

Tribute to Jean-Yves Jaffray

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1 Introduction

It is very rare to find in a single person both the qualities of a remarkable scientific mind and of a wonderful human being. With this tribute to Jean-Yves Jaffray, we hope to convince the reader of his outstanding creativity and vision, of the coherence of his work, and of its relevance for some topics in decision theory that are currently under lively debate.

As a scientist, Jean-Yves Jaffray can be characterized by one main insight and one main concern. His main insight is that a sound decision theory must explicitly use *all* the information available to the decision maker. This information about events must further be treated in a strictly objective manner. In the models he proposed as a result, objective information can be disentangled from subjective attitudes with respect to this information.¹ For that purpose, before asking how to represent preferences, one must wonder how to treat and represent the *given* information.

Jean-Yves' *main concern* in designing his models is that they must be tractable, implementable and testable. This leads him to emphasize the simplicity of the models he puts forward, including the way the arrival of new information is modeled, and to develop experiments to test them. This adherence to objectivity together with his concern for implementable models fits well with Jean-Yves' applied work in statistics and computer science that we will not review here.

After some words on his early contributions, we will discuss the way he addressed different questions linked to Decision Theory : How to describe information (or lack of information) on events? How to model decisions in this framework? How to evaluate decisions? How to update in the presence of new

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¹For a discussion of this idea, see Giraud and Tallon, (2010), and Wakker (2010).

information ? How to test each model experimentally ? We will discuss these questions in turn.

2 Early contributions : Jean-Yves and Utility Theory

Jean-Yves started his research in the 70's by studying Utility Theory. His Ph.D. thesis (Jaffray, 1974a), written under the supervision of Jean Ville, a disciple of Borel, is entitled "Existence, Propriétés de Continuité, Additivité de Fonctions d'Utilité sur un Espace Partiellement ou Totalement Ordonné". As stated in the title, he generalized and extended several results of Debreu and others on utility theory. He published three papers from his Ph.D. thesis: Jaffray (1974b) in *Journal of Mathematical Psychology*, Jaffray (1975a) in *Econometrica* and Jaffray (1975b) in *Journal of Mathematical Economics*.

At the same time, he was discovering, by studying Savage, what will become his main field of research : "Decision under Uncertainty". He first wrote a paper (Guesnerie and Jaffray, 1973), revisiting Savage, providing some new pedagogical proofs and refinements. Then he took a completely new direction, departing from Savage's approach, based on strong ideas: the type and quantity of available information matters and has to be explicitly taken into account in each model. For instance, in the canonical 3 colors urn of Ellsberg (Ellsberg, 1961), there is some objective information, a point that should not be discarded. Thus, before giving an axiomatization of preferences, one first has to focus on the available information and find appropriate tools to represent it.

3 Jean-Yves and decision theory under non-probabilized uncertainty

Under risk (a situation of probabilized uncertainty), the starting point to represent the available information is a probability space (S, \mathcal{A}, P) . The problem is more complex in a situation of non-probabilized uncertainty, when the given information does not pin down a unique probability distribution. To better understand the problem, Jean-Yves began by studying an extreme case : the case of total uncertainty also known as *Complete Ignorance*.

3.1 Decision under Complete Ignorance

Under complete ignorance, Not only does P have to be removed since there is not enough information to single out such a probability distribution, but the construction of the set of states S itself, is not obvious. This issue has been addressed by Jaffray in a series of papers by the beginning of the eighties (Cohen and Jaffray, 1980, 1983, 1985).

What does complete ignorance mean? Under complete ignorance, we have no information whatsoever on S and we are simply left with events in \mathcal{A} . The

starting point of the aforementioned papers is an *atomless* Boolean algebra of events, from which the sets of states of nature can be retrieved but in a non-unique way. Hence, the first axioms proposed by Jean-Yves in his work, (Cohen and Jaffray, 1980 and 1983) impose some consistency on the reconstruction of S from different partitions of the sure event; then, some rationality axioms of monotonicity and transitivity of the strict preference relation are stated, allowing for strict preference in the case of dominance.²

(i) As a first order approximation, the characterized decision criterion takes into account only the extremal possible outcomes of each decision, a decision criterion akin to Arrow and Hurwicz's criterion (Arrow and Hurwicz, 1972):

$$V(d) = \alpha m_d + (1 - \alpha)M_d$$

where m_d and M_d are the infimum and supremum outcomes of decision d , and α is the (Hurwicz) index of Pessimism-Optimism under complete ignorance.

(ii) In Jean-Yves' construction, effects linked to events also come into play although only *in the second order*. This contrasts with Arrow-Hurwicz's construction, which requires transitivity of indifference, and thereby excludes these second order effects. Moreover, contrary to Arrow-Hurwicz's model, these models can be compatible with the independence axiom, but, then, the only criteria left are maxmin and maxmax.

This study was seen by Jean-Yves as a preliminary step before studying the intermediate case of imprecise information on events.

3.2 Between Risk and Complete Uncertainty: Imprecise Risk

After studying complete ignorance, Jean-Yves turned to the study of a more frequent situation of uncertainty where there exists some objective information on the likelihood of events, but not as precise as a probability distribution. His approach can be broken down into three steps (i) How to describe imprecise information on events? (ii) How to model decisions in this framework? (iii) How to evaluate decisions?

To describe imprecise information, he introduced the notions of "Imprecise Risk" and its special case of "Regular Uncertainty" (Chateauneuf and Jaffray 1989 and 1995, Jaffray 1989a and 1989b). The basic idea was to bridge the gap between various extant representations of non-probabilizable uncertainty.

3.2.1 Representing imprecise information

Jean-Yves started from the assumption that information on events is given by a set \mathcal{P} of probability distributions. In his view, the appropriate tools to describe \mathcal{P} are the lower envelope of \mathcal{P} , denoted f , or alternatively the upper envelope

²In a recent paper, (Jaffray and Jeleva, 2010, this issue), Jean-Yves uses again some of these axioms of complete ignorance to model preferences on a set of "partially analyzed acts".

of \mathcal{P} , denoted F ,³ and especially the Möbius transform⁴ of f , denoted ϕ (these concepts were defined by Dempster, 1967 and by Shafer, 1976).

More precisely, for a set \mathcal{P} of probabilities distributions, the lower envelope f is defined as $f = \inf_{P \in \mathcal{P}} P$, the upper envelope F is defined as $F = \sup_{P \in \mathcal{P}} P$, the core of f is the set of probability distributions that dominate f , i.e. $core(f) = \{P, \text{probability} / P \geq f\}$ and the Möbius transform ϕ is defined, for all $B \subset S$ (for a finite set S), by $\phi(B) = \sum_{A \subseteq B} (-1)^{|B \setminus A|} f(A)$, the dual formula being $f(B) = \sum_{A \subseteq B} \phi(A)$.

Chateauneuf and Jaffray (1989) characterized, through properties of their Möbius transforms, monotone capacities (on finite sets S) of finite or infinite order and offered both new findings and easier proofs of some classical results on lower envelopes and Möbius transforms (even though the paper is technical, it received many citations).

Jean-Yves more specifically focused on all situations in which \mathcal{P} is generated by a *random set*, in which case f is not only convex (or 2-monotone) but also a *belief function*, also called a (normalized) *totally monotone capacity*. Such situations are particularly tractable. Let us note that a belief function can be characterized by the fact that its Möbius transform ϕ is positive, i.e. for all $B \subset S$, $\phi(B) \geq 0$.

Moreover, when f is a *belief function*, we can write, for any $A \in \mathcal{A}$

$$f(A) = \sum_{B \in \mathcal{C}} \phi(B) e_B(A) \text{ with } \phi(B) \geq 0, \sum_{B \in \mathcal{C}} \phi(B) = 1,$$

where ϕ is the Möbius transform of f and e_B , called the *elementary belief function*⁵ associated with B is defined by $e_B(A) = 1$ if $A \supset B$ and $e_B(A) = 0$ if not. \mathcal{C} is the (finite) focal set of $f : \mathcal{C} = \{B \in \mathcal{B} / \phi(B) \neq 0\}$.⁶ Jaffray and Wakker (1993) showed that belief functions are appropriate if and only if some principles of complete ignorance hold.

The formula above is still true for a convex capacity, except that, in that case, some $\phi(B)$ may be negative.

Now, as Jean-Yves himself stated, in Jaffray (1988): "*Dempster-Shafer theory is of little interest for decision analysts in the absence of a complementary decision model*", and he was one of the first to fill the gap between imprecise information representations and decision evaluations.

3.2.2 Modeling decisions : "Imprecise Risk"

Jaffray (1989a and 1989b) defined a situation of "imprecise risk" by the following two properties:

³Both are linked in the sense that, for all A in Ω , $F(A) = 1 - f(\bar{A})$. F is called the *conjugate of f* (or the *dual of f*).

⁴also called Möbius inverse.

⁵also called *unanimity game*.

⁶Note the analogy, when P is a probability distribution, with : $P(A) = \sum p(x_i) \delta_{x_i}(A)$ with $p(x_i) \geq 0$, $\sum p(x_i) = 1$, where $\delta_{x_i}(A) = 1$ if $x_i \in A$ and $= 0$ if not (Dirac measure).

1. there exists a true probability that is only known to belong to a certain set \mathcal{P} of probability distributions ;
2. the lower envelope f of \mathcal{P} characterizes \mathcal{P} :

$$\mathcal{P} = \text{core}(f) = \{P/P \geq f\} \text{ and } f = \inf_{P \in \mathcal{P}} P ;$$

Jean-Yves also defined a particular case called "regular uncertainty" where, moreover, the lower envelope f is *convex*.

3.2.3 Evaluating decisions

When it comes to modeling decisions, the starting point is a preference relation on the set \mathcal{D} of decisions, i.e., mappings from (S, \mathcal{A}, f) into \mathbb{R} , equipped with its Borel σ -algebra \mathcal{B} . For each d , define the image capacity f_d by setting, for each $B \in \mathcal{B}$, $f_d(B) = f[d^{-1}(B)]$. Jean-Yves proved, in Jaffray (1989a), that the f_d are convex whenever f is convex and, in Jaffray (1989b), that they are belief functions whenever f is a belief function as well. The preferences on the set of decisions can thus be replaced by preferences on the set of capacities $\{f_d\}$.⁷

Jean-Yves then had, at this point, the 3 ingredients to build his original decision model :

- The properties of "Regular Uncertainty" as stated in Chateauneuf and Jaffray (1989 and 1995), Jaffray (1988, 1989a and 1989b);
- The representation of preferences in a situation of *complete ignorance*, as stated in Cohen and Jaffray (1980), which includes the e_B , elementary belief functions ;
- The following observation : *the set of convex capacities is a "mixture set" in the sense of Herstein-Milnor (Herstein and Milnor, 1953) and thus von Neumann-Morgenstern's theorem can be used.*

Jaffray (1989a, 1989b, 1991a and 1991b), by adding a *dominance axiom*, proposed a general decision criterion under a situation of regular uncertainty with several special cases and some enlightening discussions.⁸ More precisely, combining all the ingredients, he proceeded as follows.

First, when f_d is a *belief function*, the decision criterion is :

$$V(f_d) = \sum_{B \in \mathcal{C}} \phi_d(B) V(e_B) \text{ and since there is } \textit{complete ignorance} \text{ on each } B,$$

the formula can be written as:

$$V(f_d) = \sum_{B \in \mathcal{C}_d} \phi_d(B) [\alpha(m_B, M_B) u(m_B) + (1 - \alpha(m_B, M_B)) u(M_B)]$$

where ϕ_d represents the Möbius transform of f_d , \mathcal{C}_d is the (finite) focal set of f_d i.e.: $\mathcal{C}_d = \{B \in \mathcal{B} / \phi_d(B) \neq 0\}$,⁹ u the utility of the outcomes and α the

⁷The procedure is the same as under *risk*, where, with any decision (or random variable) X , is associated the corresponding probability distribution F_X .

⁸The paper by Gul and Pesendorfer (2008) is the corresponding "Savage-style" representation of this "vNM-style" representation of Jean-Yves.

⁹Let us recall that, when f_d is a belief function, the ϕ_d are ≥ 0 .

(Hurwicz) index of Pessimism-Optimism under complete ignorance that, here, can depend on the extremal outcomes of d .

Then, Jean-Yves generalized the decision criterion to the case of regular uncertainty, where f is only *convex*, using the same formula, except that some $\phi_d(B)$ can be negative.

In the particular case where α happens to be constant, the criterion can be written as

$$V(f_d) = \alpha \inf_{P \in \text{core}\mathcal{P}} E_P u(f_d) + (1 - \alpha) \sup_{P \in \text{core}\mathcal{P}} E_P u(f_d)$$

This result was obtained by 1989.

The natural following question, in the case of regular uncertainty, deals with updating. How can a new piece of information modify a given set of probability distributions? Or said differently, how should a convex lower envelope be updated?

3.3 Attitude toward new information

Unlike the case of additive measures, for nonadditive measures, no updating rule satisfies all the desirable properties.¹⁰ Jaffray (1992) studied the effect of Bayesian conditioning for a belief function, when this belief function is understood as the lower envelope of compatible probability distributions.

Jean-Yves obtained the following results. First, he gave another proof of the result of Fagin and Halpern that the lower envelope of all Bayesian conditionals is still a belief function. Then, going beyond Fagin and Halpern (1991), he developed an explicit expression for the mass function for the lower envelope (this was in fact the key argument drastically simplifying the rest of the proof). Second, he showed that, in general, the resulting "conditional" lower envelope *does not characterize the set of all conditionals*. Finally, considering only events E such that $f(E) > 0$ and letting \mathcal{P}^E be the set of Bayesian conditionals of the elements of \mathcal{P} , Jean-Yves proved that the lower envelope f^E characterizes \mathcal{P}^E (i.e. $\text{core}(f^E) = \mathcal{P}^E$) for all such E *if and only if f is "almost additive"*, i.e.:

$$f(A \cap B) > 0 \text{ and } f(A \cup B) < 1 \text{ implies } f(A \cap B) + f(A \cup B) = f(A) + f(B)$$

Further results related to a convex lower envelope (more general than a belief function) can be found in *Jean-Yves' last paper* "Regular updating" (Chateauneuf, Gajdos and Jaffray, 2010), where it can be seen, in particular, that the condition above is less restrictive than it may seem to be at first glance.

¹⁰Two main updating rules have been proposed : the *full Bayes updating rule* (Fagin and Halpern 1991, Jaffray 1992) using the infimum of all the conditional probability distributions and the *maximum likelihood updating rule* of Dempster-Shafer (Dempster,1967 ; Shafer, 1976) and Gilboa-Schmeidler (1993) using the infimum of the conditional probability distributions *only* for the probability distributions giving the maximum probability to the realized event.

3.4 Dynamic decision making

Jean-Yves also had deep thoughts about dynamic consistency issues in non-EU models (Jaffray 1994 , Nielsen and Jaffray 2006).

Always in order to preserve an operational aspect of the model, Jean-Yves proposed, in Nielsen and Jaffray (2006), a procedure that involves a rolling back of the decision tree and selects a non-dominated strategy. A simulation confirmed the computational tractability of the model.

4 Links between Choquet Expected Utility and Jean-Yves' model under uncertainty

Jaffray and Philippe (1997) and Philippe, Debs and Jaffray (1999) provided an important result linking Choquet Expected Utility (CEU) and Jaffray's model under regular uncertainty or, more precisely, giving a behavioral foundation to the capacity in the CEU model, under the following assumption.

Assumption: The DM is a CEU(c, u) maximizer such that his capacity c can be written as $c = \alpha f + (1 - \alpha)F$ where α is a constant, f a capacity and F its conjugate.

In this case, the CEU criterion takes the form :

$$CEU(f_d) = \sum_{B \subset \mathcal{C}_d} \phi_d(B) [\alpha u(m_B) + (1 - \alpha)u(M_B)].$$

where ϕ_d is the Möbius transform of f_d , and m_B and M_B are the extremal outcomes in B and u is the utility of outcomes.

When f is *convex*, this is exactly Jean-Yves' criterion under regular uncertainty. That is, such a CEU maximizer *acts as if* he was able to locate the probabilities of the d -events in *subjective probability intervals* and his preferences are represented by:

$$CEU(f_d) = \alpha \inf_{P \geq f} E_P u(d) + (1 - \alpha) \sup_{P \geq f} E_P u(d).$$

Moreover, Jean-Yves showed that the preceding assumption first is not very restrictive¹¹ and second can be tested.

5 Experimental results

In Jean-Yves' view, a theory without empirical evidence was useless. So, it was important for him to confront the model with real individual choices. As early as the beginning of the eighties, when experimental economics was far from being widespread, he began a series of experiments *under risk and under complete uncertainty, for gains and for losses*. The purposes of these experiments were to:

¹¹The assumption is satisfied when the DM has an overall CEU criterion under uncertainty and consistently adopts Expected Utility under (objective) imprecise risk.

- assess individual attitudes under different types of uncertainty, especially in Cohen, Jaffray and Saïd (1987) ;
- explore the link between attitudes towards gains and losses in Cohen, Jaffray and Saïd (1987) ;
- test different decision models in Cohen and Jaffray (1988) ;
- test different updating rules in Cohen, Gilboa, Jaffray and Schmeidler (2000).

The experiments in Cohen, Jaffray and Saïd (1987), show that attitudes under risk and under complete uncertainty are not correlated (a result that also appears in Cohen, Tallon and Vergnaud, 2010).

More surprisingly, the same experiments show that attitudes towards gains and towards losses are not correlated.¹² Let us note that Kahneman and Tversky who found a "reflection effect" between gains and losses in their 79 paper, using only a between subjects design (whereas Jean-Yves' used a *within* subjects design) proposed then, in their 92 paper, two different weighting functions for gains and losses in Cumulative Prospect Theory, based on this result.

Finally, Jean-Yves found, in Cohen and Jaffray (1988), that the certainty effect is more important than the transformation of probability distribution in $(0, 1)$ and, in Cohen, Gilboa, Jaffray and Schmeidler (2000), that the Full-Bayes updating rule is used by 2/3 of the subjects, the others mostly using the maximum likelihood updating rule.

6 Conclusion

It is a pleasure to see Jean-Yves' intellectual "grandchildren" continuing to read his papers, extracting precious scientific thought nuggets to work with, and pursuing research in the spirit he initiated, but there is still work to be done ...

Jean-Yves' papers are :

- clearly and efficiently written. Jean-Yves' rigorous mind also shows in his writing style: in his papers, there is not a single word more than necessary! We hope this short overview has given you the desire to read some of them more in depth.
- not so easy to find. We can help you find them: A web-site will soon be available at : <http://jaffray.lip6.fr>

Several generations of researchers are indebted to Jean-Yves for their scientific vocation. His seminal works will certainly remain a major source of inspiration for future generations in decision theory.

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¹²Result confirmed by Abdellaoui and Munier (1992).

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