B} = \max_{x \in X}(\mu_{\widetilde{A}}(x), \mu_{\widetilde{B}}(x)) \tag{3.9}$$
$$\widetilde{A} \cap \widetilde{B} = \min_{x \in X}(\mu_{\widetilde{A}}(x), \mu_{\widetilde{B}}(x)) \tag{3.10}$$

$$\alpha\widetilde{A} = \begin{cases} (\alpha a_A, \alpha b_A, \alpha c_A, \alpha d_A) & \text{if } \alpha > 0 \\ (\alpha d_A, \alpha c_A, \alpha b_A, \alpha a_A) & \text{if } \alpha \leq 0 \end{cases} \tag{3.11}$$

Other operations like multiplication and division have also been studied. For more details regarding fuzzy arithmetic, we refer readers to [Dubois and Prade, 1988].

In practice, different interpretations of the membership functions μ can be made:

– It is possible to represent an occurrence possibility with $\mu(X = v)$; *i.e.*, the possibility that variable X is instantiated with value v.

- We can express similarity degree with $\mu(Zeineb\ is\ young)$ that represents the truth of the information "*zeineb* is young". Young is represented by a fuzzy profile.

- It is also possible to express preferences with $\mu(X = v)$ that represents the satisfaction degree when variable X is equal to value v.

The possibility theory is based on fuzzy subsets. It was introduced by [Zadeh, 1978] to provide a mean to take into account the uncertainties associated with the occurrence of events. Since we have chosen a fuzzy model, we will use the possibility theory to transpose uncertainty in data into uncertainty in workload.

With a possibility function, we can represent both imprecision and uncertainty. For example, we represent the fact that we do not know precisely and with total certainty the purchasing duration; experts say that it is completely possible that the needed components exist in stock. Otherwise the purchasing takes generally 1 to 6 weeks, but can take 0 to 7 weeks in extreme cases, covering different scenarios such as the components exist somewhere in the world, or are obsolete, and so must be manufactured. Figure 3.3 represents such a possibility function.

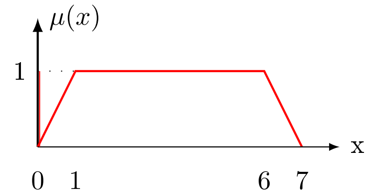


Figure 3.3: Representation of purchasing duration

The classical concept of set is limited for representing vague knowledge, and probability theory is not able to represent subjective uncertainty and ignorance, however, fuzzy logic and the theory of possibility overcome these difficulties. The main drawback of fuzzy representation is the subjective way for interpreting results.

The possibility theory introduces both a possibility measure (denoted Π) and a necessity measure (denoted N). Let P to be a set (fuzzy or not), and $\widetilde{A}$ is a fuzzy set attached to a single valued variable x. The *possibility* of the event "$x \in P$", denoted by $\Pi(x \in P)$, evaluates the extent to which the event is "possibly" true. It is defined as the degree of intersection between $\widetilde{A}$ and

P by the minimum operation:

$$\Pi(x \in P) = \sup_u \min(\mu_{\widetilde{A}}(u), \mu_P(u)) \tag{3.12}$$

The dual measure of *necessity* of the event "$x \in P$", denoted by $N(x \in P)$, evaluates the extent to which the event is "necessarily true". It is defined as the degree of the inclusion ($\widetilde{A} \subset P$) by the maximum operation:

$$N(x \in P) = \inf_u \max(1 - \mu_{\widetilde{A}}(u), \mu_P(u)) = 1 - \Pi(x \in P^c) \tag{3.13}$$

where P^c is the complementary of P ($\mu_{P^c}(u) = 1 - \mu_P(u)$).

Let τ be a variable in the fuzzy interval $\widetilde{A}$ and t be a real value. To measure the truth of the event $\tau \leq t$, equivalent to $\tau \in (-\infty; t]$, we need the couple $\Pi(\tau \leq t)$ and $N(\tau \leq t)$ representing the fact that $\tau \leq t$ is respectively possibly true and necessarily true (see Figure 3.4). Thus:

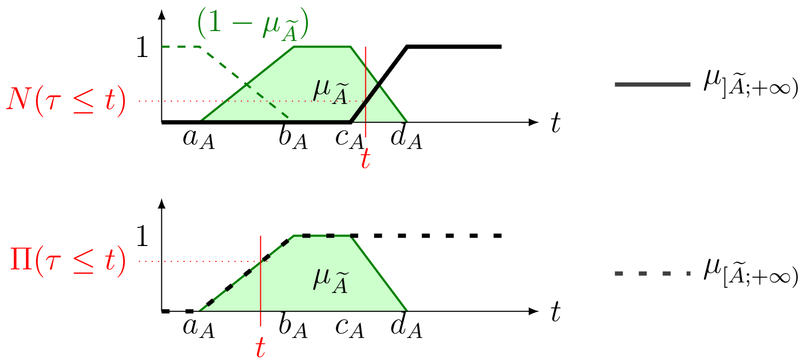


Figure 3.4: Possibility and Necessity of $\tau \leq t$ with $\tau \in \widetilde{A}$.

$$\Pi(\tau \leq t) = \sup_{u \leq t} \mu_{\widetilde{A}}(u) \mu_{[\widetilde{A};+\infty)}(t) = \sup_u \min(\mu_{\widetilde{A}}(u), \mu_{(-\infty;t]}(u)) \tag{3.14}$$

$$N(\tau \leq t) = 1 - \sup_{u > t} \mu_{\widetilde{A}}(u) = \mu_{]\widetilde{A};+\infty)}(t) = \inf_u \max(1 - \mu_{\widetilde{A}}(u), \mu_{(-\infty;t]}(u)) \tag{3.15}$$

Let τ and σ be two variables in respectively fuzzy intervals $\widetilde{A}$ and $\widetilde{B}$, and t be a real value. Based on the aforementioned possibility theory, we define a useful measure corresponding to the truth of the event "t between τ and σ"

by the couple $\Pi(\tau \leq t \leq \sigma)$ and $N(\tau \leq t \leq \sigma)$:

$$\begin{aligned}\Pi(\tau \leq t \leq \sigma) &= \mu_{[\widetilde{A};\widetilde{B}]}(t) = \mu_{[\widetilde{A};+\infty)\cap(-\infty;\widetilde{B}]}(t)\\ &= \min(\mu_{[\widetilde{A};+\infty)}(t), \mu_{(-\infty;\widetilde{B}]}(t))\\ &= \min[\sup_u \min(\mu_{\widetilde{A}}(u), \mu_{(-\infty;\tau]}(u)), \sup_v \min(\mu_{\widetilde{B}}(v), \mu_{[\tau;+\infty)}(v))]\end{aligned} \tag{3.16}$$

and

$$\begin{aligned}N(\tau \leq t \leq \sigma) &= \mu_{]\widetilde{A};\widetilde{B}[}(t) = \mu_{]\widetilde{A};+\infty)\cap(-\infty;\widetilde{B}[}(t)\\ &= \min(\mu_{]\widetilde{A};+\infty)}(t), \mu_{(-\infty;\widetilde{B}[}(t))\\ &= \min[\inf_u \max(1-\mu_{\widetilde{A}}(u), \mu_{(-\infty;\tau]}(u)),\\ &\qquad \inf_v \max(1-\mu_{\widetilde{B}}(v), \mu_{[\tau;+\infty)}(v)))]\end{aligned} \tag{3.17}$$

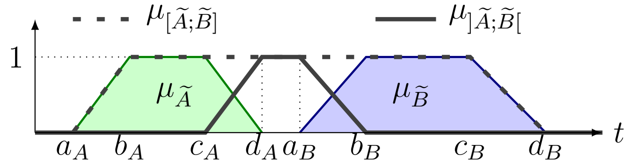


Figure 3.5: Necessity and possibility of t being between $\widetilde{A}$ and $\widetilde{B}$.

Figure 3.5 presents the possibility and necessity membership functions for an event t to be between fuzzy intervals $\widetilde{A}$ and $\widetilde{B}$. These necessity and possibility expressions are to be exploited in Section 5 to define the necessity and possibility of a task to be present between its starting and finishing time.

Fuzzy sets can be used to model uncertain and imprecise informations e.g. uncertain macro-task processing times and task's durations. A state of the art about fuzzy planning and scheduling is provided in section 3.4.

3.2.3 Bridges between fuzzy sets and probability

For some industrial problems, part of the imprecise information is probabilistic and the remainder is subjective. Hence, both probabilistic and possibilistic approaches are to be involved, which makes the calculation hard, and the global approach neither trivial nor unique [Baudrit et al., 2006]. Many authors studied the propagation of heterogeneous uncertainty models. One of the solutions provided in this field is the homogenization of probabilistic and possibilistic models throughout bridges from possibilistic to probabilistic and vice-versa [Baudrit, 2005] *e.g.* the pignistic transformation from possibility to probability and the inverse of the pignistic transformation from probability to possibility [Dubois et al., 1993]. This research field is recent and still not well developed. Moreover, information is lost while transforming probability distribution to possibility distribution, and gain of information is not allowable in the other sense [Dubois and Prade, 2006]. Despite, the homogenization technique has proven its effectiveness for many problems such as risk assessment, but combination of possibilistic and probabilistic approaches are out of our scope, because the uncertainty that we deal with can be completely covered with a unique modelling approach. Hence, motivated by the possibilistic approach (see section 3.2.4), a complete new fuzzy modelling and solving techniques are provided for tactical and operational planning problems. However, in section 4.2.5 a new theoretical modelling of stochastic tactical planning problem is provided, which can be applied to other applications where probability distributions of macro-tasks work contents are statistically available.

3.2.4 Motivation to use fuzzy/possibilistic approach

To model uncertainties in scheduling issues, both fuzzy sets and probability are considered [Dubois et al., 1995, Hillier, 2002, Herroelen and Leus, 2005].

The study of uncertainty started probably in 1654 by Pascal et Fermat with the development of modern concepts of probability theory [Tannery and Henry, 1894]. Nevertheless, this theory cannot deal with subjective and imprecise knowledge. In fact, the application of probability theory needs statistical data, which is not always available, to evaluate uncertainty. Moreover, within probability theory, to express the lack of certitude in an event is equivalent to determining the certitude in the contrary event, but sometimes, it is more reliable to say that nothing is sure while no rich information is available.

The study of imprecision and subjective uncertainty came far later in 1965 when Zadeh [Zadeh, 1965] proposed Fuzzy set theory as alternative to probability theory. This new theory is also a generalization of classical set theory. In fact, it is based on the idea that vague notions without clear limits such as “old”, “near”, “short” can be modelled by a gradual number called “fuzzy subsets”. The representation of vagueness and imprecisions became consequently possible thanks to fuzzy logic. The possibility theory is then provided by Zadeh [Zadeh, 1978] to deal with a non probabilistic uncertainty that is modelled with fuzzy logic. This new theory treats uncertainty and imprecision with the same formalism. This modelling is used in this thesis and the corresponding formalism is detailed in section 3.2.2. Many statisticians, based on the work of Finetti [1992], are convinced that probability is sufficient to deal with uncertainties and thus fuzzy logic is unnecessary. On the other hand, Bart [1990] claims that probability is a sub-theory of fuzzy logic. Moreover, [Zadeh, 1995], the creator of fuzzy logic and possibilistic approach, claims that probability theory and fuzzy logic are complementary rather than competitive and that possibility theory is the alternative to probability.

Fuzzy modelling is judged more appropriate when few and imprecise information is available [Chen, 2000], which is the case in helicopter maintenance domain [Masmoudi and Haït, 2010]. In fact, Probability requires statistical data that do not systematically exist and cannot represent subjective uncertainty and full or partial ignorance. However, fuzzy logic and theory of possibility overcome these difficulties. Moreover, in many cases and due to the central limit theorem, the assumption of Gaussian distribution is quite satisfied. In other cases, scientists used to take strong hypothesis and estimated the real distribution equivalent or similar to a mastered probability distribution such as exponential, beta and normal, to be able to make simulations. Otherwise, the probability formulations like convolution product become too complex to apply. This fact is highly criticized as we add knowledge that is not known [Bart, 1990]. Based on fuzzy set modelling, the possibility theory offers an alternative to this problem by providing bounders to a set of possible probability distributions instead of a crisp one. Moreover, with fuzzy modelling and possibilistic approach, we respect exactly what experts know about uncertainty; even small hypotheses are also taken to define the entire fuzzy profile. Nevertheless, fuzzy arithmetics are easy to manipulate whatever the complexity of the profile considered. This fact makes the fuzzy/possibilistic approach powerful enough to keep the attention of many authors in different research domains, and particularly in planning and scheduling (see section 3.4).

Many papers in literature deal with the comparison between fuzzy/possibilistic and stochastic approaches [Dubois and Prade, 1993, Mauris, 2007, Nikolaidis et al., 2004]. Though, using fuzzy logic or probability is still a subject with high controversy.

3.3 Planning and scheduling approaches under uncertainties

Several techniques are provided in literature to deal with uncertainties in planning and scheduling. These techniques are classified by [Mehta and Uzsoy, 1999] into 4 categories: reactive, predictive-reactive, robust, and knowledge based approaches. Davenport and Beck [2000] suggested another classification to 3 categories: proactive, reactive and proactive-reactive. Aloulou [2002] and Marmier [2007] considered the Davenport approach more complete and covering all possible approaches. In fact, reactive and predictive-reactive and knowledge based approaches as explained in [Mehta and Uzsoy, 1999] are equivalent to the reactive approach as explained by Davenport and Beck. Then the robust approach as explained in [Mehta and Uzsoy, 1999] is equivalent to the proactive approach as explained by Davenport and Beck. Finally, proactive-reactive approaches are only considered in the Davenport and Beck classification. This latter definition is the one we will detail in this section.

3.3.1 Proactive approach

The proactive approach consists of anticipating perturbations before they really happen and count uncertainties while realising the initial planning or scheduling. It aims at making predictive planning and scheduling more robust [Davenport and Beck, 2000]. The idea of a proactive approach is to integrate a robust margin for uncertain activities (date, duration or processing time) into the planning and scheduling to absorb uncertainties during the execution. The critical chain/ buffer management approach [Goldratt, 1997] is the proactive approach that is the most used in industry. Modelling uncertainties with probability and/or fuzzy sets while realizing the initial planning are among the most used proactive techniques in literature [Bidot, 2005].

3.3.2 Reactive approach

Reactive approaches are applied during the execution [Davenport and Beck, 2000]. They are may be based on an initial predictive planning. The reactivity is on the accounting of additional information or updates and their integration into the planning or scheduling that can be within different ways. Based on [Mehta and Uzsoy, 1999], we distinguish between the totally reactive approach that does not consider an initial predictive planning, and the predictive-reactive approach that considers a deterministic initial planning without considering the variation and a reactive algorithm that provide a new solution once a new perturbation occurs.

The performance of a totally reactive approach is relatively poor because the planning is only known once realised. Hence, in several situations reactive and proactive approaches are to be considered at the same time, thus, we talk about proactive-reactive approach.

3.3.3 Proactive-reactive approach

Proactive-reactive approach consists of coupling reactive and proactive techniques. Generally static planning and scheduling can not absorb all uncertainties and perturbations, hence a dynamic approach coupling proactive and reactive approaches is necessary [Davenport and Beck, 2000].

The global approach starts by providing a baseline schedule using a proactive approach based on knowledge (objective or subjective) of uncertainty and possible disruptions. When disruptions occurs during the execution, a reactive procedure is called to modify the baseline in response to the corresponding disruptions and produce a new so-called realized schedule [Aytug et al., 2005]. The study of robustness and stability refers to the difference between the baseline and the realized schedule. To get information about the study of robustness and how we can provide a robust schedule, we refer readers to [Herroelen and Leus, 2005, Leus, 2003].

The scenario based technique is considered as a proactive-reactive approach. It consists of establishing a flexible set of scenarios that cover all possible perturbations. Flexibility can be considered in time or task sequence [La, 2005] or also in resource assignment [flexibilité GOThA, 2002] and hence a large panel of scenarios can be defined. According to which scenario occurs, the corresponding on shelf schedule is executed [Aloulou, 2002]. To get a deep study of robustness and flexibility we refer readers to [flexibilité GOThA, 2002].

3.4 State of the art about fuzzy planning and scheduling

Herroelen and Leus [2005] proposed a review of methods that have been used in literature for scheduling under uncertainty: Stochastic project, Stochastic Gert-network, fuzzy project, robust(proactive), contingent, and sensitivity analysis scheduling. Motivated by the fuzzy approach, we devote this section to the state of the art about fuzzy planning and scheduling.

Since the early 90s, fuzzy logic became a very promising mathematical approach to model production and project management problems characterized by uncertainty and imprecision. Wong and Lai [2011] provide a non exhaustive survey of 402 journal papers leading with 17 applications of fuzzy set theory in production and operations management. Among the considered applications, they find fuzzy scheduling with 8.44% , capacity planning with 30.27%, aggregate planning with 9.18% and project management with 2.73% as percentage of applications by application area. In this section, a non exhaustive overview about fuzzy planning and scheduling is provided. We will distinguish between production and project planning although they have some similarities.

3.4.1 Fuzzy Pert technique

The Critical Path Method is one of the project scheduling specificities. The majority of the research on the project scheduling topic has been devoted to fuzzy PERT [Guiffrida and Nagi, 1998]. As explained before, the PERT technique is composed of two steps; the forward and the backward propagations. The generalization of the PERT technique to fuzzy parameters is a complex task. The forward propagation is done using fuzzy arithmetic, leading to fuzzy earliest dates and a fuzzy end-of-project event. Unfortunately, backward propagation is no longer applicable because uncertainty would be taken into account twice. Chanas et al. [2002] study the criticality of tasks within fuzzy project. Dubois et al. [2003] show that the boundaries of some fuzzy parameters like the tasks' latest dates and floats are reached in extreme configurations. Fortin et al. [2005] justify the problem complexity and propose some algorithms to calculate the tasks' latest dates and floats. These parameters are considered in this thesis to deal with the scheduling problem.

3.4.2 Fuzzy planning problem

Applications of fuzzy logic to production planning are not widely used; a fuzzy modelling of delivery dates and of the resource capacities has for instance been suggested in [Watanabe, 1990], whereas imprecise operation durations and preferences at tactical level of production are considered in [Inuiguchi et al., 1994]. In [Fargier and Thierry, 2000], a fuzzy representation of imprecise ordered quantities is proposed. In [Grabot et al., 2005], a fuzzy MRP is provided based on the formalism of possibility distributions described in [Dubois and Prade, 1988]. Recently, fuzzy requirement plans have been expressed in terms of quantities by periods [Guillaume et al., 2011]. In tactical project planning, Masmoudi et al. [2011] proposed simulated annealing to provide a robust solution for a fuzzy capacity planning problem. This latter work is to be the content of chapter 4.

3.4.3 Fuzzy scheduling problem

The study of a fuzzy model of resource-constrained project scheduling has been initiated in [Hapke et al., 1994, Hapke and Slowinski, 1996]. Slowinski and Hapke [2000] gather important work in fuzzy scheduling. Many techniques were generalized to fuzzy parameters, particularly the parallel SGS technique, and the resource levelling technique.

Leu et al. [1999] and Masmoudi and Haït [2011b] employ a genetic algorithm and fuzzy set theory to develop a resource leveling model under uncertainty. They consider a fuzzy profile to represent the uncertain activity duration. Leu et al. [1999] apply different alpha-cuts (called acceptable risk levels) on all activity durations and keep for each alpha-level the two deterministic problems corresponding to all lower(optimistic) and all upper (pessimistic) bounds. Then, for each deterministic problem, they apply deterministic CPM techniques to get the margin of each activity and apply a deterministic GA-based approach to solve the problem. Finally, for each alpha in $[0, 1]$ they get a solution for the two corresponding deterministic (pessimistic and optimistic) problems. On the other hand [Masmoudi and Haït, 2011b] apply a generalization of the Pert technique per interval provided by [Dubois et al., 2005] to fuzzy activities durations to get the fuzzy activities margins. Then based on the fuzzy modelling of resource usage provided in [Masmoudi and Haït, 2010], they proposed a fuzzy Genetic algorithm to solve the complete problem and provide only one fuzzy solution instead of multiple deterministic solutions.

Hapke and Slowinski [1996] and Masmoudi and Haït [2011a] employ priority heuristic method and fuzzy sets to develop a resource-constrained project scheduling problem under uncertainty. Hapke and Slowinski [1996] consider the uncertainty in time parameters and apply alpha-cuts on the fuzzy Gantt chart obtained by using a new parallel SGS that is based on the fuzzy Pert technique provided in [Dubois and Prade, 1988]. They generate twice as many deterministic workload plans as the number of alpha-cuts chosen in interval $[0, 1]$. They correspond to the lower(optimistic) and upper(pessimistic) bounds of each alpha-level activity durations. On the contrary, Masmoudi and Haït [2011a] have recently provided completely fuzzy parallel SGS algorithm in which uncertainty is considered in both time parameters and resource usage. Based on a new modelling of fuzzy workload provided in [Masmoudi and Haït, 2010], they provide only a couple of fuzzy workload plans instead of a cumbersome result with multiple deterministic plans.

The work provided in [Masmoudi and Haït, 2011b,a] is to be the content of chapter 5.

Chapter 4

Project planning under uncertainties for helicopter maintenance

Contents

4.1 Introduction

At the tactical level of planning, a project is viewed as a set of macro-tasks with both precedence and resource constraints. A macro-task may require specific skills to be completed (*e.g.* mechanical skills). Rough cut capacity planning, performed at the tactical level, aims at allocating the specific workforce, on a periodic basis, in order to complete the macro-tasks within their time windows (TW) with a minimum cost. The deterministic resource loading problem is NP-hard. Integrating uncertainties increases the complexity of the problem.

Two approaches can be considered simultaneously or separately to solve the tactical planning problem: the time driven approach and the resource driven approach. The former aims to minimize overcapacity cost (overtime, hiring and subcontracting capacity cost) and the latter aims to minimize the cost incurred by projects lateness. To deal with uncertainty, we will introduce a robustness criterion. This concept, which is also called stability, has gained the interest of several researchers in operational [Leus, 2003] and tactical [Wullink et al., 2004] planning. Hans [2001] has proposed a Branch-and-Price technique to solve the deterministic RCCP. In this chapter, the Hans' model will be generalized to uncertain parameters to cope with fuzzy and stochastic multi-project tactical planning problem.

A main issue of project planning for aircraft maintenance is to account for uncertainties. Wullink et al. [2004] have extended the deterministic Hans' model to develop a scenario-based approach based on a discretization of the stochastic work content. They consider a time driven problem and also introduce different robustness objective functions. In this chapter[1], we represent uncertainties with fuzzy numbers and stochastic distributions. We have developed a simulated annealing algorithm to solve the non-deterministic RCCP problem, where uncertainty is mainly reflected into the objective function. Moreover, contrary to Wullink *et al.*, uncertainty is modelled with continuous distributions. Next section outlines the fuzzy and stochastic modelling of the RCCP problem under uncertainty, even though for the following computation section, we consider only the fuzzy RCCP problem.

4.2 RCCP Problem under uncertainty

The Rough Cut Capacity Planning is applied at the negotiation stage with customer. It consists of studying the impact of a project-acceptance on the resource capacity and provide a feasible and competitive project delivery date.

Applying RCCP approach to uncertain projects results in a network of

[1] This chapter is based on joint work with Dr. Erwin Hans and partially reproduce the content of the paper [Masmoudi et al., 2011]

work packages with rough estimates of resource requirements (in man-hours as explained in the previous section) and minimum durations. This aforementioned definition of the RCCP result is shared by many authors [de Boer, 1998, Wullink, 2005, Masmoudi et al., 2011]. The minimum durations are a result of technical constraints such as available working space and expected precedence relations between activities at a lower level. A feasible tactical plan respects all these technological restrictions in addition to the decided projects' release and due dates.

Without loss of generality, a multiple project capacity planning problem can be modelled as a single project capacity planning problem. This single project contains all macro-tasks from all projects. A tactical planning (RCCP) consists of allocating macro-tasks portions of work contents to time periods (*e.g.* 50 hours in period 3, 100 hours in period 4) in order to determine the required capacity and reliable projects release and due dates.

In this chapter, we deal with the RCCP problem in MROs domain and the management of HMVs is particularly considered. The PUMA helicopter HMV, for example, lasts about 6 months. Hence, the planning horizon for a helicopter maintenance is set to 12 months in order to cover the overall delay of a project. This situation can be compared to ETO manufacturing environment [Hans, 2001]. The modelling of uncertain inputs for the RCCP problem is provided in the next section. After that, a deterministic modelling of the RCCP problem is provided and then generalized to fuzzy and stochastic areas.

4.2.1 Uncertain project release date

Figure 4.1 presents an example of an equipment inspection date determination from helicopter exploitation assumptions, calendar limits and flight hours (top) or flight cycle (bottom) limits, for example 30000 hours or 15000 cycles by 10 years. From the update, flight hours evolve in a range going from no exploitation to the physical limits of the aircraft, through pessimistic and optimistic exploitation values. Intersections of these lines with calendar and flight hours limits define the four points a_H, b_H, c_H and d_H of the trapezoidal fuzzy number $\widetilde{H}$, inspection date according to flight hours. It is the same for flight cycles.

For a single equipment, the fuzzy inspection date is the minimum between fuzzy dates involved by flight hours ($\widetilde{H}$) and cycles limits ($\widetilde{C}$):

$$\min(\widetilde{H}, \widetilde{C}) = (\min(a_H, a_C), \min(b_H, b_C), \min(c_H, c_C), min(d_H, d_C)) \quad (4.1)$$

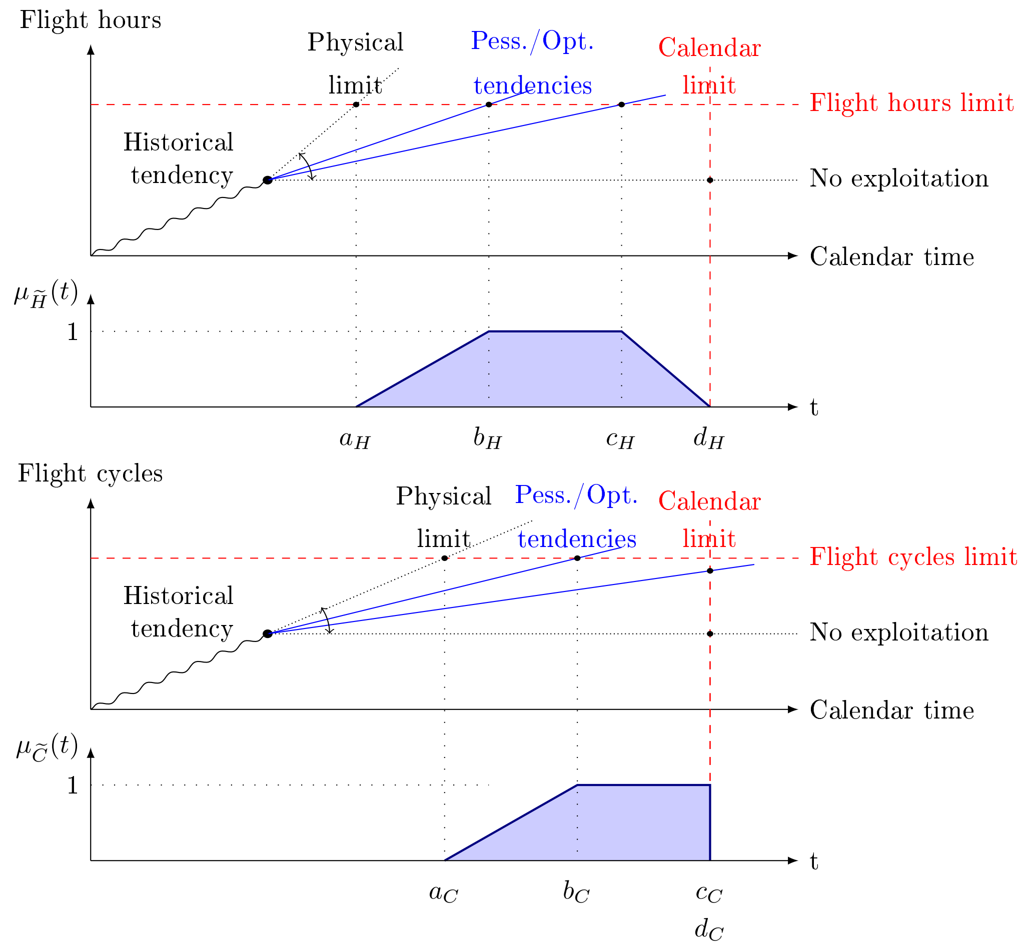


Figure 4.1: Fuzzy inspection release date

The HMV fuzzy release date $\widetilde{Rd}$ is the minimum between fuzzy inspection dates of the critical equipments and the helicopter itself. The uncertainty in this date decreases along the time, as information on helicopter utilization increases.

The HMV starting and due dates are fixed 6 to 8 weeks ahead after a negotiation between the MRO stakeholders and the customer (helicopter owner). The deterministic starting date S is chosen among the possible values of the fuzzy release date $\widetilde{Rd}$.

Deterministic project release and due dates are required to make the tactical planning. Hence, for projects with fuzzy release dates (not yet negotiated), a degree of risk is considered by the MRO planner. This degree of risk is quantified by the possibility and necessity of the event "$S \geq \widetilde{Rd}$".

4.2.2 Uncertain macro-task work content

At tactical level, uncertainty in macro-task work content is mainly due to additional tasks generated by unexpected corrective maintenance. These additional tasks (work and delays) can represent one third to one half of the total project workload. They generally appear during the structural inspection macro-tasks, but the whole project is impacted.

Procurement for corrective maintenance may introduce delays in planning. As the equipments to be purchased are not known before inspection, we consider different scenarios: the equipment is available on site; or at a European supplier; or at a foreign supplier; or it may be found after some research; or it is obsolete and must be manufactured again. According to the information on the helicopter (age of the aircraft, conditions of use, etc...), some scenarios can be discarded from the beginning (*e.g.* new helicopter ⇒ no obsolescence) and others at the end of the major inspection macro-tasks and hence macro-tasks workloads are refined.

Macro-tasks work contents are established by asking experts. Rommelfanger [1990] proposes a 6-point fuzzy number to represent the expert knowledge. In this work, however, we will still consider 4-point fuzzy numbers within a trapezoidal profile.

Each macro-task work content is divided into portions that are allocated to the time periods between the macro-task's starting and finishing dates [Masmoudi et al., 2011]. Let us consider a macro-task A with a fuzzy work content $\widetilde{P}_A = (120, 180, 240, 300)$ present between period 3 and period 5. One third of the work content corresponds to resource type $1(\upsilon_{A1} = 1/3)$ and two thirds correspond to resource type $2(\upsilon_{A2} = 2/3)$. We choose to carry out the three quarters of the macro-task A at period $3(Y_{A3} = 3/4)$ and the other quarter at period $4(Y_{A4} = 1/4)$. Table 4.1 shows the macro-task and its different work content portions.

Table 4.1: Fuzzy macro-task Resource portions

Macro-task	Resource type	Period 3	Period 4
A	1	(30,45,60,75)	(10,15,20,25)
A	2	(60,90,120,150)	(20,30,40,50)

Analogously, we can consider a stochastic distribution to model the macro-task work content *e.g.* using a beta distribution we get $\widetilde{P}_A = (120, 300, 2, 2)$ with $\beta = \alpha = 2$. To obtain the different work content portions we apply classical mathematics (see Table 4.2).

Table 4.2: Stochastic macro-task Resource portions

Macro-task	Resource type	Period 3	Period 4
A	1	(30, 75, 2, 2)	(10, 25, 2, 2)
A	2	(60, 150, 2, 2)	(20, 50, 2, 2)

4.2.3 Deterministic RCCP

We consider a set of projects (index $j \in 1, \ldots, n$) composed of macro-tasks (b, j), $b \in 1, \ldots, n_j$, linked by precedence constraints. A project is constrained by its release and due dates, and so are its macro-tasks. The work content of macro-task (b, j) is denoted p_{bj} and its minimum duration ω_{bj}. To perform a macro-task, several skills may be needed. A resource group (index $i \in 1, \ldots, I$) is associated to each skill. The fraction of macro-task work content p_{bj} performed by resource group i is denoted υ_{bji}, so that $\sum_i \upsilon_{bji} = 1 \, \forall b, j$. Finally, we consider a planning horizon discretized into $T + 1$ periods (index t). Variable Y_{bjt} represents the fraction of the work content of macro-task (b, j) executed in period t.

In order to model the calendar and precedence constraints, Hans [2001] uses the concept of *order plan*. We can transpose it to project planning. A *project plan* $a_{j\pi}$ specifies for each macro-task (b, j) the periods in which it is allowed to be performed. A project plan $a_{j\pi}$ is a vector of 0-1 values $a_{bjt\pi}$ $(b = 1, \ldots, n_j,\ t = 0, \ldots, T)$ where $a_{bjt\pi} = 1$ if macro-task (b, j) is allowed to be performed in period t, 0 otherwise. A *feasible project plan* is a project plan that respects release and due dates as well as precedence constraints. Hence, to ensure consistency, variables Y_{bjt} can be greater than 0 if and only if $a_{bjt\pi} = 1$. The vector Y_j of variables Y_{bjt}, $b = 1, \ldots, n_j$, $t = 0, \ldots, T$ is called *project schedule*. A plan is defined as a set of elements $P_{bjt} = p_{bj}.Y_{bjt}$ that specify the amount of the work content (in hours) of macro-task (b, j) executed in period t.

A tactical plan is defined by parameters $W_{it} = \sum_j p_j \upsilon_{ji} Y_{jt}$ corresponding to the total workload by resource group i $(i \in 1, \ldots, I)$ to be executed in period t $(t = 0, \ldots, T)$.

We present below the adaptation of Hans' model to our multi-project RCCP problem. Hans used a Branch and Price technique, and so divided the global RCCP problem into a master problem and pricing problem. We provide only the master problem to understand the approach, given that we will use simulated annealing to solve the problem. To get more informations about the Hans' model we refer readers to [Hans, 2001, Wullink, 2005].

$$Objective : Min(Cost) \tag{4.2}$$

S.t.:

$$\sum_{\pi \in \Pi_j} X_{j\pi} = 1 \quad \forall j \tag{4.3}$$

$$Y_{bjt} - \frac{\sum_{\pi \in \Pi_j} a_{bjt\pi} X_{j\pi}}{\omega_{bj}} \leq 0 \quad \forall b, j, t \tag{4.4}$$

$$\sum_{t=0}^{T} Y_{bjt} = 1 \quad \forall b, j, t \tag{4.5}$$

$$\sum_{b,j} p_{bj} \upsilon_{bji} Y_{bjt} \leq \kappa_{it1} + O_{it} + H_{it} + S_{it} \ \forall i, t \tag{4.6}$$

$$O_{it} \leq \kappa_{it2} - \kappa_{it1} \quad \forall i, t \tag{4.7}$$

$$H_{it} \leq \kappa_{it3} - \kappa_{it2} \quad \forall i, t \tag{4.8}$$

$$all \quad variables \geq 0 \tag{4.9}$$

$$X_{j\pi} \in \{0, 1\} \quad \forall j, \pi \tag{4.10}$$

Where:
κ_{i1t}: regular capacity available on resource i in week t.
κ_{i2t}: regular and overtime capacity on resource i in week t.
κ_{i3t}: regular, overtime and hiring capacity on resource i in week t.
O_{it}: number of overtime hours on resource i in week t.
H_{it}: number of hired hours in week t.
S_{it}: number of subcontracted hours in week t.

The implicit objective function 4.2 is to be specified in Section 4.2.4 and Section 4.2.5 with uncertainty consideration based on fuzzy sets modelling and probability modelling. Constraints 4.3 and 4.10 make sure that exactly one project plan is selected for each project. Constraints 4.4 guarantee a minimum duration (ω_{bj}) for macro-task (b, j) and make sure that the project schedule (Y_{bjt}) and the project plan ($\sum_{\pi \in \Pi_j} a_{bjt\pi} X_{j\pi}$) are consistent. Constraint 4.5 make sure that all work is done. Constraints 4.6, 4.7 and 4.8 make sure that capacity limits are respected. Constraint 4.6 will particularly be generalized to fuzzy and stochastic processing time to integrate uncertainties into the model.

Uncertainty is considered for activities processing times as confirmed in [Elmaghraby, 2002]. Hence, we will provide a fuzzy modelling and a stochastic modelling of the processing time instead of a crisp one ($p_{bj} \to \widetilde{p}_{bj}$). Fuzzy and stochastic modelling are studied separately in what follows and corresponding objective functions are defined to be integrated in a generalization of the previous model to solve RCCP problem under uncertainties. We refer to these problems as Fuzzy RCCP and Stochastic RCCP, respectively.

4.2.4 Fuzzy RCCP

The macro-task work content is an uncertain quantity vaguely defined by experts; for example, the expert says that the macro-task (b, j) needs on average 100 to 140 hours, but 80 to 160 hours is possible in extreme cases. Therefore, the macro-task (b, j) works content is represented by a 4-points fuzzy number ($\widetilde{p}_{bj} = [80, 100, 140, 160]$). The same definition is available for all uncertain macro-tasks.

As processing times are to be modelled with fuzzy profiles, a tactical plan is defined by parameters $\widetilde{W_{it}} = \sum_{b,j} \widetilde{p}_{bj} \upsilon_{bji} Y_{bjt}$, corresponding to the total workload by resource group i $i \in 1, \ldots, I)$ to be executed in period t ($t = 0, \ldots, T$) (Figure 4.2), while $\widetilde{W_{it}}$ are fuzzy numbers calculated using fuzzy mathematical operations explained in chapter 3 ($\widetilde{W_{it}} = (a_{W_{it}}, b_{W_{it}}, c_{W_{it}}, d_{W_{it}})$).

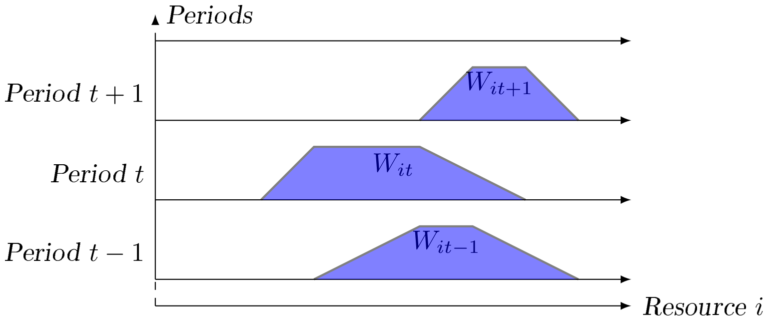


Figure 4.2: Partial fuzzy workload plan

Let L_{it} be the capacity limit of resource i at period $t = 0, \ldots, T$. To check if the fuzzy workload exceeds the capacity limit or not, we consider the possibilistic approach. In fact, to measure the truth of event $\widetilde{W_{it}} < L_{it}$, we need the couple $\Pi(\widetilde{W}_{it} < L_{it})$ and $N(\widetilde{W}_{it} < L_{it})$. Thus:

$$\begin{aligned}\Pi(\widetilde{W}_{it} < L_{it}) &= 1 - N(L_{it} \leq \widetilde{W}_{it}) \\ &= \sup_{u<L_{it}} \mu_{\widetilde{W}_{it}}(u) = \sup_u \min(\mu_{\widetilde{W}_{it}}(u), \mu_{(-\infty, L_{it}[}(u)) \quad (4.11)\\ N(\widetilde{W}_{it} < L_{it}) &= 1 - \Pi(L_{it} \leq \widetilde{W}_{it}) \\ &= 1 - \sup_{u \geq L_{it}} \mu_{\widetilde{W}_{it}}(u) = \inf_u \max(1 - \mu_{\widetilde{W}_{it}}(u), \mu_{(-\infty, L_{it}[}(u)) \quad (4.12)\end{aligned}$$

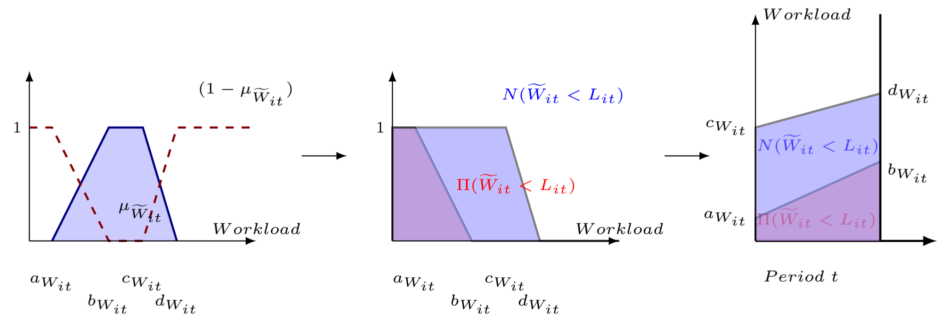


Figure 4.3: How to get a fuzzy load by period using the Necessity and possibility measures.

Figure 4.3 shows the way to represent a fuzzy load by period using the necessity and possibility measures. This representation is similar to the one proposed by [Grabot et al., 2005] to model uncertainty in orders in MRP.

Let N_{it} and Π_{it} be the values of the workload membership function intersection with the capacity limits: $\forall i, t\ N_{it} = N(\widetilde{W}_{it} < L_{it})$ and $\Pi_{it} = \Pi(\widetilde{W}_{it} < L_{it})$ $(\forall i, t)$ with N and Π the possibility and necessity measures respectively. Expressions N_{it} and Π_{it} are calculated as follows:

$$N_{it} = \begin{cases} 0 & \text{if} L_{it} < c_{W_{it}} \\ \frac{L_{it} - c_{W_{it}}}{d_{W_{it}} - c_{W_{it}}} & \text{if } L_{it} \in [c_{W_{it}}, d_{W_{it}}] \\ 1 & \text{if} L_{it} > d_{W_{it}} \end{cases} \quad (4.13)$$

$$\Pi_{it} = \begin{cases} 0 & \text{if } L_{it} < a_{W_{it}} \\ \frac{L_{it} - a_{W_{it}}}{b_{W_{it}} - a_{W_{it}}} & \text{if } L_{it} \in [a_{W_{it}}, b_{W_{it}}] \\ 1 & \text{if } L_{it} > b_{W_{it}} \end{cases} \quad (4.14)$$

It is common in literature to use the term "*credibility*" that is in some way a combination of the possibility and the necessity. Liu and Liu [2002] proposed the following simple expression $Cr_{it} = \frac{1}{2}(N_{it} + \Pi_{it})$ $(\forall i, t)$. However, it is possible to consider other expressions of Credibility like $Cr_{it} = \beta N_{it} + (1-\beta)\Pi_{it}$) $(\beta \in [0, 1])$ giving different weights to possibility and necessity. The possibility Π_{it}, necessity N_{it}, and consequently Cr_{it} are to be minimized while dealing with optimization of the capacity planning problem [Masmoudi et al., 2011].

Once the representation of uncertainty is done, the model in Section 4.2.3 can be completed by one or more fuzzy objective functions to finally obtain a fuzzy RCCP model. In the following sections some fuzzy objectives are provided and will be analysed on a real problem case in Section 4.4.2.

4.2.4.1 Fuzzy cost expectation

The non regular capacity (overtime, hiring and subcontracting) is fuzzy because it represents the difference between the fuzzy workload and the different capacity limits. The objective function to minimize the costs of the use of non regular capacity is:

$$\widetilde{E} = \sum_{i=1}^{I}\sum_{t=0}^{T}(\varsigma_{i1}\widetilde{O}_{it} + \varsigma_{i2}\widetilde{H}_{it} + \varsigma_{i3}\widetilde{S}_{it})$$

where:
ς_{i1}, ς_{i2}, and ς_{i3} specify the costs of using one hour of non regular capacity (overtime $\widetilde{O}_{it}$, hiring $\widetilde{H}_{it}$, and subcontracting $\widetilde{S}_{it}$, respectively).

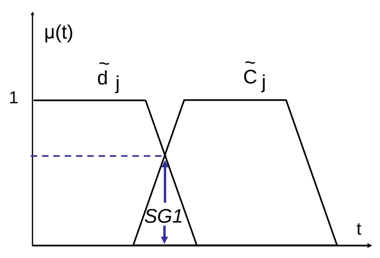
Defuzzification is one of the easiest ways to solve the fuzzy problem within a deterministic way. The defuzzification is studied in the literature and Dubois and Prade [1987] proposed a simple formula for trapezoidal profiles. This latter is used to provide the following defuzzification of the fuzzy cost expectation:

$$E = \sum_{i=1}^{I}\sum_{t=0}^{T}\sum_{m=1}^{l} q_m(\varsigma_{i1}O_{itm} + \varsigma_{i2}H_{itm} + \varsigma_{i3}S_{itm}) \tag{4.15}$$

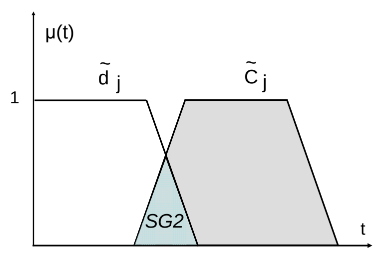
Hence, the cost expectation is a weighted sum of the costs for the various work content values (weights q_m). For example, with trapezoidal fuzzy numbers, we have $l = 4$. We use a credibility expression, and consider $q_1 = q_2 = \frac{(1-\beta)}{2}$ and $q_3 = q_4 = \frac{\beta}{2}$ with $\beta > \frac{1}{2}$ to give more importance to the necessity profile.

4.2.4.2 Fuzzy robustness functions

Fuzzy tactical planning is a new concept that we develop in this thesis. Therefore, no robustness functions are available in literature for FRCCP. Project scheduling deals partially with projects lateness; difference between project j completion time (C_j) and due date (d_j). In the fuzzy scheduling literature, the robustness functions are called satisfaction grades. Some authors calculate the satisfaction grade using an approach based on possibility measure [Song and Petrovic, 2006] and others calculate it based on intersection area [Chen and Hwang, 1992] (see figure 4.4).



(a) Using possibility measure



(b) Using area of intersection

Figure 4.4: Satisfaction Grade of completion time.

The two satisfaction grades are calculated respectively as follows:

$$SG1 = \Pi_{\widetilde{C}_j}(\widetilde{d}_j) = \sup_j \min(\mu_{\widetilde{C}_j(t)}, \mu_{\widetilde{d}_j(t)}) \tag{4.16}$$

$$SG2 = (area\widetilde{C}_j \cap \widetilde{d}_j)/(area\widetilde{C}_j) \tag{4.17}$$

For FRCCP problem, we propose to measure robustness as the eventuality of a plan to exceed a capacity limit i.e. compare the fuzzy workload to the available capacity. Inspired from the $SG1$, we provide a first robustness expression:

$$R_1 = \frac{\sum_{t=0}^{T} \sum_{i=1}^{I} \sum_{p=1}^{3} \varsigma_{ip}(\beta N_{ipt} + (1-\beta)\Pi_{ipt})}{(T+1)(\sum_{i=1}^{I} \sum_{p=1}^{3} \varsigma_{ip})} \tag{4.18}$$

Where N_{ipt} and Π_{ipt} are the values of the workload membership function intersection with the capacity limits κ_{ipt} ($p \in \{1, 2, 3\}$): $N_{ipt} = N(\widetilde{W}_{it} < \kappa_{ipt})$ and $\Pi_{ipt} = \Pi(\widetilde{W}_{it} < \kappa_{ipt})$ ($\forall i, p, t$) with N and Π respectively the possibility and necessity measures (see Figure 4.5). The weighted sum $\beta N_{ipt} + (1-\beta)\Pi_{ipt}$ expresses the *credibility* of $\widetilde{W}_{it}$ being under the limit κ_{ipt}. Expressions N_{ipt} and Π_{ipt} are calculated as follows:

$$N_{ipt} = \begin{cases} 0 & \text{if} \kappa_{ipt} < c_{W_{it}} \\ \frac{\kappa_{ipt} - c_{W_{it}}}{d_{W_{it}} - c_{W_{it}}} & \text{if } \kappa_{ipt} \in [c_{W_{it}}, d_{W_{it}}] \\ 1 & \text{if} \kappa_{ipt} > d_{W_{it}} \end{cases} \tag{4.19}$$

$$\Pi_{ipt} = \begin{cases} 0 & \text{if } \kappa_{ipt} < a_{W_{it}} \\ \frac{\kappa_{ipt} - a_{W_{it}}}{b_{W_{it}} - a_{W_{it}}} & \text{if } \kappa_{ipt} \in [a_{W_{it}}, b_{W_{it}}] \\ 1 & \text{if } \kappa_{ipt} > b_{W_{it}} \end{cases} \tag{4.20}$$

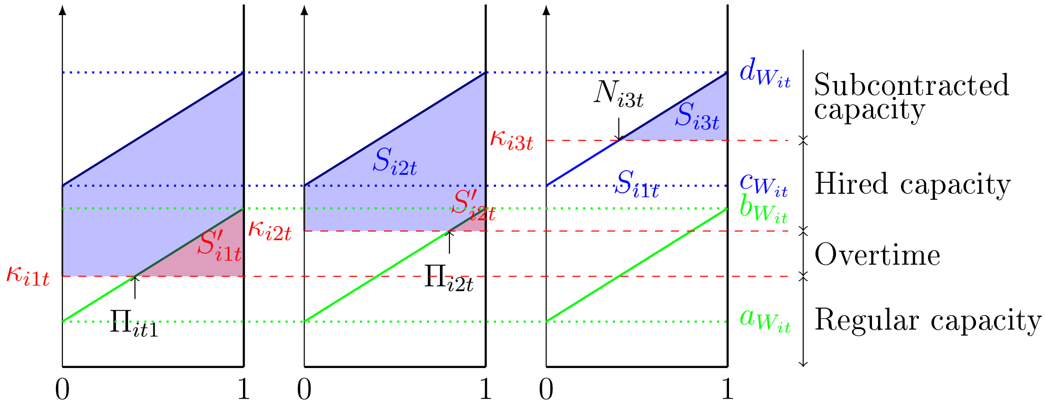


Figure 4.5: Fuzzy distribution and robustness coefficients

Inspired from $SG2$, a second fuzzy robustness function can be provided based on intersection area. This function accounts for the necessary and potential excess value of workload over the capacity limit, represented by surfaces S_{ipt} and S'_{ipt}, whereas the previous one relies on necessity and possibility of excess. Figure 4.5 shows the robustness coefficients N_{ipt}, Π_{ipt}, S_{ipt} and S'_{ipt}.

The second robustness objective function using intersection area is defined as follows:

$$R_2 = \frac{\sum_{t=0}^{T} \sum_{i=1}^{I} \sum_{p=1}^{3} \varsigma_{ip} (\frac{\beta}{S_{ipt}+1} + \frac{1-\beta}{S'_{ipt}+1})}{(T+1)(\sum_{i=1}^{K} \sum_{p=1}^{3} \varsigma_{ip})} \tag{4.21}$$

The fuzzy workload $\widetilde{W}_{it}$ at period t is equal to $\sum_{bj} Y_{bjt} \upsilon_{bji} \tilde{p_{bj}}$. Let $\tilde{W}_{it} = [a_{W_{it}}, b_{W_{it}}, c_{W_{it}}, d_{W_{it}}]$, the areas S_{ipt} and S'_{ipt} (see Figure 4.5) are calculated as follows:

While dealing with RCCP problem, the planner is looking for a planning that is robust for at least the first periods of the planning horizon and that remains robust as long as possible. Hence, we reward early robustness more

```
for t = 0 → T do
  for i = 1 → I do
    for p = 1 → 3 do
      if d_{W_it} < κ_ipt then
        S_ipt = 0
      else if c_{W_it} > κ_ipt   or   c_{W_it} = d_{W_it} then
        S_ipt = (c_{W_it} + d_{W_it})/2 − κ_ipt
      else
        S_ipt = (d_{W_it} − κ_ipt)^2 / (2(d_{W_it} − c_{W_it}))
      end if
      if b_{W_it} < κ_ipt then
        S'_ipt = 0
      else if a_{W_it} > κ_ipt   or   a_{W_it} = b_{W_it} then
        S'_ipt = (a_{W_it} + b_{W_it})/2 − κ_ipt
      else
        S'_ipt = (b_{W_it} − κ_ipt)^2 / (2(b_{W_it} − a_{W_it}))
      end if
    end for
  end for
end for
```

Figure 4.6: How to calculate S'_{ipt} and S_{ipt} to get the robustness function R_2

than late robustness [Wullink, 2005]. The aforementioned robustness functions can be easily modified to be time related. Nevertheless, this concept will not be taken into account in this work.

The aforementioned robustness functions R_1 and R_2 are both non-linear. Hence, it is difficult to integrate into an LP-based algorithm like the B&Price approach of Hans [2001], and the linear programming based heuristics of Gademann and Schutten [2005]. On the other hand, within the simulated annealing heuristic that will be provided later, these different objective functions are accepted.

4.2.5 Stochastic RCCP

The macro-task work content is an uncertain quantity. We suppose that it can be modelled with a probability distribution instead of fuzzy profile. But, which distribution is appropriate? In project management, and particularly when applying stochastic PERT/CPM techniques, the beta distribution is the most frequently used because it is bounded, positive, continuous, uni-modal, and multi-shaped. To get the probabilistic workload profile that is defined by parameters $\widetilde{W}_{it}$, we should sum up the fractions (by period and by resource type) of macro-tasks' workload probability distributions (see section 4.2.2).

Analogously to the way presented for fuzzy load (see figure 4.3), we propose a way to model stochastic load through the rotation of the probability cumulative function (see figure 4.7).

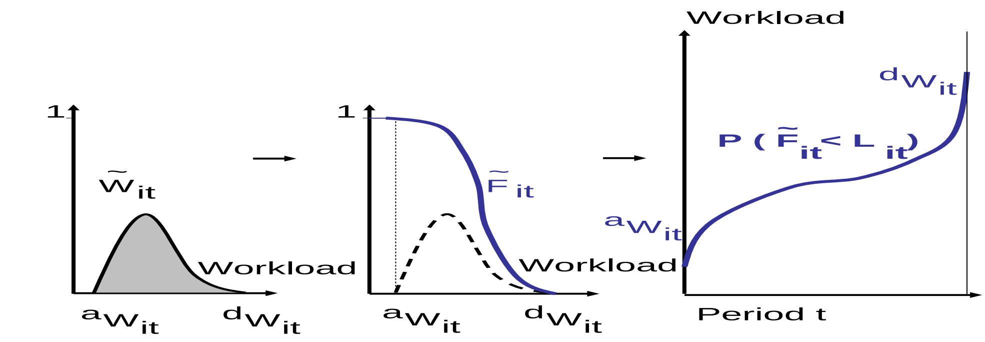


Figure 4.7: How to get a stochastic load by period

Figure 4.8 shows the two possible models; Possibility vs probability distributions. Hence, the aforementioned couple possibility $(\Pi(t))$ and necessity$(N(t))$ profiles are changed by the cumulative probability distribution function $(P(t))$.

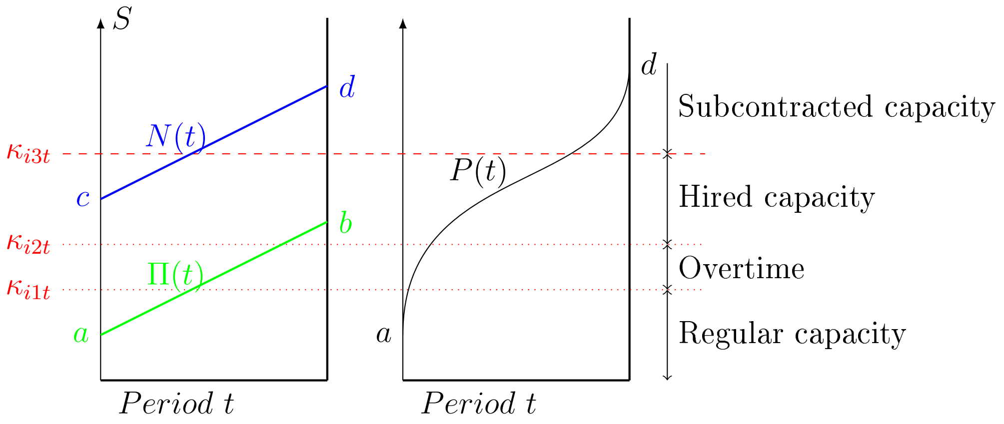


Figure 4.8: Fuzzy vs stochastic workload distribution

4.2.5.1 Stochastic expectation and variance evaluation

In stochastic scheduling, authors analyse the expectation and the variance as objectives to find optimal solutions [Subhash et al., 2010]. According to our knowledge, this idea had never been used to solve stochastic planning problems. The beta distribution is considered in this section to show the result of our study, however any other distribution can be used. Table 1.1 in section 1.3 shows a PUMA HMV project with macro-tasks work contents defined with beta distributions. Let $\widetilde{p}_{bj} = [a_{bj}, d_{bj}, \alpha_{bj}, \beta_{bj}]$ be the work content of the macro-task (b, j) that is represented with a beta distribution. The expectation and the variance are calculated as follows:

$$E(\widetilde{p}_{bj}) = a_{bj} + (d_{bj} - a_{bj})\frac{\alpha_{bj}}{\alpha_{bj} + \beta_{bj}} \tag{4.22}$$

$$V(\widetilde{p}_{bj}) = (d_{bj} - a_{bj})^2 \frac{\alpha_{bj}\beta_{bj}}{(\alpha_{bj} + \beta_{bj})^2(\alpha_{bj} + \beta_{bj} + 1)} \tag{4.23}$$

A Workload plan is defined by $\widetilde{W_{it}} = \sum_{bj} \widetilde{p_{bj}} \upsilon_{bji} Y_{bjt}$ $(\forall\ i, t)$. Hence, the expectation is calculated as follows:

$$\begin{aligned} E(\widetilde{W}_{it}) &= E(\sum_{bj} \widetilde{p}_{bj}\upsilon_{bji}Y_{bjt}) = \sum_{bj} E(\widetilde{p}_{bj})\upsilon_{bji}Y_{bjt} \\ &= \sum_{bj}(a_{bj} + (d_{bj} - a_{bj})\frac{\alpha_{bj}}{\alpha_{bj} + \beta_{bj}})\upsilon_{bji}Y_{bjt} \end{aligned} \tag{4.24}$$

And the formulation of variance is as follows:

$$\begin{aligned} V(\widetilde{W}_{it}) &= V(\sum_{bj} \widetilde{p}_{bj}\upsilon_{bji}Y_{bjt}) = \sum_{bj} V(\widetilde{p}_{bj})\upsilon^2_{bji}Y^2_{bjt} \\ &= \sum_{bj}(d_{bj} - a_{bj})^2 \frac{\alpha_{bj}\beta_{bj}}{(\alpha_{bj} + \beta_{bj})^2(\alpha_{bj} + \beta_{bj} + 1)}\upsilon^2_{bji}Y^2_{bjt} \end{aligned} \tag{4.25}$$

Minimizing the variance or the expectation can be considered as objective function. By different ways, we can minimize an expression that combines the variance and the expected value [Subhash et al., 2010]:

$$Obj = min(w1 \sum_{it} E(\widetilde{W}_{it}) + w2 \sum_{it} V(\widetilde{W}_{it})) \tag{4.26}$$

$$Obj = max(= \frac{T - \sum_{it} E(\widetilde{W}_{it})}{\sum_{it} V(\widetilde{W}_{it})}) \tag{4.27}$$

4.2.5.2 Stochastic robustness evaluation

Let $\widetilde{F}_{it}$ be the complementary cumulative function of the distribution $\widetilde{W}_{it}$. Hence, a stochastic workload is the set of cumulative functions $\widetilde{F}_{it}$. The robustness function can be defined as follows:

$$R = \frac{1}{K(T+1)(\sum_{p=1}^{3} \varsigma_p)} \sum_{t=0}^{T} \sum_{i=1}^{K} \sum_{p=1}^{3} \varsigma_p P_{ipt} \tag{4.28}$$

where $P_{ipt} = P(\widetilde{F}_{it} < \kappa_{ipt})$.

There are a lot of estimations to simplify formulations within beta distribution for example [Browning and Yassine, 2010, Dimitri, 1988]. Nevertheless, the use of continuous distributions is still computationally too heavy. In fact, the use of convolution production to get cumulative profiles (per period, per resource type and per macro-task) is too complex and strongly influences the running times. Hence, instead of studying continuous distributions, only the couple expectation and variance can be considered [Subhash et al., 2010].

4.3 Solving RCCP algorithms

The RCCP problem is proven to be NP hard [Kis, 2005]. Hence, solving the RCCP problem to optimality in the deterministic case may be unrealistic for big instances [Hans, 2001]. Moreover, the problem is more complex while dealing with uncertainties [Wullink, 2005]. Hence, several heuristics are provided in [de Boer, 1998, Gademann and Schutten, 2005]. Below, we provide a generalization of existing algorithms: the exact Branch and Price provided in [Hans, 2001] and one of the LP-based heuristics proposed in [Gademann and Schutten, 2005]. Then, a new simulated annealing procedure is provided for the non-deterministic RCCP problem.

4.3.1 Generalization of existing algorithms

Hans [2001] proposes an exact branch&Price algorithm to solve the RCCP problem within the resource driven technique. Branch and price technique is useful when coping with large-scale IP problems. It integrates Branch&Bound and Column Generation methods. The ILP problem is first relaxed. Column generation is done at each Branch&Bound tree node to solve the LP relaxation. To check optimality, a sub-problem called pricing problem is solved to identify columns to enter the basis. If such columns are found, the LP is re-optimized. Branching occurs when no more columns are candidate to enter the basis and the LP solution does not satisfy integrality conditions [Barnhart et al., 1998].

In [Hans, 2001], and according to the model shown in section 4.2.3 the feasible project plans $a_{j\pi}$ are the binary columns that are used as input for the model. Binary variable $X_{j\pi}$ takes value 1 if project plan $a_{j\pi}$ is selected for project j, 0 otherwise. Hence the variables of the master problem are the project plan selection variables $X_{j\pi}$ and the project schedule variables Y_{jt}. The determination of feasible project plans according to calendar and precedence constraints is done in the sub-problem. The linear programming relaxation of this ILP is obtained by replacing (19) by $X_{j\pi} \geq 0(\forall j, \pi \in \Pi_j)$. The optimization of the given LP is done by performing column generation on a restricted LP, in which for each project j, a subset $\tilde{\Pi}_j$ of feasible columns Π_j is considered. The pricing algorithm generates other columns $a_{j\pi}$ for project j and adds them to $\tilde{\Pi}_j$ when possible. After optimizing the LP, the branch&Bound is performed in conjunction with column generation to find an optimal solution to the ILP.

de Boer [1998] provides several heuristics to deal with RCCP problem and considers both time driven and resource driven techniques. Gademann and Schutten [2005] provide several LP based heuristics and compare them with the heuristics of de-Boer and the Hans' B&Price technique. Among the heuristics provided in the aforementioned references, we will consider the one denoted $H_{feas(basic)}$ in [Gademann and Schutten, 2005]. This heuristic is a time driven technique and generally provides very good results. It is based on a steepest-descent step within the Simplex method for evaluating the neighbours of a set S of time windows. An initial feasible set S is generated by a basic primal heuristic denoted H_{basic} [Gademann and Schutten, 2005]. Next, we look for neighbours and accept the first one that leads to an improved schedule. The local search is continued until no more improvement is found.

4.3.2 Simulated Annealing

In this section, we provide a simulated annealing procedure to successively modify project plans and project schedules in order to improve the objective function. The aforementioned fuzzy objective functions are introduced into the RCCP model. Simulated annealing [Kirkpatrick et al., 1983] is a useful fast local search heuristic, frequently used for scheduling problems [van Laarhoven et al., 1992]. We consider the original scheme of the SA. The initial solution, with objective e_1, is chosen at temperature $T = T_{initial}$. Holding T constant, the initial solution is perturbed and the change in objective Δ_e is computed. For a minimization problem, if the change in objective function is negative then the new solution is accepted. Else, it is accepted with a probability given by the Boltzmann factor $exp - (\Delta_e/T)$. This process is repeated N times to give good sampling statistics for the current temperature, and then the temperature is decremented by $(1 - alpha)\%$ and the entire process repeated until the stop criterion $T = T_{stop}$.

Perturbation consists of choosing a new solution in the neighbourhood of the current one. For the RCCP problem, we saw that a solution is defined by a project plan $a_{j\pi}$ and a project schedule Y_j (see section 4.2.3). A neighbour is then either a solution with the same project plan and a modified project schedule, or a solution with a neighbour project plan and its associated project schedule. Gademann and Schutten [2005] use a LP-based local search heuristic to improve a feasible solution. An improved feasible plan is obtained by dual LP information, solving the LP problem according to this plan then gives the new schedule.

In our simulated annealing scheme, we propose to use both kinds of neighbours. A feasible project plan $a_{j\pi}$ is defined by the set of intervals $[S_{bj}, C_{bj}]$ (referred as *Allowed To Work* (ATW) in [Gademann and Schutten, 2005]) where S_{bj} is the starting interval of macro-task (b, j) and C_{bj} is its completion interval. In the following, ES_j is the earliest start interval of macro-task (b, j), $succ(bj)$ are the successors of macro-task (b, j), and $pred(bj)$ are its predecessors. Variables Y_{bjt} are used for the project schedule, heuristically defined by spreading the work content over the allowed periods. We consider, as objective functions, the expected cost and robustness expressions presented in Section 4.2.5.1. The heuristic proceeds as follows:

- **Step1**: Initialize with a feasible set of ATW windows ($S_{bj} = ES_{bj}$ and $C_{bj} = min(S_{succ(bj)} - 1)$) with a uniform spread of each activity workload through its ATW.

- **Step2**: Local modification 1: We randomly modify the project schedule (see below).

- **Step3**: Local modification 2: We randomly modify the project plan (see below).

- **Step4**: Keep the best solution in memory. If some termination criterion is met then stop, else go to Step2.

Step2 starts with choosing the period t that has the greater minimum value of workload $\widetilde{W}_{it}$. Among all macro-tasks present in this period, we select the macro-task (b, j) that has the maximum positive slack time. Then, the fraction of the macro-task workload in period t (Y_{bjt}) is spread uniformly through $[S_{bj}, t-1] \cup [t+1, C_{bj}]$. Note that a random selection of the period and then a random selection of a macro-task provides better results while computation time is not limited.

Step3 starts with randomly choosing the way to modify the ATW windows by increasing or decreasing either start or completion times by 1. Below, the 4 possible neighbourhoods are explained in detail.

The first possible neighbourhood is to increase a starting time: we choose the macro-task (b, j) having the minimum positive local slack time$(C_{bj} - S_{bj} - \omega_{bj})$. Randomly choosing a macro-task with a positive local slack time provides better results and the combination between random and guided selection is the best. Once the macro-task has been selected, we apply the following modifications:

- $Y_{bjS_{bj}}$ is spread uniformly into $S_{bj} + 1$ and C_{bj}.
- S_{bj} is increased by 1.
- $C_{Pred_{bj}}$ is also increased by 1 if all successors start at least at S_{bj}.

The second possible neighbourhood is to decrease a completion time: we choose the macro-task (b, j) having the minimum positive local slack time$(C_{bj} - S_{bj} - \omega_{bj})$. To randomly choose a macro-task having a positive local slack time provides better results and the combination between random and guided selection is the best. Once the macro-task has been selected, we apply the following modifications:

- $Y_{bjC_{bj}}$ is spread uniformly into S_{bj} and $C_{bj} - 1$.
- C_{bjt} is decreased by 1.
- $S_{Succ(bj)}$ is also decreased by 1 if all predecessors finish at most at C_{bj}.

The third possible neighbourhood is to decrease a starting time: we choose the macro-task j having the minimum positive free slack time$(S_{bj} - S_{Pred_{bj}} - \omega_{Pred_{bj}})$. To randomly choose a macro-task having a positive local free slack time provides better results and the combination between random and guided selection is the best. Once the macro-task has been selected, we apply the following modifications:

- $Y_{Pred_{bj}C_{Pred_{bj}}}$ is spread uniformly into $S_{Pred_{bj}}$ and $C_{Pred_{bj}} - 1$.
- S_{bj} is decreased by 1.
- $C_{Pred_{bj}}$ is also decreased by 1 if all successors start at least at S_{bj}.
- $S_{Succ_{Pred_{bj}}}$ is decreased by 1 if all predecessors finish at most at $C_{Pred_{bj}}$. This modification is selected randomly.

The fourth possible neighbourhood is to increase a completion time: we choose the macro-task (b, j) having the minimum positive free slack time($S_{Succ(j)} - S_{bj} - \omega_{bj}$). To randomly choose a macro-task having a positive local free slack time provides better results and the combination between random and guided selection is the best. Once the macro-task has been selected, we apply the following modifications:

- $Y_{Succ_{bj} S_{Succ_{bj}}}$ is spread uniformly into $S_{Succ(bj)} + 1$ and $C_{Succ(j)}$.
- C_{bj} is increased by 1.
- $S_{Succ_{bj}}$ is also increased by 1 if all predecessors finish no later than C_{bj}.
- $C_{Pred_{Succ_{bj}}}$ is increased by 1 if all successors start at least at $S_{Succ_{bj}}$. This modification is selected randomly.

Contrary to the Branch&Price algorithm, Simulated Annealing accepts both linear and non linear objective functions. The simulated Annealing parameters are chosen in a generic way respecting the rule of acceptance ratio (accepted solutions/N for the first iteration) that should be greater than 95%. The use of design of experiments is imaginable as further work to fix parameters while completion time is limited.

4.4 Computations and comparisons

Instances from the navy ships maintenance domain [Hans, 2001] are considered to validate our simulated annealing algorithm in comparison with the algorithms provided by the Dutch team resumed in [Hans, 2001, Gademann and Schutten, 2005, de Boer, 1998]. To be able to make comparisons, only the expectation objective function that is linear is considered. Once simulated annealing is validated with a linear objective, an application to the helicopter maintenance domain is provided with different objectives functions-linear and non linear- such as expectation and robustness functions.

4.4.1 Validation of the simulated annealing procedure

To validate our SA, we consider several instances from navy ships maintenance domain that we got from the Dutch group [Hans, 2001]. We consider projects with 10, 20 and 50 macro-tasks, then we consider 1 to 3 projects in parallel. Table 4.3 contains the result of simulation for simple projects. We use * when

Table 4.3: Simulated annealing vs B&Price and LP-based heuristic

Instances			Exact		Heuristic		SA	
			value	time(sec)	value	time(sec)	value	time(sec)
Projects with 10 macro-tasks	One project	rccp129	17403*	0.38	17403	0.32	17440	5285.51
		rccp219	14648*	7.41	14891	0.43	14978	5781.95
		rccp309	28409*	38,31	28857	0.77	28752	8264.92
		rccp399	12402	> 1321.15	12414	1.41	12514	7167
		rccp489	25606	> 403.71	25790	4.04	25710	12177.22
	Two projects	rccp127en128	19704*	5.16	19704	1.27	20051	6684.09
		rccp217en218	4742	> 3452.4	14540	3.95	14777	8459.07
		rccp307en308	–	–	14479	5.54	14929	8762.97
		rccp397en398	–	–	35184	10.75	37278	12973.1
		rccp487en488	–	–	35714	16.43	35642	14393
	Three projects	rccp127en128en129	20342*	42.77	20342	2.19	21669	8675.56
		rccp217en218en219	–	–	13195	12.42	14111	10220.3
		rccp307en308en309	–	–	21708	21.73	24260	13231.25
		rccp397en398en399	–	–	25211	38.54	30034	15861.21
		rccp487en488en489	–	–	36644	55.56	32880	19599.91
Projects with 20 macro-tasks	One project	rccp159	18829*	1.01	18829	0.6	18977	7763.02
		rccp249	–	–	18644	2.47	19105	10399.73
		rccp339	–	–	12414	3.46	12581	7071.51
		rccp429	–	–	26696	7.49	24771	12705.73
		rccp519	–	–	26580	10.62	21797	14479.98
	Two projects	rccp157en158	15854	> 10142.76	15851	6.04	16387	13404.92
		rccp247en248	–	–	28747	18.87	30688	18314.84
		rccp337en338	–	–	32454	37.48	29587	22990.68
		rccp427en428	–	–	20036	84.59	13523	74883.09
		rccp517en518	–	–	20923	91.87	21576	25263.09
	Three projects	rccp157en158en159	–	–	15290	9.41	16014	18641.84
		rccp247en248en249	–	–	24038	50.57	26111	24843.32
		rccp337en338en339	–	–	24527	155.37	26544	30655.01
		rccp427en428en429	–	–	19265	243.23	22064	26957.45
		rccp517en518en519	–	–	12305	461.22	15201	33258.35
Projects with 50 macro-tasks	One project	rccp189	26249	> 857.36	26248	15.17	26431	28584.52
		rccp279	–	–	12504	46.85	12571	20727.02
		rccp369	–	–	11909	70.45	11568	22349.19
		rccp459	–	–	15600	128.41	14519	23271.3
		rccp549	–	–	16152	177.24	17126	28530.14
	Two projects	rccp187en188			17502	51.93	17888	30868.82
		rccp277en278	–	–	11804	556.35	13085	34785.2
		rccp367en368	–	–	12044	381.68	13237	39925.51
		rccp457en458	–	–	9039	1335.46	14416	53396.72
		rccp547en548	–	–	9545	2986.79	15084	52485.94
	Three projects	rccp187en128en129	–	–	22342	209.29	23495	71719.83
		rccp277en278en279	–	–	9088.42	2166.8	10007	51061
		rccp367en368en369	–	–	6809	3634.17	7857	57724.04
		rccp457en458en459	–	–	7422	3652.98	12161	65041
		rccp547en548en549	–	–	8004	3640.22	12910	70628.58

an optimal solution is found, and – when no competitive solution is achieved even after an excessive computation time.

The Hans' Branch and Price technique provides optimal solutions for very simple instances. The LP-based heuristic of Gademan and Schutten is the most effective in terms of computation time.

Table 4.4: Simulated annealing vs LP-based heuristic for big instances

Instances			Heuristic		SA	
			value	time(sec)	value	time(sec)
Projects with 10 macro-tasks	One project	rccp485	14716	2.61	15151	8443.51
		rccp486	45746	1.25	49297	13591.59
		rccp487	40020	2.53	40304	10333.9
		rccp488	23251	5.23	23279	11089.36
		rccp489	25790	3.62	25582	12391.71
	Two projects	rccp485en486	38073	23.23	42561	16973.39
		rccp486en487	53926	17.86	57190	16969.39
		rccp487en488	35714	16.67	35981	13908.99
		rccp488en489	30628	14.14	25610	15772.75
		rccp485en489	21494	17.87	22674	15824.02
	Three projects	rccp485en486en487	37364	82.09	44368	20717.81
		rccp486en487en488	40878	81.29	43629	21685.43
		rccp487en488en489	36644	60.72	33764	19587.95
		rccp485en487en489	32985	37.12	31456	18733.79
		rccp486en488en489	35363	60.23	39135	22056
Projects with 20 macro-tasks	One project	rccp515	19838	11.01	19988	13990
		rccp516	13417	21.16	12105	11799.47
		rccp517	22315	16.17	23007	17016.25
		rccp518	26349	1497	25421	15600.49
		rccp519	26580	9.49	21216	14554.27
	Two projects	rccp515en516	13069	66.07	14829	19648.16
		rccp516en517	19072	85.51	15446	25989.88
		rccp517en518	20923	78.92	21770	25537.81
		rccp518en519	20541	76.43	21107	23746.13
		rccp515en519	16313	77.51	17689	20880.3
	Three projects	rccp515en516en517	13449	215.76	12716	31918.06
		rccp516en517en518	14720	158.37	15855	32444.7
		rccp517en518en519	12305	345.93	14989	32252.93
		rccp515en517en519	9849	435.4	13199	32465.18
		rccp516en518en519	14034	209.61	15362	29198.99
Projects with 50 macro-tasks	One project	rccp545	12649	193.53	12003	24234.91
		rccp546	17250	268.43	18166	29267.98
		rccp547	18176	256.66	18760	31529.23
		rccp548	12146	198.86	12751	31271.65
		rccp549	16152	151.86	16881	28399.91
	Two projects	rccp546en547	14785	2433.39	18830	54733.52
		rccp547en548	9545	3286.39	13592	52915.72
		rccp548en549	7843	2520.86	11006	48358.46
		rccp545en549	12676	2101.43	14059	49885.64
	Three projects	rccp485en486en487	12594	3691.91	16741	76680
		rccp486en487en488	13379	3637.55	13390	72963
		rccp487en488en489	7979	3637.63	13773	74335.68
		rccp485en487en489	9870	3656.83	13037	72795
		rccp486en488en489	6774	3639.39	9381.1	68597

For the majority of the instances, the LP-based heuristic and the SA are competitive in terms of solutions. For more complex instances, we exclude exact techniques from comparison as they do not provide optimal solutions any more (see Table 4.4).

We notice a big difference in computation time between our SA algorithm and the LP-based heuristic for simple and complex instances, which is due to the use of different computation methods as we use Matlab to model our SA which is a matrix oriented language and Gademann and Schutten used Delphi which is much more efficient than Matlab in native computations. We expect to significantly reduce the computation time while converting our Matlab code to Delphi code, and thus comparison will be more realistic with a competitive computation time and objective values.

It is apparent that the simulated annealing is very competitive for many instances. Moreover, we know that the more time the algorithm takes, the better is the result. Hence, for very complex instances, the increase of SA parameters is necessary to improve the convergence of the algorithm. Table 4.5 shows the result of simulation for several big instances considering different parameter values in comparison with the LP-based heuristic.

Table 4.5: Results using different parameters; application to big instances

Instances	SA(T=100,N=70,α =0.997)		SA(T=150,N=100,α =0.998)		Heuristic	
	value	time(sec)	value	time(sec)	value	time(sec)
rccp546en547	18830	54733.52	13886	116786.74	14785	2433.39
rccp547en548	13592	52915.72	11835	119550.31	9545	3286.31
rccp545en546en547	16741	76680.57	10819	163302.51	12594	3691.91
rccp546en547en548	13390	72963	9834	111873.41	13379	3637.55

The study of SA algorithm parameters is beyond the scope of this thesis. Nevertheless, the use of Design of experiments is expected to improve the performance of the SA algorithm with the selection of best parameters values [Pongcharoen et al., 2002] (to be studied in future work).

4.4.2 Computations; application to helicopter maintenance

The simulated annealing is applied to the real HMV specified in chapter 1 from the helicopter maintenance domain. For computation, we consider that the regular capacity limit, the overtime capacity limit and the hiring capacity limit are fixed in the strategic level of planning and equal to 20 (20 hours per day equivalent to 100 hours per week). Hence, $\kappa_{i1t} = 20$, $\kappa_{i2t} = 40$ and $\kappa_{i3t} = 60$. All workload exceeding κ_{i3t} is subcontracted. The weighting coefficient β is chosen equal to 0.6. We consider $\varsigma_{i1} = 20$, $\varsigma_{i1} = 50$, $\varsigma_{i1} = 100$

the unit costs of respectively overtime, hiring and subcontracted hours. We consider that the HMV due date was negotiated with the customer and fixed to 32.

The SA parameters are $alpha = 0.995$, $N = 3 * Number\ of\ macro-tasks$, $T_{initial} = 250$, $T_{stop} = 0.005$. These parameters values are chosen respecting the rule of acceptance ratio that should be greater than 0.95%.

The initial solution is obtained by applying Step 1 and the corresponding fuzzy objective values are $E = 33530$, $R1 = 90.7\%$, $R2 = 87.5\%$. From this initial solution, we successively optimize objectives E, $R1$ and $R2$. Just one objective function is introduced into the algorithm and the others are considered as secondary and calculated at the end. The simulated annealing is performed 10 times for each objective to draw more reliable conclusions. Table 4.6 shows the mean values of 10 computations for each objective.

Table 4.6: Study of the different objective functions

Objective: E	R1	R2
20592	90.2%	85.5%
Objective: R1	E	R2
92.3%	28737	89.9%
Objective: R2	E	R1
90.8%	31084	92.0%

It may be difficult to interpret these results. But, we can notice that optimizing robustness $R1$ does not lead to the best values of $R2$ and vice-versa. In addition, we can notice that $R1$ is always better than $R2$ (not only for the mean values, but also for separate simulations values) and that computation time is similar for the optimization of $R1$ and $R2$, but there is an increase of about 15% when optimizing E.

The numerical result justifies that the study of robustness functions is useful; by improving the robustness we improve the cost, but, on the contrary, by improving the cost we may worsen the robustness.

Figure 4.9 shows the convergence of the SA algorithm and the difference between the initial fuzzy workload plan and the result after the optimization. The corresponding main objective for this figure is $E = 20475$ and the secondary objectives $R1$ and $R2$ are equal to 0.90 and 0.84, respectively.

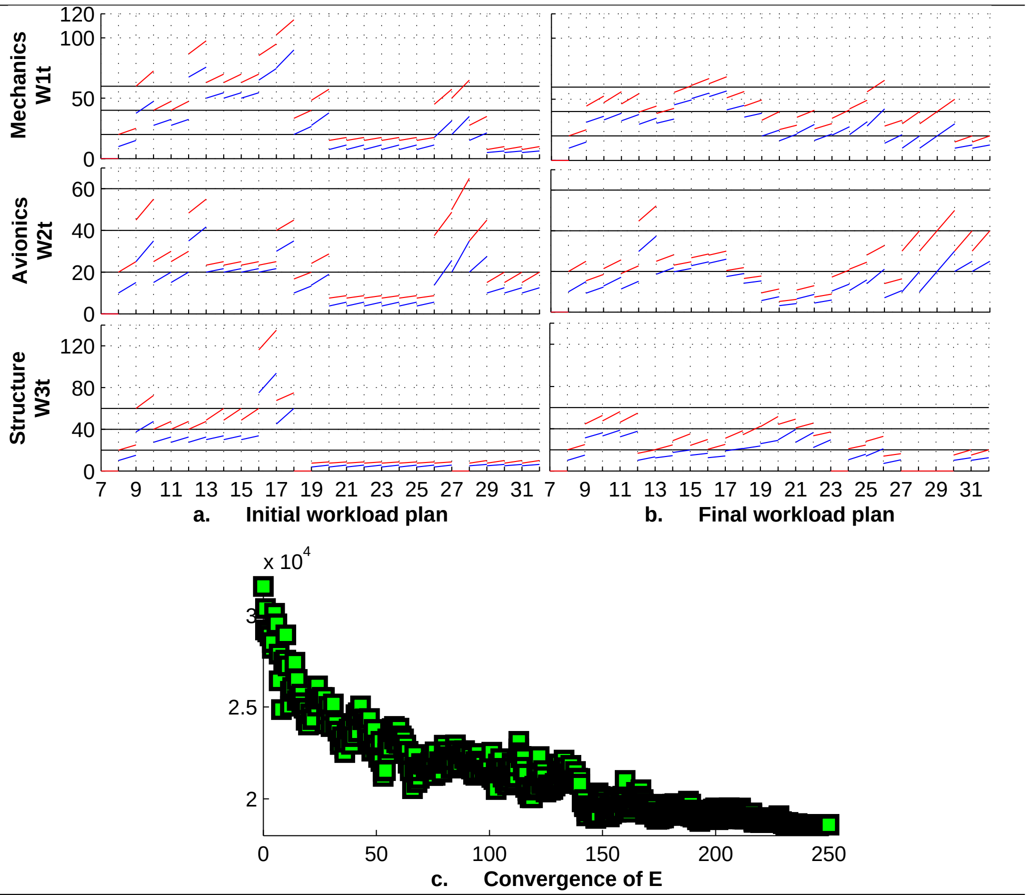


Figure 4.9: Result of the simulation: Fuzzy tactical workload plans and algorithm convergence

4.5 Conclusions

This chapter explains how an RCCP problem under uncertainty can be modelled using the fuzzy/possibilistic approach or the stochastic approach. Then, existing deterministic algorithms are, easily and without adding complexity, generalized to accommodate uncertainty. Some fuzzy and stochastic objectives functions are defined and a Simulated Annealing algorithm is provided to solve the Fuzzy and the Stochastic RCCP problem. For computation, we were interested in the fuzzy approach that corresponds more to our case of study. The Simulated Annealing algorithm is compared to the exact B&Price technique of Hans [2001] and one of the linear programming based heuristics of Gademann and Schutten [2005] using the expected value as objective function. Then, a real application from the helicopter maintenance domain is considered and a study of different fuzzy objective functions is provided.

Table 4.7: Real PUMA HMV project.

Macro-task	Id	Pred.	Duration	Processing times(fuzzy-stochastic) $[a, b, c, d]$	$[a, d, \alpha, \beta]$	Resource fraction
Waiting for the release date	A	-	8	0	0	0
First check	B	A	1	[60, 90, 120, 150]	[60,150,3,4]	1/3-1/3-1/3
Removal structure and mechanics	C	B	3	[160, 180, 220, 260]	[160,260,2,5]	1/2-0-1/2
Removal avionics	D	B	3	[120, 160, 200, 240]	[120,240,2,3]	1/4-1/2 -1/4
Supplying procedure for finishing	E	C	14	0	0	0
Mechanical inspection I	F	C	5	[360, 390, 420, 450]	[360,450,3,4]	2/3- 1/3-0
Supplying to assembling	G	C	7	0	0	0
Supplying to structural inspection	H	C	2	0	0	0
Subcontracted structure-cleaning	I	C	1	0	0	0
Subcontracted avionic repairs	J	D	3	0	0	0
Structural inspection I	K	I	3	[160, 180, 260, 320]	[160,320,3,4]	1/4-0-3/4
Structural inspection II	L	H-K	1	[120, 160, 180, 200]	[120,200,3,4]	1/4-0-3/4
Subcontracted painting	M	L	1	0	0	0
Mechanical inspection II	N	F	1	[90, 120, 150,180]	[90,180,2,3]	2/3-1/3-0
Assemble helicopter parts	O	G-J-M-N	1	[120, 180, 240, 280]	[120,280,3,4]	1/2-1/4-1/4
Finishing before fly test	P	E-O	1	[40, 80, 120, 160]	[40,160,2,5]	1/2-1/2-0
Test before delivering	Q	P	1	[40, 60, 80, 100]	[40,100,3,4]	1/2-1/2-0
Possible additional work	R	Q	2	[80, 100, 120, 160]	[80,160,3,4]	1/4-1/2-1/4

Chapter 5

Project scheduling under uncertainties for helicopter maintenance

Contents

5.1 Introduction

Project scheduling consists of generating a feasible schedule that respects precedence and resource constraints, and achieves specific organizational objectives. The schedule should serve as a baseline for execution. Most of the scheduling problems are known to be NP-Hard. Many uncertainties can affect the scheduling problem and hence increase its complexity [Bidot, 2005]. In many cases, the real environment is imprecise or/and partially known. In other areas, we do not know precisely whether some events will occur and, if yes, for how much time. Elkhayari [2003] groups uncertainties into three subsets: uncertainties in tasks, uncertainties in resources, and temporal uncertainties. To deal with uncertainties in scheduling, several techniques are provided in literature. Davenport and Beck [2000] classify these techniques into three categories: proactive, reactive, and proactive-reactive approaches. Proactive scheduling try to cope with uncertainty in creating a flexible (or robust) schedule as a baseline schedule that does not necessarily need hight modifications while perturbations occurs. On the other hand, reactive approach is based on the idea to revise and re-optimize the baseline schedule when an unexpected event occurs. In project scheduling, Herroelen and Leus [2005] distinguish between five main approaches to deal with scheduling under uncertainty: reactive scheduling, stochastic project scheduling, stochastic project networks, fuzzy project scheduling and proactive/robust scheduling. Particularly, the fuzzy project scheduling, based on the assumption that task durations rely on human estimations, is used while theory of probabilities is not compatible with the decision-making situation *e.g.* lack of historical data [Bonnal et al., 2004, Herroelen and Leus, 2005].

Resource management is a prerequisite to get a successful scheduling. Two major techniques; resource constrained scheduling (RCS) and resource leveling (RL), are employed for managing resources in a scheduling process [Kim et al., 2005a]. As far as we know, the resource management issue is not studied in the fuzzy scheduling field. In fact, only fuzzy dates and intervals are treated and resource workload is always generated consequently by a deterministic way [Hapke and Slowinski, 1996, Leu et al., 1999].

In this chapter, to manage resources within a fuzzy way, we propose to keep uncertainty at all steps of the solving procedures. Hence, firstly, we exploit the fuzzy/possibilistic approach to model a new concept that we call fuzzy workload. Secondly, based on this modelling concept, two techniques RCS and RL are generalized to fuzzy parameters. We refer to these problems as the fuzzy resource leveling problem (FRLP) and the fuzzy Resource

Constrained project Scheduling problem (FRCPSP). A genetic algorithm and greedy algorithm are provided to solve FRLP and FRCPSP, respectively. We apply these modelling and solving techniques to the instance of civil helicopter Maintenance activity that were provided in chapter 1.

5.2 Fuzzy task modelling

The project dates and durations are represented by trapezoidal fuzzy numbers. Let $\widetilde{S}(a_S, b_S, c_S, d_S)$ be the fuzzy start date of a task T, and $\widetilde{D}(w, x, y, z)$ its duration. Let $\widetilde{F}(a_F, b_F, c_F, d_F)$ be its finish date with $\widetilde{F} = \widetilde{S} + \widetilde{D}$. Let C be the number of operators assigned to the task T. Once starting time and finishing time of all tasks are defined, in literature, several deterministic resource workload plans are established by applying alpha-cuts (see Figure 5.1).

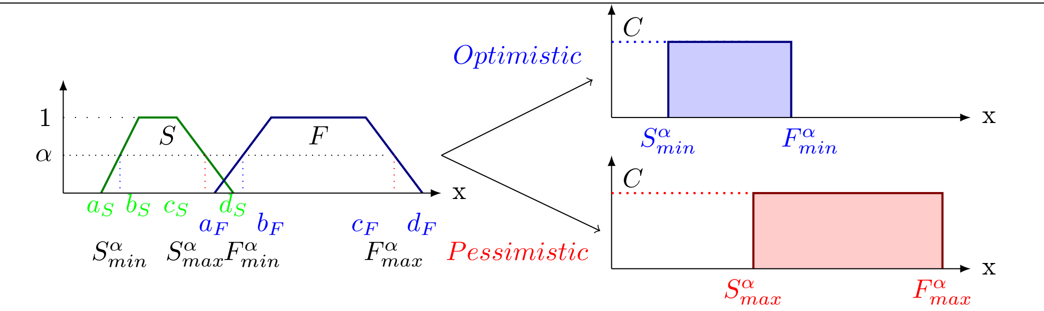


Figure 5.1: Alpha-cuts and deterministic workloads.

In this section, we provide a new technique to deal with fuzzy resource planning. Instead of applying alpha-cuts on a fuzzy Gantt to get deterministic resource plans, both Gantt and workload plan are considered fuzzy.

Inspired from last expressions in section 3.2.2, we can define $]\widetilde{S}; \widetilde{F}[$ (respectively, $[\widetilde{S}; \widetilde{F}]$), the domain where the task T presence is necessarily (respectively, possibly) true. They represent the truth of the event "t between the starting and the finishing date of T". Associated membership functions, $\mu_{]\widetilde{S};\widetilde{F}[}(t)$ and $\mu_{[\widetilde{S};\widetilde{F}]}(t)$ are respectively denoted $N(t)$ and $\Pi(t)$.

We can distinguish three different configurations depending on the intersection degree between fuzzy start and finish dates (see Figure 5.2): a configuration without overlap ($d_S \leq a_F$), a configuration with small overlap ($d_S > a_F$ and $c_S \leq b_F$) and a configuration with large overlap ($c_S > b_F$).

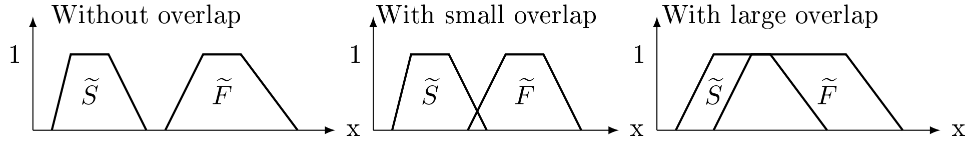


Figure 5.2: Different configurations: with and without overlap.

Each configuration is studied separately within two modelling (symmetric and non symmetric) distribution of the workload to be used in the scheduling optimization algorithms. Building a relevant resource usage profile for a task with fuzzy dates and durations is not straightforward. Most of the time, the problem parameters are fixed in order to obtain a deterministic configuration. This leads to a scenario based approach [Hapke and Slowinski, 1996] where various significant scenarios may be compared in a decision process: lower and upper bounds, most plausible configuration, etc.

In this chapter we propose to build task resource usage profiles in a way that keeps track of uncertainty on start and finish dates. Hence, the profile reflects the whole possible time interval while giving a plausible repartition of workload according to the duration parameter value. To achieve this aim, the resource usage profiles are defined as projections onto the workload space of the task presence distributions.

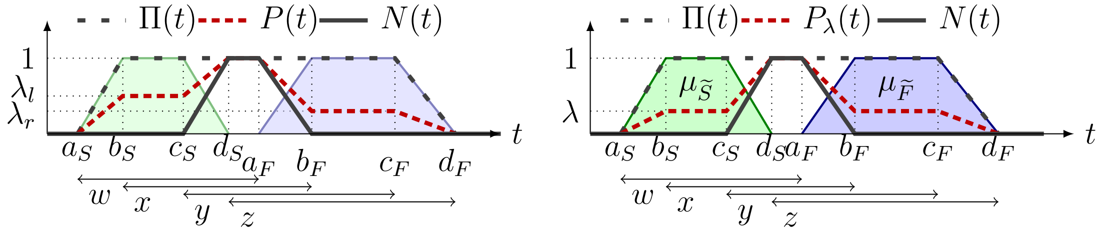
5.2.1 Configuration without overlap

In the configuration without overlap between the starting date $\widetilde{S}$ and the finishing date $\widetilde{F}$ (see Figure 5.3), we can identify the following intervals of possibility and necessity:

$$
\begin{array}{lcll}
[d_S; a_F] & : & \Pi = 1 & N = 1 \\
[c_S; d_S] \text{ and } [a_F; b_F] & : & \Pi = 1 & N \geq 0 \\
[b_S; c_S] \text{ and } [b_F; c_F] & : & \Pi = 1 & N = 0 \\
[a_S; b_S] \text{ and } [c_F; d_F] & : & \Pi \geq 0 & N = 0 \\
[0; a_S] \text{ and } [d_F; +\infty[& : & \Pi = 0 & N = 0
\end{array}
$$

Then we characterize the probability of task T presence as a distribution $P(t)$ situated between the possibility and the necessity profiles: $N(t) \leq P(t) \leq \Pi(t)$.

We propose a parametric piecewise linear distribution to represent the probability of the presence of task (dashed lines on Figure 5.3).



(a) General distribution: Non symmetric (b) Particular distribution: Symmetric

Figure 5.3: Presence of a task: No overlap configuration.

Both symmetric and non symmetric distributions are considered and will be used to establish resource requirement. The symmetric distribution is a particular case, and thus the non symmetric distribution which is the general one is represented by a compound function depending on different intervals of possibility and necessity:

$$P(t) = \begin{cases} \frac{\lambda_l}{b_S - a_S}(t - a_S) & \text{if } t \in [a_S; b_S] \\ \lambda_l & \text{if } t \in [b_S; c_S] \\ \frac{1}{d_S - c_S}\left((1-\lambda_l)t + \lambda_l d_S - c_S\right) & \text{if } t \in [c_S; d_S] \\ 1 & \text{if } t \in [d_S; a_F] \\ \frac{1}{b_F - a_F}\left((\lambda_r - 1)t + b_F - \lambda_r a_F\right) & \text{if } t \in [a_F; b_F] \\ \lambda_r & \text{if } t \in [b_F; c_F] \\ \frac{-\lambda_r}{d_F - c_F}(t - d_F) & \text{if } t \in [c_F; d_F] \\ 0 & \text{otherwise,} \end{cases} \tag{5.1}$$

where parameters λ_l and λ_r, varying from 0 to 1, makes profile $P(t)$ evolve from $N(t)$ ($\lambda_l = \lambda_r = 0$) to $\Pi(t)$ ($\lambda_l = \lambda_r = 1$).

Suppose that the resource requirement of the task is r. Resource workload then lies in $[r.w, r.z]$, according to the task duration. Figure 5.4b presents the resource profiles $L_N(t)$ and $L_\Pi(t)$, projections of the necessity and possibility presence distributions.

We define the "equivalent durations" D_N and D_Π of the areas covered by resource profiles $L_N(t)$ and $L_\Pi(t)$:

$$r.D_N = \int_0^{+\infty} L_N(t)dt = r\,(b_F - c_S + a_F - d_S)/2 \tag{5.2}$$

$$r.D_\Pi = \int_0^{+\infty} L_\Pi(t)dt = r\,(d_F - a_S + c_F - b_S)/2 \tag{5.3}$$

If D_N and D_Π do not match with task extreme durations w and z, the profiles must be modified so that resource workload belongs to $[r.w, r.z]$. If

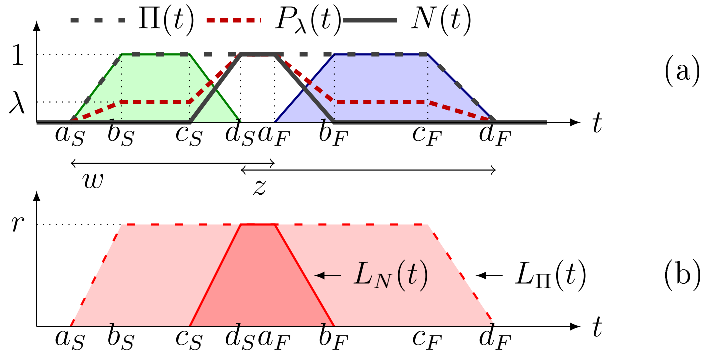


Figure 5.4: Configuration without overlap: presence distributions (a) and resource profiles (b).

$D_N < w$ or $z < D_\Pi$, the projection of probability presence distribution $P_\lambda(t)$ is used to define a minimal or maximal value of λ. The link between task duration D and profile parameter λ, in case of symmetric distribution, is given by the following formula that expresses the equivalence of resource requirement:

$$\begin{aligned} r.D &= \int_0^{+\infty} r.P_\lambda(t)dt = \int_0^{+\infty} (\lambda.L_\Pi(t) + (1-\lambda)L_N(t))dt \\ &= \lambda.r.D_\Pi + (1-\lambda)r.D_N \\ &= \lambda.r\,(d_F - a_S + c_F - b_S)/2 + (1-\lambda)r\,(b_F - c_S + a_F - d_S)/2. \end{aligned} \tag{5.4}$$

In general case where distribution is non symmetric, the link between the task duration and the profile is as follows:

$$\begin{aligned} r.D &= \int_0^{+\infty} r.P(t)dt \\ &= r.\lambda_l(\frac{d_S - b_S}{2} + \frac{c_S - a_S}{2}) + r.\lambda_r(\frac{c_F - a_F}{2} + \frac{d_F - b_F}{2}) \\ &\quad + r.(\frac{a_F - d_S}{2} + \frac{b_F - c_S}{2}) \end{aligned} \tag{5.5}$$

In case of symmetric distribution, the range of λ may be reduced from $[0,1]$ to $[\lambda_{\min}, \lambda_{\max}]$ if $D_N < w$ and $z < D_\Pi$. From the previous relations, we can deduce:

$$\text{If } D_N < w,\ \lambda_{\min} = (w - D_N)/(D_\Pi - D_N)$$
$$\text{If } z < D_\Pi,\ \lambda_{\max} = (z - D_N)/(D_\Pi - D_N).$$

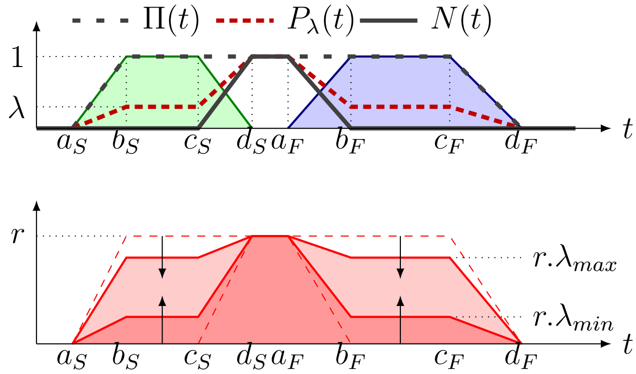


Figure 5.5: Resource profiles: restriction to λ_{min} and λ_{max} in order to match with extreme workloads.

Figure 5.5 shows an example of restricted extreme profiles.

Let us consider the particular case of a task with a fuzzy duration, but a deterministic starting date ($a_S = b_S = c_S = d_S = s$, Figure 5.6). If we choose $D = z$, there is only one possible position for the task, between s and d_F. So the resource chart is fixed, rectangular shaped. One can remark that in this case, the projection of the probability distribution is not able to represent the resource consumption: even with $\lambda = 1$, the resource workload would be underestimated (Figure 5.6). Indeed, the surface of profile $L_\Pi(t)$ is $r.(c_F - s + d_F - s)/2 = r.(y + z)/2$, lower than $r.z$.

For any duration D so that $(y+z)/2 < D \leq z$, the area of resource profile $L_\Pi(t)$ is too small to represent the resource workload. To cope with this problem, we modify the resource profile: in place of points (s, s, c_F, d_F), the new profile is defined by the points (s, s, c'_F, d_F), where $c'_F = c_F + \max(0, 2D - z - y)$. Hence, while $D \leq (y+z)/2$, the initial profile is used and $\lambda \leq 1$, then the new profile is used. When $D = z$, the rectangular profile is reached. A similar modification can be done for the minimal duration, when the area of the projected necessity distribution is greater than $r.w$.

These modifications can be generalized to the case with fuzzy dates and duration. Then the profiles, if needed, are modified on both sides. The extended maximal profile, defined by (a_S, b'_S, c'_F, d_F), is used when $D_\Pi < D \leq z$. Values b'_S and c'_F are:

$$b'_S = b_S - 2(D - D_\Pi)\frac{b_S - a_S}{b_S - a_S + d_F - c_F} \tag{5.6}$$

$$c'_F = c_F + 2(D - D_\Pi)\frac{d_F - c_F}{b_S - a_S + d_F - c_F} \tag{5.7}$$

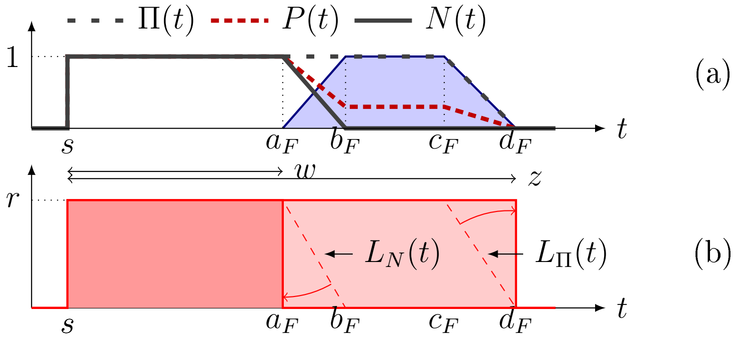


Figure 5.6: Case of a deterministic start date: presence distributions and maximal resource profile.

The reduced minimal profile, defined by (c'_S, d_S, a_F, b'_F), is used when $w \leq D < D_N$. Values c'_S and b'_F are:

$$c'_S = c_S + 2(D_N - D)\frac{d_S - c_S}{d_S - c_S + b_F - a_F} \tag{5.8}$$

$$b'_F = b_F - 2(D_N - D)\frac{b_F - a_F}{d_S - c_S + b_F - a_F} \tag{5.9}$$

Figure 5.7 shows an example of modified extreme profiles.

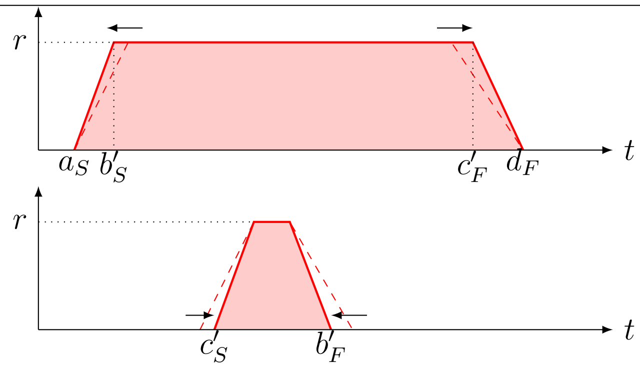


Figure 5.7: Resource profiles: extension of maximal profile and reduction of minimal profile in order to match with extreme workloads $r.w$ and $r.z$.

5.2.2 Configuration with small overlap

For the small overlap configuration (as in the previous configuration), the general distribution is also represented by a compound function (dashed line on Figure 5.8):

$$P(t) = \begin{cases} \frac{\lambda_l}{b_S - a_S}(t - a_S) & \text{if } t \in [a_S; b_S] \\ \lambda_l & \text{if } t \in [b_S; c_S] \\ \frac{1}{d_S - c_S}\left((1-\lambda_l)t + \lambda_l d_S - c_S\right) & \text{if } t \in [c_S; \alpha] \\ \frac{1}{b_F - a_F}\left((\lambda_r - 1)t + b_F - \lambda_r a_F\right) & \text{if } t \in [\alpha; b_F] \\ \lambda_r & \text{if } t \in [b_F; c_F] \\ \frac{-\lambda_r}{d_F - c_F}(t - d_F) & \text{if } t \in [c_F; d_F] \\ 0 & \text{otherwise.} \end{cases} \quad (5.10)$$

where the higher point $(\alpha,\, \beta)$ is calculated as follows:

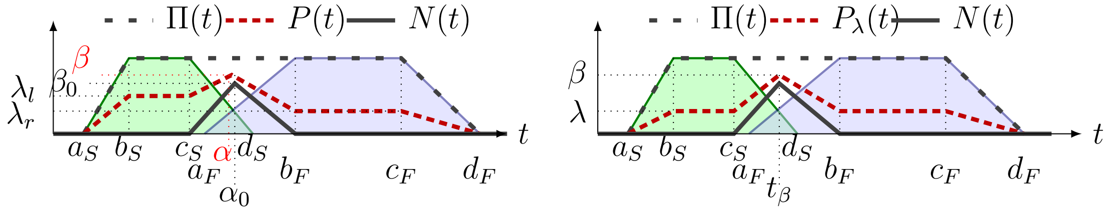
$$\alpha = \frac{(b_F - a_F)(\lambda_l d_S - c_S) + (d_S - c_S)(\lambda_r a_F - b_F)}{(b_F - a_F)(\lambda_l - 1) + (d_S - c_S)(\lambda_r - 1)} \quad (5.11)$$

$$\beta = \frac{(b_F - \lambda_r a_F)(\lambda_l - 1) + (\lambda_l d_S - c_S)(\lambda_r - 1)}{(b_F - a_F)(\lambda_l - 1) + (d_S - c_S)(\lambda_r - 1)} \quad (5.12)$$

And particularly while $\lambda_l = \lambda_r = \lambda$:

$$\alpha = \alpha_0 = \frac{d_s.b_f - a_f.c_s}{(b_f - c_s) + (d_s - a_f)} \quad (5.13)$$

$$\beta = \frac{(b_f - c_s) + \lambda(d_s - a_f)}{(b_f - c_s) + (d_s - a_f)} \quad (5.14)$$



(a) Non symmetric (b) Symmetric

Figure 5.8: Presence of a task: small overlap configuration

The point (α_0, β_0) corresponds to the maximum value of the necessity profile (peak). The value β varies in a range $[\beta_0, 1]$ and the value α varies in a range $[a_F, d_S]$ along with parameters λ_l and λ_r.

The areas of the projected necessity and possibility distributions are:

$$r.D_N = \int_0^{+\infty} r.N(t)dt = r.\,\beta_0 \frac{b_F - c_S}{2} = r\frac{(b_F - c_S)^2}{2(d_S - a_F + b_F - c_S)} \quad (5.15)$$

$$r.D_\Pi = \int_0^{+\infty} r.\Pi(t)dt = r.(d_F - a_S + c_F - b_S)/2 \tag{5.16}$$

If $r.D_N$ is lower than the minimal workload $r.w$ (respectively, $r.D_\Pi$ greater than the maximal workload $r.z$) we use the projection of the presence probability distribution and determine $\lambda_{\min}$ (respectively, $\lambda_{\max}$). Given D so that $D_N < D < D_\Pi$,

$$r.D = \int_0^{+\infty} r.P_\lambda(t)dt = \lambda.r.D_\Pi + (1-\lambda)r.D_N. \tag{5.17}$$

In general case where distribution is non symmetric, the link between the task duration and the profile is given by the following formula:

$$\begin{aligned} r.D &= \int_0^{+\infty} r.P(t)dt \\ &= r.\lambda_l(\frac{c_S+\alpha}{2} - \frac{a_S+b_S}{2}) + r.\lambda_r(\frac{d_F+c_F}{2} - \frac{\alpha+b_F}{2}) + r.\beta(\frac{b_F-c_S}{2}) \end{aligned} \tag{5.18}$$

In case of symmetric distribution, when $D_N < w$, $\lambda_{\min} = (w - D_N)/(D_\Pi - D_N)$ and when $D_\Pi > z$, $\lambda_{\max} = (z - D_N)/(D_\Pi - D_N)$.
The extended maximal profile, defined by (a_S, b'_S, c'_F, d_F), is used when $D_\Pi < D \leq z$. It is the same extended profile as the one of no overlap configurations. The reduced minimal profile, defined by (c'_S, d_S, a_F, b'_F), is used when $w \leq D < D_N$. Values c'_S and b'_F are:

$$c'_S = \theta.c_S + (1-\theta)d_S \tag{5.19}$$
$$b'_F = \theta.b_F + (1-\theta)a_F \tag{5.20}$$

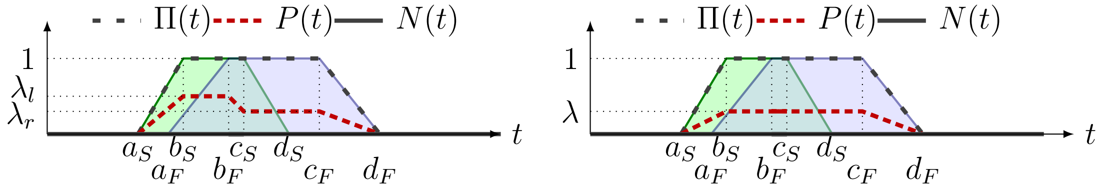
where $\theta = (1-\beta_0)/(1-\beta')$ and

$$\beta' = \frac{\sqrt{r^2D^2 + 2(d_S - a_F)r.D} - r.D}{d_S - a_F} \tag{5.21}$$

5.2.3 Configuration with large overlap

For the large overlap configuration (as in previous configurations), the general distribution is also represented by a compound function (see Figure 5.9):

$$P(t) = \begin{cases} \frac{\lambda_l}{b_S - a_S}(t - a_S) & \text{if } t \in [a_S; b_S] \\ \lambda_l & \text{if } t \in [b_S; b_F] \\ \frac{1}{b_F - c_S}((\lambda_l - \lambda_r)t + \lambda_r b_F - \lambda_l c_S) & \text{if } t \in [b_F; c_S] \\ \lambda_r & \text{if } t \in [c_S; c_F] \\ \frac{-\lambda_r}{d_F - c_F}(t - d_F) & \text{if } t \in [c_F; d_F] \\ 0 & \text{otherwise.} \end{cases} \tag{5.22}$$



(a) Non symmetric (b) Symmetric

Figure 5.9: Presence of a task: Large overlap configuration

The necessity presence distribution is $N(t) = 0 \quad \forall t$. The areas of the projected necessity and possibility distributions are:

$$r.D_N = \int_0^{+\infty} r.N(t)dt = 0 \tag{5.23}$$

$$r.D_\Pi = \int_0^{+\infty} r.\Pi(t)dt = r.(d_F - a_S + c_F - b_S)/2 \tag{5.24}$$

If the minimal workload $r.w$ is greater than zero (respectively, $r.D_\Pi$ greater than the maximal workload $r.z$) we use the projection of the presence probability distribution and determine $\lambda_{\min}$ (respectively, $\lambda_{\max}$). Given D so that $0 < D < D_\Pi$,

$$r.D = \int_0^{+\infty} r.P_\lambda(t)dt = \lambda.r.D_\Pi \tag{5.25}$$

In general case where distribution is non symmetric, the link between the task duration and the profile, is given by the following formula:

$$r.D = \int_0^{+\infty} r.P(t)dt = r.\lambda_l(\frac{c_S + b_F}{2} - \frac{a_S + b_S}{2}) + r.\lambda_r(\frac{d_F + c_F}{2} - \frac{c_S + b_F}{2})) \tag{5.26}$$

In case of symmetric distribution, when $w > 0$, $\lambda_{\min} = w/D_\Pi$ and when $D_\Pi > z$, $\lambda_{\max} = z/D_\Pi$.

The extended maximal profile, defined by (a_S, b'_S, c'_F, d_F), is used when $D_\Pi < D \leq z$. It is the same extended profile as the one of no overlap configurations. The minimal profile is never reduced.

In this section we have studied the resource workload for a fuzzy task and provided symmetric and non symmetric fuzzy distribution for the three possible configurations depending on the degree of intersection between the starting and finishing times. These modelling approaches will be useful to solve fuzzy scheduling problem.

5.2.4 Fuzzy tasks pre-emption

Pre-emption can be a way to add more flexibility to solve scheduling problem. In case of deterministic projects, pre-emption is provided by cutting macro-tasks into elementary work parts [de Boer, 1998]. Obviously, the elementary duration value is unique in the deterministic case and is equal to 1. Thus, any deterministic duration is a multiplication of 1. In the same way, any trapezoidal fuzzy number $\widetilde{A} = [a, b, c, d]$ is equal to a unique linear combination of the elementary numbers $\widetilde{I}_0$=[1, 1, 1, 1], $\widetilde{I}_1$=[0, 1, 1, 1], $\widetilde{I}_2$=[0, 0, 1, 1] and $\widetilde{I}_3$=[0, 0, 0, 1], listed from the most necessary to the less possible equal to 1 (see Figure. 5.10) [Masmoudi and Haït, 2011a]:

$$\widetilde{A} = a\widetilde{I}_0 + (b - a)\widetilde{I}_1 + (c - b)\widetilde{I}_3 + (d - c)\widetilde{I}_4 \tag{5.27}$$

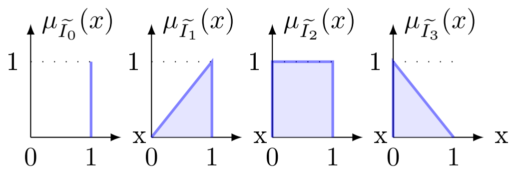


Figure 5.10: Elementary trapezoidal fuzzy numbers ([Masmoudi and Haït, 2011a])

The decomposition formula (5.27) is applied to tasks fuzzy durations in an AOA graph. The elementary arcs are assigned in the order of their possibility to be equal to 1. Thus, the $\widetilde{I}_0$ are assigned first, then the $\widetilde{I}_1$, after that the $\widetilde{I}_2$ and finally the $\widetilde{I}_3$ (see Figure 5.11).

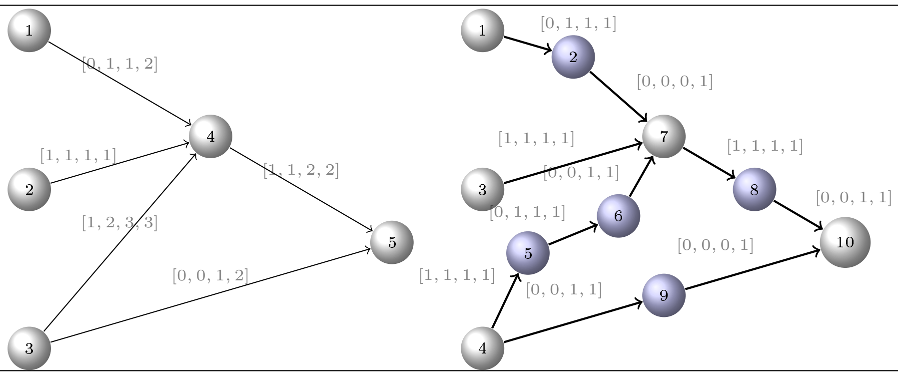


Figure 5.11: Fuzzy AOA network; before and after preemption.

For example, the duration of task (34) on the left graph (before pre-emption) is equal to [1, 2, 3, 3]. According to the formula (5.27), we have

$\widetilde{34} = \widetilde{I}_0 + \widetilde{I}_1 + \widetilde{I}_2$. Thus, the task (34) can be replaced in the right graph (after pre-emption) by (45), (56) and (56) with $\widetilde{45} = \widetilde{I}_0$, $\widetilde{56} = \widetilde{I}_1$ and $\widetilde{67} = \widetilde{I}_2$.

5.3 Fuzzy RL problem

In this section, firstly, a model for deterministic resource leveling problem is proposed. Then, a genetic algorithm that support this model is provided, and finally generalized to fuzzy parameters.

Many exact and heuristic techniques were developed to solve resource leveling problems [Zhao et al., 2006, Easa, 1989]. In multi-projects context, the Resource Leveling Problem can be defined as a set of tasks with precedence constraints and predetermined durations. A schedule is defined by a set of tasks starting times. Let n be the total number of tasks and let N be the number of projects to schedule and n_j the number of tasks in project j ($n = \sum_{j=1}^{N} n_j$). A schedule is defined by the set $S = (S_{11}, S_{21}, ..., S_{n_1 1}, ..., S_{ij}, ..., S_{1N}, ..., S_{n_P N})$ where S_{ij} is the starting time of task i from project j ($Card(S) = n$). The lower and upper bounds of each value S_{ij} are respectively the Earliest Starting time (ES_{ij}) and the Latest Starting time (LS_{ij}) of task i from project j. These parameters are defined by applying the Critical Path Method. The objective of the resource leveling technique is to smooth resources utilization which can be mathematically expressed as follows:

$$L : min \sum_{k=1}^{K} \sum_{t=1}^{T} [\sum_{j=1}^{N} \sum_{i=1}^{n_j} r_{kijt} - r_k^*]^2 \tag{5.28}$$

where:
L: The resource leveling index indicates the sum of squared differences between period resource usage and average resource usage.
r_{kijt}: The partial resource k demand of the activity i from the project j at the period of time t.
D: The projects duration.
K: The number of resource types.
r_k^*: average of resource k per period ($r_k^* = [\sum_{t=1}^{T} \sum_{j=1}^{N} \sum_{i=1}^{n_j} r_{kit}]/D$)

5.3.1 Genetic Algorithm description

Since 1975, the Genetic Algorithm has proved its effectiveness for complex problems like particularly the multi-projects and multi-objectives scheduling problems [Kim et al., 2005b]. The Genetic Algorithm (GA) is particularly studied in this chapter. A GA is a search heuristic that follows the natural evolutionary process. The strength of GA is that it represents the main computational intelligence approach that cope with big instances. The technique of GA is quite known, thus to get more complete information about the Genetic Algorithm technique we refer readers to [Goldberg, 1989].

The genetic algorithm procedure followed in this chapter to cope with resource leveling problem is presented below in pseudo-code form:

```
Apply Fuzzy CPM/PERT technique;
Parametrize the Genetic algorithm;
Generate initial population P_0 of n_pop candidates;
Initialize generation counter t ← 0;
while Stopping criteria not satisfied  do
  Evaluate the current population;
  Select best candidates using Roulette wheel method;
  The best m_n candidates from selected candidates are
  identically kept for P_{t+1} and the other candidates are
  reproduced based on Elitist method until the population
  P_{t+1} is completely generated,
  Crossover m_k candidates (from n_pop \ m_n) randomly at one
  or more random position(s),
  Mutate m_d candidates (from n_pop \ m_n) randomly, by
  mutating gmut random genes per candidate,
  Increment current population: P_t ← P_{t+1},
  Increment generation counter: t ← t + 1,
end while
```

Figure 5.12: A Genetic Algorithm procedure for resource leveling problem

The issue of applying Genetic Algorithm is to define an appropriate form of chromosome. In resource leveling problem, the well-appropriate form is the one considering starting times of tasks as decision variables being coded as genes values (see Figure 5.13). Thus, the sequence of the tasks in the chromosome corresponds to the sequence of tasks project by project sorted by their Id number. Each gene value is equal to a possible starting time of the corresponding task. The starting time of each task T_{ij} is chosen randomly in its domain rate respecting precedence constraints.

The fitness function needed to evaluate chromosomes is chosen equally to the resource leveling index L defined in 5.28. There was no need in our algorithm to modify the equation to cope with the fact that GA is traditionally designed to solve problems of seeking maximums.

The adopted selection technique is the roulette wheel method that we combine with Elitist method [Goldberg, 1989] in order to improve selection

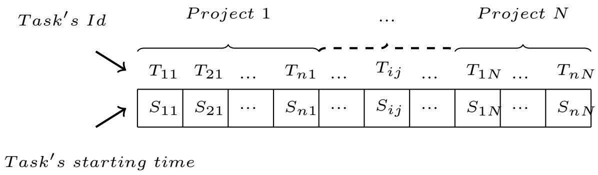


Figure 5.13: Chromosome representation in Multi-project resource leveling

efficiency. Thus, the probability of selection for a chromosome k is proportional to the ratio $f_k / \sum_{j=1}^{n_{pop}} f_j$, where f_k is the fitness value of the chromosome k and n_{pop} is the population size. According to the Elitist method, the best chromosomes of the current generation are kept and preserved into the next generation.

The GA operators are uniform 1-point crossover and uniform mutation. The crossover starts with randomly selecting a cut point and parent's chromosomes. The right parts of the chromosomes are swapped and hence child are generated (see Figure 5.14). Some children generated with this way do

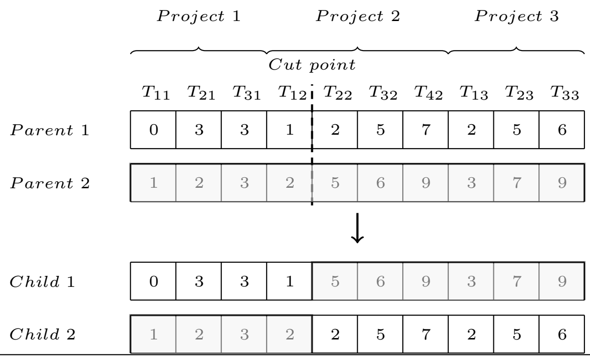


Figure 5.14: Uniform 1-point crossover

not satisfy precedence constraints. To deal with this situation, a reparation technique is applied (see Figure 5.15).

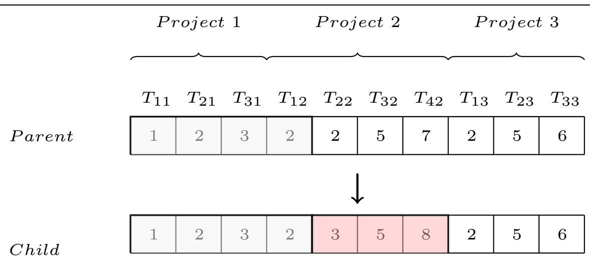


Figure 5.15: Reparation after crossover

Let k be the one-cut-point value and the task T_{ij} is the corresponding task of the gene k. All genes values of the successors of k must be checked to deal with precedence constraints. Hence, the task $k+1$ is the first task to be checked if its part of the project j, else no reparation is needed. The formula of reparation is the following:

$$S_{lj} = max(S_{lj}, max_{p \in pred(T_{lj})}(S_{pj} + D_{pj}))\forall l \in [i+1, n] \qquad (5.29)$$

Where:
$pred(T_{lj})$: The set of predecessors of task T_{lj}.
D_{pj}: The duration of the task T_{pj}.

The mutation consists in randomly replacing at least one gene with a random real value within the specified range of the corresponding task's starting time (see Figure 5.16).

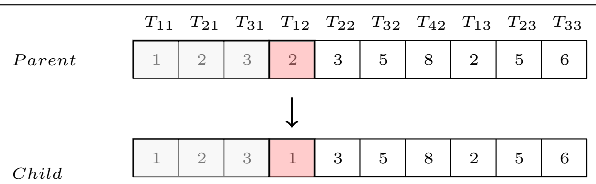


Figure 5.16: Uniform mutation

Let k be a selected gene to mutate and the task T_{ij} is its correspondent task. The new value of the gene is chosen randomly between the maximum finishing time of predecessor tasks ($max_{p \in pred(T_{ij})}(S_{pj}+D_{pj})$) and the minimum starting time of successor tasks ($min_{p \in succ(T_{ij})}(S_{pj})$) minus D_{ij} the duration of T_{ij}.

5.3.2 Fuzzy GA for FRLP

Resource Leveling technique for Fuzzy Scheduling Problem is studied in some recent papers [Zhao et al., 2006, Leu et al., 1999] where genetic algorithm is adapted to projects with fuzzy time parameters. The idea in these papers is to make a different α-cuts on tasks durations to obtain pessimistic and optimistic scenarios for each α-cut, and then apply deterministic Genetic Algorithm to each scenario to find the corresponding best plan.

In this section, a new vision of fuzzy resource leveling is provided. The Genetic Algorithm developed in section 5.3.1 copes well with deterministic Mutli-projects and Multi resources scheduling problems. To be generalized to fuzzy parameters, some useful hypothesis and extensions are suggested, where the main idea is to run just one couple of fuzzy Genetic Algorithm instead of numerous deterministic ones.

A trapezoidal fuzzy number is numerically represented by 4 deterministic values. Genetic algorithm becomes very heavy in computation when considering 4 numbers for each fuzzy decision variable. To deal with this problem only one value is considered and then the encoding and decoding of each solution (chromosome) is done according to the principle of linearity (see Figure 5.17) which appears logical in our case.

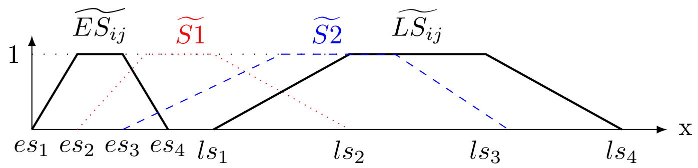


Figure 5.17: Linearity hypothesis

Let $\widetilde{ES}_{ij}= [es_1, es_2, es_3, es_4]$ the Earliest Starting time and $\widetilde{LS}_{ij}= [ls_1, ls_2, ls_3, ls_4]$ the Latest Starting time of task T_{ij}. To generate a possible Starting time $\widetilde{S}_{ij} = [s_1, s_2, s_3, s_4]$, we choose randomly a value of s_4 between es_4 and ls_4. Let $\beta = (s_4 - ls_4)/(es_4 - ls_4)$. Thus, $\widetilde{S}_{ij}$ is simply calculated according to the principle of linearity within $s_i = \beta es_i + (1-\beta) ls_i \ \forall i \in \{1, 2, 3\}$. In Figure 5.17 two examples of possible starting times are shown; $\widetilde{S1}$ with $\beta = 1/3$ and $\widetilde{S2}$ with $\beta = 2/3$.

Some algorithms in [Fortin et al., 2005] are provided to calculate fuzzy latest starting times and fuzzy total floats. However, no algorithms are provided in the same framework to calculate fuzzy latest finishing times. As these parameters are necessary for our study, the following formula is provided to calculate them:

$$\widetilde{LF}_{ij} = \min(\widetilde{LS}_{ij} + \widetilde{D}_{ij}, \min(\widetilde{LS}_{succ(ij)}), \widetilde{Dd}_(j)) \qquad (5.30)$$

where:
$\widetilde{LF}_{ij}$: The fuzzy Latest Finishing time of task T_{ij}.
$\widetilde{Dd}_j$: The fuzzy duedate of the project j.

As latest starting times are calculated within the consideration of extreme configuration as explained in [Dubois et al., 2003], the value of $\widetilde{LS}_{ij} + \widetilde{D}_{ij}$ can exceed the range domain of $\widetilde{LF}_{ij}$. In fact, the duration $\widetilde{D}_{ij}$ of the task T_{ij} is not necessarily totally in the range of the extreme configurations provided by the forward propagation. Thus, the formula (5.30) provides meaningful computable results respecting precedence constraints. Considering the same explanation, the finishing time is calculated as follows:

$$\widetilde{F}_{ij} = \min(\widetilde{S}_{ij} + \widetilde{D}_{ij}, \widetilde{LF}_{ij}) \qquad (5.31)$$

Once starting and finishing times are calculated for each task, the fuzzy workload is established as explained in section 5.2. Thus, for each solution (chromosome), the corresponding fuzzy fitness $\widetilde{L}$ is calculated as follows:

$$\widetilde{L} : min \sum_{k=1}^{K} \sum_{t=1}^{T} [\sum_{j=1}^{N} \sum_{i=1}^{n_j} \widetilde{r}_{kijt} - \widetilde{r}_k^*]^2 \tag{5.32}$$

where:
$\widetilde{r}_k^* = [\sum_{t=1}^{T} \sum_{j=1}^{N} \sum_{i=1}^{n_j} \widetilde{r}_{kit}]/\widetilde{D}$

Many defuzzification techniques are provided in literature [Fortemps, 1997, Dubois and Prade, 1987] to cope with fuzzy rules particularly while using Genetic Algorithm [Sanchez et al., 2009]. In this study, the considered defuzzification technique is the mean value proposed by Dubois and Prade [1987]. Hence, Let $\widetilde{L} = [a_L, b_L, c_L, d_L]$ be a fuzzy value, and $\bar{L}$ its mean value, thus $\bar{L}$ = $(a_L + b_L + c_L + d_L)/4$. Moreover $\widetilde{D}$ is always projected to the maximum value of the projects duration.

Section 5.5 contains an application of the fuzzy genetic algorithm to the helicopter maintenance activity.

5.4 Fuzzy RCPS problem

The Schedule Generation Schemes (SGS) are the core of many heuristics for the RCPSP. The so-called serial SGS performs activity incrementation and the parallel SGS performs time incrementation [Kolish and Hartmann, 1999]. In both procedures, tasks are ranked in some order and scheduled according to resources availabilities. Hapke and Slowinski [1996] have proposed a parallel scheduling procedure for fuzzy projects. It is based on fuzzy priority rules and fuzzy time incrementation. The parallel procedure that we propose mainly differs from the latter on the resource availability test.

5.4.1 Fuzzy priority rules

Priority heuristics using crisp or fuzzy time parameters were found efficient by many researchers either for one project or multi-project scheduling [Kolish and Hartmann, 1999, Browning and Yassine, 2010, Hapke and Slowinski, 1996].

It is useful to perform parallel scheduling with a set of rules instead of one as the computational complexity is generally low [Hapke and Slowinski, 1993]. Some rules that appears to be good in minimizing makespan are presented in

Table 5.1.

Table 5.1: Priority rules giving good results in makespan minimisation

Rule	Name	Formula
EST	Early Start Time[1]	$min(\widetilde{E}_j^s)$
LIS	Least Immediate Succesors[1]	$\min(\|S_j\|)$
EFT	Early Finish Time[1]	$min(\widetilde{E}_j^f)$
MIS	Most Immediate Succesors[1]	$\max(\|S_j\|)$
LST	Late Start Time[123]	$min(\widetilde{L}_j^s)$
MTS	Most Total Successors[23]	$\max(\|\overline{S_j}\|)$
LFT	Late Finish Time[123]	$min(\widetilde{L}_j^f)$
GRD	Greatest Resource Demand[1]	$\widetilde{p}_j \sum_{k=1}^{K} r_{jk}$
MINSLK	Minimum slack[123]	$min(\widetilde{f}_j)$
SASP	Shortest Activity from Shortest Project[3]	$min(\widetilde{p}_{jl})$
MAXSLK	Maximum slack[3]	$max(\widetilde{f}_j)$
LALP	Longest Activity from Longest Project[3]	$max(\widetilde{p}_{jl})$
SPT	Shortest Processing Time[123]	$min(\widetilde{p}_j)$
GRPW	Greatest Rank Positional Weight[123]	$max(\widetilde{p}_j + \sum_{i \in S_j} \widetilde{p}_i)$
LPT	Longuest Processing Time[13]	$min(\widetilde{p}_j)$
LRPW	Least Rank Positional Weight[1]	$min(\widetilde{p}_j + \sum_{i \in S_j} \widetilde{p}_i)$

Where:

[1]: used by [Hapke and Slowinski, 1996] for a Fuzzy RCPSP,

[2]: used by [Kolish and Hartmann, 1999] for Deterministic RCPSP,

[3]: used by [Browning and Yassine, 2010] for Multi-projects RCPSP (RCMPSP),

$\tilde{p}_j$: duration,

$\widetilde{L}_j^f$: last finishing,

$\widetilde{E}_j^f$: Earliest finishing,

$\widetilde{L}_j^s$: last starting,

$\widetilde{E}_j^s$: Earliest starting,

$\widetilde{f}_j$: margin,

r_{jk}: is the requirement for resource R_k,

S_j: direct successors,

$\overline{S_j}$: total successors.

Many other interesting rules could be used, like the Minimum Worst Case Slack (*MINWCS*), the Minimum Total Work Content(*MINTWC*) and some dynamic and combined rules that are presented in [Browning and Yassine, 2010].

5.4.2 Parallel and serial tasks

If tasks were independent, the sum of their resource profiles would give the overall workloads. However, when considering a precedence constraint between two tasks, their workload profiles may not overlap because the constraint expresses the fact that the two tasks can not be performed simultaneously.

Let us consider two tasks A and B so that A precedes B. Their resource consumptions are denoted r_A and r_B. We assume that the starting date of B is equal to the finish date of A (e.g. in case of forward earliest dates calculation). This means that between the start date of A and the finish date of B, an activity will occur successively induced by A then B. So between the necessity peaks of A and B, we can affirm that an activity will necessarily occur, induced by A or B. This necessary presence of A or B is projected onto the resource load space using the minimal resource requirement $\min(r_A, r_B)$, associated to the pseudo task $A \vee B$ starting at $\widetilde{S}_A$ and finishing at $\widetilde{F}_B$ (Figure 5.19).

The projected necessity and possibility load profiles of the sequence $A \to B$ are defined as follow:

$$L_{N(A\to B)}(t) = \max(r_A.N_A(t), r_B.N_B(t), \min(r_A, r_B).N_{A\vee B}(t)) \tag{5.33}$$

$$L_{\Pi(A\to B)}(t) = \max(r_A.\Pi_A(t), r_B.\Pi_B(t)) \tag{5.34}$$

The probability workload profile is more complex to define. A constructive way can be provided; firstly the distribution of A is defined and then the distribution of B is deduced respecting resources and precedence constraints. Let us consider A without predecessors. Hence, we can assign to A its symmetric distribution while $\lambda_A^l = \lambda_A^r = \lambda_A$. For B we apply the following checks:

if $r_B\lambda_B > max(r_B, r_A) - r_A\lambda_A^r$ **then**
  $\lambda_B^l = (max(r_B, r_A) - r_A\lambda_A^r)/r_B$
else if $r_B\lambda_B < min(r_B, r_A).N(A \vee B) - r_A\lambda_A^r$ **then**
  $\lambda_B^l = (min(r_B, r_A).N(A \vee B) - r_A\lambda_A^r)/r_B$
else
  $\lambda_B^l = \lambda_B$
end if
$\lambda_B^r = f(\lambda_B^l, D_B)$

Figure 5.18: Workload modelling for two directly successive tasks

Where D_B is the duration of B, f is a function deduced from 5.5, 5.18, and 5.26, and λ_B is the parameter value of task B distribution while considering $\lambda_B^l = \lambda_B^r$.

Once probabilistic distributions of A and B are defined respecting resource and precedence constraints, the sum of the two distributions corresponds to the total probabilistic workload:

$$L_{P(A \to B)}(t) = r_A.P_A(t) + r_B.P_B(t) \tag{5.35}$$

Figure 5.19 shows the workload while $r_A = 2$, and $r_B = 1$. The integration of these profiles, considering updates made by the formula (5.35), gives the total workload.

The aforementioned approach can be easily generalized to multi-tasks within the frame of a fuzzy scheduling technique such as Parallel SGS explained in the following section.

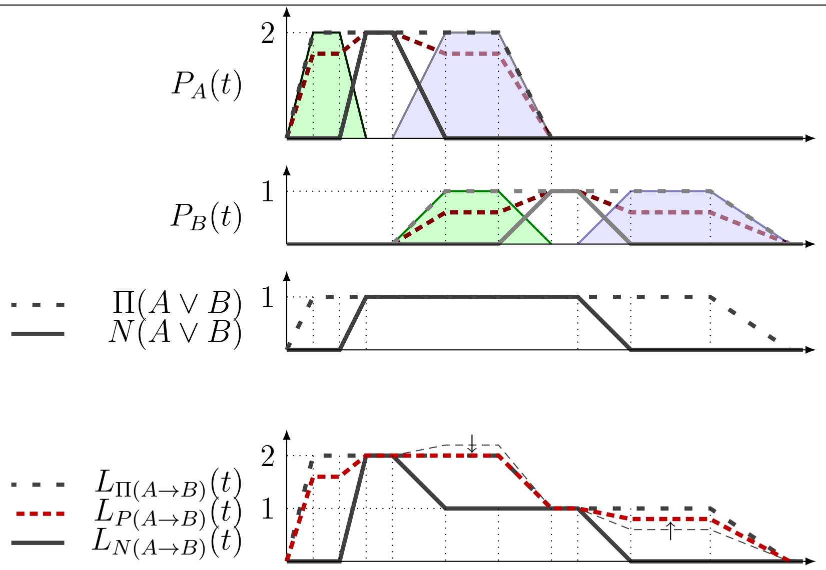


Figure 5.19: Fuzzy continuous workload plan for two successive tasks.

5.4.3 Fuzzy Greedy Algorithm: parallel SGS

Hapke and Slowinski [1996] provide a generalization of parallel SGS to fuzzy area. They use weak and strong fuzzy inequalities to compare fuzzy numbers and provide a direct tasks sequence respecting both resources and precedence constraints. They establish the workload by applying alpha-cuts as mentioned before.

Let n (index $j = 1..n$) be the number of tasks to schedule. Within a loop, we calculate each task's j distribution parameters (λ_j^l then λ_j^r) task by task within a new parallel SGS technique based on the new fuzzy workload modelling provided in this chapter:

```
Choose a priority rule;
Initialize Es̃_j; the earliest starting time of task j (∀j),
using the CPM technique;
Initialize t̃ = t̃_0; the begin of the scheduling horizon (e.g.
t̃ = 0);
Initialize the total resources availabilities at all
scheduling periods;
repeat
  Compose the set Av(t̃) of available tasks at t̃
  for each j from Av(t̃), in the order of the priority list
  do
    calculate the corresponding symmetric probabilistic
    distribution P_j
    if the symmetric probabilistic distribution P_j does not
    fit period by period the resources availabilities then
      calculate a new P_j with a asymmetric shape
      considering the minimum possible value of the left
      parameter (λ_j^l)
      if the configuration P_j fits the resource
      availabilities then
        schedule j with corresponding starting and finishing
        dates,
        integrate the distribution P_j into the workload
        plan and update the total resources availabilities,
        update the earliest starting time of all successors
        of j,
      end if
    end if
  end for
  if all tasks from Av(t̃) are scheduled  then
    t̃ = max(t̃, l̃(t̃))
  else
    t̃ = max(t̃, a_t + 1)
  end if
until all tasks are scheduled
```

Figure 5.20: Fuzzy Parallel SGS technique for resource leveling problem

Where:
$Av(\widetilde{t})$ is the set of tasks whose defuzzification value of Earliest Starting time Es are less or equal to the defuzzification value of $\widetilde{t}$ ($Es_j \leq t$, $\forall j \in Av(\widetilde{t})$).
$\widetilde{l}(\widetilde{t})$ is the least value among the earliest starting times of tasks from $A(\widetilde{t})$ and the finishing times of tasks from $S(\widetilde{t})$.

$A(\widetilde{t})$ is the set of tasks that are not yet scheduled and whose immediate predecessors have been completed by $\widetilde{t}$.
$S(\widetilde{t})$ is the set of tasks present in $\widetilde{t}$; a task j is considered present in $\widetilde{t}$ when $S_j \leq t \leq F_j$ (S_j and F_j are the defuzzifications of starting and finishing time of j, respectively.

The aforementioned algorithm is to be run as much time as the priority rules number; we talk about multi priority rule method [Boctor, 1990]. Better procedures based on Parallel SGS and called multi pass methods [Kolish and Hartmann, 1999] can be studied, but this is out of scope of this thesis.

Comparing to the algorithm provided by Hapke and Slowinski [1996], the structure of the fuzzy parallel SGS that is shown in Figure 5.20 is the same. However, there are two major differences: First, the possibility to schedule a task is checked according to the resource requirement and resource availability which are deterministic in the Hapke and Slowinski's algorithm and fuzzy in our algorithm. Second, to generalize the parallel SGS dynamic time progression [Kolish and Hartmann, 1999] to fuzzy consideration, Hapke and Slowinski [1996] consider weak and strong inequalities to compare fuzzy dates and make the adequate incrementation. In our approach, the same progression technique is considered. However, according to our fuzzy workload consideration, an additional specific time progressing technique is proposed for a situation where a task is available for scheduling but still not yet scheduling for a resource availability problem.

Consequently, our algorithm presents several advantages in comparison to the Hapke and Slowinski's algorithm: First, with the use of alpha-cuts, in Hapke and Slowinski's approach, only few possible deterministic scenarios are generated, because it is impossible to apply all alpha-cuts on different tasks for combinatory explosion reason. On the other hand, working with fuzzy charge distribution, in our algorithm, all possible scenarios are covered. Second, the full solution, in Hapke and Slowinski's approach, is a sort of an accumulation of many pessimistic and optimistic work load plans, and a $3D$ representation is needed to show different scenarios which is judged by Slowinski "too cumbersome". On the other hand, the full solution, in our approach, is represented by few workload plans; the optimistic and the pessimistic and the mean that is particularly considered in Figure 5.20, which is more understandable by the stakeholders (planners).

5.5 Computations; application for helicopter maintenance

We consider the scheduling application provided in chapter 1 with activity durations modelled with fuzzy sets (see Table 5.2).

The earliest plan without consideration of resources constraints is shown in Figure 5.21.

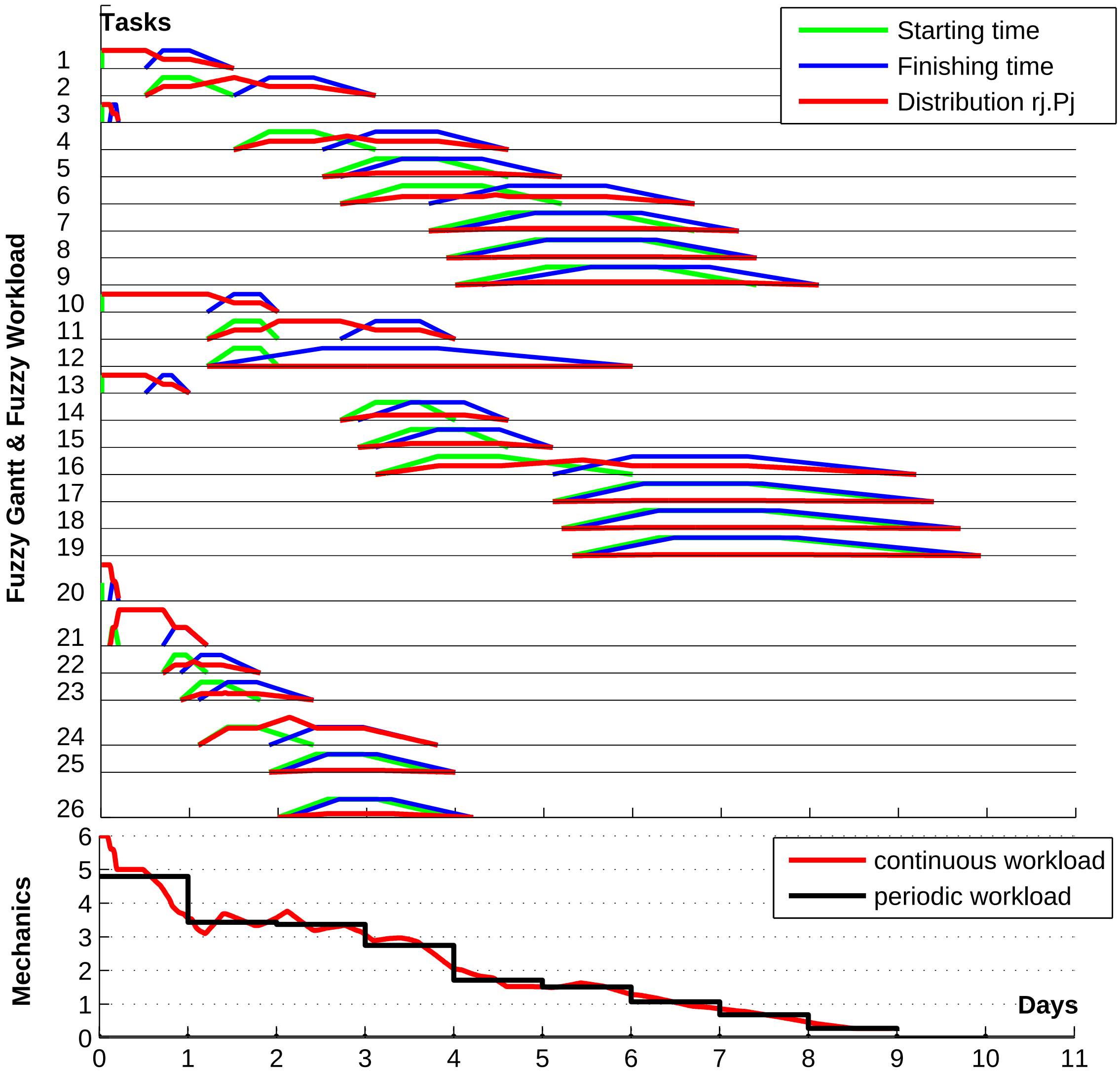


Figure 5.21: Earliest workload plan; without resource consideration

Table 5.2: Real mechanical tasks from a PUMA HMV.

Part name	Taks Id	Id	Pred.	Experts	Equipments	Duration (days)
Main Rotor	Put off Muff	1	-	1	-	[0.5, 0.7, 1, 1.5]
	Put off bearings	2	1	1	-	[1, 1.2, 1.4, 1.6]
	Put off flexible components	3	-	1	-	[0.1, 0.13, 0.17, 0.2]
	Clean	4	2-3	1	Cleaning machine	[1, 1.2, 1.4, 1.5]
	Non-destructive test	5	4	1	Testing equipment	[0.2, 1.3, 0.5, 0.6]
	Assemble components	6	5	1	-	[1, 1.2, 1.4, 1.5]
	Check water-tightness	7	6	1	-	[0.2, 0.3, 0.4, 0.5]
	Touch up painting	8	7	1	-	[0.1, 0.13, 0.17, 0.2]
	Tight screws	9	8	1	-	[0.3, 0.5, 0.6, 0.7]
Propeller	Put off axial compressor	10	-	1	-	[1.2, 1.5, 1.8, 2]
	Put off centrifugal compressor	11	10	1	-	[1.5, 1.6, 1.8, 2]
	Purchase	12	10	0	-	[0, 1, 2, 4]
	Put off turbine	13	-	1	-	[0.5, 0.7, 0.8, 1]
	Clean	14	11-13	1	Cleaning machine	[0.2, 0.4, 0.5, 0.6]
	Non-destructive test	15	14	1	Testing equipment	[0.2, 0.3, 0.4, 0.5]
	Assemble components	16	12-15	1	-	[2, 2.2, 2.8, 3.2]
	Touch up painting	17	16	1	-	[0.1, 0.13, 0.16, 0.2]
	Tight screws	18	17	1	-	[0.12, 0.17, 0.2, 0.3]
	Test	19	18	1	Test Bench	[0.12, 0.17, 0.2, 0.23]
Hydraulic System	Evacuate oil	20	-	2	-	[0.1, 0.13, 0.16, 0.2]
	Put off servos	21	20	2	-	[0.6, 0.7, 0.8, 1]
	Clean	22	21	1	Cleaning machine	[0.2, 0.3, 0.4, 0.6]
	Non-destructive test	23	22	1	Testing equipment	[0.2, 0.3, 0.4, 0.6]
	Assemble then remove joints	24	23	2	-	[0.8, 1, 1.2, 1.4]
	Test	25	24	1	Test Bench	[0.1, 0.13, 0.16, 0.2]
	Tight screws	26	25	2	-	[0.1, 0.13, 0.16, 0.2]

The result of the SGS and the GA are shown in Figure 5.22, and Figure 5.23, respectively; and Figure 5.24 shows the convergence of the GA. To get these results, we considered 3 Mechanics as capacity limit for the parallel SGS, and 10 *days* as the due date of all tasks for the Genetic Algorithm.

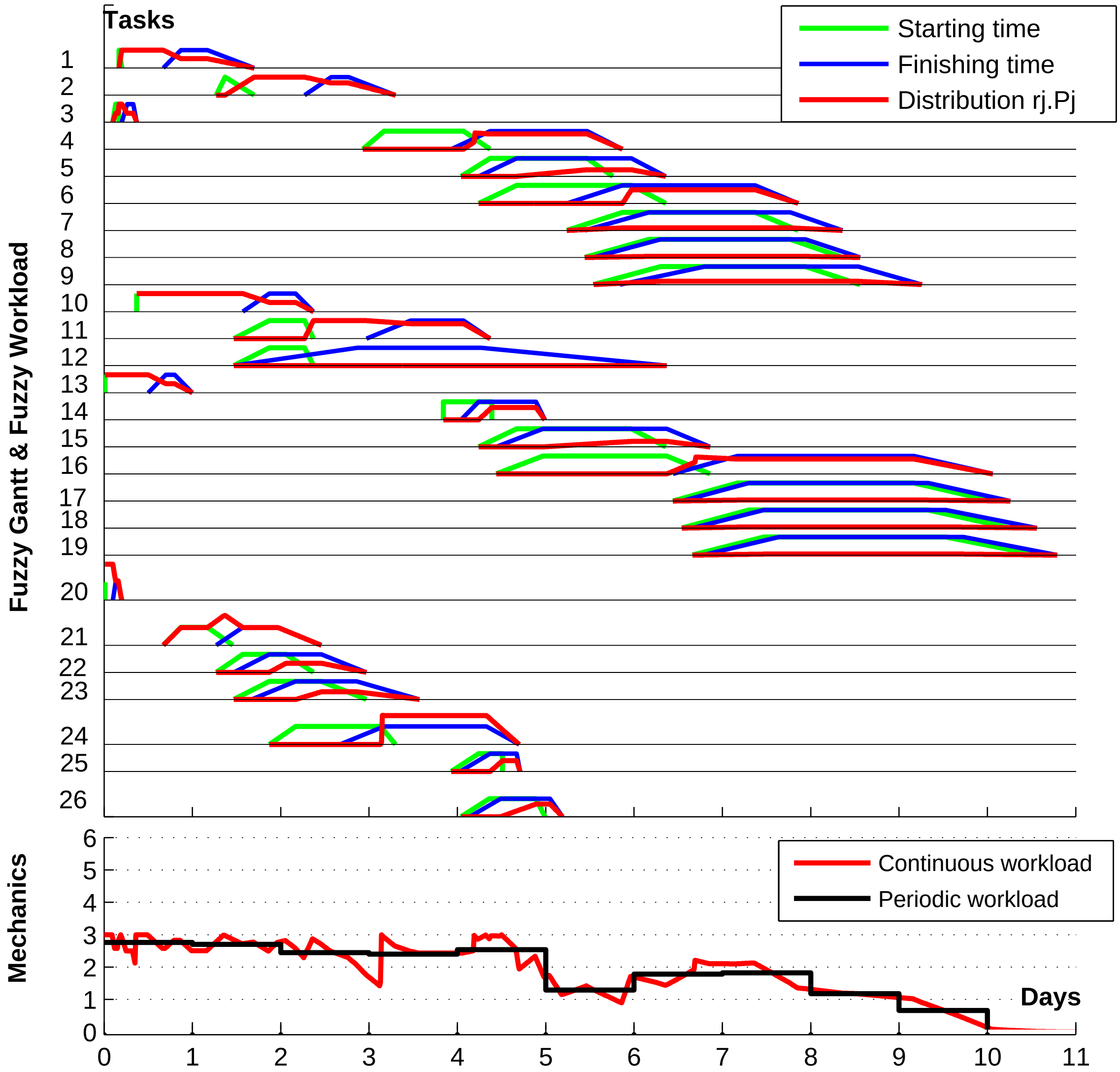


Figure 5.22: The workload plan; result of the Parallel SGS (rule LRPW)

The result of each of these algorithms is a sequence of fuzzy tasks, that is robust according to the objective function that can be minimizing the resource usage per periods or minimizing the projects makespans.

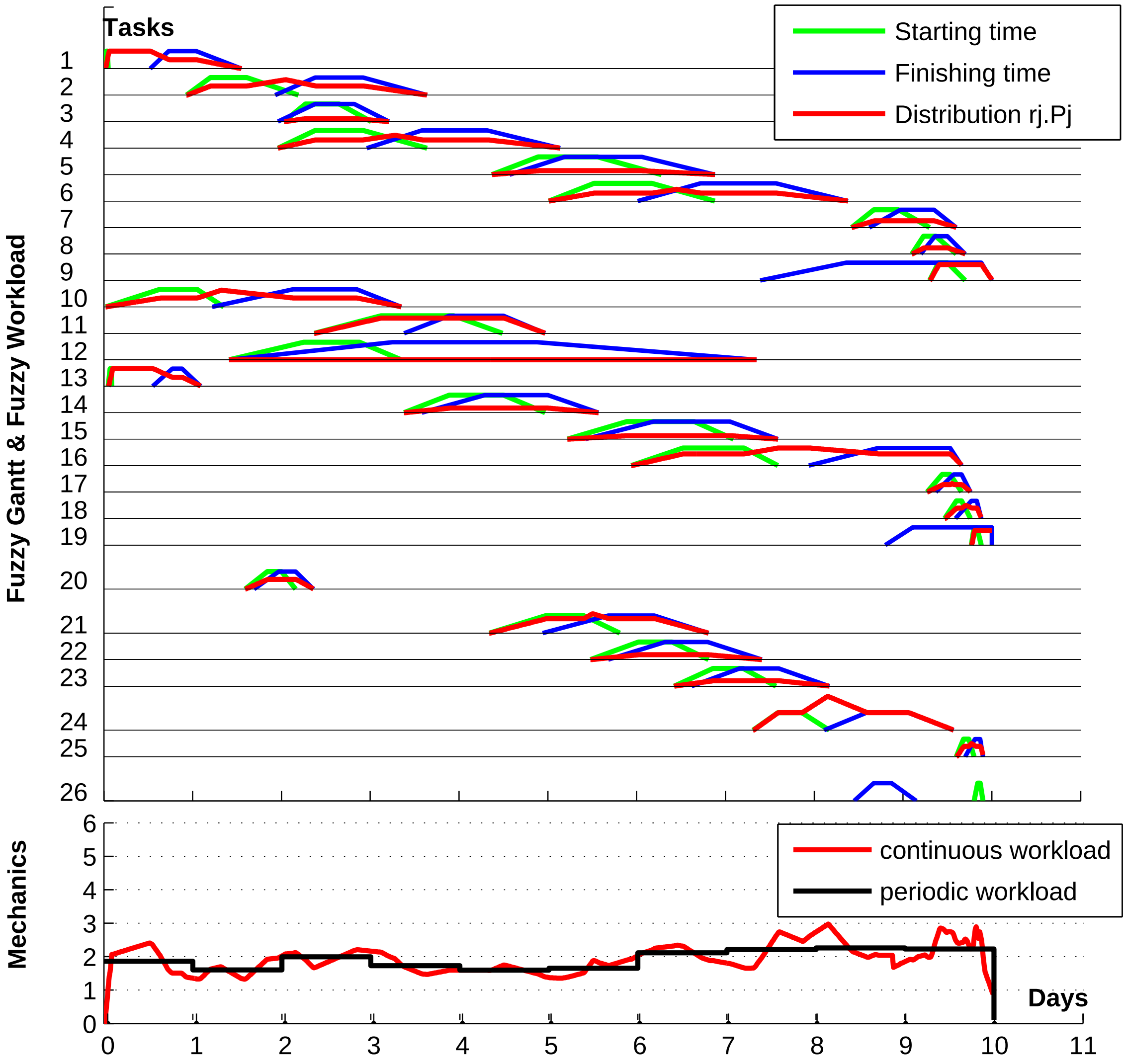


Figure 5.23: The workload plan; result of the GA

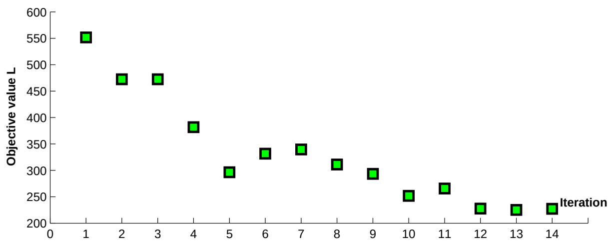


Figure 5.24: Convergence of the GA

We can check the optimal scheduling for different decisions on the capacity limits (per resource and per period) and the projects due dates, using the resource driven (parallel SGS) and the time-driven (RL) techniques. These two techniques can be used iteratively and repetitively within a close loop until we get a good compromise which minimizes the cost and satisfies the customers. The iterative use of the two techniques is out of scope of this thesis.

5.6 Algorithm validation

The new modelling concept we have provided for fuzzy project scheduling is completely different to what already exists in literature, but, the algorithms we have provided are basically a generalization to fuzzy area of existing deterministic algorithms such as the parallel SGS of Kolish and Hartmann [1999] and the GA for RLP of Leu et al. [2000]. Hence, we have just added a layer of specific treatments to support the fuzzy modelling of uncertain data (see figure 5.25).

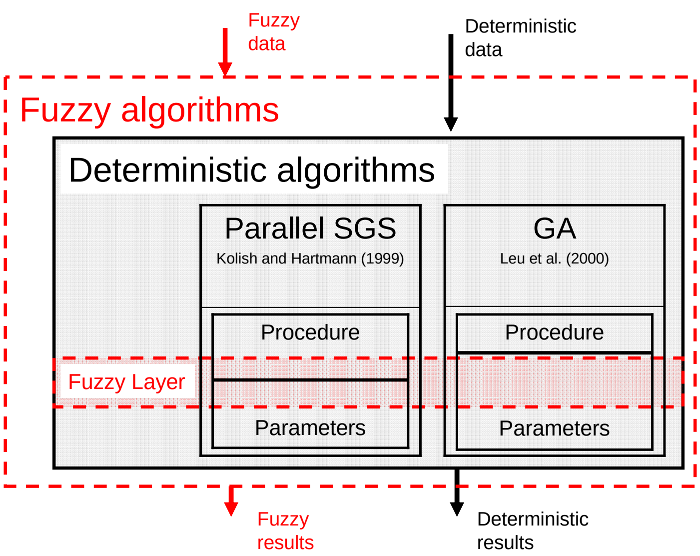


Figure 5.25: From deterministic to fuzzy scheduling algorithms

Drira et al. [2011] have made a GA algorithm for a specific application in a fuzzy area. To assess the performance of their algorithm, they compare it to existing algorithms with deterministic data, because all existing algorithms in the literature that deal with the same application case are deterministic. The question is: suppose the algorithm is proved good for deterministic area, is it automatically good too for fuzzy area, and vice versa?

To apply our algorithms to deterministic area in order to check their performances, first we should eliminate the fuzzy layer, otherwise non necessary additional treatments are considered. By doing that, we are back to exactly the initial deterministic algorithms. Notice that more computation time is expected because we are using Matlab tool that is matrix oriented language (see section 4.4). We conclude that, the deterministic validation technique as presented by Drira et al. [2011] can not be carried out for our fuzzy scheduling algorithms, because the modelling approaches of data that we have provided are completely new and the algorithms, on the contrary, are just a generalization of existing ones to fuzzy data.

5.7 Conclusion

In this chapter, we have presented a fuzzy model for project scheduling problems. A method to establish a resource workload is proposed for the operational level of planning. The provided model is applied to the helicopter maintenance domain. Based on this modelling approach, we provided a Genetic Algorithm to solve Fuzzy Resource Levelling Problem [Masmoudi and Haït, 2011b], and a Parallel SGS to solve Fuzzy RCSPS problem [Masmoudi and Haït, 2011a]. These two techniques (resource leveling and resource scheduling) can be applied simultaneously within a decisional loop handling projects due dates and production capacity simultaneously *i.e.* we can increase/decrease a project due date and apply resource leveling technique or increase/decrease the production capacity and apply Resource scheduling technique [Kim et al., 2005a]. Unfortunately, being limited by the lack of real data from MROs, the industrial validation of our modelling approach and the algorithms are still not yet accomplished.

Conclusion and Further research

This thesis deals with project management under uncertainties. It is a study within the framework of an R&D project, called Hélimaintenance R&D1, that consists of developing a complete logistical support to improve helicopter maintenance.

A considerable gain is expected while optimizing the MRO activity. However, to the best of our knowledge, this subject is under developed until now. This thesis explains the complexity of this activity. It is particularly focused on the planning and scheduling problem, taking into account the specificities of the helicopter maintenance domain. The main question this thesis answers to is how to optimize the helicopter maintenance and deal with uncertainties inherent to the activity within a proactive approach.

This thesis starts with an introduction to the context of our study and then a survey of significant subjects that are related to our work, in particular existing uncertainty modelling approaches such as the fuzzy/possibilistic theory, and existing models and algorithms for operational and tactical planning. Next, two different approaches are proposed to model uncertainty for tactical and operational levels of planning. Based on these modelling approaches, several algorithms (Simulated Annealing, Genetic Algorithm and Parallel SGS) are developed to solve the Fuzzy Rough Cut Capacity Problem (FRCCP), the Fuzzy Resource Leveling Problem (FRLP) and the Fuzzy Resource Constraint Project Scheduling Problem (RCPSP), respectively. It is also expected to generalize other problems and algorithms to fuzzy consideration within the use of these modelling approaches.

This thesis is a contribution to the development of a *Decision Support System* (DSS) for the management of a civil MRO. However, a lot of work still need to be carried out in order to get an operational DSS *e.g.* the spare part management. Moreover, the interaction between different levels of planning within a reactive proactive decisional scheme is to be carried out after the validation of the provided modelling approaches and algorithms. Finally, we will have to deal with the development problem of interaction between the DSS and the other components of the complete integrated system (see Section 1.1).

Once all that problems are solved, we will get an original framework that permits to improve the process by taking into account different data updates (Return of experiments and result of prognostics), providing (re-)planning and (re-)scheduling (optimization algorithms), and thus minimizing the activity cost.

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Curriculum Viate

Malek MASMOUDI was born on August 27, 1982 in Tunis, Tunisia. In 2001 he obtained his Baccalaureate at Borj El Baccouch and El attarine hight school in Ariana. From 2001 to 2003 he studied mathematics and physics at the preparatory institute of Tunis in Montfleury. From 2003 to 2005 he studied industrial engineering at Ecole Nationale d'Ingénieurs de Tunis in El Manar, and moved to France in September 2005 for a double diploma. From 2005 to 2007 he studied logistics engineering at Ecole Nationale Supérieur des Arts et Industries Textile in Roubaix. From September 2006 to Mars 2007, he did his graduation assignment under the supervision of Pr. Besoa Rabenasolo and Dr. Pierre Douillet. In September 2007, he graduated after completing his End of Studying thesis, entitled "Taux de retours entre constat et prévision". The content of this thesis was extended few years later and published in international conference proceeding. From 2007 to 2008, he did his research Master under the supervision of Pr. Jean Pierre Bourey. From Mars 2008 to September 2008, he did his graduation assignment at INRETS in Villeneve d'Ascq, under the supervision of Dr. Mohamed Ghazel and Pr. Armand Toguyeni. In September 2008, he graduated after completing his Master thesis intitled "Evaluation des systèmes complexe : Application au contrôle-commande ferroviaire". The content of this thesis was extended few months later and published in two international conference proceedings.

On October 14, 2008 he started as a PhD student under the supervision of Dr. Alain Hait at SupAero in Toulouse. He joined on a research project called "Helimaintenance R&D 1". The contribution to this project resulted in this thesis, entitled "Tactical and operational project planning under uncertainties: application to helicopter maintenance". From September 2011, he is employed as a research and teaching assistant in ATER position at the university of Saint-Etienne.

Tactical and operational planning under uncertainties: application to helicopter maintenance

Abstract: This thesis is a study within the framework of the Helimaintenance project; a European project approved by the French aerospace valley cluster that aims to establish a center for civil helicopter maintenance which is also able to make R& D projects in the field.

Maintenance has attracted more and more interest in the aerospace industry, especially for critical systems. In particular, aircraft maintenance cost becomes an important issue. Managing aircraft maintenance center is a complex activity that can be viewed as multi-project management, where each visit on an aircraft is a project that should be carried out with the minimum cost-duration. According to experts in the field, the highest difficulty comes from uncertainties that disturb the activity and cause continuous perturbations in the planning at high and low levels.

According to our knowledges the problem of tactical planning for civil aeronautical maintenance center has never been studied. Moreover, in contrary to the scheduling project problem, the uncertainties in tactical project planning problem has not been well studied in literature.

Our work consists of integrating uncertainties into both tactical and operational multi-resources, multi-projects planning and dealing with Rough Cut Capacity Planning, Resource Leveling Problem and Resource Constraint Project Scheduling Problem under uncertainties.

This thesis provides a modelling of the periodic workload on tactical level considering uncertainties in macro-tasks processing times, and a modelling of the continuous workload on operational level considering uncertainties in tasks durations. Uncertainty is modelled within a fuzzy/possibilistic approach instead of a stochastic approach since very limited data is available in our case of study. Three types of problems are referred in this study which are the Fuzzy Rough Cut Capacity Problem (FRCCP), the Fuzzy Resource Leveling Problem (FRLP) and the Fuzzy Resource Constraint Project Scheduling Problem (RCPSP).

Moreover, a genetic algorithm and a greedy algorithm (Parallel SGS) are provided to solve the FRLP and FRCPSP problems, respectively. A simulated Annealing is provided to solve the FRCCP problem.

Keywords: Project management, helicopters maintenance, planning, scheduling, uncertainty, probability, fuzzy sets, possibility, genetic algorithm, simulated annealing, parallel SGS, RCCP, RCPSP, RLP,